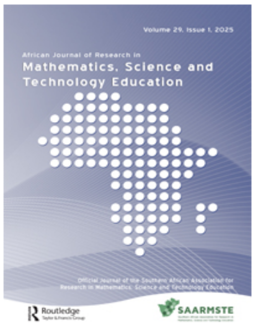

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Intervening to Improve Additive Relations Mathematising in Home Language Classrooms

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In this paper, we detail the outcomes of a small-scale intervention study that aimed to support the mathematising processes of Grade 2 South African learners when working with additive relations problems. We report on the mathematising shifts that were evident in pre- and post-test responses prior to and after a sequence of 13 intervention lessons. The intervention drew from Askew's *Big books of word problems* teaching resources, which were based on Realistic Mathematics Education and Cognitively Guided Instruction. The intervention involved intervention and control groups located within isiZulu speaking classes in one Johannesburg township school. The outcomes indicated greater shifts for the intervention group in their extent of awareness of structure and progression towards use of a more formal mathematics register. The results suggest the need for more focus on supporting learners with the setting up of models, and attunement to the association found between demarcating quantities within models and overall outcomes.

Keywords: *Realistic Mathematics Education; word problems; additive relations; intervention; mathematisation; sense-making*

1. Introduction

Research in the teaching and learning of mathematics at the primary school level in South Africa shows consistently that there are difficulties with both sense-making of word problems (Sepeng, 2014) and in the inefficient counting-based strategies that children use (Ensor et al., 2009). Learner difficulty with word problems has also been identified as a recurrent weakness in the Annual National Assessments diagnostic reports for Foundation Phase (Department of Education, 2012, 2013, 2015). Several researchers, nationally (e.g. Sepeng, 2014) and internationally (e.g. Greer, 1997), have argued that difficulties with word problems stem from the fact that sense-making is not primarily elicited from learners, with teachers, instead, focusing directly on the mathematical calculations needed to solve word problems.

Supporting sense-making within pedagogy involves promoting attention to the structural relationships between the given quantities in word problem situations. A wide variety of international research presents evidence of learners who answer school mathematics word problems with no sense of the situations described in these problems (Verschaffel et al., 1994). Existing literature points to suggestions for dealing with these difficulties. Work within the Realistic Mathematics Education (RME) tradition suggests that starting from realistic situations with which learners are familiar and which

[†]Co-author, Thulelah Blessing Takane, sadly passed away prior to the article's publication.

engage learners' natural propensity to set up models of situations can be used to support problem-solving (Peled & Balacheff, 2011). Other authors (Askew, 2004), including those in the RME tradition (Barnes & Venter, 2010; Gravemeijer, 1994), have argued that word problems that present situations in everyday language that are real, realistic or imaginable by learners can help learners to see mathematical sense-making as linked to sense-making in everyday life, and through this, support mathematical problem-solving. It was this approach to word problems that framed the intervention, and the analysis of its outcomes is presented in this paper.

Word problems provide a useful vehicle for introducing additive relationships in ways that promote what RME theory describes as 'horizontal mathematisation' (HM) and 'vertical mathematisation' (VM). Horizontal mathematisation involves the setting up of models of problem situations and VM then involves the devising of calculation strategies linked to the HM-based model that can be used to solve the problem. Barnes and Venter (2010) have described learners' progress from using informal concrete strategies to describe and solve contextual problems into using mathematical language and suitable algorithms.

In South Africa, much of the more recent attention has been on a push for progression beyond calculation strategies based on counting in ones towards more efficient calculation strategies based, in particular, on the base 10 structure of the number system (Venkat et al., 2022). This attention was driven by ongoing evidence of learners failing to move beyond inefficient counting-based approaches well into the middle grades (Schollar, 2008) coupled with evidence of instruction that allowed this to happen through the provision of unstructured counting-based resources (Ensor et al., 2009). The intervention at the centre of this paper was designed with the intention of focusing on both sense-making of word problems through the use of familiar situations and encouraging moves to more efficient calculation strategies.

In this paper we detail the ways in which the intervention made use of the approach promoted in Askew's (2004) *Big book of word problems* (*BBWP*) instructional resources. The *BBWP* drew on the classification of additive situations and the hierarchies of difficulty offered in Carpenter et al.'s (1999) writing within the work on Cognitively Guided Instruction (CGI) theory, and on RME's attention to using familiar situations to support sense-making and formalising (Freudenthal, 1973).

We begin this paper by discussing the literature base on word problems in CGI, and then outline RME theory which framed the broader doctoral study from which this paper emerged (Takane, 2021). We then detail the intervention and discuss the research methodology including the data sources and analytical approaches. This leads into the findings that emerged from the first author's attempts to promote both sense-making and efficient calculation strategies.

2. Additive Relations Word Problems and Cognitively Guided Instruction

This study is located within the specific domain of additive relations problems—problems that refer to situations underpinned by an additive structure. Additive relations word problems involve addition or subtraction situations expressed in narrative forms. Some research studies have considered word problems relating to addition as distinct from word problems relating to subtraction (van den Heuvel-Panhuizen & Treffers, 2009). For example, there is the research that has identified and labelled two different models of subtraction: take-away vs. difference. The 'take-away' model can be conceptualised as 'remove from a given set' and the 'difference' model can be conceptualised as 'comparing given sets' (van den Heuvel-Panhuizen & Treffers, 2009). Other research has placed emphasis on the inverse relationship between addition and subtraction, which necessarily meant that these operations were no longer examined independently but rather the structural relationship between them became a focus (Schmittau, 2004). This led to the introduction of the phrase 'additive relations', thus considering both operations simultaneously.

While the desire is to promote moving between the two operations simultaneously, the research evidence also points to subtraction problems being more difficult for learners to solve than addition problems (Carpenter et al., 1999). In this study addition was the starting point of the intervention in order to smoothly acquaint the learners with the sense-making of the word problems presented to them. This point about the difficulty of the problems leads to the discussion on CGI (Carpenter et al., 1999) which provides a framework for progression.

The CGI framework is premised on the view that children bring to school informal knowledge of mathematics that can serve as the basis for developing formal mathematics. The CGI framework was developed because of findings that learners intuitively solve word problems by modelling the action and relations described in them. Carpenter et al. (1999) classified problems by focusing on the types of action or relation described in the problems presented to learners. This produces an initial listing of problem situations in order of difficulty: join, separate, part–part–whole and compare, where join/separate problems both relate to dynamic change situations based on addition/subtraction respectively. Problem situations can vary based on the position of the unknown: result unknown, change unknown, start unknown, with these also listed in order of difficulty. In part–part–whole problems, the unknown is either the whole set or one of the subsets.

While the broader study focused on the full range of problems in Carpenter et al.'s classification, in this paper, we focus on problems drawn from the easiest levels: change—result unknown and part–part–whole—whole unknown problems, as these were particularly illuminative of changes in HM based on two emergent aspects of change in this study, detailed in the next section.

2.1. *Modes of Representation and Extent of Structuring*

Ensor et al. (2009) provide a trajectory of progression from concrete to abstract modes of representation. This framework incorporates the following:

- concrete modes (entailing manipulation of physical objects such as counters);
- pictorial modes (iconic, involving images of everyday objects, or indexical, involving generic, rather than realistic depictions of context, e.g. tally marks);
- symbolic modes (number-based, numerals used to represent quantities, or syntactic, use of mathematical notation to represent relationship statements).

This framework provided a language useful for describing one aspect of the progression seen in learners' models in this study, with pictorial and symbolic forms seen in the empirical dataset.

A second aspect of progression that emerged as important in this study related to the 'extent of structuring'. Schöner and Benz (2017) showed that perceiving quantity in terms of separate sets is more advanced than perceiving in ones. This fits well with Venkat et al.'s (2019) notion that structural awareness is based on the idea of seeing mathematical relationships. Mulligan and Mitchelmore's (2009) study places this distinguishing of quantities within a broader problem-solving hierarchy in which the first level of awareness of structure is evident in the absence of construction of an appropriate model. The next level is the construction of an appropriate model. The third level becomes the demarcation of given quantities within an appropriate model, with the inclusion of a mathematical symbolic operation. Mulligan and Mitchelmore argued that the extent of awareness of structure in emergent models appeared to be an important marker for learners who calculate more efficiently. Awareness of structure can be seen in all modes of representation.

3. Theoretical Framework

3.1. *Mathematising within the context of Realistic Mathematics Education*

Realistic Mathematics Education theory underpinned the broader study. Realistic Mathematics Education's view is that for mathematics to be of human value it should be connected to reality, stay close to learners and be relevant to society (Freudenthal, 1971). Realistic Mathematics Education's view of 'mathematics as a human activity' implies that learners should be given the opportunity to reinvent mathematics by mathematising— mathematising subject matter from reality and mathematising mathematical subject matter (Freudenthal, 1971). In both cases, the subject matter that is to be mathematised should be experientially real for the learners. Thus, RME rests on three key concepts denoted as *mathematics as a human activity*, *mathematising* and *re-invention*. Further, within RME there are three design principles denoted as *guided re-invention*, *didactical phenomenology* and *self-developed or emergent modelling.* These design principles help in designing a possible learning route together with a set of potentially useful instructional activities that fit this learning route.

The objective of the RME design principles is to inform the design of support materials by trying to construe learning paths along which learners could reinvent their own mathematics. According to Freudenthal (1971), re-invention is often known as discovery or re-discovery of the mathematics by the learners. The principle of *guided re-invention* involves presenting learners with appropriate contextual problems that offer them opportunities to develop informal, highly context-specific solution strategies. Gravemeijer (2004) asserts that according to the re-invention principle, the goal is not to teach learners solution strategies in the form of ready-made techniques when teaching addition and subtraction up to 100; instead, learners should be helped to develop similar solution methods through their own activity.

Guided re-invention and didactical phenomenology were complemented with what Treffers (1987) called progressive mathematisation—a process by which learners begin by mathematising subject matter from reality and then switch to analysing their own mathematical activity. As noted already, he also distinguished between HM and VM concepts, which cover, respectively, the sense-making necessary for setting up initial models of situations and the progressions involved in increasingly formal working and efficient calculation of missing values.

The second principle, which is *didactical phenomenology*, suggests identifying different possible activities to support classroom engagement in progressive mathematisation. Situations presented through these activities must have the potential to evoke solution procedures that can be a basis for construction of formal mathematics. Gravemeijer (2004) recommends the number line as a preferred model as he suggests it is instrumental in helping learners make a connection between the cardinal (line segment) and ordinal (point) aspects, and allows for learners' emergent modelling. The number line model was discussed and used alongside number sentences in the intervention lesson sequence.

Askew's (2004) *BBWP* brought RME and CGI research together within the author's formulation of problem sets designed on the basis of the problem classes stipulated within CGI, with the pedagogical approach focused on learners discussing, designing and sharing initial models of problem situations (HM) that can feature as the bases for formalising and devising calculation strategies (VM). In this study, the design of the *BBWP* was such that themes, contexts and language used could be aligned to translate/adapt word problems that were close to the learners which is one of RME's key values. Second, *BBWP* offered guidance on designing problems based on the CGI categories and for teaching with them in ways that allowed for discussion of models of these situations and solution

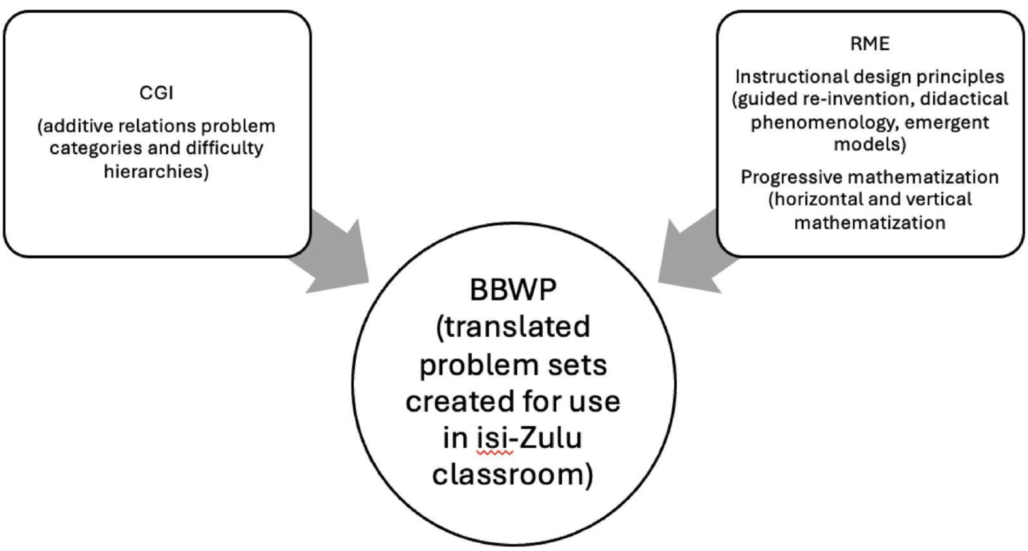


Figure 1. The merging of theories underlying the research methodology

strategies that reflected CGI principles. In the broader study, we used the *BBWP* as a source for creating sets of isiZulu additive word problems based on the same structure. Figure 1 presents an overview of the merger of RME and CGI in the *BBWP* as the key resource that was used to design the intervention at the heart of this study.

This merger made use of *BBWP*'s bringing together of the categories of additive word problems and the combinations of theoretical analyses of additive situations and empirical analyses of learners' problem-solving that underlay CGI's identification of problem hierarchies of difficulty, with RME's attention to teaching for sense-making and progression. The *BBWP* resources then formed the base for the creation of translated (and sometimes edited for contextual familiarity) and piloted sets of additive word problems in isiZulu.

4. Methodology

The broader study adopted a broadly qualitative approach with some quantitative analysis incorporated in it. It stemmed from an interpretivist view as we aimed to 'interpret the meaning of social action and thereby give a causal explanation of the way the action proceeds and the effects which it produces' (Weber, 1978: 7). The research was carried out at a quintile 1 (low economic status) government township primary school in Gauteng. In the study school, there were two isiZulu Grade 2 classes (alongside classes where other South African languages served as the language of learning and teaching) that the school and teachers agreed could be constituted as an intervention and a control group. The intervention class was composed of 38 isiZulu speaking learners, but absences meant that matched pre- and post-test data for analysis was available for $n = 28$ matched learners, and $n = 27$ matched learners for analysis in the control class against 37 registered learners. The selection of the intervention group was premised on the teacher being willing to observe during the intervention and to get involved where necessary especially with regards to language dynamics. Learners in the control class continued with their normal lessons and received no intervention but were included in writing the pre- and post-tests. Thirteen lessons informed by the approach and structure of the *BBWP* resource were conducted across 3 months twice a week. Within each lesson, activity was broken up into three parts. In Activity 1, contextual problems were presented to the learners and discussed to guide the learners towards the use models that made sense to them to solve the problems. This section involved pair and whole-class discussion. The second part of the lesson entailed individual activity to promote independent working and to assess the understanding of individual learners. The third part of the lesson entailed a wrap up discussion with the class to elicit and establish what had been learned during the lesson.

The main instruments of the broader study consisted of the following:

- written pre- and post-tests that included some repeated questions administered prior to and at the end of the intervention lesson sequence with the intervention and control classes;
- task-based interviews conducted after the pre- and post-tests with a cross-attainment subsample of learners; and
- learners' written working from the intervention lessons, which offered illustrations of their developing mathematising;

The written pre- and post-tests were administered to ascertain the mathematising processes of learners prior and after the intervention. As in the intervention, the design of pre- and post-test items followed the RME principle of word problems set in contexts that would make sense, and be familiar, to the learners. Given the evidence of weaknesses in reading skills in the Foundation Phase in South Africa, the questions were read out to the class by the first author with the class teacher present, and the learners were then given time to answer each question before moving on to the next question.

In the broader study, the first author noted that there were no statistically significant differences in mean scores in the pre-test common items between the intervention and control groups, but that post-test differences significantly favoured the intervention group (Takane, 2021). In this paper, we draw on examples of learners' working from the pre-/post-tests and their ongoing work to illustrate how we analysed the pre- and post-test data for instances of the key categories of modes of representation and the extent of structuring visible in the distinguishing (or not) of the given quantities. These themes were most broadly seen—as already noted—in items that have been described as being at the easiest level in earlier CGI work: change result unknown and part–part–whole—whole unknown problems. Across the pre- and post-test, the three common items in which relatively high facility (roughly half the learners getting the item correct) on the pre-test was seen were the following:

Q1. uThuli unamaswidi kashokoledi angu-9. uSam unamaswidi kastrobheri angu-6. Mangaki amaswidi wabo ehlangene?
(Part–part–whole—whole unknown: Thuli has 9 chocolate sweets. Sam has 6 strawberry sweets. How many sweets do they have altogether?)

Q2. Bekunamabhodlela angu-12 etafuleni. uMama wasusa angu-7. Mangaki amabhodlela asele?
(Separate—result unknown: there were 12 bottles on the table. Mom removed 7. How many bottles are left?)

Q3. Abafana abangu-12 namantombazane angu-8 adla amaswidi. Zingaki izingane ezidla amaswidi?
(Part–part–whole—whole unknown: 12 boys and 8 girls are eating sweets. How many children are eating sweets?

In Carpenter et al.'s (1999) view, although part–part–whole—whole unknown problems are 'static' problems where there is no action in the situation (contrasting with the dynamic nature of join and separate problems), when directly modelling these problems, learners solve them similarly to join and separate—result unknown problems because the structure of the problem provides known numbers that can be operated with to find the missing 'result'.

The analysis of change between the pre- and post-test in this paper is based on progressions to increasingly abstract modes of representation and extent of attention to distinguishing quantities. We look also at the relationship between these changes and overall problem-solving competence.

Analysis of student responses on these three questions proceeded based on the following markers noted from the literature and expanded to reference the data:

- the mode of representation used (pictorial/some formal symbolic working/entirely formal symbolic working);
- whether the given quantities were distinguished or not; and
- whether the correct answer had been produced or not.

These categories produced the analytical summary table shown in Figure 2. In Figure 3, we offer illustrations from learners' working to exemplify key categories. In the Figure 2 summary table, the move from pictorial representations to increasingly formal symbolic representations implies one direction of move on progression on the horizontal axis. On the vertical axis, moves towards demarcating the given quantities were tracked as a second marker of progression, but it was important that this indicator took into account both the 'appropriateness' of the model (i.e. whether the model shown could be interpreted as an appropriate representation of the problem situation) and whether the answer that the student went on to produce was correct. To deal with this, inappropriate models were counted together with instances where no model was included, and where there was either no answer or an incorrect answer. In the findings that follow, we present a quantitative overview of the prevalence of each category in the pre- and post-test for the intervention and control groups and go on to discuss the findings and implications.

In the instances where no model was included in a response, it was obviously not feasible to code for the mode of representation. However, RME theory on progression notes that 'models of' situations created in HM ideally, over time, come to feature as 'models for' situations—for example, knowing that an empty number line can be used to solve additive relations problems. Because of this, responses with no model/correct answer feature at the upper end of the hierarchy, whereas no model/incorrect answer responses feature at the lower end. The underlying assumption here is that 'no model/correct answer' responses indicate an internalisation of an appropriate model that is available for use within

			Increasingly abstract modes of representation		
			Pictorial	Some formal working	Entirely formal working
Increasing attention to structure, in distinguishing of given quantities (taking correct/incorrect models/answers into account	No model/inappropriate model: no answer or incorrect answer				
	Appropriate model: quantities not distinguished	Incorrect answer			
		Correct answer			
	Appropriate model: quantities distinguished	Incorrect answer			
		Correct answer			
	No model: correct answer				

Figure 2. Analytical summary table

mental calculation, although the answer may still be evaluated using inefficient unit counting. The illustrations in Figure 3 make it clear that symbolic working usually implies the demarcation of quantities in ways that are not 'in-built' in pictorial working. Additionally, in our analysis, some learners included multiple models, with different modes of representation in the working space. Where this was the case, all their models were counted. This meant that the number of responses was usually larger than the number of matched learners involved ($n = 28$ in the intervention group and $n = 27$ in the control group).

In the Findings section that follows, the outcomes of analysis of student responses are presented with frequency counts and percentages of the prevalence of each category of the summary table across pre- and post-tests for the intervention and control groups. This presentation of findings is followed by a commentary on what we see as the key findings of importance from this work.

5. Findings

In this section, we present summary tables for each of the three 'high facility' problems introduced earlier for the intervention and control groups. In each case, a short overview of key shifts follows the table with a longer discussion in the concluding section.

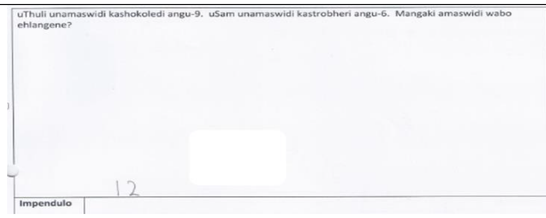
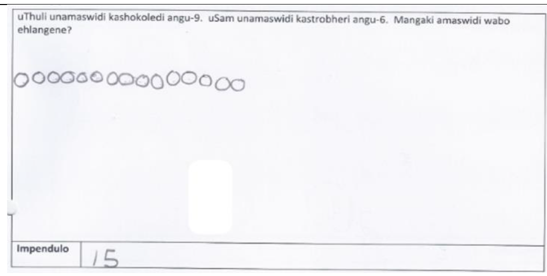
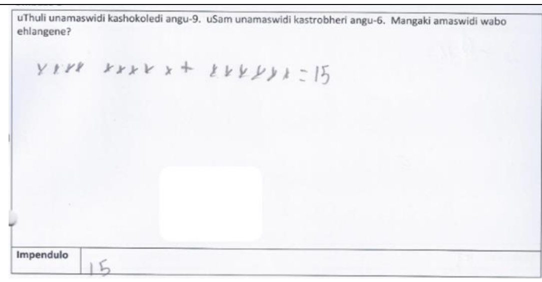
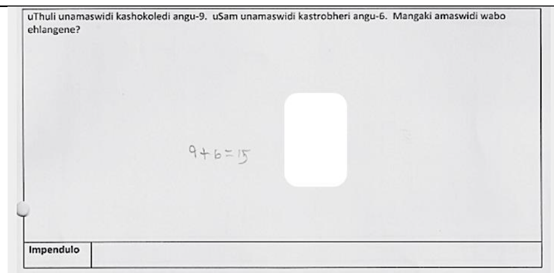
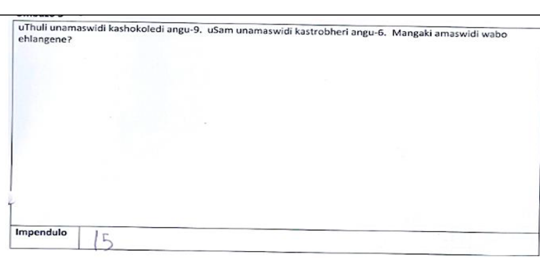
Category	Illustration
No overt model: incorrect answer	
Appropriate model (pictorial): quantities not distinguished—correct answer	
Appropriate model: (some formal): quantities distinguished—correct answer [inclusion of '+' sign here qualifies as some formal notation]	
Appropriate model (entirely formal): quantities distinguished—correct answer	
No model—correct answer	

Figure 3. Illustrations of key categories from student working on a part–part–whole—whole unknown problem: 9 + 6 = ?

Table 1. Summary pre- and post-test outcomes for intervention and control groups on part–part–whole—whole unknown problem: 9 + 6 = ?

Q1: 9 + 6=? Pre (%) **Post (%)**	Intervention	Pictorial	Some formal working	Entirely formal working	Control	Pictorial	Some formal working	Entirely formal working
No model/ inappropriate model: no answer/ incorrect answer	5/29 (17) **0/33 (0)**				2/27 (7) **3/27 (11)**			
Appropriate model: quantities not distinguished	Incorrect answer Correct answer	2/29 (7) **0/33 (0)** 1/29 (3) **1/33 (3)**			Incorrect answer Correct answer	2/27 (7) **2/27 (7)** 2/27 (7) **0/27 (0)**		
Appropriate model: quantities distinguished	Incorect answer Correct answer	0/29 (0) **0/33 (0)** 3/29 (10) **11/33 (33)**	0/29 (0) **0/33 (0)** 3/29 (10) **11/33 (33)**	0/29 (0) **0/33 (0)** 4/29 (14) **6/33 (18)**	Incorrect answer Correct answer	1/27 (4) **0/27 (0)** 15/27 (56) **16/27 (59)**	0/27 (0) **0/27 (0)** 2/27 (7) **0/27 (0)**	1/27 (4) **0/27 (0)** 0/27 (0) **1/27 (4)**
No model: correct answer	11/29 (38) **4/33 (12)**				2/27 (7) **5/27 (19)**			
Overall correct	Pre, 22/29 (76%); **post, 33/33 (100%)**				Pre, 21/27 (78%); **post, 25/27 (93%)**			

The summary of responses in Table 1 shows that, in both the intervention and control groups, there was improvement in overall performance on this item. However, the patterns of change from pre- to post-test in the models devised to produce answers varied between the intervention and control groups. In the intervention group, there was a substantial move into presenting an appropriate model: the no model/inappropriate model category which comprised 5 incorrect and 11 correct answers/29 responses in the pre-test, shifted to 4 correct answers/33 responses in the post-test. In the control group, there were 2 incorrect and 2 correct answers/27 responses with no model or an incorrect model in the pre-test, with this group increasing in size to 3 incorrect answers and 5 correct answers/27 responses in the post-test. This suggests that the intervention's encouragement to work with models in mathematising was successful. A second key difference between the intervention and control classes was in the appropriate model: quantities distinguished category. In the control group, there was no horizontal shift towards the use of more formal working between the pre- and post-test. The ongoing use of pictorial representations tended to suggest the ongoing use of unit counting. In contrast, in the intervention group, there was a marked horizontal shift into working that included, at least, some elements of a formal symbolic register (from 7/29 responses in the pre-test to 17/33 responses in the post-test. In the intervention group, there was also an increase in demarcating given quantities within the pictorial representations category—from 6/29 responses in the pre-test to 12/33 responses in the post-test. Taken together, this suggested that the improvement in performance for the intervention group was built on increased use of more formal modes of representation, and better demarcation of quantities within the pictorial representation category.

At the headline level for the next item (12 + 8 = ?), Table 2 shows that, the gain in overall correct answers on this item for the intervention class was particularly marked (45–97%) in comparison with a much more modest improvement for the control class. Beneath this headline, a similar pattern of improvement underpinned the intervention classes' improved outcomes—a horizontal

Table 2. Summary pre- and post-test outcomes for intervention and control groups on part–part–whole—whole unknown problem: 12 + 8 = ?

Q1: 12 + 8=? Pre (%) **Post (%)**	Intervention	Pictorial	Some formal working	Entirely formal working	Control	Pictorial	Some formal working	Entirely formal working
No model/ inappropriate model: no answer/ incorrect answer	12/29 (41) **1/30 (3)**				2/27 (7) **2/27 (7)**			
Appropriate model: quantities not distinguished	Inc ans	1/29 (3) **0/30 (0)**			Incorrect answer	2/27 (7) **0/27 (0)**		
	Corr ans	0/29 (0) **1/30 (3)**			Correct answer	2/27 (7) **5/27 (19)**		
Appropriate model: quantities distinguished	Inc ans	3/29 (10) **0/30 (0)**	0/29 (0) **0/30 (0)**	0/29 (0) **0/30 (0)**	Incorrect answer	1/27 (4) **1/27 (4)**	0/27 (0) **0/27 (0)**	1/27 (4) **0/27 (0)**
	Corr ans	0/29 (0) **11/30 (37)**	3/29 (10) **8/30 (27)**	3/29 (10) **5/30 (17)**	Correct answer	15/27 (56) **12/27 (44)**	2/27 (7) **0/27 (0)**	0/27 (0) **1/27 (4)**
No model: correct answer	7/29 (24) **4/30 (13)**				2/27 (7) **6/27 (22)**			
Overall correct	Pre, 13/29 (45%); **post, 32/33 (97%)**				Pre, 21/27 (78%); **post, 24/27 (89%)**			

Table 3. Summary pre- and post-test outcomes for intervention and control groups on separate—result unknown problem: 12—7 = ?

Q1: 12–7=? Pre (%) **Post (%)**	Intervention	Pictorial	Some formal working	Entirely formal working	Control	Pictorial	Some formal working	Entirely formal working
No model/ inappropriate model: no answer/ incorrect answer	7/28 (25) **0/31 (0)**				1/27 (4) **4/27 (15)**			
Appropriate model: quantities not distinguished	Incorrect answer	1/28 (4) **2/31 (7)**			Incorrect answer	4/27 (15) **2/27 (7)**		
	Correct answer	0/28 (0) **1/31 (3)**			Correct answer	3/27 (11) **1/27 (4)**		
Appropriate model: quantities distinguished	Incorrect answer	1/28 (4) **0/31 (0)**	2/28 (7) **0/31 (0)**	1/28 (4) **0/31 (0)**	Incorrect answer	1/27 (4) **0/27 (0)**	0/27 (0) **0/27 (0)**	0/27 (0) **0/27 (0)**
	Correct answer	6/28 (21) **19/31 (61)**	2/28 (7) **2/31 (7)**	1/28 (4) **4/31 (13)**	Correct answer	14/27 (52) **14/27 (52)**	0/27 (0) **0/27 (0)**	1/27 (4) **1/27 (4)**
No model: correct answer	7/28 (25) **3/31 (10)**				3/27 (11) **5/27 (19)**			
Overall correct	Pre, 16/28 (57%); **post, 26/31 (84%)**				Pre, 21/27 (78%); **post, 21/27 (78%)**			

shift into increased use of more formal modes of representation and increases in the extent to which given quantities are distinguished within pictorial representations—with neither of these shifts seen in control class responses.

On the 12 - 7 = ? item, Table 3 shows that, the intervention group started with a lower facility than the control group but ended up outperforming them on the post-test on this item. While there was a less marked shift into more formal working in these responses than on the other two items, the increase in the extent of distinguishing of given quantities in pictorial modes was somewhat more pronounced and associated substantively with the improvement in outcomes. The control group showed very little change in their ways of working on this item in the three-month interim period between the pre- and post-test.

6. Conclusion

In this paper, we set out to report on the shifts in mathematising processes of the intervention and control groups within a small-scale intervention study. The overall results reflect that in the post-test, the intervention group substantially outperformed the control group in all three items of the extent of awareness of structure. This pattern of outperformance was reflected in all of the other common items on the pre- and post-tests as well, as reported in the broader study (Takane, 2021). Also of note is that the intervention group's greater willingness to construct models of problem situations was carried through into the more difficult 'part unknown' problems in the broader study post-tests as well, and to a much greater extent than was seen in the control group's pre- to post-test change.

The findings suggest that underlying the outperformance of the control group by the intervention group, there was increased working with more formal modes of representation and greater awareness of the usefulness of demarcating the given quantities, even when working with pictorial forms. This association between the demarcation of quantities and the production of correct answers backs up the findings of both Mulligan and Mitchelmore (2009) and Schöner and Benz (2017). My study's findings lend support to distinguishing given quantities as an important step within successful problem solving.

In this study, my pedagogic attention was primarily on helping learners to set up models of additive situations and then using models like the empty number line to efficiently calculate answers. The former required extensive attention to discussing situations, acting them out and then setting up models. The focus on increasingly abstract, or formal, modes of representation and on demarcating the given quantities emerged as findings of interest in learners' responses. The ongoing relatively broad prevalence of pictorial models within the expansions in learners' willingness to produce models suggests more success with the model set-up goal than the efficient calculation goal. Overall, our findings suggest that more time and attention to initial sense-making and setting up of models with learners may well be required in order to provide a stronger base upon which later progressions into more efficient calculation can be built. The findings also point to the importance of paying attention to features like distinguishing quantities as small, but important parts in developing learners' competence with additive relations problem-solving.

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No potential conflict of interest was reported by the author(s).

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