Fractal dynamics and wavelet analysis: Deep volatility and return properties of Bitcoin, Ethereum and Ripple

Valerio Celeste[a], Shaen Corbet[b], Constantin Gurdgiev[a,c*]

[a] *Trinity Business School, Aras an Phiarsaigh, Trinity College, Dublin 2, Ireland*

[b] *DCU Business School, Dublin City University, Dublin 9, Ireland*

[c] *Middlebury Institute for International Studies at Monterey, California*

[*] *Corresponding Author: gurdgic@tcd.ie*

Abstract

The substantial volatility and growth in cryptocurrencies valuations between 2009 and the end of 2017 strongly suggest that both long memory and price volatility and return spillovers should be present in these assets' dynamics. To date, literature on the major cryptocurrencies price processes does not address jointly and comprehensively their fractal properties, long memory and wavelet analysis, that could robustly confirm the presence of fractal dynamics in their prices, and confirm or deny the validity of the Fractal Market Hypothesis as being applicable to the cryptocurrencies. This research shows that Bitcoin prices exhibit long term memory, although its trend has been reducing overtime. In fact, assessing Bitcoin, Ethereum and Ripple across the period between 2016 and 2017, focusing solely on the period prior to the crash of 2018, we can conclude that Bitcoin was better described by a random walk, showing signs of markets maturity emerging, in contrast, other cryptocurrencies such as Ethereum and Ripple present evidence of a growing underlying memory behaviour.

Keywords: Efficient Market Hypothesis; Fractal Market Hypothesis; Cryptocurrencies; Wavelet Coherence; Continuous Wavelet Transform; Hurst Exponent.

1. Introduction

In recent years, significant price volatility and dramatic price appreciation (prior to Q1 2018) in cryptocurrencies has led to a sharp increase in media interest, consequently attracting a growing number of investors that view cryptocurrencies as a new financial asset class suitable for a buy-and-hold strategy and to speculate on. As the largest cryptocurrency by market capitalisation, Bitcoin has been extensively analysed from both technical and investor perspectives[1] (Delfin-Vidal and Romero-Meléndez [2016]). The frequency distribution of the BTC/USD (Bitcoin to US dollar $) exchange rate has presented strong evidence of kurtosis, where extremely high volatility events, often several standard deviations from the mean, occur with an increased empirical probability than would be inconsistent with the Normal distribution. Secondly, Bitcoin returns appear to show a fractal property called self-similarity, thus providing similar patterns in its distribution independent of the time scale assessed. Thirdly, albeit Bitcoin's returns seem to be uncorrelated with each other, periods of high and low volatility clusters do exhibit a trend, therefore suggesting that there exists the presence of long memory and persistence in Bitcoin's volatility. When considered together, these features represent a violation of the EMH[2] (see Cunningham [2000]; Mandelbrot [1998]; Cont [2001]; and Taylor [2007]). Aslanidis et al. [2019] found three key results when analysing the behaviour of conditional correlations among main cryptocurrencies, stock and bond indices, and gold, using a generalised DCC class model. Firstly, correlations among cryptocurrencies are positive, albeit varying across time. Secondly, the authors find that correlations between cryptocurrencies and traditional financial assets are negligible.

In an attempt to provide a more realistic description of the market for cryptocurrencies, while attempting to avoid the imposition of constraints regarding the statistical and distributional properties of price returns, Peters [2015] developed the Fractal Market Hypothesis (FMH hereafter). FMH assumes that market stability is preserved when the agents trading in the markets make self-similar decisions that span across different investment time horizons, providing market liquidity (Kristoufek [2013b]). When this self-similar structure breaks down, namely when the long-term

[1]In general, there is no consensus in the markets and amongst regulatory authorities as to the preferred classification of cryptocurrencies as an asset class. Some jurisdictions define cryptocurrencies as commodities, others as currencies, and a number of jurisdictions, cryptocurrencies are treated as general assets, without any specific definition. Furthermore, the Director of the Division of Corporation Finance at the Security Exchange Commission (SEC) offered guidance to suggest, cryptocurrencies can be treated by some regulatory authorities as belonging to two different asset classes, simultaneously. Crypto-tokens issued in the primary market constitute an act of securities sale, while tokens traded in the secondary markets constitute currency-related transactions. As we consider the price of each cryptocurrency expressed in the U.S. dollars, it is more natural to view these assets as fiat currencies traded in the markets. This view has a natural link to monetary theory dimension as relevant to the traditional forex markets. While this dimension is not the subject of the present study, a good summary of monetary theory as applied to cryptocurrencies can be found in Kumar and Smith [2017].

[2]The EMH is an investment theory that states it is impossible to 'beat the market' because asset market efficiency causes existing asset prices to always incorporate and reflect all relevant information. According to the strong version of the EMH, assets always trade at their fair value in the market, making it impossible for investors to either purchase undervalued assets or sell assets for inflated prices. As such, it should be impossible to outperform the overall market through expert asset selection or market timing, and the only way an investor can possibly obtain higher returns is by purchasing riskier investments.

investors either stop trading or shorten their investment time horizon, the market becomes unstable leading to periods of high volatility (Roch [2011]). The dominance of a single time horizon in the FMH setting would therefore undermine the market liquidity and cause severe corrections[3] (e.g. flash crashes[4] and short-lived price collapses), as well as large and longer-term events, such as those experienced in the Global Financial Crisis of 2008-2009 (Kristoufek [2013a]).

Bitcoin, given its de-localized structure, presents evidence of a violation of the EMH assumptions and possesses behavioural traits that appear to be consistent with the FMH. Thus, FMH provides a unique opportunity to obtain a deeper understanding as to how these new markets for cryptocurrencies function. To the best of our knowledge, to date, only Delfin-Vidal and Romero-Meléndez [2016] has attempted to systemically analyse the FMH-consistent behaviour of Bitcoin. No study to date has either combined all aspects of technical FMH analysis nor attempted to extend FMH framework to the analysis of the largest cryptocurrencies (by market capitalization) beyond the Bitcoin. There are two particular assumptions made by the FMH, namely the 4th and the 5th, which necessitate particular attention[5]. First, prices reflect combination of short-term technical trading and long-term fundamental valuation. Thus, short-term price changes are likely to be more volatile than long term trades. While the underlying long run trend reflects changes in expected earnings driven by the economic and investment environment, or fundamentals, short-term trends are more likely to arise as the result of crowd behaviour. Second, if a security has no tie to the economic cycle, then there will be no long-term trend. Trading, liquidity and short-term information will dominate.

[3]While the present paper does not focus on flash-crashes and short-lived price collapses, our results indicate the need for continued research into extreme volatility events that are common in the cryptocurrencies markets. Specifically, our fractal and wavelet analysis shows some evidence of clustering of volatility around large-scale events in Bitcoin, Ethereum and Ripple.

[4]A flash crash is a very rapid, deep, and volatile fall in security prices occurring within an extremely short time period. A flash crash frequently stems from trades executed by black-box trading, combined with high-frequency trading, whose speed and interconnectedness can result in the loss and recovery of billions of dollars in a matter of minutes and seconds.

[5]The FMH, proposes the following assumptions: 1) The market is stable when is consists of investors covering a large number of investment horizons; 2) The information set is more related to market sentiment and technical factors in the short-term than in the long-term. As investment horizons increase, long-term fundamental information tends to dominate; 3) If an event occurs that makes the validity of fundamental information questionable, long-term investors will either stop participating in the market or begin trading based on the short-term information set. When the overall investment horizon of the market declines to a uniform level, the market therefore becomes unstable; 4) Prices reflect a combination of short-term technical trading and long-term fundamental valuation. Therefore, short-term price changes are likely to be more volatile than long-term trades. Short-term trends are more likely to be the result of crowd behaviour; and 5) If a security has no tie to the economic cycle, then there will be no long-term trend. Trading, liquidity and short-term information will dominate

We aim to address these gaps in the literature by developing a comprehensive FMH-consistent analysis of the three largest cryptocurrencies in the markets across the main dimensions of FMH. To achieve this, we shall firstly test cryptocurrency prices' time series using the Rescaling Range Analysis (R/S Analysis) and V-statistic as originally described by Peters [2015]. This enables identification of a Hurst process excluding random walk price behaviour, therefore confirming the presence of long memory in the time series. We use daily prices data for three largest (by market capitalization) cryptocurrencies, Bitcoin, Ethereum and Ripple. We derive daily returns from the daily price data for R/S and wavelet analysis. To add robustness to our selected methodology, we utilise Continuous Wavelet Transform (CWT), Cross Wavelet Transform (XWT) and Wavelet Coherence (WTC) to test as to whether Bitcoin, Ethereum and Ripple share areas of common power in the time-frequency space while analysing how they co-vary over time. These methods, combined with fractal analysis help establish the full extent of evidence concerning FMH nature of Bitcoin, Ethereum and Ripple across both dynamic fractal domain and time-frequency power spectrum analysis, providing a novel contribution to the literature on cryptocurrencies and expanding our understanding of the FMH.

The rest of the paper is organized as follows: Section 2 provides a comprehensive literature review of the EMH, the FMH and the growth of the market for cryptocurrencies to date. In Section 3, we present the theoretical framework of rescaling range analysis, wavelet transform analysis and their further modifications. Section 4 presents the results of our fractal and wavelet analysis, while Section 5 concludes.

2. Previous literature

2.1. The growth of the crytocurrency market

Bitcoin is a peer-to-peer digital asset, which claims to be decentralised and independent of monetary authority influence (Nakamoto [2008]). Böhme et al. [2015] provide a detailed description of the technology behind Bitcoin, including: the blockchain, mining, mining pools, transaction fees and wallets. Kroll et al. [2013], provides a detailed description of the mining process. Miners add verified transactions to a publicly distributed ledger, or blockchain, and are incentivised to do so by the reward of transaction fees and new bitcoins. A coin such as Bitcoin, is a single application built upon its underlying blockchain. A protocol such as Ethereum allows smart contracts to be executed upon its blockchain, and in doing so, allows decentralised applications to be built upon it. Any application built upon a blockchain, is intrinsically linked with that blockchain. A token issued upon the Ethereum blockchain (referred to as an ERC-20 token) requires Ether to execute the smart contracts that allow this token/application to function. In this way, as the token grows, so does the underlying protocol, and any income generated at the application layer, is distributed not only among its own token holders, but back to the underlying protocol itself[6].

Dyhrberg [2016] used GARCH models to classify Bitcoin as a form of financial asset, and to determine its role in the market. In doing so, the author questions if Bitcoin is more similar to US dollars (a currency and medium of exchange) or Gold (a commodity used as a store of value and for hedging). Dyhrberg determined that Bitcoin possesses similarities to both: it displays medium of exchange characteristics and reacts to Federal funds rates, but also reacts to the same variables as Gold, displays hedging capabilities, and exhibits a symmetric reaction to news. Peng et al. [2018] providesd an evaluation of the predictive performance of the volatility of three cryptocurrencies and three currencies with recognized stores of value using daily and hourly frequency data. Their results showed that SVR-GARCH models managed to outperform GARCH, EGARCH and GJR-GARCH models with Normal, Student's t and Skewed Student's t distributions. Gronwald [2014] empirically analysed Bitcoin prices which are found to be strongly characterised by price movements consistent with that of immature markets. Bouoiyour and Selmi [2016] examine the volatility of Bitcoin and ask whether this is an indication that the Bitcoin market is maturing. Using multiple-threshold and asymmetric-power GARCH models, the authors compare returns over two periods (2010-2014 and 2015-2016). The authors determine 2010-2014 to have been an 'explosive process' and observe Bitcoin volatility persistence to have fallen in 2015. In addition, the authors find Bitcoin prices to be asymmetrically driven. Corbet et al. [2017] examined the reaction of a broad set of digital assets to US Federal Fund interest rate, quantitative easing policy announcements, and the volatility spillovers generated as a result. The study provided some links between the cryptocurrencies valuations and monetary theory. Through the classification of cryptocurrencies into three broad categories denoted as currencies, protocols and decentralised applications, the authors present evidence of differing volatility reactions, indicating a diverse market in which not

[6] As such, Ethereum token holders benefit not only from the growth of Ethereum, but from the growth of any application built upon the platform, as a growth in the token, inherently causes the platform itself to grow

all cryptocurrencies are comparable to Bitcoin. Corbet et al. [2018] had originally found that cryptocurrencies may offer diversification benefits for investors with short investment horizons. Katsiampa [2018] found evidence of interdependencies in the cryptocurrency market, while it is shown that the two cryptocurrencies' conditional volatility and correlation are responsive to major news. Zhang et al. [2018] constructed a value-weighted Cryptocurrency Composite Index (CCI) and show that CCI and Dow Jones Industrial Average are persistently cross-correlated.

Bariviera et al. [2017] compare the dynamics of Bitcoin and traditional currency returns while focusing on the long-range memory of returns to find that Bitcoin volatility appears to be reducing over time. Fry and Cheah [2016] uncover evidence of a spillover from Ripple to Bitcoin, which exacerbates price decreases in Bitcoin, raising concerns about the long-term sustainability of Bitcoin with regards to increased competition from rival cryptocurrencies. The authors also examine the effect of a number of external events on the Bitcoin market (a technical software glitch[7] in 2013; the closure of the Silk Road website[8] in 2013) and find that they brought about an end in the speculative bubble, a scenario which was repeatedly observed during the dot-com bubble[9]. The volatility of the cryptocurrencies has been also analysed by Blau [2017], Katsiampa [2017] and Pieters and Vivanco [2017], to name but a few. Cryptocurrency exchanges have experienced a number of substantial crashes. Corbet et al. [2018] and Corbet et al. [2019] listed a number of significant issues that have faced broad cryptocurrencies, with regulation being identified as a prime source of pricing shocks and market volatility. Examples include the early 2018 decision by both China and South Korea to ban cryptocurrency trading, leading to broad price collapses across cryptocurrency exchanges. There have been further cases involving the collapse and hacking of exchanges, including the most famous example of Mt. Gox, however, the authors also consider the use of cryptocurrencies for the

[7]In March 2013, the Bitcoin blockchain temporarily split into two independent chains with differing rules on how transactions were accepted. For six hours two bitcoin networks operated at the same time, each with its own version of the transaction history. The core developers called for a temporary halt to transactions, sparking a sharp sell-off. Normal operation was restored when the majority of the network downgraded to version 0.7 of the bitcoin software. The Mt. Gox exchange briefly halted Bitcoin deposits and the exchange rate briefly dipped by 23% before recovering to previous level in the following hours.

[8]Silk Road was an online black market and the first modern darknet market, best known as a platform for selling illegal drugs. As part of the dark web, it was operated as a Tor hidden service, such that online users were able to browse it anonymously and securely without potential traffic monitoring. The website was launched in February 2011. In October 2013, the Federal Bureau of Investigation (FBI) shut down the website

[9]The dot-com bubble occurred in the late 1990s and was characterised by a rapid rise in equity markets fuelled by investments in Internet-based companies. During the dot-com bubble, the value of equity markets grew exponentially, with the technology-dominated NASDAQ index rising from under 1,000 to more than 5,000 between 1995 and 2000. The dot-com bubble grew out of a combination of the presence of speculative or fad-based investing, the abundance of venture capital funding for start-ups despite the failure of dot-coms to turn a profit.

illegal transfer of wealth which is been investigated by international regulators including that of the SEC in the United States.

2.2. Efficient Market Hypothesis (EMH) and Fractal Market Hypothesis (FMH)

The EMH was developed during the middle of the last century with the pioneering work of Osborne [1959] who empirically showed that stock prices follow a random walk. Eugene Fama [1965] consolidated earlier theories of market efficiency, developing the EMH, where the price of an asset fully reflects all the information available. Thus, EMH implies that it is impossible for markets participants to consistently outperform the market on risk-adjusted basis, as noted by Malkiel [1973][10]. Since the development of the EMH as the main theoretical framework for market analysis in the 1970s, 1980s and into the 1990s, researchers developed a large body of empirical evidence suggesting that EMH does not hold in either its strict or weaker forms. This evidence has prompted two developments in modern finance and economics. First, the emergence of behavioural finance as a field that offers fundamental-linked alternatives to the EMH (Kahneman [2014]); and secondly, the development of the FMH (Peters [2015]) as the key framework for analysing markets dynamics. Crucially, as argued by a number of authors (Kahneman [2014]), behaviourally anchored concepts of herding and referencing enable FMH-type dynamics. The FMH is based directly on empirically observed properties of the financial markets, as opposed to theoretical foundations, as is the case with EMH (Rachev et al. [1999] and Weron and Weron [2000]). Phillips and Gorse [2018], while investigating the price drivers of cryptocurrency markets using wavelet coherence analysis find evidence of medium-term positive correlations between online factors and prices which strengthen significantly during bubble-like regimes of the price series. The authors state that this explains why these relationships have previously been seen to appear and disappear over time. Secondly, the authors find that such short-term relationships between the chosen factors and price appear to be caused by particular market events (such as hacks / security breaches), and are not consistent from one time interval to another. While Corbet et al. [2018] found evidence of that Bitcoin has been in a consistent "bubble-phase" since breaching $1,000, there is evidence that some cryptocurrencies are reacting to their own internal dynamics such as mining difficulty and hashrate.

[10] Roberts [1967] extended Fama [1965] theory of informational efficiency in the markets to introduce a distinction between weak and strong form, which became the core of Fama [1970] taxonomy. While this paper does not pursue an in-depth review of the EMH, it is important to acknowledge that, since its formulation, the EMH has been subject to significant definitional and empirical challenges and modifications.

Two of the most relevant properties of the fractal geometry, as applied to the financial markets analysis are self-similarity and Non-Integer Dimension. In presence of fractal dynamics, each scale looks similar but not identical to the others, making data scale-invariant or self-similar. Put differently, there is no unique scale from which the others derive. Secondly, fractals show a Non-Integer Dimension. In the Euclidean geometry, dimension, namely how objects fill its space and how the object scales (Peters [2015]), are represented by integers[11]. In fractal geometry, dimensions are likely to be a non-integer number. As described by Feder [2013], D_s can be expressed as:

$$D_s = -\frac{lnN}{ln\ \mathrm{r}(N)} \tag{1}$$

The Hausdorff-Besicovitch dimension D equals D_s for self-similar fractals. Fractal geometry applied to financial markets means dealing mainly with time series and calculating time series' fractal dimension may not be so immediate. However, as pointed out by Hurst et al. [1969], there is a connection between the Hurst Exponent of a financial time series and its fractal dimension:

$$D = 2 - H \tag{2}$$

The fractal dimension of a financial time series measures how jagged it is and its value spans from 1 to 2. When $1 < d < 1.5$, financial time series is less jagged, more persistent and exhibits more long memory. For $1.5 < d < 2$, financial time series is more jagged and shows tendency to reverse more frequently than a random walk. Higher Hurst exponents would therefore imply a less volatile time series and a smoother and easily identifiable trend (Mandelbrot [1998]). Since we shall calculate the Hurst exponents for Bitcoin, Ethereum and Ripple, information about their fractal dimension will be provided. FMH transposes the concept previously described of self-similarity and local randomness as opposed to global determinism, to financial markets. The latter of these concepts can be traced back to the role of agents with different time horizons in ensuring market stability. EMH states that securities are publicly traded approximately at their fair value, but nothing is said about the level of liquidity. In his book setting out the main tenets of the FMH, Peters [2015] clarifies the implication of liquidity on market stability:

[11]Zero dimension for points, one dimension for lines and curves, two dimensions for planes and three dimensions for solids

1. In a liquid market it is possible to buy or sell an underlying security close to the price the market considers fair
2. Traders with different time horizons can trade among them efficiently
3. Supply and demand are balanced and the market is stable

In other words, a stable market needs to be a liquid one. If liquidity is adequate, the market is in equilibrium and the prices are assumed to be close to their fair value. If not, as the 2008 credit crunch proved, in a case of liquidity shortages, agents are willing to take any price they can get, irrespective of a fair value. The concept of liquidity is therefore the corner stone of FMH in explaining the presence of outliers in the normal distribution in a scale-invariant manner. Peters [2015] uses the FMH framework to argue that currencies valuations in the markets are not necessary related to their underlying economic fundamentals. Being traded in pairs, the main influencing factors are the respective interest rates in the two different countries and the activity of the central bank in manipulating its own currency buying or selling in the market according to predefined targets of monetary policy rather than profit objectives. Moreover, currencies are different from other traded securities like stocks and bonds, since there are no dividends or coupons to collect. In simple terms, currency trades are a zero-sum game and can be described as a 'pure' trading market. Exchange rates pairs of USD/JPY, JPY/GBP, USD/GBP and DEM/USD were assessed with evidence indicating that the long memory Hurst exponent is locally not Brownian but Brownian on average, and indeed a decay of the Hurst exponent as a function of time between 'regulatory acts' as shown by Ausloos and Ivanova [2000], Ausloos and Ivanova [2001a], Ausloos and Ivanova [2001b] and Ivanova and Ausloos [2002]. Furthermore, in all of them, the absence of a break in the visual representation of the plot of the V-Statistic led Peters [2015] to conclude that currencies have no stable relationship with their respective economies and that technical information and crowd behaviour tend to dominate. In other words, currencies appear to be an example of a 'True Hurst Process' characterised by infinite memory as opposed to bonds and stocks that may show long but still finite memory.

3. Methodology

3.1. Re-scaled range analysis

Financial market returns are considered not to be independent and identically distributed, therefore a non-parametric tool must be used in order to analyse market structures and dynamics. R/S

Analysis, is a robust non-parametric methodology developed by Hurst [1956] to distinguish between random and non-random series, the persistence of a trend as well as the existence and duration of cycles (Peters [2015]). Bariviera et al. [2017] had previously analysed the informational efficiency of the Bitcoin market to find that firstly, the R/S method is prone to detect long memory, whereas DFA method can discriminate more precisely variations in informational efficiency across time, while daily returns exhibit persistent behaviour in the first half of the period under study, whereas its behaviour is more informational efficient since 2014. To calculate the Hurst exponent, we first convert the financial time series of M price observations in a series of $N = M - 1$ logarithmic returns to represent the volatility of the price series:

$$V_i = log(P_i/P_{i-1}) \tag{3}$$

The times series of N observations is then divided into A_{ka} continuous sub-groups of length n, such as A x $n = N$, where each subgroup I_a constructed with $a = 1, 2, 3, ..., A$ and each element in I_a is labelled N_{ka} such that $k = 1, 2, 3, ..., n$. For each I_a of length n, we calculate the mean as:

$$\mu_a = \frac{1}{n}\sum_{k=1}^{n} N_{k,a} \tag{4}$$

We then create $Y_{k,a}$ which represents the deviations from the mean previously de-trended by subtracting the mean from each element of each sub-group:

$$Y_{k,a} = \sum_{k=1}^{n}(X_{i,a} - \mu_a) \tag{5}$$

The range is calculated by subtracting the minimum value from the maximum value of $Y_{k,a}$ within each sub-group I_a:

$$R_{I_a} = max(Y_{k,a}) - min(Y_{k,a}), \tag{6}$$

where, $l \leq k \leq n$. The standard deviation S_{I_a} is defined as:

$$S_{I_a} = \sqrt{\sum_{k=1}^{n}(N_{k,a} - \mu_a)^2} \tag{7}$$

Each range R_{I_a} is then normalised by dividing by S_{I_a}. The rescaled range of each I_a is therefore equal to R_{I_a}/S_{I_a}. Since we have A contiguous subgroups of length n, the average R/S value for length n is equal to:

$$(R/S)_n = \frac{1}{n}\sum_{a=1}^{a} R_{I_a}/S_{I_a} \tag{8}$$

Hurst [1956] applied this function to series that are in Brownian motion as follows:

$$(R/S)_n \sim cn^H \tag{9}$$

It is therefore possible to estimate the Hurst exponent H executing a linear regression considering the logarithmic versions of Hurst's equation:

$$log(R/S)_n = log(c) + Hlog(n) \tag{10}$$

It is important to note that the R/S analysis does not require that the underlying values be normally distributed, but just independent. The Hurst exponent therefore ranges between 0 and 1. If $H = 0.5$, the underlying time series follows a random walk. If $0.5 < H < 1$, it therefore implies that the underlying time series shows persistence and therefore long memory. If $0 < H < 0.5$, the underlying time series presents evidence of anti-persistence, thereby exhibiting negative correlations and generating reversals that occurs far more frequently than that of the random walk.

3.2. Correcting the Hurst Exponent

Since its development and its diffusion thanks to the work of Mandelbrot and Wallis [1969], R/S Analysis has been used to test for long memory. Over the years, the measure have shown varying degrees of success and effectiveness, especially in determining long-term memory in stock returns data (Willinger et al. [1999]). The authors note that for small samples, the original Hurst approach described above, shows a significant deviation from the slope representing Brownian Motion, hence theoretical values given by R/S Analysis are often approximated to:

$$E\left(\frac{R}{S}\right)_N = \begin{cases} \frac{N-\frac{1}{2}}{N}\frac{\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{\pi}.\Gamma\left(\frac{n}{2}\right)}\sum_{i=1}^{n-1}\sqrt{\frac{n-1}{i}}, n \leq 340 \\ \frac{N-\frac{1}{2}}{N}\frac{1}{\sqrt{n\frac{\pi}{2}}}\sum_{i=1}^{n-1}\sqrt{\frac{n-1}{i}}, n > 340 \end{cases} \tag{11}$$

where $\Gamma(x)$ is the Euler Gamma Functions. The formula was implemented by Peters [2015] generate improved results while dealing with issues relating to small sample sizes. In this paper, we study the behaviour of these assets that are relatively new to the markets. As the result, available data covers only a short period of time[12]. We estimate both classical and corrected statistics, and use the latter in our analysis.

3.3. V-statistics and cycles

The V-Statistic, originally developed by Hurst [1956] to test his results for stability, has proved to be a useful tool for establishing existence of any cycle in the underlying time series. It also tells us the cycle length. The V-Statistic is computed as follows:

$$V_n = \frac{(R/S)_n}{\sqrt{n}} \tag{12}$$

Plotting the V-Statistic and the Log of number of days, in case of the R/S scaling with the square root of time, the result should be an horizontal line. On the other hand, in presence of long term memory ($H > 0.5$) or anti-persistence ($H < 0.5$) the resulting graph would be up-trending or down-trending respectively. The V-Statistic will be used to confirm the validity of the Hurst Exponents obtained and looking for presence of bounds, or finite memory, in the financial time series. If the V-chart flattens out, that subsequent break in the memory process is found to have been dissipated and a cycle has been completed.

3.4. Continuous wavelet transform analysis

Continuous Wavelet Transform Analysis (CWT), is the second tool adopted for this study. The strength of this tool is its ability to assess the underlying information in the time as well as the frequency domain, providing information about signal's time and distribution evolution across different frequency time scales. A wavelet $\psi_{u,s}(t)$ is a real-valued square integral function with a scale s, location u and time t, which can be defined as:

$$\psi_{u,s}(t) = \frac{\psi\left(\frac{t-u}{s}\right)}{\sqrt{s}} \tag{13}$$

[12] 2011-present for Bitcoin, and 2015-present for Ethereum and Ripple

Any time series can be subsequently obtained back from its wavelet transform if the following condition where $\Psi(f)$ is admissible:

$$C_{\Psi} \int_{0}^{+\infty} \frac{|\Psi(f)|^{2}}{f}, df < +\infty \tag{14}$$

where $\Psi(f)$ is the Fourier transform of a wavelet. The CWT $W_x(u,s)$ is obtained by the projection of the conjugate $\psi'(.)$ of the wavelet $\psi(.)$ on the series $x(t)$ so that:

$$W_x(u,s) = \int_{-\infty}^{+\infty} \frac{x(t)\psi'\left(\frac{t-u}{s}\right))}{\sqrt{s}} \tag{15}$$

where $\psi^*(.)$ is a complex conjugate of $\psi(.)$. The continuous wavelet transform decomposes the series into frequencies that can be used to reconstruct the original series so that there is no information loss while the energy of the examined series is maintained, This can be explained through:

$$x(t) = \frac{\int_{0}^{+\infty} \int_{-\infty}^{+\infty} W_x(u,s)\Psi_{u,s}(t)duds}{s^2 C_{\Psi}} \tag{16}$$

$$\|x\|^2 = \frac{\int_{0}^{+\infty} \int_{-\infty}^{+\infty} |W_x(u,s)|^2 \, duds}{s^2 C_{\Psi}} \tag{17}$$

where $|W_x(u,s)|^2$ is the wavelet power at scale $s > 0$. The CWT divides and reconstructs the underlying series into frequencies so that there is no loss in the information and the energy contained in the series is maintained. There is a considerable number of different wavelets available, following (Percival and Walden [2000]), we shall opt for the Morlet Wavelet, which provides good results in localization balance between time and frequency (Aguiar-Conraria et al. [2008]). Morlet Wavelet with a central frequency of ω_0 is defined as:

$$\psi(t) = \frac{e^{i\omega_0 t - \left(\frac{t^2}{2}\right)}}{\pi^{1/4}} \tag{18}$$

According to Grinsted et al. [2004], a value of $\omega_0 = 6$ ensures finally a well-balanced result between time and frequency location.

3.5. Cross wavelet transform analysis

The CWT can be further generalised into the Cross wavelet transform (XWT) $W_x(u,s)$ and $W_y(u,s)$ in order to study the relationship of two differing time series $x(t)$ and $y(t)$ across time and scales.

$$W_{xy}(u,s) = W_x(u,s) \times W_y^*(u,s) \tag{19}$$

Since the XWT is usually quite complex, the cross wavelet power $|W_{xy}(u,s)|$ is frequently adopted to measure the relationship between two different time series, showing regions of the time-frequency space where both of the series present high power, which is interpreted as localised covariance between the latter. However, CWP results are limited, since they do not have any bound. Therefore, to solve the issue, the wavelet coherence, $R^2_{xy}(u,s)$ and a smoothing operator S have been introduced and are calculated as:

$$R^2_{xy}(u,s) = \frac{\left|S\left(\frac{1}{S}W_{xy}(u,s)\right)\right|^2}{S\left(\frac{1}{S}\left|W_x(u,s)\right|^2\right)S\left(\frac{1}{S}\left|W_y(u,s)\right|^2\right)} \tag{20}$$

$R^2_{xy}(u,s)$ ranges between 0 and 1, and can be considered as the localised squared correlation between two different time series across time and frequency. However, since we use the square coherence, information inherent to the direction of the correction between $x(t)$ and $y(t)$ is lost. For this reason, a phase difference is introduced, where Φ and Ω represent the real and imaginary operators respectively.

$$\varphi_{xy}(u,s) = tan^{-1}\left(\frac{\Phi\left|S\left(\frac{1}{s}W_{xy}(u,s)\right)\right|}{\Omega\left|S\left(\frac{1}{s}W_{xy}(u,s)\right)\right|}\right) \tag{21}$$

The graphical result of the function is an arrow, if it points west (east) in the chart, it means that in that point the underlying time series are negatively (positively) correlated. Finally, if the arrow points south (north), the first series leads the second by a ratio of $\pi/2$ and vice versa. However, the result is often a combination of both, for instance with the arrows pointing in direction of south-west, meaning that there is negative correlation with the first series leading the second one.

4. Data and Results

4.1. Data

For the purpose of this paper, daily price time series for Bitcoin, Ethereum and Ripple have been assessed. All the data were downloaded from https://www.cryptocompare.com/. All returns were calculated in log form. Bitcoin's price history under investigation spanned from 1 January 2011 to 31 December 2017, for a total of 2,557 observations. XRP and ETH, due to their more recent inceptions have been investigated from 21 January 2015 7 August 2015 until 31 December 2017, for a total of 1076 and 878 observations respectively.

4.2. Rescaled range analysis

We begin our analysis by calculating the Hurst exponent which is designed to detect the presence of long memory. Since all the time series have been also divided into yearly subsets to assess development over time, we calculate both the 'classical' Hurst exponent (un-adjusted) as well as the 'adjusted' exponent. The latter is used to correct the significant deviations from the slope representing Brownian Motion when R/S Analysis is performed on small samples.

The resulting overall H coefficients, which are reported in Table 1, provide interesting evidence that allows us to draw the conclusion that Bitcoin (BTC), Ethereum (ETH) and Ripple (XRP) may show quite different price behaviours. As also concluded by Bariviera et al. [2017], since its inception, Bitcoin's Hurst Exponent has in fact decreased over time. Starting, on the adjusted basis, from 0.58440 in 2011, BTC shows a cyclical pattern of alternate persistence/anti-persistence across the whole period assessed. The H oscillation has indeed reduced starting from 2014, showing values converging toward a smaller range around 0.50. On the other hand, assessing the whole time series available for BTC, a H value of 0.61317 clearly depicts the historical presence of persistency in its underlying time series. Furthermore, looking across results for 2015-2017, while BTC shows the same pattern of up-down movements in H as shown in previous years, we note that ETH and XRP show instead increasing Hurst exponent measurements over time. Considering the whole time series available, ETH shows an overall H value of 0.55820, presenting therefore evidence of slight persistence, while XRP, with $H = 0.4750$, is the only one showing anti-persistent features. The lack of Hurst exponent stability across different years of cryptocurrencies evolution is, in our view, consistent with the lack of markets maturity for cryptocurrencies across the years. However, as the Hurst exponents presented in Table 1 show some sensitivity to time intervals and to the fact that, by their nature, two of the three cryptocurrencies considered did not exist prior to 2015 (XRP and ETH were released in January and August 2015 respectively), to control for the latter problem, we estimate the comparable Hurst exponents for the three cryptocurrencies over the period 2016-2017, as presented in the last column in Table 1. Assessing the period 2016-2017, BTC can be described by a random walk while ETH and XRP with $H > 0.5$ show presence of persistence. Our 2016-2017 results, therefore, can lead to different conclusions. Bitcoin has been maturing and markets for BTC are becoming more efficient (similar to Bariviera et al. [2017]) and BTC/USD price development might also be led by underlying cryptocurrency fundamentals emerging overtime. In contrast, ETH and XRP still show presence of persistence, suggesting that price dynamics may be instead dominated by short-termism and crowd behaviour. However, it is worth noting that the data for cryptocurrencies used to calculate the exponents is inherently lower quality due to fragmented nature of exchanges and significant changes in regulatory environments altering the nature of cryptocurrencies markets on the ongoing basis. As the result, it is informative to consider overall Hurst exponent in our analysis.

Insert Table 1 about here

Theoretically, the presence of long memory implies that today's returns are correlated with future returns, regardless of the time scale considered. We have also found that Bitcoin's time series shows a fractal dimension of 1.38683, implying a less rugged time series and a smoother and easily identifiable trend compared to that of a random walk. Ethereum, as discussed earlier, differs from Bitcoin as it follows an increasing and unbounded supply. ETH's inception in mid-2015 at 3$ was characterised by a strong correction which led the price constantly down for 3 months before reverse and stabilise at 1$ toward the end of 2015. This strong correction may be the explanation for the resulting anti-persistence with $H = 0.45757$. 2017 tremendous explosion in price is also well captured by a $H = 0.53220$. However, albeit Ethereum's price behaviour, with an overall fractal dimension of 1.4418, shows slight presence of persistence, compared to BTC, is de facto better described by a random walk.

Ripple, as already discussed, has instead a fixed number of tokens supplied. Considering that the company holds more than 60% of the whole supply, inventors' fear of a potential flood of billions of new tokens may act as an explanation as to why there is evidence of strong anti-persistence, with H values of 0.31784 and 0.40300 in 2015 and 2016 respectively, implying that Ripple's returns revert more frequently compared to a random walk process. As with BTC and ETH, also XRP presents evidence of persistence in 2017 with H = 0.57740, which may be attributed to Ripple altering its policy on the limits of supply of the tokens to the market hence improving overall ripple's predictability by the investors[13]. Finally, the overall $H = 0.4754$ and a consequent fractal dimension of 1.5246, lead to the conclusion that Ripple's price behaviour is the only one to show features of anti-persistence generating reversals that occurs far more frequently than expected by a random walk.

4.3. Evidence from V-statistics

The V-Statistic is the ratio between the R/S_n and the square root of time, which is presented in Figure 2. In presence of a time series that shows persistence (anti-persistence) the ratio will be up-trending (down-trending), while in the case of random walk the ratio will be essentially flat. Furthermore, every break in the V-Statistic plot, namely where the V-chart flattens, may imply that the memory process within the time series is dissipated, thereby representing the hypothetical existence of hidden market cycles in time series' movements.

Insert Figure 1 about here

Looking at the V-Statistics, we may conclude that either BTC's price data available is not enough or, as observed by Peters (1994) assessing the returns of the most traded currency pairs, BTC's overall price history would be another example of 'True Hurst Process' (Peters [2015]), where the positive slope, along with the absence of a break in the plot suggests that there is no crossover to longer term fundamental valuation and technical information continues to dominate all investment horizons, therefore showing infinite memory and no stable relationship with economic fundamentals, where crowd behaviour and technical information dominate its price behaviour hence excluding a buy-and-hold as the optimal strategy. Similar conclusion is not warranted for Ethereum and Ripple mainly given to the shorter time horizon available.

[13]In May 2017, Ripple announced that the company will permanently remove 55 billion XPR from the market, placing it in a cryptographically-secured escrow account. As stated by Ripple in 2017, "By securing the lion's share of our XRP, investors can now mathematically verify the maximum supply of XRP that can enter the market".

4.4. Wavelet analysis

As explained in methodological section above, we have approached the time series under analysis using the Continuous Wavelet Transform (CWT) and the different techniques based on it, namely Cross Wavelet Transform (XWT) and Wavelet Coherence (WTC). As a first step, we shall repeat the analysis provided by Delfin-Vidal and Romero-Meléndez [2016] on Bitcoin, who used Wavelet Analysis to confirm the FMH's assumption that unstable markets present evidence of dominance sourced from short-term investors.

As suggested by Torrence and Compo [1998], to ensure significance of the results, the wavelet power generated is tested against the Null Hypothesis of a red noise, resulting in the thick black line surrounding the areas with high energy. Moreover, since the CWT applied on finite-length time series suffers from border distortion, a Cone of Influence (COI) separates the portion where the inference is reliable from the pale one, where it is not (Grinsted et al. [2004]).

Insert Figure 2 about here

As concluded by Delfin-Vidal and Romero-Meléndez [2016], Figure 2 attests that regions of highest power are those characterised by high volatility. Important and significant power is also detected at lower frequencies, implying that in those periods, namely June 2011, April and November 2013 and February 2014, investors with multiple time horizons contributed to the volatility generated. However, what should capture the attention of the reader, is the complete lack of power in the second half of the chart, corresponding to the period between the second half of June 2014 and the end of 2017.

Insert Figure 3 about here

Figure 3 depicts the wavelet power spectrum of Bitcoin daily returns from June 2014 to the end of 2017. Over the time horizon under assessment, it is again possible to observe a clear dominance at high frequency during the moments of high volatility. This is particularly evident back to January 2015, when BTC, after losing more than 25% from \$221.29 to \$164.92 on the 14th, rebounded back by circa 27% to \$209.78 on the following day, marking daily volatilities of 0.0889 and 0.0559 respectively. Excluding the extreme event experienced on February 26th 2014, when BTC made

+336%, going from \$135.78 to \$593.14, the highest daily volatility recorded was experienced on June 11th 2011 with 0.2457. During the strong rally of the last half of 2017, the highest volatility occurred on July 20th 2017 when BTC, scoring a +25.5%, from \$2282.58 to \$2866.0 experienced a daily volatility of 0.0499. The lack of this information in the previous daily power spectrum, along with the results of the R/S Analysis previously assessed, confirms the structural reduction in Bitcoin's volatility throughout the years, may also correspond to a structural change in the investors' time horizon's role in times of high volatility. The latter aspect may be predominantly driven by traders whose strategies are influenced by small price movements compared to those adopted by investors with longer investment time horizons and hence greater risk tolerance.

Insert Figures 4 and 5 about here

In Figures 4 and 5, we applied the CTW to ETH and XRP. In moments of high volatility, we notice dominance of power in the high frequencies in both the time series. Furthermore, unlike BTC, the steep price increase shown by ETH and XRP between April and May, and the consequent price development up to the end of 2017, was correctly captured by the power spectra. Looking at its propagation over different time scales, we can also notice that those regions of high energy at high frequencies, coinciding with moments of high volatility, acted as triggers for the initial up-trend consequently reinforced by investors with lower time horizons. This is depicted by the energy spreading over lower frequencies.

The final step was to analyse how the different cryptocurrencies are correlated with each other. Starting from the respective couple of CWTs, the resulting XWT and WTC were calculated. The XWT allow us to spot regions in the time-frequency space where the instruments analysed show high common power and its related phase relationship, while WTC give us information about the local correlation between two different time series and how it changes over time. Figures 6, 7 and 8 present evidence of the regions where BTC, ETH and XRP, taken in pairs, share common energy. The arrows give information about the phase and lag-lead behaviour of their relationship.

Insert Figures 6, 7 and 8 about here

Figure 6 depicts the regions of common energy between Bitcoin and Ethereum. Albeit the presence of statistically significant energy in several regions in the period assessed going from the

7 August 2015 to the end of 2017, the behaviour of the arrows in the first half of the chart does not allow us to distinguish a clear relationship between them; however, something changed starting from august 2016, where the areas of common power across the different frequencies shows a positive phase (right arrows) between BTC and ETH. In-phase areas of common power which characterise 2017 price developments also depicts how BTC led ETH at lower frequencies in the spectrum.

Figures 7 and 8, show the same relationship between BTC/XRP and ETH/XRP respectively. Despite the shared energy at high frequencies during the period from April 2017 and the end of 2017, neither Bitcoin nor Ethereum seem to show any clear relationship with Ripple. According to the evidence obtained, we may therefore infer that while BTC and ETH, starting from the beginning of 2017, converged showing evidence of in-phase shared energy, Ripple does not show any clear signs of common behaviour with its counterparts. Nonetheless, we notice that ETH and XRP also show in-phase common power on the lowest part of the spectrum, although a significant amount of this information is near or outside the COI, making it not fully reliable.

Insert Figures 9, 10 and 11 about here

In Figures 9, 10 and 11 we investigate as to how the pairwise valuations co-vary over time. Looking at Figure 9, depicting the relationship between BTC and ETH, starting from August 2016, we see how in moments of high energy, the arrows suggest presence of positive correlation with BTC slightly leading ETH. Figure 10, depicts the same relationship between BTC and XRP. It shows far less regions in which they correlate. The energy depicted at the right edge of the chart, which represents the turmoil between June and the end of 2017, exhibits positive correlation led by BTC at higher frequencies. Finally, in Figure 11, we see the relationship between ETH and XRP. The pair of cryptocurrencies shows alternating regions of positive and negative correlation. ETH seems to lead at high frequencies in moments of high volatility. Looking at the very bottom of the chart, we also observe signs of long term positive correlation with ETH leading XRP. However, this information shows near or outside the COI and cannot be considered highly reliable. Our results appears to be in agreement with that presented previously by Phillips and Gorse [2018]. When concluding, the authors found that such short-term relationships between their selected factors and prices appear to be caused by particular market events, which are stated to be represented by events such as hacks and security breaches, which are not consistent from one time interval to another.

To conclude, assessing the three cryptocurrencies' relationship using Wavelet Transform Coherence, we still conclude that are BTC and ETH have the highest degree of correlation, sharing far more common areas of positive correlation and energy on high frequencies, especially in events of high volatility. XRP, on the other hand, seems to be correlated with ETH, and is led mainly by ETH, in the short, as well as in the in the long term.

4.5. Implications for investors

The results presented above are of significant importance to investors in cryptocurrencies and to the broader investment markets participants for a number of reasons.

Our rescaled range analysis shows that BTC, ETH and XRP exhibit distinct price behaviours. The markets for BTC are shown, in the latter periods covered by our analysis, to hold promise of improved efficiency and increasing potential for fundamentals-based asset pricing, although our V-statistic estimates imply that maturation trend is still weak. This highlights the increasing importance of BTC as a 'store of value' and 'long-only' asset, similar in some ways to gold and other precious metals. These findings also confirm a popular (especially since the end of 2018) view that BTC is a distinct asset from other cryptocurrencies. In contrast to BTC, ETH and XRP continue to exhibit presence of persistence, with both cryptocurrencies price dynamics appearing to be dominated by speculative short-termism and crowd herding. In the longer run, our V-statistic analysis indicates that despite improvements mentioned above, BTC shows no stable relationship with economic fundamentals so far and that crowd behaviour and technical information continue to dominate its price behaviour hence excluding a buy-and-hold as the optimal strategy for pricing in broader financial markets risks. This problem is even more pronounced for other cryptocurrencies.

Our wavelet analysis is of further utility to investors by extending the results from Delfin-Vidal and Romero-Meléndez [2016] for BTC to XRP and ETH, and including coverage of the more mature period of BTC and other cryptocurrencies history from 2016 through 2017. The key finding here is that price dynamics of cryptocurrencies were still dominated by short-term speculators, as opposed to buy-and-hold investors, and that the period of 'maturation' of BTC and the emergence of ETH and XRP offers little divergence from this trend. In investment and trading risk management space, our findings suggest that lower liquidity risks and higher impact of short-termism in the major cryptocurrencies pricing are coincident with periods of higher volatility, presenting an added problem for fundamentals-focused long term investors.

Our final set of contributions of relevance to investors relates to the analysis of correlations between the currency pairs. We identify periods of high and low common power in the time-frequency space, as well as provide analysis of the changes in these relationships over time. We show that from August 2016 there is a positive relationship between BTC and ETH, BTC and XRP, and ETH and XRP. BTC leads ETH and XPR at low frequencies, providing some support to the investors' view that BTC offers strongest opportunities for capitalising on the price dynamics of the broader cryptocurrencies asset class. Interestingly, from April 2017, our findings suggest that at higher frequencies, investors do need to consider diversifying their cryptocurrency portfolios to include XRP, while dominance of BTC over ETH remains. In addition, we also show evidence of sensitivity of some pairwise correlations to the general markets environment, and in particular evidence of ETH leading XRP in period of cryptocurrency markets turmoil and in the wake of major security breaches. Given that ETH itself is led by BTC, we can conclude that investors can benefit from major risk management and investment properties of the crypto assets by focusing predominantly on BTC as the preferred asset allocation in the asset class.

Our findings present an important addition to the literature and offer investors a more clear perspective on the strategic choices of cryptocurrency assets to be included in their portfolios, time horizon features of these cryptocurrencies and the hedging/risk management properties of BTC, ETH and XRP. In simple terms, investors pursuing purely speculative and/or trading gains are best served by focusing their attention on ETH and XRP, while investors interested in longer-term store of value and/or fundamentals-driven investing can be better served by pursuing BTC allocations.

5. Conclusions

In this paper we investigated dynamics and fractal properties of the price behaviour of BTC, ETH and XRP using Rescaled Range and Wavelet Analysis. Several interesting findings have emerged. First, the results lead to the conclusion that Bitcoin, assessed over the entire history of its existence, with H = 0.61417, presents evidence of long memory. BTC presents evidence of cyclical persistency/anti-persistency behaviour throughout the whole sample, which is not found with regard to Ethereum and Ripple so far. Furthermore, this cyclical behaviour has been also smoothing overtime, showing values ranging toward smaller deviations from 0.50. The reducing H over time can be explained by a maturing market for BTC and a change in agents structure with an increased number of investors with a long term investment horizon sensitive to the opportunity cost of selling or spending it, especially during the last up-trend in 2017, providing incentives to select a 'mine-and-hold strategy'.

We also used the V-statistic to test the goodness of fit of our results. Where a flat slope implies Brownian motion, assessed overall, BTC's slope coefficient of 0.2275 and unbounded memory allowed us to conclude that Bitcoin may be best described as a 'True Hurst Process', presenting no stable relationship with economic fundamentals and infinite memory, where crowd behaviour and technical information tend to dominate its price development.

Using R/S Analysis and V-Statistic to compare how BTC, ETH and XRP behaved across 2016-2017, showed how BTC, with H=0.50040 resembled a random walk, while ETH and XRP, with H=0.58100 and H=0.52640 respectively, showed a growing underlying memory behaviour, which may be mainly attributed to recent price increases and still immature markets.

Globally, we have also found clear evidence of high energy at high frequencies under periods of high volatility. We make no conclusion as to the causality direction in this observation. In fact, high Hurst statistic can correspond to persistence during rising and falling trend prices. However, while previous research was mainly focused on how financial markets break down, our power spectra show instead how FMH's third assumption holds also when markets increase substantially in value. In our case, we have found that volatility and momentum in market values are located at the beginning of the up-trends, acting as actual triggers for further price appreciations. Power spectra also show the energy spreading from high to lower frequencies, which well depicts an idea of a cascade effect, that is, where the new trend, starting from short-time traders operating on technical information and news, is subsequently strengthened by the involvement of investors with longer time horizons.

Overall, our results show that all three major cryptocurrencies are exhibiting evolving dynamics that can be attributed to the emerging nature of this asset class and lower maturity of markets for these assets. Bitcoin clearly leads these processes, as indicated by our findings, and offers investors the strongest opportunity for fundamentals-based long term investment, as well as the stronger risk management properties (correlations dynamics) of the three assets. On the other hand, investors interested in capturing more speculative or higher frequency trading returns, notwithstanding liquidity risks involved, should pay more attention to evolution of Ethereum and Ripple. We suggests that future research should focus on continuing tracking evolution of the cryptocurrencies dynamics for further signs of markets maturation. In addition, fruitful direction for future research can involve looking into causal drivers for differences in dynamic properties of various cryptocurrencies, including the relationship between monetary policies and the broader monetary-economic environments, and the cryptocurrencies price dynamics.

Bibliography

Aguiar-Conraria, L., N. Azevedo, and M. J. Soares (2008). Using wavelets to decompose the time-frequency effects of monetary policy. *Physica A: Statistical Mechanics and its Applications 387*(12), 2863–2878.

Aslanidis, N., A. F. Bariviera, and O. Martínez-Ibañez (2019). An analysis of cryptocurrencies conditional cross correlations. *Finance Research Letters 31*, 130–137.

Ausloos, M. and K. Ivanova (2000). Introducing false eur and false eur exchange rates. *Physica A: Statistical Mechanics and its Applications 286*(1-2), 353–366.

Ausloos, M. and K. Ivanova (2001a). Correlations between reconstructed eur exchange rates versus chf, dkk, gbp, jpy and usd. *International Journal of Modern Physics C 12*(02), 169–195.

Ausloos, M. and K. Ivanova (2001b). False euro (feur) exchange rate correlated behaviors and investment strategy. *The European Physical Journal B-Condensed Matter and Complex Systems 20*(4), 537–541.

Bariviera, A. F., M. J. Basgall, W. Hasperué, and M. Naiouf (2017). Some stylized facts of the Bitcoin market. *Physica A: Statistical Mechanics and its Applications 484*, 82–90.

Blau, B. M. (2017). Price dynamics and speculative trading in bitcoin. *Research in International Business and Finance 41*, 493–499.

Böhme, R., N. Christin, B. Edelman, and T. Moore (2015). Bitcoin: Economics, Technology, and Governance. *Journal of Economic Perspectives 29*(2), 213–238.

Bouoiyour, J. and R. Selmi (2016). Bitcoin: A beginning of a new phase? *Economics Bulletin 36*(3), 1430–1440.

Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. *Quantitative Finance 1*(2), 223–236.

Corbet, S., C. Larkin, B. Lucey, A. Meegan, and L. Yarovaya (2017). Cryptocurrency reaction to fomc announcements: Evidence of heterogeneity based on blockchain stack position. *Available at SSRN*, November 18, 2017.

Corbet, S., B. Lucey, A. Urquhart, and L. Yarovaya (2019). Cryptocurrencies as a financial asset: A systematic analysis. *International Review of Financial Analysis 62*, 182–199.

Corbet, S., B. Lucey, and L. Yarovya (2018). Datestamping the bitcoin and ethereum bubbles. *Finance Research Letters 26*, 81–88.

Corbet, S., A. Meegan, C. Larkin, B. Lucey, and L. Yarovaya (2018). Exploring the dynamic relationships between cryptocurrencies and other financial assets. *Economics Letters 165*, 28–34.

Cunningham, L. A. (2000). From Random Walks to Chaotic Crashes: The Linear Genealogy of the Efficient Capital Market Hypothesis. *SSRN Electronic Journal*.

Delfin-Vidal, R. and G. Romero-Meléndez (2016). The fractal nature of bitcoin: Evidence from wavelet power spectra. In *Trends in Mathematical Economics: Dialogues Between Southern Europe and Latin America*, pp. 73–98.

Dyhrberg, A. H. (2016). Bitcoin, gold and the dollar - A GARCH volatility analysis. *Finance Research Letters 16*, 85–92.

Fama, E. F. (1965). Random Walks in Stock Market Prices. *Financial Analysts Journal 21*(5), 55–59.

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The journal of Finance 25*(2), 383–417.

Feder, J. (2013). *Fractals*. Springer Science & Business Media.

Fry, J. and E. T. Cheah (2016). Negative bubbles and shocks in cryptocurrency markets. *International Review of Financial Analysis 47*, 343–352.

Grinsted, A., J. C. Moore, and S. Jevrejeva (2004). Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics 11*(5/6), 561–566.

Gronwald, M. (2014). The economics of Bitcoin: Market characteristics and price jumps. *CESifo Working Paper* (5121).

Hurst, H., R. Black, and Y. Simaika (1969). Long-term storage. an experimental study, constable, london (1965). *Google Scholar*.

Hurst, H. E. (1956). The problem of long-term storage in reservoirs. *International Association of Scientific Hydrology. Bulletin 1*(3), 13–27.

Ivanova, K. and M. Ausloos (2002). Are eur and gbp different words for the same currency? *The European Physical Journal B-Condensed Matter and Complex Systems 27*(2), 239–247.

Kahneman, D. (2014). Thinking Fast and Slow. *Igarss 2014* (1), 1–5.

Katsiampa, P. (2017). Volatility estimation for Bitcoin: A comparison of GARCH models. *Economics Letters 158*, 3–6.

Katsiampa, P. (2018). Volatility co-movement between bitcoin and ether. *Finance Research Letters*.

Kristoufek, L. (2013a). Bitcoin meets google trends and wikipedia: Quantifying the relationship between phenomena of the internet era. *Scientific reports 3*, 3415.

Kristoufek, L. (2013b). Fractal markets hypothesis and the global financial crisis: wavelet power evidence. *Scientific reports 3*, 2857.

Kroll, J. a., I. C. Davey, and E. W. Felten (2013). The Economics of Bitcoin Mining, or Bitcoin in the Presence of Adversaries. *The Twelfth Workshop on the Economics of Information Security (WEIS 2013)* (Weis), 1–21.

Kumar, A. and C. Smith (2017). Reserve bank of new zealand analytical notes.

Malkiel, B. G. (1973). A Random Walk Down Wall Street. *Foundations 13*, 464.

Mandelbrot, B. B. (1998). Fractals and scaling in finance: Discontinuity, concentration, risk. *Physics Today*, 551.

Mandelbrot, B. B. and J. R. Wallis (1969). Computer Experiments With Fractional Gaussian Noises: Part 1, Averages and Variances. *Water Resources Research 5*(1), 228–241.

Nakamoto, S. (2008). Bitcoin: A Peer-to-Peer Electronic Cash System. *Www.Bitcoin.Org*, 9.

Osborne, M. F. M. (1959). Brownian Motion in the Stock Market. *Operations Research 7*(2), 145–173.

Peng, Y., P. H. M. Albuquerque, J. M. C. de Sá, A. J. A. Padula, and M. R. Montenegro (2018). The best of two worlds: Forecasting high frequency volatility for cryptocurrencies and traditional currencies with support vector regression. *Expert Systems with Applications 97*, 177–192.

Percival, D. B. and A. T. Walden (2000). Wavelet Methods for Time Series Analysis. *Cambridge Series in Statistical and Probabilistic Mathematics*, 1–620.

Peters, E. E. (2015). Fractal market analysis : Applying chaos theory to investment and economics. *Wiley Finance Editions*, xviii, 1–315.

Phillips, R. C. and D. Gorse (2018). Cryptocurrency price drivers: Wavelet coherence analysis revisited. *PloS one 13*(4), e0195200.

Pieters, G. and S. Vivanco (2017). Financial regulations and price inconsistencies across Bitcoin markets. *Information Economics and Policy 39*, 1–14.

Rachev, S. T., A. Weron, and R. Weron (1999). Ced model for asset returns and fractal market hypothesis. *Mathematical and computer modelling 29*(10-12), 23–36.

Roberts, H. V. (1967). Statistical versus clinical prediction of the stock market. *Unpublished manuscript, CRSP, University of Chicago, May.*.

Roch, A. F. (2011). Liquidity risk, price impacts and the replication problem. *Finance and Stochastics 15*(3), 399.

Taylor, S. (2007). Asset Price Dynamics, Volatility, and Prediction. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*(4), 1187–1189.

Torrence, C. and G. P. Compo (1998). A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society 79*(1), 61–78.

Weron, A. and R. Weron (2000). Fractal market hypothesis and two power-laws. *Chaos, Solitons & Fractals 11*(1-3), 289–296.

Willinger, W., M. S. Taqqu, and V. Teverovsky (1999). Stock market prices and long-range dependence. *Finance and Stochastics 3*(1), 1–13.

Zhang, W., P. Wang, X. Li, and D. Shen (2018). The inefficiency of cryptocurrency and its cross-correlation with dow jones industrial average. *Physica A: Statistical Mechanics and its Applications 510*, 658–670.

Table 1: Classical and Corrected Hurst exponents by year and Fractal Dimension

Token	2011	2012	2013	2014	2015	2016	2017	Total Sample	2016/2017
Bitcoin	0.58440	0.46010	0.59050	0.47900	0.52850	0.49560	0.53000	0.61317	0.50040
	(0.67670)	(0.54180)	(0.67930)	(0.56360)	(0.61810)	(0.58320)	(0.61710)	(0.66271)	(0.57540)
								*1.38683	*1.4996
Ethereum	-	-	-	-	0.45757	0.51380	0.53220	0.55820	0.58100
	-	-	-	-	(0.59574)	(0.59740)	(0.61980)	(0.63490)	(0.65830)
								*1.4418	*1.419
Ripple	-	-	-	-	0.31784	0.40300	0.57740	0.47540	0.52640
	-	-	-	-	(0.41170)	(0.58350)	(0.67110)	(0.54900)	(0.60630)
								*1.5246	*1.4736

Note: The Table shows the H values resulting for the R/S Analysis by year and on the entire time series. The values in brackets represent the 'classical' Hurst exponent calculation, while the values denoted with * are the resulting fractal dimension calculated as $dim = 2 - H$.

Figure 1: V-Statistic compared with Brownian motion

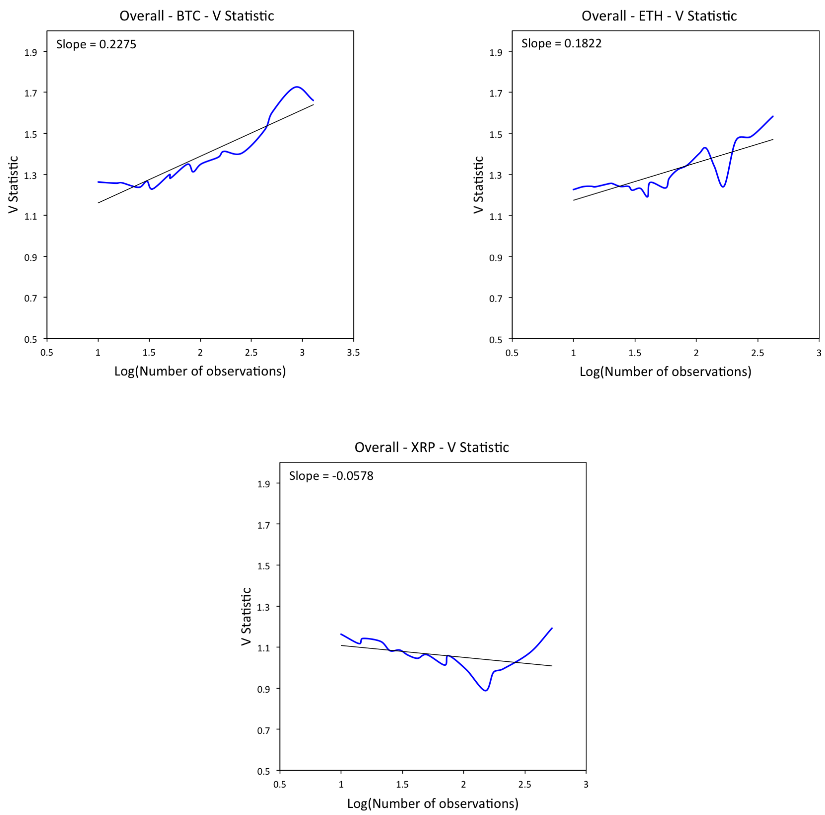

Figure 2: Wavelet power spectrum of daily Bitcoin returns

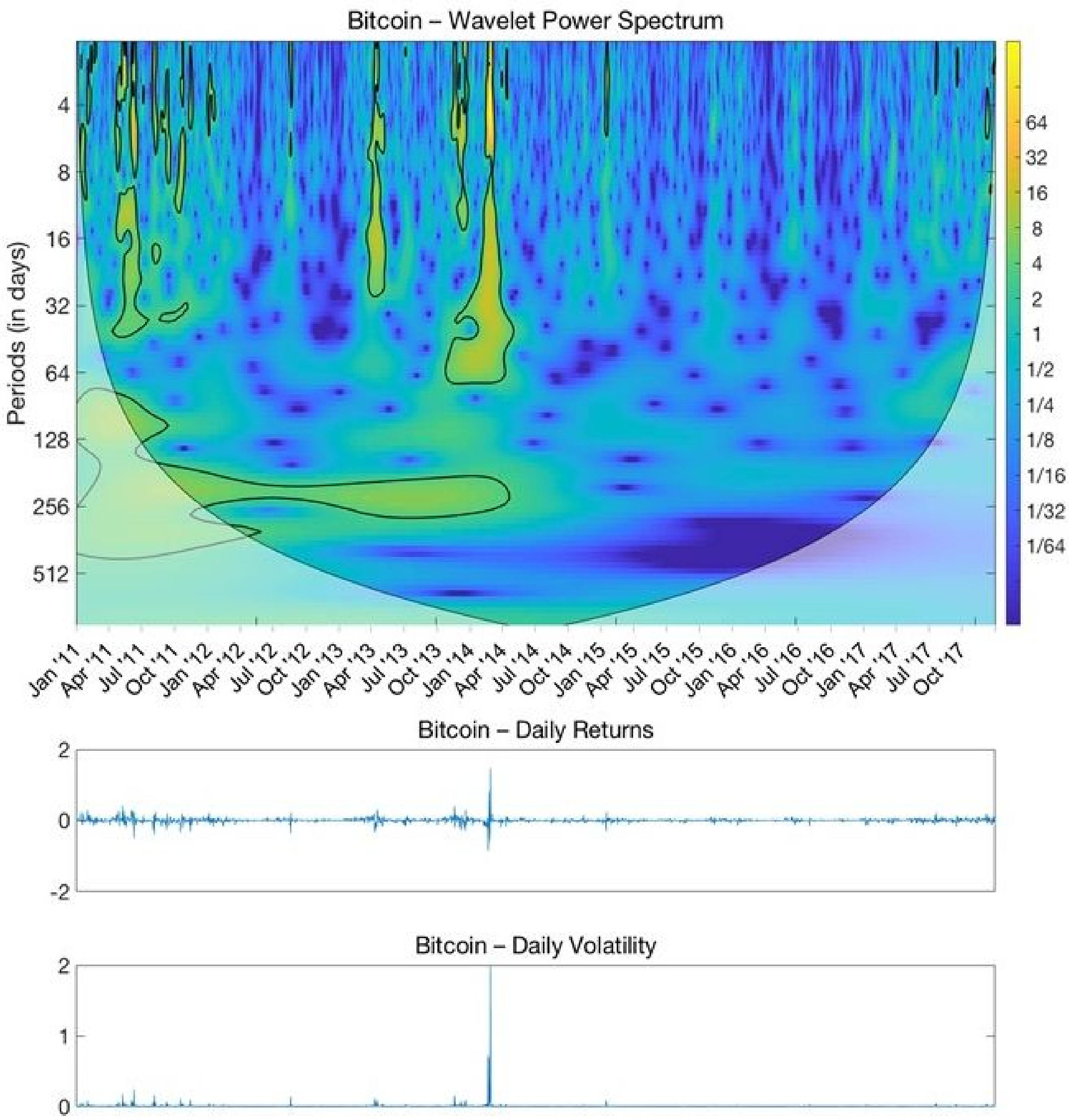

Figure 3: Wavelet power spectrum of daily Bitcoin returns from June 2014 to Dec 2017

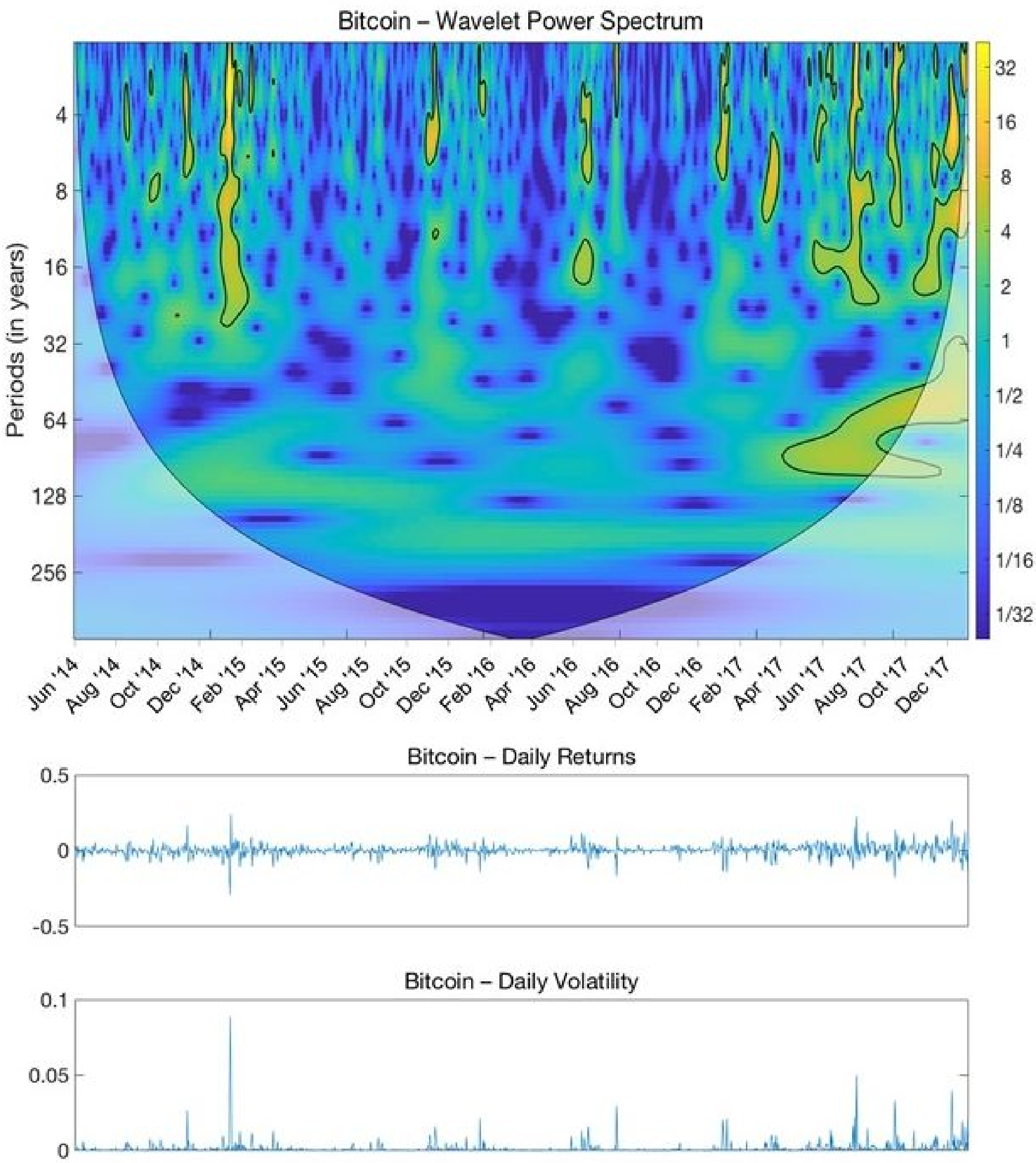

Figure 4: Wavelet power spectrum of daily Ethereum returns

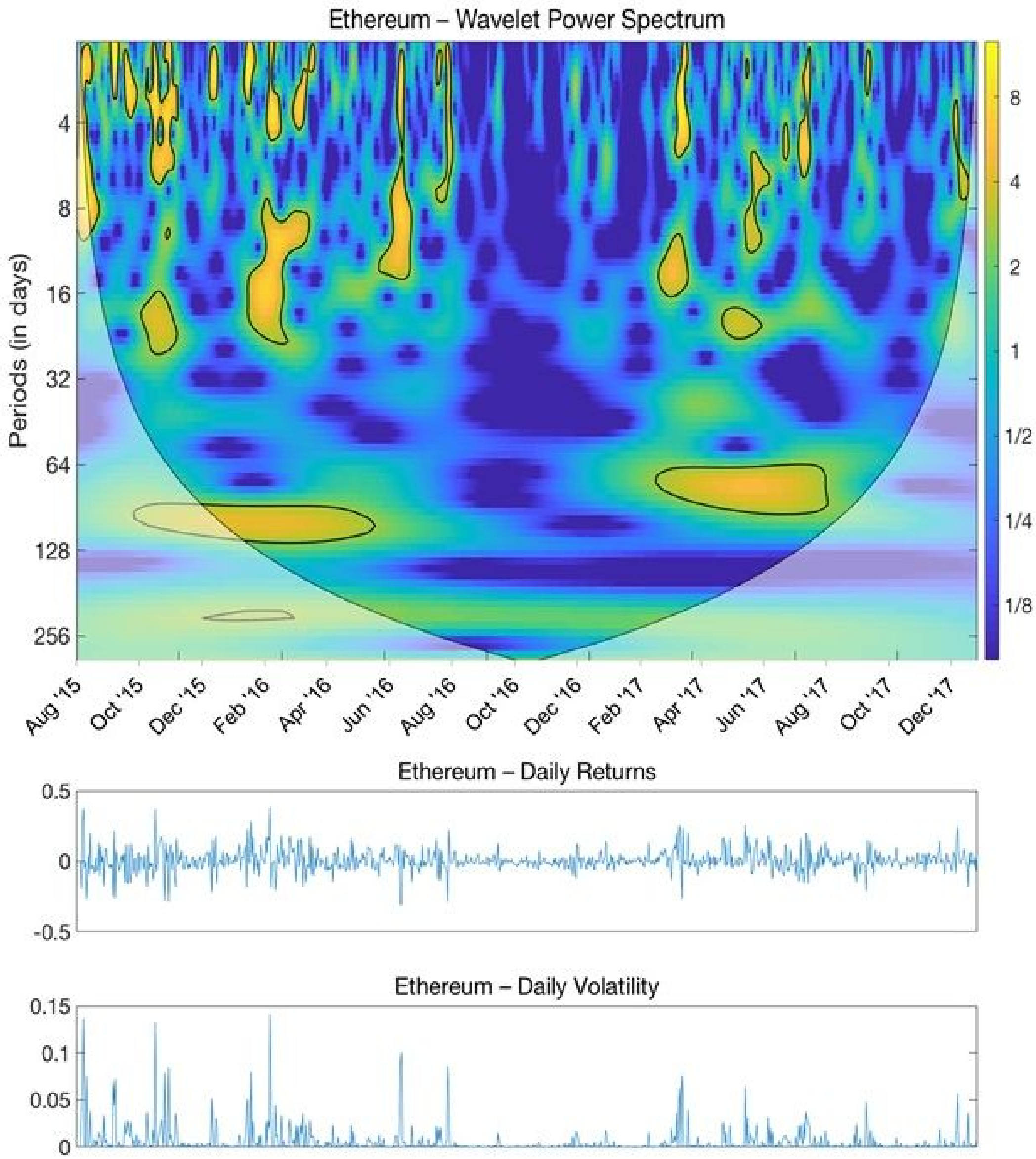

Figure 5: Wavelet power spectrum of daily Ripple returns

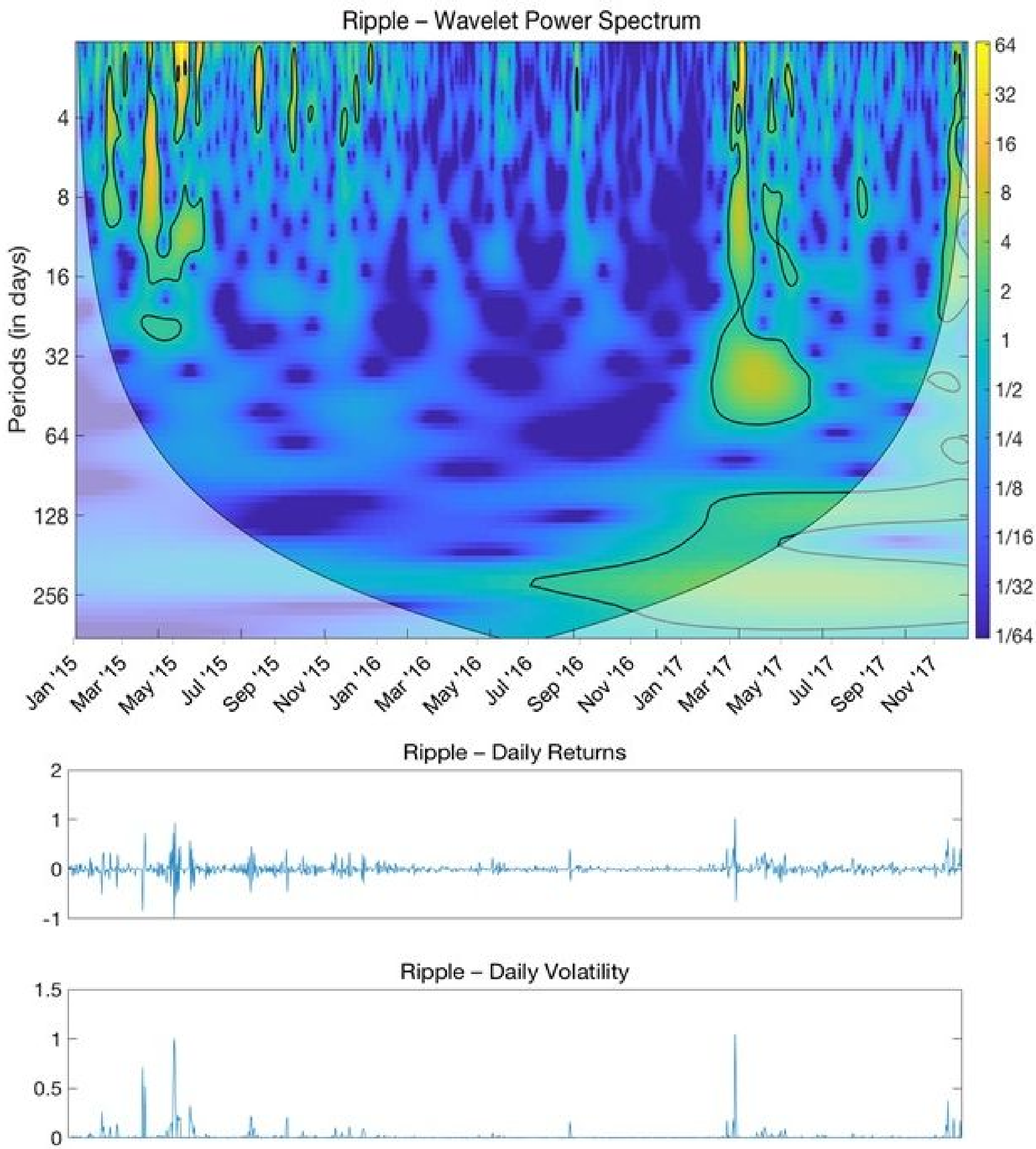

Figure 6: Bitcoin and Ethereum cross wavelet transform

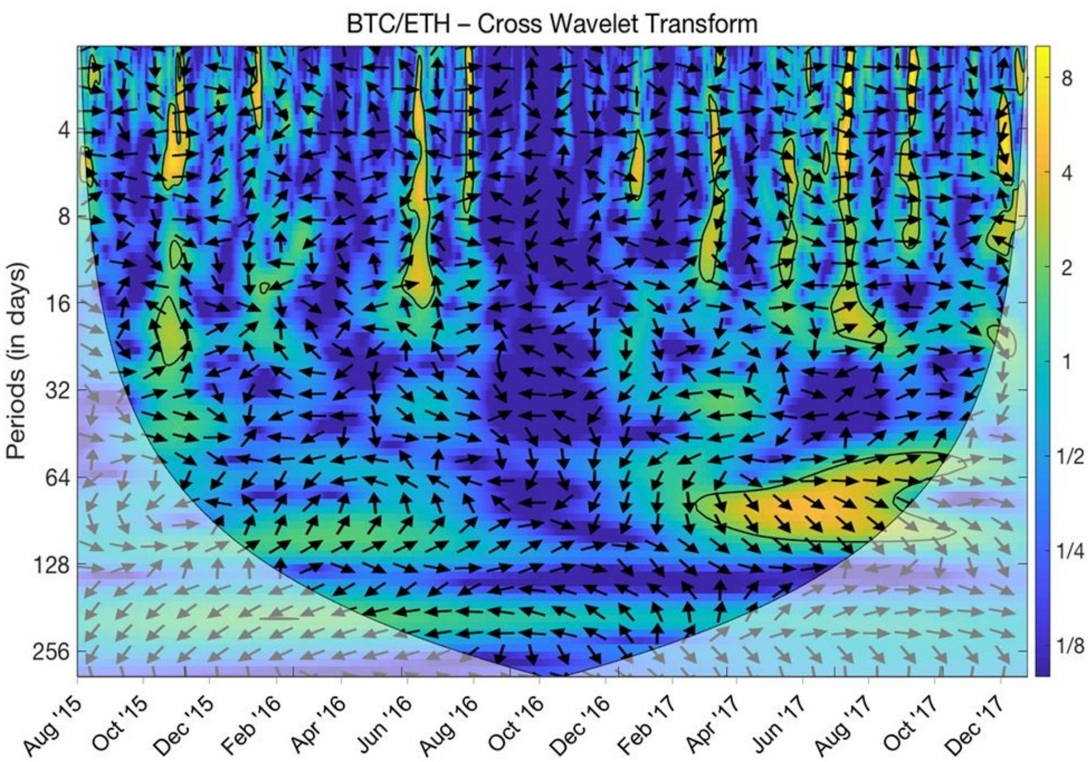

Figure 7: Bitcoin and Ripple cross wavelet transform

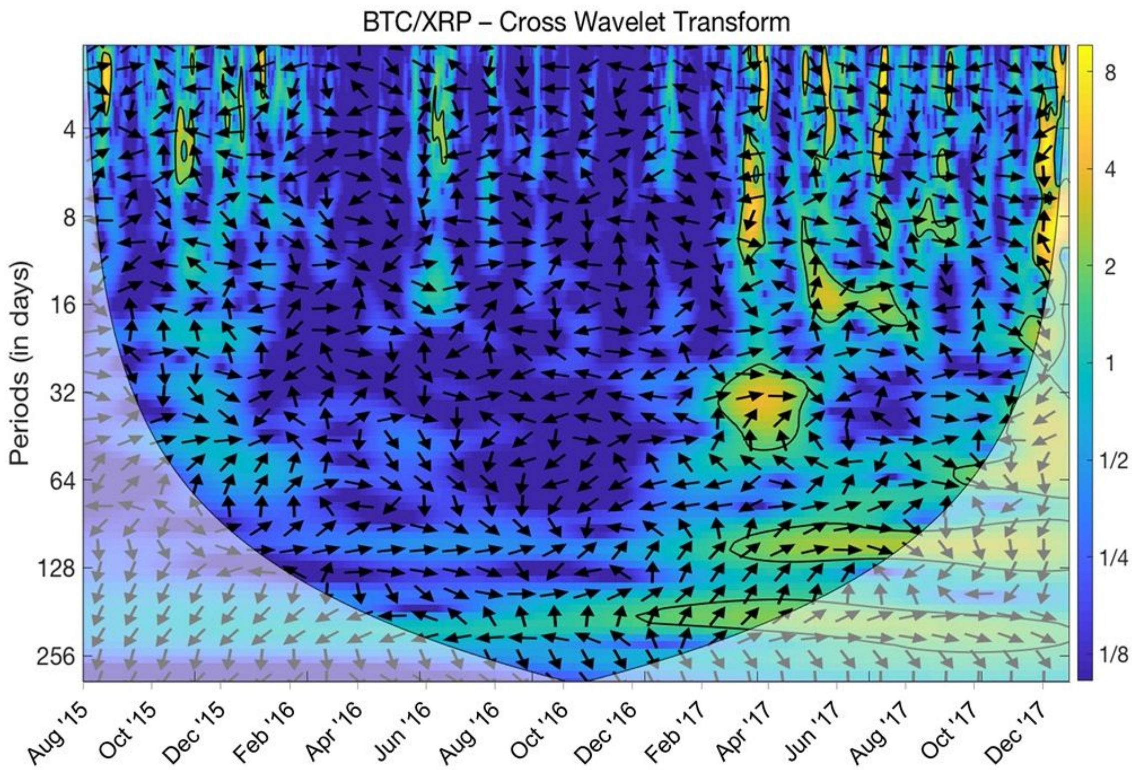

Figure 8: Ethereum and Ripple cross wavelet transform

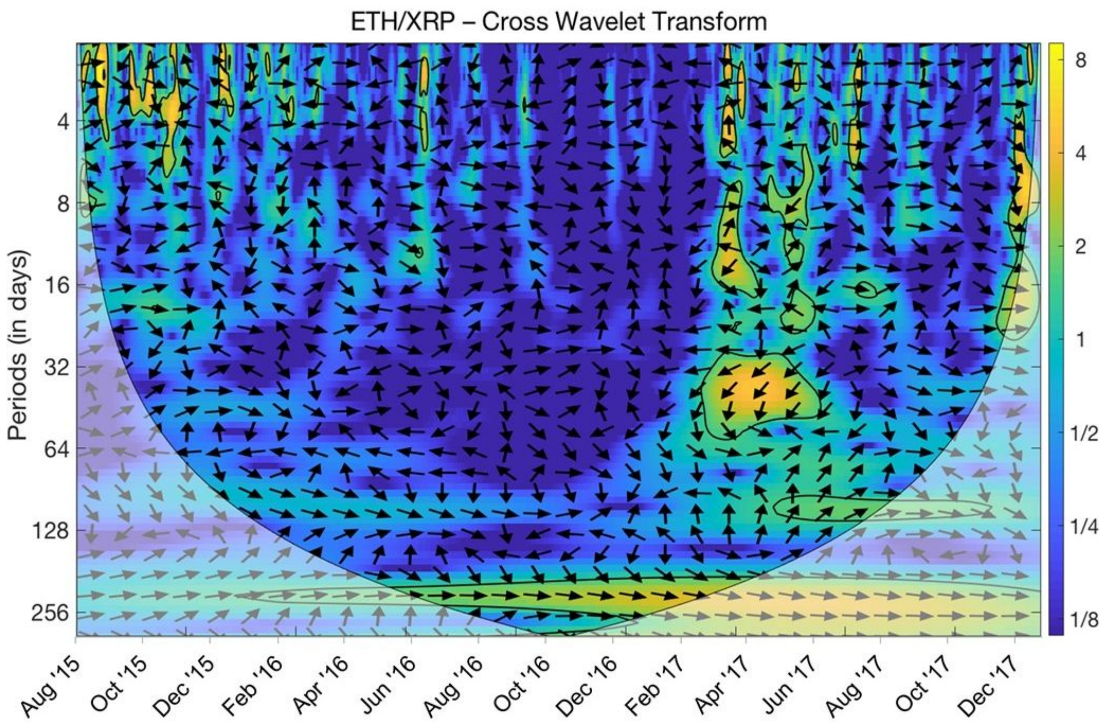

Figure 9: Bitcoin and Ethereum wavelet coherence

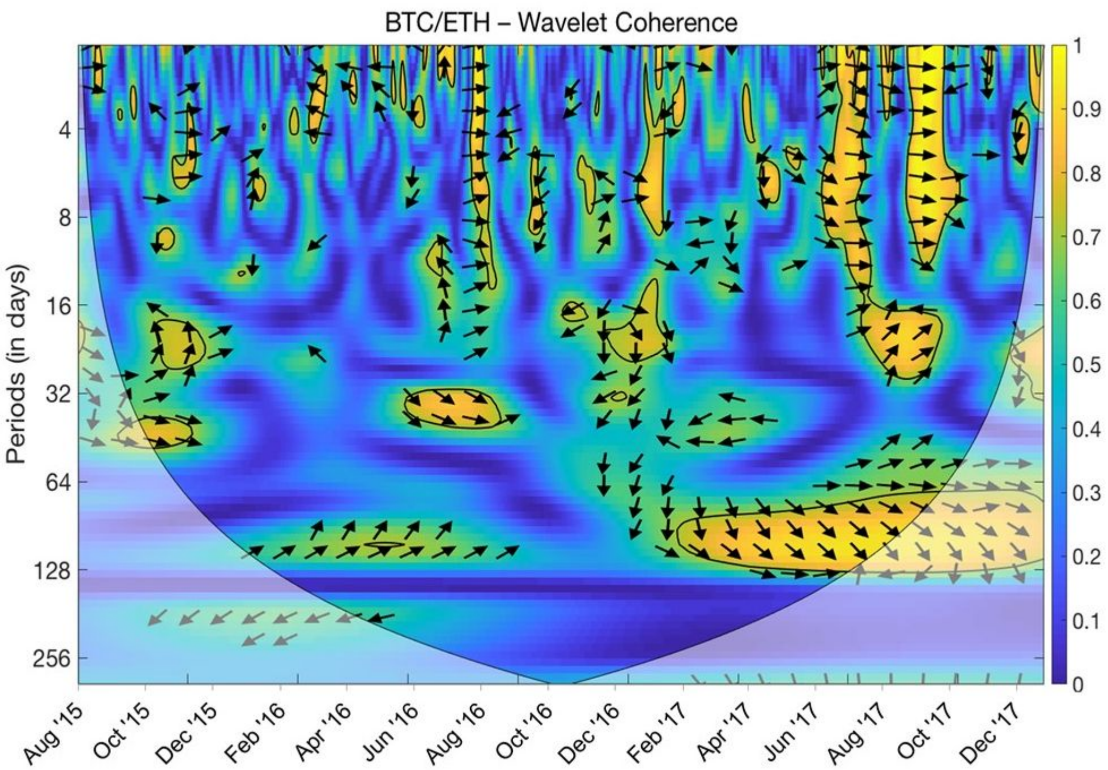

Figure 10: Bitcoin and Ripple wavelet coherence

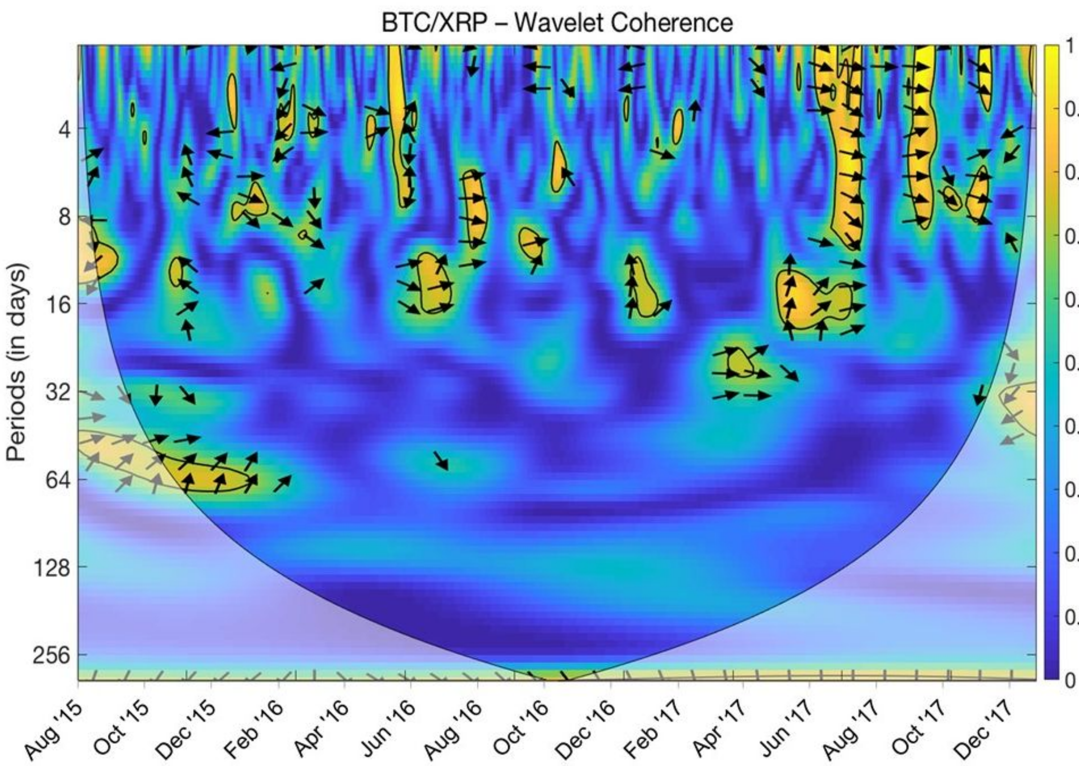

Figure 11: Ethereum and Ripple wavelet coherence