Developing and assessing student’s conceptual understanding of electrostatics in upper secondary Physics

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Declaration

I hereby certify that this material which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work, that I have exercised reasonable care to ensure the work is original and does not to the best of my knowledge breach any law of copyright, and has not been taken from the work of others, save and to the extent that such work has been cited and acknowledged within the text of my work.

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Publications.

1. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. (2016) *Developing student understanding of attraction between charged and uncharged objects in a lower secondary setting.* School Science Review, No. 363, pg 101-108.

2. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. (2016) *Helping students explore concepts related to the electric field at upper secondary level science education.* Proceedings of GIREP-EPEC 2015, pg 195 – 201.

3. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. (2017) *Developing second level student's understanding of the inverse square law, and electric fields.* Proceedings of GIREP-ICPE-EPEC 2017. (Currently under review).

Conference presentations.

1. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *The application of tutorial based worksheets to enhances student understanding of static electricity and magnetism at lower second level education.* SMEC, 2014.

2. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *Helping students explore concepts related to the electric field at upper secondary level science education.* GIREP-EPEC, 2015.

3. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *Progress and difficulties in student's understanding of vector and field concepts in electrostatics: A qualitative study of a small group of upper secondary students*. SMEC, 2016.

4. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *Developing second level student's understanding of the inverse square law, and electric fields.* GIREP-ICPE-EPEC, 2017.

Abstract

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Developing and assessing student's conceptual understanding of electrostatics in upper secondary Physics

This thesis presents research studies carried out with upper secondary level physics students (n=14) over a timeframe of four months, with the aim of promoting their conceptual understanding of the electrostatic concepts, Coulomb's law, electric fields, and work and potential difference. Teaching and learning materials were developed that adopted Inquiry Based Learning (IBL) with structured inquiry tutorials and Multiple Representations (MR) approaches to examine the extent of conceptual change in the student's understanding of these topics. The research utilizes a case study methodology, with various sources of evidence recorded. These sources include pre-test and post-test comparison, retrieval of student artefacts used during lessons, audio recordings of student's discussions during lessons, teaching and learning interviews and teacher field notes.

In the research design, vector concepts, the inverse square law and field line representations were identified as concepts that students need to be familiar with, to develop their understanding of electrostatics. The students completed structured inquiry tutorials for these topics in the context of mechanics. The findings of these studies show that conceptual change generally occurred in the student's understanding, for these three concepts, with evidence of conceptual extinction and extension occurring. However, the findings of these studies show students encountered difficulties in transferring their understanding of vectors, the inverse square law and field lines to the electrostatic context. By revisiting these concepts during the electrostatic tutorial lessons, evidence of conceptual exchange and extension was observed in the student's understanding of Coulomb's law, the electric field and work and potential difference.

The research presents evidence that the use of structured inquiry tutorials and multiple representations can be an effective approach in promoting conceptual change in student's understanding of electrostatics in upper secondary physics. The overall findings of this research suggest that this approach may have significant benefit for the teaching and learning of other physics / science topics at both upper secondary and lower secondary levels.

Chapter 1. Introduction

As a branch of science, physics is a human endeavour to study the nature of energy, matter and their interactions. It allows for the construction of models, theories and laws that explain observable phenomena on scales as small as the sub-atomic to as large as galaxies and beyond. Physics helps generate fundamental knowledge needed for technological progression that can directly be used to develop new products or influence the progression of other disciplines. In Ireland, physics and physic-trained people underpin a range of sectors from medical technologies to ICT and web-services (Institute of physics, 2012). There are many branches of physics; the ones encountered by upper secondary school students in Ireland are optics, thermodynamics, mechanics, electricity and electromagnetism with introductions to quantum and particle physics. These branches of physics can give students a fundamental foundation that they can use as a platform for further study in more advanced fields of science, technology, engineering and mathematical fields.

Electrostatics concepts are important domains of science teaching that deserve attention. In Ireland, the Chief Examiner's Reports (2013; 2010; 2009; 2008; 2005a; 2005b) for physics at senior level and science at junior level show that students have above-average difficulties with electrostatics. If possible, they tend to avoid questions relating to electricity, as seen in the reports. As the domains of static and current electricity are intrinsically linked, the teaching and learning materials developed in this thesis aim to develop student's understanding of electrostatics concepts. The expected outcome of this approach is that students can transfer their understanding of these concepts to concepts in current electricity, such as the behaviour of current, potential-difference and resistance in circuits. This can aid the development of the student's understanding of these domains, instead of treating electrostatics and current electricity as two separate and exclusive phenomena.

The use of multiple representations of these physics concepts has been recognised as beneficial to learners. Hestenes (1996) explains that a complete understanding of a model requires a student to be able to coordinate information between multiple representations to complete their understanding. Although this research does not employ modelling methodology explicitly, the use of a structured inquiry approach is used to support student's development of models. The utilisation of multiple representations in this work aims to develop similar gains in the student's understanding of electrostatics to those that a modelling methodology would aim to produce. Jackson, *et al*., (2008) note that the use of a modelling method involves students developing conceptual understanding through graphical and diagrammatic representations, before moving onto the algebraic treatment of problem-solving. In the design of the teaching and learning materials in this research, this instruction method is employed in numerous lessons, to aid student's conceptual development.

Using multiple representations in developing education materials can help achieve other goals. Student difficulties that may not be apparent in one representation may easily be assessed in others. McDermott, *et al.*, (1986) showed students can have a relatively good grasp of concepts, but numerous issues of interpretation can cause confusion when interpreting graphs. This inability to successfully translate between representations can hinder student's ability to completely understand a concept. Kozma (2003) notes that that experts are fluid in their transitions between different representations when problem solving, while novices typically use one or two. Students focus on surface features of the concept and don't develop understanding at a deep level. This suggests that helping students translate between representations could enable them to develop expert–like understanding, and tackle problems using multiple methods.

1.1. Context and background

This thesis concerns the teaching of electrostatics in the Irish secondary system. Irish secondary education comprises a Junior Certificate and a Leaving Certificate programme. The Junior Certificate programme is a three-year programme, in which students typically aged 12-15 take ten subjects, including Science. In some schools, Science is mandatory, in others it is optional. At the time this research was completed, Junior Certificate students completed the 2003 science syllabus (NCCA, 2003). The Leaving Certificate programme is a two-year programme, in which students typically aged between 16-18 take seven subjects. Leaving Certificate Physics is optional to all students who undertake the programme, subject to availability of facilities and suitably qualified teachers. Leaving Certificate Physics follows the 1999 syllabus (NCCA, 1999), which is briefly discussed in this section. The physics course is set at two levels; ordinary level and higher level. The difference between these two levels is the following:

- Ordinary level consists of a defined set of concepts. Higher level consists of the ordinary level concepts, additional concepts and a particle physics or applied electricity optional section.
- Ordinary level provides an overview of physics and its applications, while at higher level, there is a deeper, more quantitative treatment of physics.
- At ordinary level, equations must be used, while at higher level, some equations must be used and derived. Calculations for higher level are more challenging that those found at ordinary level.

(NCCA, 1999)

As electrostatics concepts are the focus of this research, Table 1.1 provides an extract from the physics syllabus and a discussion of the extract and a commentary of an examination question, shown in Figure 1.1, from the state exams commission (SEC, 2015) is presented.

In recent years, the teaching of Science and Physics in Ireland has undergone changes in both classroom syllabus and examination style. First examined in 2002, the Leaving Certificate Physics Syllabus, reduced the content to be covered in mechanics and electrostatics. This was to accommodate for the introduction of a new section on modern physics. The Junior Certificate Science syllabus (NCCA, 2003), first examined in 2006, also reduced the amount of content to be covered, to allow time for an investigative approach in the delivery of the syllabus. This was justified under the rationale that it allowed for the allocation of class time to allow for an investigative method of learning, but it has been seen that typically, traditional methods of teaching still prevailed in classrooms (Wemyss, 2009).

In developing teaching and learning materials for this research, the learning outcomes for the students, as listed in the Leaving Certificate Physics syllabus (NCCA, 1999) were used to consider what concepts should be addressed. The syllabus lists the topics to be completed in the Leaving Certificate Physics course, the depth of treatment the topics are to be taught to, and suggested activities and examples of science and technology that are appropriate for the topics. There is no prescribed methodology suggested for the teacher to use in completing the syllabus. For instance, there is no indication of suggested explanatory models to understand the concepts underpinning Coulomb's law and the electric field. The Leaving Certificate Physics syllabus is designed to be completed over two academic years, in approximately 180 hours of class contact time. This contact time generally breaks down to just under three hours a week to include laboratory work, teaching and learning of content and concepts and practice solving numerical problems in physics. The syllabus is assessed in a three-hour written examination, which is drafted by the State Examinations Commission and is corrected anonymously. The Physics examination papers give little weight to student's conceptual understanding of the content covered. Typically, students are given mathematical questions which are typically solved using algorithmic problem-solving methods. Any questions that are conceptual tend to revolve around the direction of an electric field in a given setup, with one or two charges, or explaining how Coulomb's law is an example of an inverse square relationship. Table 1.1 presents an extract of the electric field section of the Leaving Certificate Physics syllabus. Items denoted in bold indicate that they are part of the higher-level physics course only.

Topic	Depth of treatment	Suggested activities	Science, Technology and Society
Static Electricity			
1. Force between charges.	Coulomb's law - $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$ - As an example of an inverse square law. **Forces between collinear charges.**	**Appropriate calculations.**	
2. Electric fields.	Idea of lines of force. Vector nature of electric field to be stressed. **Definition of electric field strength.**	Demonstration of field patterns, using oil and semolina, or other method. **Appropriate calculations – collinear charges only.**	Precipitators Xerography Hazards: effect of electric fields on integrated circuits.
3. Potential difference	Definition of potential difference: work done per unit charge to move a charge from one point to another. Definition of the volt. Concept of zero potential.	Appropriate calculations.	

Table 1.1 Extract from Leaving Certificate Physics syllabus (NCCA, 1999) detailing static electric learning outcomes.

When considering student understanding of potential and potential difference, it is defined in the syllabus as work done when moving a unit of charge from one place to another in an electric field. However, it has been shown that students at third level have difficulties in the understanding of concept related to work (Doughty, 2013). This has also been seen to be a topic of difficulty to students exploring potential difference (Hazelton, 2012), so it is not unreasonable to speculate that some of the difficulties could be allayed by helping students to develop a conceptually sound understanding of work and potential at second level. Figure 1.1 presents an extract from the 2015 Leaving Certificate Physics examination paper, which illustrates a number of these points.

Define electric field strength. (6)

Both Van de Graaff generators and gold leaf electroscopes are used to investigate static electricity in the laboratory.

Draw a labelled diagram of a gold leaf electroscope.
Describe how it can be given a negative charge by induction. (20)

A Van de Graaff generator can be used to demonstrate point discharge.

Explain, with the aid of a labelled diagram, how point discharge occurs.
Describe an experiment to demonstrate point discharge. (18)

The polished spherical dome of a Van de Graaff generator has a diameter of 40 cm and a charge of +3.8 μC.

What is the electric field strength at a point 4 cm from the surface of the dome? (12)

Figure 1.1. Electrostatics question from Leaving Certificate Physics examination, 2015. (SEC, 2015)

The initial question requires students to define electric field strength, in which they need to recall that it is the force experienced per unit charge in an electric field. Acceptable answers to this question were in written word, or mathematical notation in which the variables were explained. The following pair of questions required students to sketch a diagram of a gold leaf electroscope and describe how it can be given a negative charge by induction. While the latter question can be answered by understanding the principle involved in this process, both questions can be answered by recalling rote-learned material. This also applies to the following pair of questions which required the students to describe the process of point discharge and describe an experiment to demonstrate the process. The final question required students calculate the electric field strength at a point from the surface of the surface of the Van der Graaff generator. Relevant formulae for this question are provided to the students in the examination. The student must correctly determine the distance from the point to the centre of the Van der Graaff generator, and otherwise, substitute variables into the formulae and evaluate the expression on their calculators. A student who has spent time rote-learning the theory for the initial questions and practised the substitution and calculation process could score highly on this question, without demonstrating the depth of their conceptual understanding to the examiner marking the paper.

1.2. Research approach

In this research, the researcher is also the teacher that develops the teaching and learning materials used to promote the student's understanding of electrostatics. At the time the lessons in this thesis were implemented, I had been employed as a full time second level teacher for 8 years. In this

time, I gained experience in teaching multiple cohorts of students the Junior Certificate Science, Junior Certificate Mathematics and Leaving Certificate Physics syllabi. As a teacher, I developed classroom activities that in which the students would predict the outcome of observable phenomena, share reasons for their predictions, discuss alternative outcomes with their peers. When completing quantitative exercises, I would ask students to consider whether numerical answers appeared reasonable, based on their understanding of underpinning theory they were applying. As the second level syllabi were prescriptive in the material that is to be learned, opportunities to develop lessons in which the students were given a high degree of autonomy in their learning were rare. This research provided an opportunity to develop a sequence of such lessons and gather data on the student's understanding of electrostatics.

The aim of this research is to develop a suite of research-informed and research-validated educational materials to improve student learning of electrostatics concepts, by improving their conceptual understanding of Coulomb's law, the electric field, work and potential difference. The teaching and learning materials employed during the tutorial lessons in this research are, unless referenced otherwise, of my own original design with the primary inspiration for their design coming from *Tutorials in Introductory Physics* (McDermott, *et al.*, 2003) and *Conceptual Physics* (Hewitt, 2009) In this research, the students completed tutorial lessons in groups for three/four, in which they explore different concepts. This allows opportunities for peer tuition between the students, as they discuss and debate different ideas relating to the content in the tutorials and reach conclusions that the group agrees to. The teacher gets real-time insight to student ideas about the concepts covered in the tutorial, as they circulate the classroom, reviewing the student's responses and using probing question. This is discussed in-depth within the specific electrostatics topics covered in chapters four, five and six. Analysis of student's responses and worksheets also allows the opportunity to critically evaluate the efficacy of the tutorials and for redesign of the materials. The theory underpinning this style of research is discussed in chapter 3. There has been no published research completed on the teaching and learning of these electrostatics topics in the Irish second level context, but the use of activity and inquiry learning, has been shown to be useful in the effective teaching of science in Ireland (Wemyss, 2009; Broggy, 2010; Flynn 2011).

The design of the educational materials utilises a multi-representational approach (discussed in section 2.2.3), to promote the student's development of their conceptual understanding. Representations are anything that symbolize an object, a collection of objects, interactions, etc (Rosengrant, *et al.*, 2007). Different representations can be used to display different information and can have various levels of difficulty for learners to interpret information from. Ainsworth (1999) states when learners encounter multiple representations related to a topic, the different representations in which the topic is presented allows for different functions in learning. They can show different processes, different information, constrain the learner's interpretation of the material presented and enable them to construct deep understanding. Using multiple – representations,

abstract concepts can become more accessible to learners (Dienes, 1973). Kozma (2003) notes that experts can transition from one representation to another without difficulty, while novice learners struggle with this skill. The use of multiple representations in lessons can help learners develop this skill. For instance, learners may initially struggle to understand a relationship displayed in an equation, but scaffolding using tabular data and graphs can allow students to consider common characteristics between the representations and equations and to establish the relationship in the symbolic form (Project Maths, 2011). To develop student's understanding of Coulomb's law and the electric field, the representations utilised were vectors, field lines and multiple representations of the inverse square law, which were used to develop their understanding of the properties of both topics and relate the concepts to mathematical formulae. The inverse square law, as a mathematical function, allowed for opportunities to employ the use of diagrammatic model, tabular data and graphical representations. The use of field lines, vectors, diagrammatic models and graphs were employed in the lessons for work and potential difference.

1.3. Research Questions

As presented in Sections 1.1. and 1.2, there are difficulties encountered by students developing their understanding of vector concepts, the inverse square law and field lines. To help promote student's understanding of these topics, structured inquiry with multiple representations was used. Therefore, this thesis was designed to focus on the following research questions (RQs).

RQ 1. To what extent does the use of a structured inquiry approach develop student understanding of vector concepts?

RQ 2. To what extent does the use of a structured inquiry approach develop student understanding of the inverse square law relationship, by employing multiple representations?

RQ 3. To what extent does the use of a structured inquiry approach develop student understanding of the field, when utilising field line representations?

These first three research questions address these topics outside of an electrostatic context. The lessons on these topics were completed during the mechanics section of the Leaving Certificate Physics course. The student's understanding of the application and transfer of these topics to an electrostatic context is addressed in the following two research questions.

RQ 4. To what extent does the use of a multi-representational structured inquiry approach develop student understanding of Coulomb's law and electric fields?

RQ 5. To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

Chapters 4, 5 and 6 address these questions, and discuss the considerations that were employed in drafting each of these research questions. The considerations were used to both design the instructional materials and construct a boundary to which these questions could be answered. An overview of the evidence collection methods employed to address these questions is presented in the following section.

1.4. Research Design

In designing this research, it was decided that the students would encounter vector concepts, the inverse square law and field lines outside of the context of the electrostatic topics, and then apply the concepts during the later tutorial lessons. This in line with lesson sequences for how the Leaving Certificate Physics course was delivered to student groups taught in years prior to this research. Student groups first encountered vectors and the inverse square law in mechanics, as an introductory topic and as a concept related to Newton's Universal Law of Gravitation respectively. As field lines can be applied to gravitational contexts, this representation was also explored by the participants of this research in this topic. This allowed the students to develop their understanding of the three representations in relatively familiar contexts. They then later applied the representations to electrostatics to develop their understanding of the Coulomb's law, the electric field, work and potential difference. Figure 1.2 illustrates the design of the lesson sequence employed over the course of this research.

Students commence the Leaving Certificate Physics programme, at age 15-16 years after they have completed a three-year Junior Certificate programme (depicted in blue in Figure 1.2). As part of the Junior Certificate Science syllabus, students explore forces, motion and static electricity (NCCA 2003). In the Junior Certificate Mathematics syllabus, they explored algebra operations, linear and quadratic patterns, trigonometric ratios and graphing functions (NCCA, 2012). These Junior Certificate topics lay a foundation for Leaving Certificate Physics. The prior learning of the students is explained in-depth in Section 3.5.

Figure 1.2 shows how I sequenced core syllabus topics relevant to this research. The five concepts, depicted in green, represent the topics addressed by this research. The sequence of teaching the first three of these concepts were first introduced to the students while teaching of some different

topics. The final two are specific to teaching the electrostatics section of the Leaving Certificate Physics syllabus.

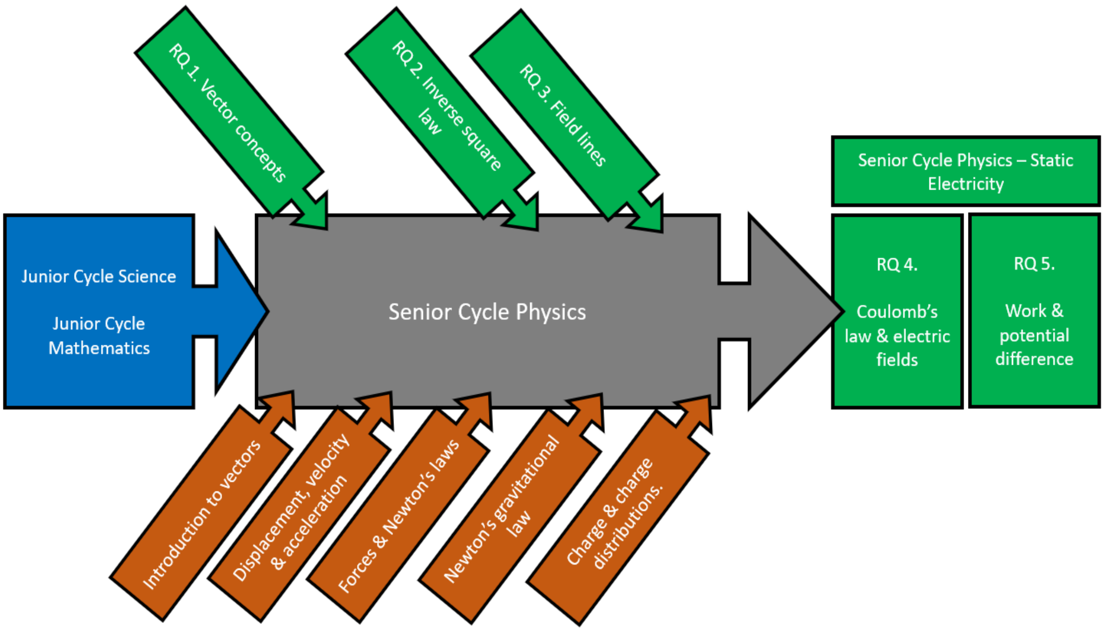


Figure 1.2. Leaving Certificate syllabus topics relevant to this research.

In my classroom, I started by defining vectors and basic mathematical operations with them. The first tutorial lesson was implemented after this initial introduction, in which students were given the opportunity to develop their understanding of vector magnitude, vector addition and vector components. This allowed for evidence collection to address RQ 1. Upon completion of the vector concepts tutorial, the students applied these concepts to the context of motion. The students explored the concepts of displacement, speed, velocity, and acceleration. They practised quantitative problems involving the three equations of linear motion. They used $s = ut + {}^{1}/_{2}\,at^2$ to experimentally calculate the acceleration due to gravity, by using apparatus designed to record the fall-time as a ball bearing is in free-fall. Upon completion of motion, I gave them a lecture-style introduction to momentum, law of conservation of momentum, forces and Newton's three laws of motion. The students were presented with definitions, formulae, videos and scenario's involving collisions between bodies in which they were to predict the outcome based on their understanding of the topics. The students also experimentally demonstrated Newton's second law and the conservation of momentum. Motion, forces and momentum are treated as 2-dimensional quantities but typically only 1-dimensional calculations are completed. The students completed various problem-solving exercises involving these quantities before being introduced to their next topic, gravitational forces, where the second tutorial lesson was implemented, the inverse square law.

This point was chosen as Newton's gravitational law is the first opportunity to apply an equation of the form $y = k\frac{1}{x^2}$ to a physics context. The students would not have used an equation of

this form in their prior formal education, so the tutorial was designed to employ a multi-representational format to allow the students to develop conceptual understanding of the inverse square law. This allowed for evidence collection to address RQ 2. After the tutorial, they applied their understanding to Newton's gravitational law and completed mathematical exercises.

Proceeding from Newton's gravitational law, the students completed a tutorial on field lines, in a gravitational context. While field line representations are not required for the Leaving Certificate Physics course for gravitational contexts, this presented an opportunity for students to use the representation before learning the electrostatics topics in this research. It also allowed for students to differentiate the behaviour of forces, acceleration, velocity and displacement in a gravitational context, and evidence was collected to address RQ 3.

In the interim between the field line tutorial and the initial lessons on charge and charge distributions, lessons on remaining mechanics topics were completed. As these topics were outside the scope of this research, they do not appear on the lesson sequence shown in Figure 1.2. When the students completed the introduction to charge and charge distributions, they completed the tutorials on the electrostatic topics. This allowed for evidence collection to address RQs 4 and 5. These topics were presented in a series of tutorial lessons. The students were required to employ their understanding of vector concepts, the inverse square law and field lines to Coulomb's law and electric fields. They also were required to apply vector concepts and field lines to work and potential difference.

This section overviewed the overall research design as the tutorial lessons were implemented into the student's Leaving Certificate Physics course. The next section details the overall structure of the thesis.

1.5. Structure of the thesis

This section briefly outlines the structure of this thesis. Following this introductory chapter, chapter two presents the theoretical basis for the research and presents the literature on developing student's conceptual understanding in physics, the use of inquiry learning in physics, difficulties encountered by students in vectors, the inverse square law, field lines and their transfer to electrostatics and potential difference, how students learn and a review of various inquiry teaching and learning materials that were used to influence the design of the tutorial lessons employed in this research. Chapter three discusses the research methodology utilised in this research. It discusses case study methodology, justifications and limitations for the use of case studies and the various sources of evidence collected for analysis are identified. This is followed by the implementation

methodology, which illustrates what a tutorial lesson looks like and why it was chosen to be used in the research lessons. The chapter finishes with an overview of the analysis methodology used and a description of the participants in the study.

Chapters 4, 5 and 6 present student's developments and understanding of vectors, the inverse square law, field lines, their application to Coulomb's law, the electric field, work and potential difference. Chapter 4 presents the results of the tutorial lessons in the topics of vectors, the inverse square law and field lines. For each of these topics, a narrative and discussion of the pre-test results, the lesson tutorials, homework assignments and the post-test results are given. These were composed using the sources of evidence presented in Chapter 3. These are followed by a discussion comparing the results, highlighting evidence of student development and persistent student difficulties. Chapter 5 uses a similar structure to present the findings from the tutorial lessons used during Coulomb's law and the electric field. In addition to the student's understanding of these topics, the chapter discusses the student's ability to transfer and apply vectors, the inverse square law and field lines to these two topics. There is also some discussion on the student's ability to transfer between vector and field line representations. Chapter 6 follows the same structure as Chapters 4 and 5 but looks at students understanding of work and potential difference in an electrostatics context.

Chapter 7 presents the final conclusions of this research, in which answers to the five research questions, presented in Section 1.4 are discussed. Implications for teaching and avenues for further research are identified.

Chapter 2. Research Basis

2.1. Introduction

In this chapter, a review of literature detailing how students learn concepts in Physics is presented. The initial section of this chapters discusses how students learn, discussing the information processing model, constructivism and the use of scaffolding in teaching and learning. This is followed by how student develop conceptual understanding and the use of Inquiry as a method to promote conceptual understanding.

The middle section of this chapter presents the theoretical framework that underpins this research. It defines conceptual change, based on the work of Hewson (1992), and describes the conditions necessary for conceptual change to occur in teaching and learning. How inquiry is employed in this research, as a method to promote conceptual change is discussed. As multiple representations are employed in this research, the use of multiple representations in teaching and learning is also discussed.

The final section of this chapter details student difficulties recorded from literature that relate this research. Student difficulties in vectors, the inverse square law, field lines, Coulomb's law, the electric, work and potential difference are detailed. Finally, a pedagogical framework is presented to inform the implementation of the teaching and learning materials over the course of the lessons for this research.

2.1.1. How students learn

Learning can be defined as the process through which relatively permanent change in behaviour or knowledge occurs because of experience (O'Donnell, *et al.*, 2009). Joyce, *et al.*, (2002) state that students learn in "human settings" which are assemblies of teachers and students in environment created for learning purposes. They state that effective schools gather students together to learn, but also engage in specific kinds of inquiry (Rutter, *et al.*, 1979; Mortimore, *et al.*, 1988 and Levine and Lezotte, 1990). As classroom engagement tend to be at the centre of education discussion, the question of how to teach is central to discussions of effective methodologies. Yet traditional "chalk and talk, drill and recite" (CTDR) and didactic teaching still dominate the methods used by teachers, and the minds of critics of education (Race and Powell, 2000; Joyce, *et al.*, 2002). This contrasts with student's perceptions of their own learning: they feel learning occurs when they engage in activities such as class debates, and little learning occurs during intervals when the teacher is talking

(Jensen and Kostarova-Unkovska, 1998). Joyce, *et al.*, (2002) suggest that, in the US, the combination of recitation and lectures contributes to approximately one third of learners being unable to complete secondary education, and approximately one fifth being unable to read and write to a standard that allows them to acquire professions that require literacy. They also suggest that evidence supports that this situation is reflected in Great Britain. In these type of classroom environments, Cooper and McIntyre (1996) suggest that students tend not to have strategies for learning, and instead adopt a passive role, in which it is the teacher's responsibility to ensure they learn, and they are "made to work" in the classroom.

Regardless of the way a teacher attempts to engage the student in leaning, the information-processing model (IPM) can explain how students learns (Huffman, 2004). The IPM describes how learners develop internal representations of the external world. It illustrates how stimuli from the environment is transferred from the sensory memory, to the short-term memory (STM) and to the long-term memory (LTM). The model is presented diagrammatically in Figure 2.1.

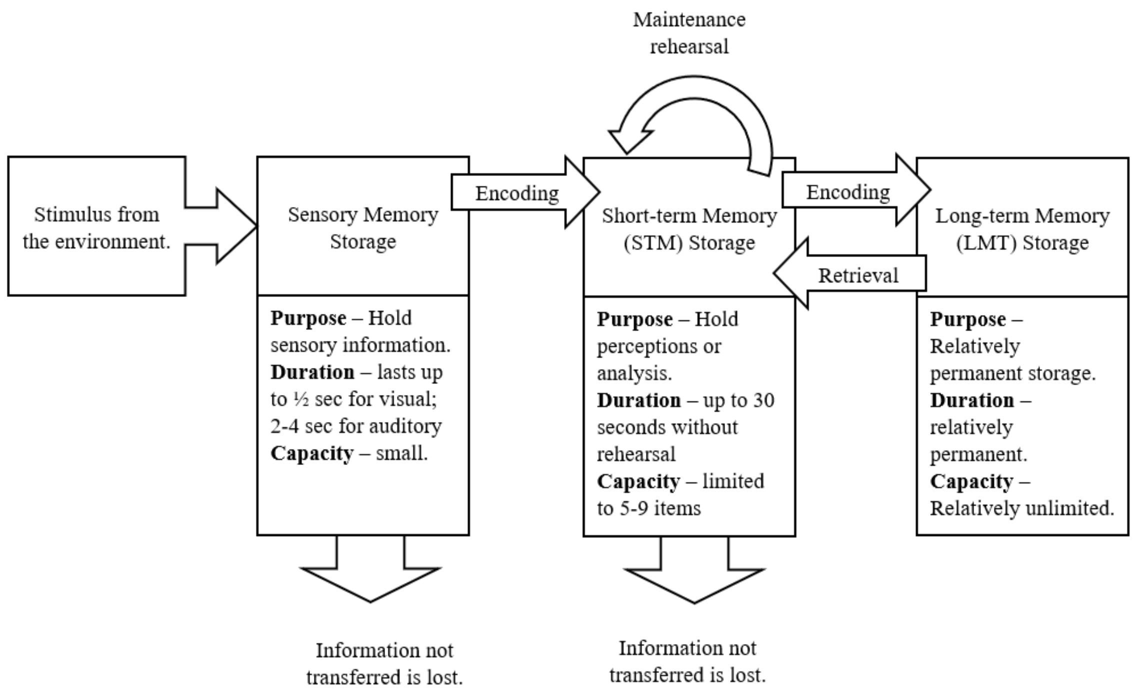


Figure 2.1. Information-processing model (O'Donnell, et al., 2009), reproduced from Huffman (2004).

Sensory memory (SM) is very brief and tends to be applied in two forms in the classroom, visual and auditory. This information is passed to the STM, where if it is not processed, it is lost. However, the information in the memory can be rehearsed to keep it active, in the STM. This information is subject to decay, when information not used is lost, and interference, when something else gets in the way of the recall. Information that is transferred to the LTM potentially has permanent and unlimited storage for the life-time of the learner. O'Donnell, *et al.*, (2009) explain how LTM can take the forms of episodic (events), semantic (verbal information), declarative (details about a structure) and procedural (details about how to do something). This model suggests varied and

complex processes occurring cognitively in the learner, but didactic teaching and CTDR can be limited in how they allow students to engage in the encoding process to transfer from SM to STM to LTM.

Reid (2009) references Yuan, *et al.,* (2006), stating that there is a consensus that STM is a function of working memory (WM). Reid (2009) expands upon this point that the STM not only retains information, but also allows the information to be worked on. Information can be retrieved from the LTM into WM and worked on, pending the capacity of the WM is not overloaded. The capacity of WM can be considered in terms of the number of items that can be processed at any one time. Reid (2009) states that people aged 16 years and older can have 7±2 items of information available to work on in their WM capacity, and processes that involve using more of these can lead to capacity overload, in which the learner can no longer effectively process all the information and will be unable to complete the task they are participating in. Over the course of their education, a learner can "chuck" information together into one item, and there is virtually no limit to how much information a person can "chuck" together to form an item. This frees up WM items for the learner to process new information and complete their task. If the learner is unsuccessful in this, their ability to develop understanding ceases as there is too much information for them to process.

Joyce, *et al*., (2002) list numerous models for learning based on the work of Jean Piaget's theory constructivism. Constructivism is a theory about how students learn, and has many sub-theories related to information-processing, the construction of knowledge from prior knowledge and new experiences and knowledge construction because of social interactions (O'Donnell, *et al*., 2009). In the theory of constructivism, Piaget (1967) discusses learners use of mental processes, called schemas, to organize their knowledge and experiences.

Through observations and experiences, children learn about the world they live in, in which prior knowledge and understanding are used as a lens through which they view new knowledge and experiences. In constructivist learning, no information learned is independent of the experience of the learner and the context in which it is learned. Hein (1991) outlines some principles of constructivist learning as the following:

- Learning is an active process, in which learners use sensory input and construct meaning out of it.
- People learn to learn, as they learn. Learning consists of both constructing meaning and constructing systems of meaning.
- Constructing meaning is a mental action. Hands on activities are not sufficient unless the mind is also engaged.
- Learning is a social activity. We are more likely to be successful in instructional efforts if we recognise the use of conversations, interactions with others and the application of knowledge as an integral part of learning.

- Learning takes time. It is not an instantaneous process and for significant learning, we need to revisit ideas, ponder them, try the out and use them.
- Motivation is a key component in learning. Unless we know "the reasons why," we may not be very involved in using the knowledge learned.
- Learning is contextual. We do not learn facts and theories in an isolated fashion but instead link them and relate them to other facts and theories.
- One needs knowledge to learn. It is not possible to assimilate new knowledge without first having knowledge to build upon.

Transmission models of learning (Mestre, 1991) tend to ignore learners' prior knowledge and experience, while constructivist models provide opportunities for students to perceive an event, and the student's perception informs what learning occurs for them. Generally, students make sense of these perceptions and assimilate them into, and extend, their mental schemata. Green and Gredler (2002) note that from Piagetian and Vygotskyian perspectives, constructivism helps learners develop logical thinking, self-regulated attention, conceptual thinking and logical memory. They note constructivist classrooms may employ student-directed experimentation, or interaction between students and subject matter concepts to develop advanced cognitive capabilities. The learners manipulate objects and ideas, which may lead to cognitive conflict between one's ideas and experimental results, and they can interact with the teacher to develop conscious awareness of, and mastery of, one's thinking and learn to think in subject matter concepts.

The degree of prior knowledge required to process an event and complexity of the event being perceived can lead to difficulty in extending a mental schema in an accurate fashion. Vygotsky (1978) presents a model for learning based on student's development that addresses this. Students have 3 zones in this model; the zone of actual development, the zone of proximal development and the zone of no development. If a student is presented with tasks that they can complete unaided, the skills required to do so are in their zone of actual development. When a student requires assistance or instruction to complete a task or develop a skill, it is said to be in the zone of proximal development. If a task or topic is in the zone of no development, this is currently beyond the capabilities of the student, and no form of instruction will aid the student. This model is useful for educators as it allows them to account of what students can do, what students are capable to doing with aid, and consider what teaching and learning needs to occur for the students to be able to complete more difficult aspects of their courses.

The zone of proximal development is of importance as this is where cognitive development occurs. To support students in this zone, scaffolding can be employed by the teacher. Scaffolding is the guidance, support and tutelage provided by a teacher during social interaction, designed to advance student's current level of skill and understanding (O'Donnell, *et al.,* 2009). Scaffolding provides support for the learner, extends the range of what the learner can do to enable

accomplishment of tasks that would otherwise not be possible. It can be reduced or removed from lessons as learners develop the skill and understanding to learn, or complete a set of tasks, on their own. Lynn, *et al.,* (2013) discusses 4 central tenets of scaffolding in science education, in which it can be used to make science accessible to the learner, make thinking visible, help students learn from others and promote autonomy and life-long learning.

Lynn, *et al.,* (2013) explain that making science, and scientific reasoning, accessible to learners using scaffolding can involve using tangible examples familiar to students, as opposed to abstract models that are employed in science. This allows learners to develop scientifically normative views and explain them using familiar contexts. Learning can be made visible as learners can be encouraged to explain their ideas to others. Multiple representations utilise various models for the students to engage with. Scaffolding can involve helping students listen to their peers, to take advantage of the collective knowledge of the group. Class discussions where students are required to respond to each-other and critique to one another can allow for individual learning, in learning to consider alternative views, expand their own knowledge and engage in effective communication with one another. Autonomy and life-long learning is promoted through scaffolding, as who students who reflect and explain their ideas learn more (Chi, Bassok, *et al.*, 1989), and gain a more robust understanding when revisiting concepts in new contexts.

2.1.2. Developing student's conceptual understanding

As discussed in Section 2.1.1, traditional methods of instruction tend to focus on transfer of facts from teacher to student, while little emphasis on students constructing their own knowledge and understanding. When discussing traditional instruction Mestre (1991) discusses the transmission model of education and notes that it is not a model of learning, but an instructional practice. He highlights that a central assumption of the instructional practice is that the message that the student receives is the same in which the teacher intended for them to learn. In this practice, difficulties in student's understanding are due to the manner the material is presented and the teacher needs to augment their presentation of the material. Roth (1990) notes that primary and secondary level education has struggled to develop student's conceptual understanding when traditional methods of teaching and learning are employed. She notes that while students in these environments are proficient in memorizing facts and procedures, they struggle to build arguments, make predictions and explain observable phenomena, both inside and outside the classroom. When considering conceptual development in a traditional classroom, she states that student's conceptions are largely invisible to both teachers and students, the instruction focused on student's learning explanations and terminology already developed by scientists and test questions revolve around repeating these ideas. Students are rarely asked to apply the concepts to explain everyday situations and the emphasis of

the learning was students developing the "right" answer rather than exploring the nature of the student's conceptions. As detailed in section 2.2.1, Roth (1990) suggests a conceptual change model of instruction that promotes the student's abilities in these areas, allowing them to overcome the difficulties that traditional instruction do not address.

To develop learner's conceptual understanding, Mestre (1991) suggests a constructivist approach, which considers the principles discussed in Section 2.1.1, which sees learning as a process of constructing knowledge and understanding. Meaningful learning occurs when students interpret and apply knowledge in novel contexts, in which the students are mentally engaged. Roth (1990) suggests that the teaching and learning materials should focus on allowing the students to develop deep understanding of a limited number of concepts, that they can apply in novel contexts, which does typically not occur in traditional classrooms. A constructivist approach allows the students to engage with new concepts and can provide opportunity for students to resolve their prior understandings/misunderstandings with new concepts. Johnston (2010) suggests strategies to successfully engage students in developing their conceptual understanding in the classroom, such as actively engaging students and providing regular feedback, focusing on the observable phenomena, explicitly exploring misconceptions, using various problem-solving skills and strategies and providing homework tasks that involve qualitative and conceptual analysis of phenomena. This sort of constructivist environment promotes students to use active learning to support their construction of knowledge and understanding. The students are not being viewed as passive recipients of knowledge but of active participants in its creation (McDermott, 1991).

Regardless of the teaching style of the teacher, students will have constructed their own models of understanding from both their formal education and their interactions and observations of the world. This prior learning can result in their models of the world not being scientifically accurate, and the students cannot be considered "blank slates" when they partake in science lessons (Knight, 2004). Students have relied on these models to explain the world for some time before entering classrooms. These models, which may contain misconceptions, can be resistant to change, so constructing a conflict between the student's model and the scientific model may be required multiple times before successful change takes effect.

The approach chosen to facilitate the student's conceptual development in this manner is structured inquiry. This approach is discussed in the Section 2.1.3.

2.1.3. Inquiry learning in Physics

To promote development of conceptual understanding as discussed in Section 2.1.2, inquiry-based learning (IBL) can be utilised (Tabak, *et al.*, 1995; Blanchard, *et al.*, 2010). There are many

definitions for IBL in science education. The National Research Council present a general definition for inquiry that does not necessarily preclude the use of any one specific teaching / learning method.

> *Scientific inquiry refers to the diverse ways in which scientists study the natural world and propose explanations based on the evidence derived from their work. Inquiry also refers to the activities of students in which they develop knowledge and understanding of scientific ideas, as well as an understanding of how scientists study the natural world.*
>
> (National Science Education Standards, 2006, p23)

An aspect of this definition is that student's experiences in a classroom setting should reflect scientists' experiences when seeking to expand the human scientific knowledge. In using inquiry in the classroom, learning involves the students thinking scientifically. While this is not an exhaustive list, inquiry can take the form of designing and critiquing experiments, performing research, engaging in scientific debate, discourse with peers, construct models, search for information, and/or some combination of these (Lynn, *et al.*, 2013).

As there as many classroom activities that can be used to engage students in inquiry, there is a wide interpretation of what inquiry is. Banchi and Bell (2008) present inquiry on a four-point scale, "limited, structured, guided and open." Limited inquiry involves following a set of predetermined instructions to arrive at a pre-determined conclusion and is typically referred to as "cook book". The aim is for students to confirm something they have already learned. Structured inquiry occurs when there is no pre-determined conclusion to the task. It is purely based on the student's construction of knowledge through whatever activity was completed, such as an investigation. Guided inquiry has no predetermined method presented to the students, and they must determine how to complete the task, as well as draw conclusions from it. In a manner, guided inquiry involves presenting students with a learning objective, such as a phenomenon to be investigated, but allowing them to complete the objective in a manner in which they see fit. Open inquiry involves giving the students no predetermined questions to answer, and instead they generate their own questions which they answer themselves.

The choice of inquiry to use in a set of lessons can depend on the outcomes the teachers wishes for their students to achieve. Tabak, *et al.,* (1995) and Blanchard, *et al.,* (2010) showed structured and guided inquiry practices are beneficial for students to develop conceptual knowledge and engaging with nature of science skills. Research into open inquiry showed that learners gained independence and autonomy over their learning, relying less on their instructors (Krystyniak and Heikkinen, 2007). Students may develop meta-cognitive reflection skills, engage in higher order thinking and improved motivation to complete investigations (Berg, *et al*., 2003). The several types of inquiry allow students to engage with several types of autonomy (Stefanou, *et al*., 2004). In guided and open inquiry, students are given an opportunity to display an elevated level of cognitive

autonomy. They can, amongst other things, discuss multiple ways to think about an issue, debate thoughts freely, re-evaluate errors and ask questions. Open inquiry, additionally, allow students to display an elevated level of procedural autonomy, such as design investigations freely, choose how to demonstrate competency and design their learning outcomes.

Inquiry learning is not accepted as best practice by all academics in the educational sphere. Clark *et al.*, (2012) argue when learners are presented with information, they should be fully instructed on what to do and how to do it, basing their argument on a review by Mayer (2004) and overloading short term memory (STM). Mayer's review identifies numerous studies since the mid twentieth century which compared unguided to guided instruction. In all cases, the results indicated that guided instruction resulted in better students gains, over discovery learning, in the tasks specified in the studies. However, the studies referenced by Mayer do not consider the various inquiry types (Banchi and Bell, 2008), and when considering these, the studies can be interpreted as utilising structured and guided inquiry. The argument that Clark, *et al.*, (2012) make regarding STM relates to the limited number of elements it can process at any one time. They state that instruction should aim to impart as much knowledge and skills into a learner's long-term memory (LTM) as they have relatively limitless access to this, as opposed to their STM. They suggest a "worked-example" approach, in which the learner uses their STM resources to develop comprehension instead of both comprehension and discovery which directs attention to storing essential information and understanding. However, they state that this approach is only fruitful for introducing new knowledge, and it can have a detrimental effect for material the learner is familiar with. Their article does not appear to address limited or structured, in which case the design of the teaching and learning materials can present learners with the introductory concepts and knowledge, to which the learners initially only need to comprehend. After this introduction, the learners can then apply the initial concepts and develop a deeper understanding in a limited or structured environment.

Rocard, *et al.*, (2007) identifies IBL as a method of best practice for implementation in school classrooms. Bevins and Price (2016) state that there is a wide range of empirical evidence that reports positive outcomes for learners in terms of achievement, enthusiasm, ownership and scientific skills development. They reject that inquiry must be entirely student driven and unsupported by the teacher. They propose that inquiry can develop a new sophisticated model, to reap further benefits. They propose a three-dimensional model to inquiry practices, as shown in Figure 2.2.

The first dimension focuses on the body of knowledge in science. This informs how scientists think about the natural world and generate questions for inquiry. The second dimension focuses on procedural knowledge. These allow for the reliable generation of data and evidence, ensure they are reliably interpreted, and the data and evidence is communicated appropriately. The third dimension focuses on learner's psychological energy, in which they engage with the inquiry process, which generates energy to create and manage authentic inquiry process. Bevins and Price propose that these

three dimensions are not intrinsically interlinked but instead the model of inquiry in made up of these dimensions as the sum of their individual parts.

An early implementation of structured inquiry as defined by Banchi and Bell, which the authors however term guided inquiry, is the Physics by Inquiry worksheets developed by McDermott, *et al.,* (1995). It comprises a series of modules across various physics topics. Each module is structured so students develop their understanding based on their own observations and encourages them to develop explanatory models. There are gaps between each narrative that each student must bridge as they complete the experiments and exercises in the module. The primary emphasis is on teaching by questioning rather than the transmission of knowledge. This helps students to see physics as a process of discovery. The student is central to the learning process, developing sound qualitative understanding, which compliments and improves student's ability to tackle quantitative problems, and overall, sets a higher standard of learning (McDermott, 2001). However, the time required to complete a module in any one topic can be quite demanding, making it inappropriate for direct implementation in a secondary level setting where multiple topics are to be taught in a relatively short amount of time.

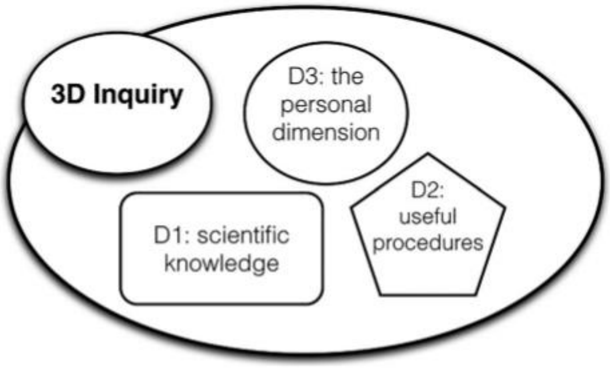


Figure 2.2. Three-dimensional model for inquiry (Bevins and Price, 2016).

Within the field of Physics Education research, there have been numerous studies in recent years (Shaffer and McDermott, 2005; Close and Heron, 2010; Heron, *et al.*, 2004; Hazelton *et al.*, 2012) show that student who learn through inquiry methods develop better conceptual understanding of physics than students who learn using traditional approaches. These approaches focused on development of mental models, student reasoning, conceptual reasoning and conceptual change.

To illustrate the efficacy, the first of these four cited studies is reviewed which employed a structured inquiry methodology for teaching and learning. When looking at student's understanding of vectors, Shaffer and McDermott (2005) showed students can have trouble with one dimensional vector subtraction when shown in a real-life context, such as collisions, and these student difficulties in vectors appeared to increase when transferred to a two-dimensional problem. Their lesson was designed to engage students with using vectors to identify changes in velocity and determine the

direction of acceleration as a result. In a separate lesson, students apply the same skills in a two-dimensional context to provide commentary about velocity and acceleration for a body in motion at points on a closed loop path. They found upon completion of the lessons, they helped not only undergraduate students transfer vector analysis from one-dimension to two-dimensions, but also the postgraduate students who acted as teaching assistants during the tutorials. This example illustrates how structured inquiry lessons can help inform instructors of student difficulties and provide an environment for students to overcome them.

2.2. Theoretical framework

This section discusses the theoretical framework that underpins this research. As a mechanism to promote conceptual change, this research employs the use of structured inquiry and multiple representations in the design and implementation of the teaching and learning material used in this work.

2.2.1. Conceptual change

Konicek-Moran and Keeley (2015) describe traditional forms of instruction as promoting "literal understanding". Literal understanding allows students to memorise and reproduce knowledge without understanding the meaning behind it, or the power to use it to argue, predict or delve deeper into the ideas involved. They argue that to do these things, a learner must develop a conceptual understanding of what they learn. Although they do not give a formal definition, Konicek-Moran and Keeley (2015, pg. 6) state that when a learner develops, or is developing their conceptual understanding, they can partake in or demonstrate any of the following attributes with their conceptual knowledge:

1. Think with a concept.
2. Use the concept in areas other to that in which they learnt it.
3. State it in their own words.
4. Find an analogy or metaphor for it.
5. Build a mental or physical model of it / to explain it.

To develop conceptual understanding, the learner must be exposed to a concept in some manner. Concepts are packages of meaning; they capture regularities, patterns, or relationships among objects, events, and other concepts (Novak and Canas, 2006). They are formed when a student engages in an intellectual function in which memory, attention, inference and language all

participate. From experience, students naturally construct their own conceptions, which may or may not be scientifically accurate, to make sense of the interactions they see in the world around them. This can also result from methods of instruction that do not aim to reduce the occurrences of students developing misconceptions (McDermott and Shaffer, 1992). Assessments and evaluation based on recall of content and solving quantitative problems can hinder development of conceptual understanding, as educators tend to teach to the test instead of focusing on their students developing coherent understanding of the material they learn in the classroom setting. For this reason, misconceptions do not tend to be addressed by traditional instruction (Dykstra, *et al.*, 1992). As students spend considerable time and energy constructing these concepts, they can develop an emotional and intellectual attachment to them, which is not easily overcome. To overcome such an attachment, a conceptual change model of instruction is proposed.

Hewson (1992) discussed conceptual change as three mechanisms, conceptual extinction, conceptual exchange and conceptual extension.

- **Conceptual extinction** is when an alternate idea is challenged to the point where it is no more, and all that remains is the new learned concept.
- **Conceptual exchange** occurs when a student is presented with a conception that challenges their current understanding. Through the learning process, the status they associated with their current understanding is lowered and the status of the new conception is raised. The original idea is still present, but the student becomes aware of its inaccuracies or limitations and disregards it in favour of the more useful robust concepts that were learned.
- **Conceptual extension** is when a person goes from not knowing an idea to knowing the idea. A person makes connections between the things they already know and extends their understanding to account for new material achieves this.

To promote conceptual change in the context of conceptual exchange and extension, the following proposed four conditions generally tend to be present:

- *There must be dissatisfaction with existing conceptions.*
- *The new conception must be intelligible*
- *The new conception must be initially plausible*
- *A new conception should suggest the possibility of a fruitful research programme.*

(Posner, *et al.*, 1982, pg. 214)

Under these conditions, student's models are effectively challenged and allowed to fail. This allows the students to adjust their models of understanding of the topic being studied. Vosniadou and Brewer (1994) consider conceptual change as it applies to a student's overall understanding of a topic and see it *"as the product of the gradual lifting of constraints, as presuppositions, beliefs, and mental*

models are added, eliminated, suspended or revised during the knowledge acquisition process". By encouraging students to evaluate their initial ideas over time and adjust it by adding the elements of scientific explanation, students are facilitated to allow their models to undergo conceptual change to create models that extend to more fields of study and possess greater explanatory power.

In some cases, cognitive conflict arises between the new knowledge and the existing schema (Piaget, 1975, Posner, *et al.*, 1982, Chan, *et al.,* 1997). This results in the student altering their schema to accommodate the new knowledge. This reorganization of mental schema is required to make sense of the world, as they learn more about the world around them. Schemata can be altered or replaced with other schemata that better explain the environment the learner is in. This results in the construction of new knowledge that furthers learner's understanding of a particular experience / concept / etc. In this view, constructivism can be understood "in terms of a shift in the location of the meaning of what is found in our environment" (Taber, 2011, pg. 40). However, the understanding constructed by the students is not always what the teacher wishes, which can lead to the construction of misconceptions, which can be difficult to deconstruct.

The following sections discuss structured inquiry as a method to promote conceptual change in the classroom, the use of various inquiry resources employed in the development of the research instruments and the use of representations as an aid to promote conceptual change.

2.2.2. Approach adopted in this research

Structured inquiry is adopted as the approach used in the research to promote conceptual change for the student's understanding of Coulomb's law, the electric field, work and potential difference. Structured inquiry generally allows students to develop conceptual understanding (Tabak, *et al.*, 1995; Blanchard, *et al.*, 2010). As seen in the pre-test discussions in chapters 4 and 5, many students produced answers consistent with difficulties recorded in literature. Therefore, structured inquiry was chosen to be utilised in this research. Section 3.3 discusses how structured inquiry was implemented in this research, through the development of tutorial lessons. To avoid confusion, the term inquiry is used as an umbrella of the inquiry types used in the materials developed, instead of identifying each type of inquiry as they arise.

In developing the materials, numerous approaches to inquiry tasks that could have been employed in the lessons were reviewed. After consideration of the research questions, learning objectives of the Leaving Certificate Physics curriculum, the Leaving Certificate Physics examination format and ability of the students, the following approaches were considered for use in the development of the teaching and learning materials designed in this research to be employed in the second level classroom: Tutorials in Introductory Physics and Conceptual Physics.

Tutorials in Introductory Physics (McDermott, *et al.*, 2003) is a set of supplementary activities to accompany lectures or a standard textbook in a standard university physics course. The emphasis is on student understanding of concepts and scientific reasoning skills, as opposed to rote learning theory or solving quantitative problems. A tutorial consists of a pre-test, worksheets, homework assignments and a post-test. The pre-test is typically given after the lecture to determine what concepts the students do understand, and what they are expected to understand at the end of the materials. This indicates student's initial conceptions that can be targeted for conceptual change. The worksheet questions are designed to guide students to construct concepts and apply them to real world situation, when contextually appropriate. These are completed in groups to allow for peer tuition when constructing answers. When students run into difficulty, the teacher uses prompt questioning to guide student thinking instead of volunteering answers themselves. While the teacher can explicitly state when a student's reasoning is diverging from what is intended, they ultimately encourage the students to find their own correct answers. Through completing the worksheet and engaging in discussions, students encounter the conditions for conceptual change to occur (Posner, *et al.*, 1982). The homework exercises are designed to reinforce what is covered in the worksheet, applying the concepts in both familiar and unfamiliar contexts, and in some cases, extend student knowledge. Questions used in the pre-test may or may not appear in the tutorial lesson, or the homework assignment, depending on the design of the materials. This can be used for comparative purposes, and to allow the students to apply their developed understanding to a previous question they may have struggled with or completed in error. The post-test can then be used to gauge any development in student understanding. To be effective, the post-test is written to emphasize the concepts and reasoning skills used in the tutorial lesson and can be used as a comparative tool to determine how a student's understanding has developed since completing the pre-test. Student responses can be individually, or as a group, compared with the pre-test responses. This allows for the identification for the extent of conceptual change that occurred, and determine if the conceptual change was extinction, exchange or extension (Hewson, 1992). Any difficulties persistent in both the pre-test and the post-test can then be redesigned in a future edition of the tutorial, with the research informed of specific difficulties encountered by students. This cycle of results-based redesign has been used to develop more robust materials that increase the number of students able to access difficult concepts (McDermott and Shaffer, 1992; Wosilait, *et al.*, 1998).

The use of materials based on Tutorials in Introductory Physics as the intervention allows for a strong targeting of specific concepts and topics (Ambrose, 2004). For this reason, the tutorial lesson format of Tutorials in Introductory Physics Tutorials in Introductory Physics (McDermott and Shaffer, 2003) address multiple topics in Physics and they could also be used as a guide to draft and develop lessons that adopt the tutorial approach in the second level context, as they were for this research is adopted in this research. However, Tutorials in Introductory Physics is aimed at the introductory undergraduate level, instead of upper secondary level. The material that is covered is

too advanced for a second level classroom; and the worksheets developed in this research were of original design, utilising the structured inquiry approach utilised in the Tutorials in Introductory Physics materials.

Another resource that was considered was Conceptual Physics (Hewitt, 2009), as the material is more accessible to a second level student, and utilises familiar everyday contexts, representations and analogies to illustrate physics concepts.

In 1964, Paul Hewitt began teaching at City College, San Francisco, in which he taught Physics to non-scientists. His approach focused on teaching concepts and relationships in physics using English words, and using little or no mathematics (Hewitt, 2011a). The approach uses analogies and imagery from real-world situations to promote student conceptual understanding of physics principles. When students explore equations, they learn to reveal information about the relationships involved and then develop the ability to manipulate formulae to substitute values in during the last step. This allows students to observe relationships that are otherwise not typically seen when the equations are represented in their standard form.

> "When problems are couched in symbols, and the numbers held for later, a student's task calls for thinking that calculators cannot supply. They think concepts."
>
> (Hewitt, 2011b, p. 264)

By giving students a solid foundation in the concepts involved in Physics, they are equipped to understand various formulae, and to make connections between the concepts of physics and their everyday world.

Exercises from the Conceptual Physics practice book, 10th edition (Hewitt, 2009), were used to develop the contexts used in exploring concepts related to the electric field and potential difference. The material presented in the practice book was further developed, to align it with the current Leaving Certificate Physics curriculum, so the students would not be at a disadvantage when they complete their terminal examinations at the end of their second level education.

2.2.3. Using representations to develop conceptual understanding

A representation describes something that symbolizes or stands for an object, a collection of objects, interactions and/or a process (Rosengrant and Etkina, 2007). Representations can come in many forms, such as words, diagrams, mathematical equations, tables, graphs, animations, simulations, etc. In this research, the use of multiple external representations (MERs) is employed as an aid in the tutorials, to promote student's conceptual development of the different electrostatic

topics. Presenting the same concepts in separate ways provides the learner with the opportunity to build abstractions about mathematical concepts (Dienes, 1973). This is a fundamental step in successful learning. Numerous studies show the benefits of using MERs (Cox and Brna, 1995; Mayer and Sims, 1994; Tabachneck, *et al.,* 1994). The use of MERs is not a "silver bullet," as others have failed to find benefits (Chandler and Sweller, 1992; Van Someren, *et al.*, 1998). This section briefly discusses design parameters to consider when using MERs, cognitive tasks related to the use of MERs and the functions of using MERs in a learning experience for learners.

Ainsworth (2006) states in designing educational materials that employ the use of MERs, the designer should consider number, information, form, sequence and translation. The number of representations must be at least two, but more can be employed. They can be presented simultaneously to the learner, or in sequence. The designer must consider the information they wish to convey to the learner. MERs allow for flexibility in how information is distributed across representations. Information can be redundant, where each representation presents specific information that is isolated from others, partially redundant, where some information is common to all representations and some in unique to each representation, or all the representations express the same information and the only difference is how the information is represented. This must influence the design of teaching and learning materials, as redundant information in representations can make it difficult to transfer between representations, relate representations together and overload their cognitive load capacity. The form of MERs considers what type of representations are chosen and how the learner will interpret how the interactions interact with each other upon completing their lesson. When considering the learner's translation when using MERs, the designer needs to consider how the learner will interpret the information and consider if the student will increase their understanding of the representational format chosen, or the domain knowledge it represents.

When leaners are presented with, or use their own MERs to complete a task, they must face and master several cognitive tasks (Ainsworth, 2006). These cognitive tasks are presented in order, but they are not necessarily the order a learner should approach them.

1. Learners must understand the form of a representation. They must know how a representation presents information, and how to use the operators of the operation. Learners can have difficulties in both learning the operators of a representation and how the operators are applied in the contexts the representation models. An inability for students to use representation operators can hinder the learners' ability to complete a task, understand the concepts and/or context they are studying or achieve the targets learning from an activity that employed MERs.
2. Learners should understand the relation between the representation and domain. Interpretation of representations is a contextualized activity. For instance, students need to be aware that the slope of a line on a distance – time graph represents the velocity, as opposed to the height of the graph (Leinhardt, *et al.,* 1990). This can be a challenging task during a learning process, as opposed to problem solving, as the learner is applying MERs upon incomplete domain

knowledge. Deciding how the learner will use the representation to model the context of the domain knowledge needs consideration. Otherwise, difficulties could be encountered by the students, and the envisioned learning using MERs may be hindered.

3. Learners need to understand how to select an appropriate representation. Learner may have the opportunity to select which representation is most appropriate to complete a task, and they consider factors such as the representation, task characteristics and outcomes and individual preferences. While students are often competent in this (diSessa, 2004), selecting appropriate representations tends to be more difficult for novices as opposed to experts (Chi, Feltovich and Glaser, 1981). Understanding, and accounting for, learners limited ability to select appropriate representations can be used to inform the design of the tasks that employ MERs.
4. Learners need to understand how to construct appropriate representations. In certain tasks, learners may need to construct their own representations. DiSessa (2004) argues that learners are good at doing this, and Grossen and Carnine (1990) showed that children solved problems more effectively when they constructed their own diagrams, rather than select from a series of pre-drawn diagrams. This can be incorporated in material design by providing learner the opportunity to model a process using a representation and have them explain how it represents the process.

When MERs are employed in a teaching and learning sequence, it is important the designer considers why they wish to employ MERs. When carefully planned and applied, such as being cognisant of the cognitive tasks involved in using MERs and their limitations, Ainsworth (1999) suggest the use of MERs allows for the employment of multiple parallel functions in learning. However, without considering the 4 cognitive tasks that Ainsworth (2006) outlines, the result of using MERs could range from fruitful to detrimental to the desired learning outcomes. To consider the functions of MERs, she developed a functional taxonomy of multiple representations that illustrates these functions, shown in Figure 2.3.

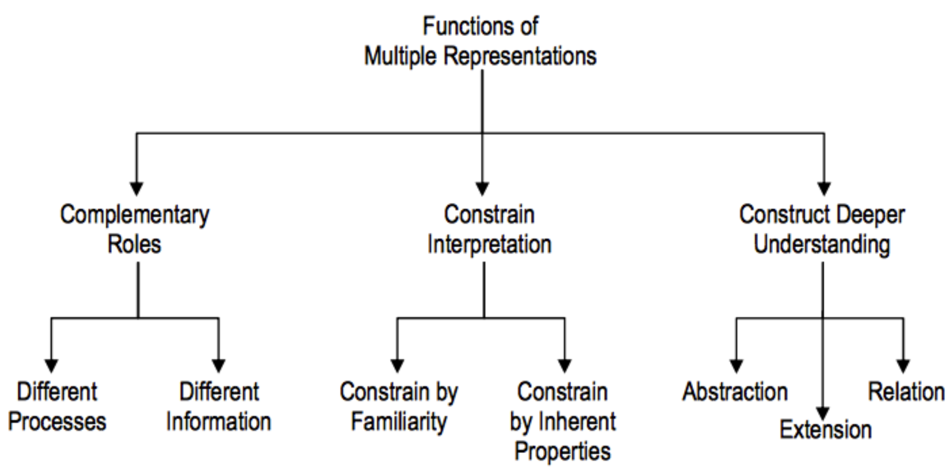


Figure 2.3. Functions of Multiple representations (Ainsworth, 1999)

To understand the complementary roles of multiple representations, consider how different information can be represented in diverse ways, as is appropriate to that information. Representing the velocity and acceleration against time is appropriate in a table or a graph, while inappropriate for representing the mass of the body. Different processes used in multiple representations allow the student to take advantage of their computational properties. Diagrams allow students to group together relevant information, graphs and tables allow students to determine values from reading off, patterns can be determined by analysis of the table and graph, while algebraic equations summarize the relationships between various variables in a given context.

Constraining information involves using the properties of one representation to focus on information taken from another. This allows the familiar representation to be used as a scaffold to show the same or similar concept in a more difficult representation. For instance, using a matching game with several types function in graph form, tables of graphs typical of those functions and general equations for those functions can give students the opportunity to identify key coefficients, variables and parameters in the equations that are characteristic of each type of function. When designed at the appropriate level, there are only a limited number of combinations and reasoning the student can arrive at, and can further be facilitated by instructional guidance.

Multiple representations can also support deeper understanding of the material covered by reinforcing concepts and information that is common to multiple representations of the same material, but also by highlighting features that is most notably prominent in a representation. Velocity – time graphs for non-uniform displaying increasing and decreasing motion can be relatively easy for students of all levels to understand and interpret, but when given various algebraic equations, with corresponding domains for time can be quite difficult to interpret for students. The use of multiple representations in this research primarily aims to employ this construction of deep understanding. Many sections of the tutorials require the students to represent information as tables, graphs, apply algebraic reasoning, and draw and interpret vector and field line diagrams.

Kozma (2003) discussed one difference between understanding concepts by novices and experts, being that experts are fluid in their transitions between representations while novices typically use one or two. Students focus on surface features of the concept and do not develop understanding at a deep level. To advance their understanding, Hestenes (1996) suggests that a complete understanding of a model of a physical system requires a student to be able to transfer between multiple representations. This ability to engage in transfer between different representations leads to increased conceptual understanding.

2.3. Overview of the research study

2.3.1. Teaching and learning electrostatics

As discussed in chapter 1, this research focuses on upper secondary level student's understanding of vectors, the inverse square law, field line representations, Coulomb's law, electric fields and work and potential difference. This section presents a review of literature related to these domains, detailing difficulties and misconceptions typically encountered by students in electrostatic forces, fields and potential difference. Section 2.1.3.1 discusses issues of vector concepts vector addition and vector components, and then difficulties of using vectors in Coulomb's law and electric fields. Section 2.1.3.2 details issues around the inverse square law, focusing on learner difficulties surrounding scaling. Section 2.1.3.3 discusses difficulties related to field lines, and how they are applied to represent electric fields. Finally, section 2.1.3.4 discusses issues related to work and potential difference.

2.3.1.1. Difficulties in understanding of vector concepts and their application to the electrostatic context

Student difficulties are encountered in the understanding of vector concepts. Flores, *et al*., (2008) showed that highlighting the vector nature of forces, and acceleration, in kinematics can increase student's ability to use vectors to solve problems that otherwise prove difficult, but that the overall improvement of vector understanding is quite a challenge. Nugyen and Meltzer (2003) showed that students have difficulties with vector addition, in cases of collinear vectors and vectors in two dimensions. Illustrations of these difficulties are shown in Figure 2.4. Difficulties seen with collinear vectors included adding vectors to form two-headed arrows, connecting vectors "tail to tail" or "tip to tip," incorrectly attempting to find the resultant between two vectors. They also showed another difficulty in student's understanding, such that when finding the superposition of two vectors, students re-orientate vectors arrows if they were not perpendicular to each-other to apply Pythagoras' theorem to determine the resultant vector. This shows an inability for students to correctly combine vectors, both diagrammatically and mathematically. Conceptual difficulties with vector addition in two dimensions include adding magnitudes as in scalar addition instead of vector addition, not taking into account direction of a vectors, or the angles between which the vectors act, not conserving vertical / horizontal components or not acknowledging their contribution to the resultant vector and using a "split the difference" algorithm, in which the resultant is always along the bisector of two vectors, regardless of their magnitude.

Students must be aware of how vectors sum to form resultant vectors, by the principle of superposition. Flores, *et al.*, (2008) showed students have difficulties applying vector concepts to forces in which they treat them as scalar quantities, in the domain of mechanics. It is reasonable to postulate these difficulties would transition to electrostatics.

As an overall aim of the research was to improve student's overall development of the electrostatics, an isolated tutorial covering vector concepts was developed. It was expected that the students could transfer their understanding to electrostatics and reduce the cognitive load on their working memory. By initially developing understanding of vector concepts, this would free up items in the student's working memory capacity, to be devoted to Coulomb's law and the electric field. Nguyen and Meltzer (2003) highlighted the common errors that were likely to occur, and these were considered in both the design of the materials, and as conceptual difficulties to identify in student responses as they progressed through the vectors teaching and learning materials.

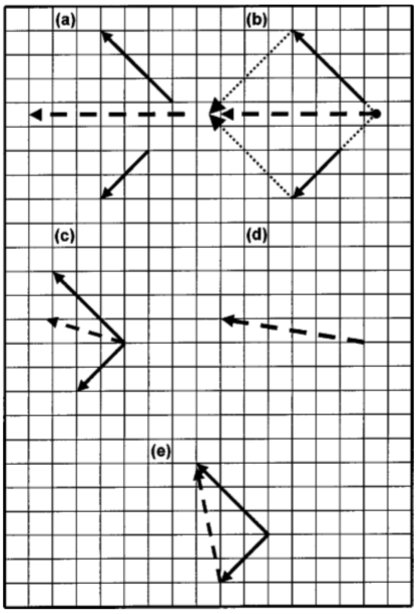


Figure 2.4. depictions of student errors during vector addition (a) zero vertical vector components (b) "split the difference" (c) incorrect parallelogram addition (d) incorrect horizontal component and (e) top – to – toe error (Nguyen and Meltzer, 2003).

Maloney, *et al.*, (2001) showed that undergraduate students can struggle with vector concepts such as superposition in an electro-static context. In question 6 of the Concept Survey of Electricity and Magnetism (CSEM), as shown in Figure 2.5 (i), students were asked to find the direction of the net force acting on the charged particle labelled B. In question 8, shown in Figure 2.5 (ii), the students were required to determine the outcome that adding a charge +Q at (b, 0) would have on the charged particle, q_1.

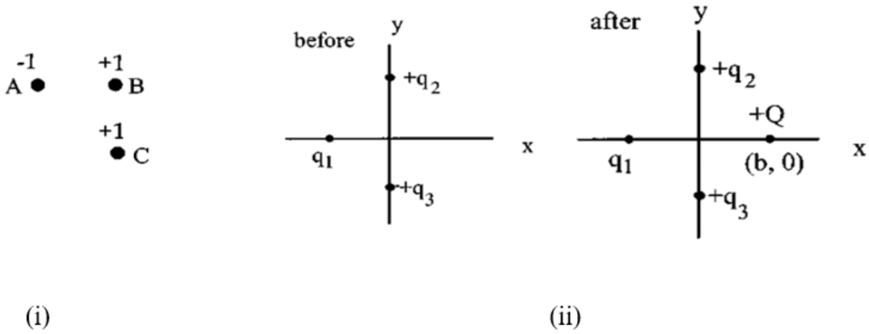


Figure 2.5. Question 6 (i) and question 8 (ii) from the CSEM (Maloney, et al., 2001).

These questions rely on the understanding of vector magnitude, vector direction and the superposition of vector quantities, and their application to electric forces and fields. They noted that in question 6, during the post-test, 33% of students in an algebra-based course and 27% in a calculus-based course could not determine the superposition of the force acting on the charged particle, B. They also reported that noted, in question 8, 47% and 34% of students respectively could not determine how the addition of a new charged particle would affect the direction of the force felt by q_1. The majority of incorrect answers suggested there would be a change in the magnitude and direction of the force, suggesting student struggled to differentiate between the quantities, and not consider the interaction of vertical and horizontal components.

2.3.1.2. Difficulties in understanding of the inverse square law and its application to the electrostatic context

Coulomb's law is considered the fundamental law of electrostatics. It is the first mathematical treatment that students in Leaving Certificate Physics engage with, after completing charge transfer and demonstrate some charge phenomena qualitatively. The Leaving Certificate Physics syllabus (NCCA, 1999) outlines that students should appreciate it is an example of an inverse square law, without explicitly defining what constitutes as an appreciation. Marzec (2012) and Arons (1997) notes that students find it difficult to understand unless exposed to it repeatedly. Maloney, *et al.*, (2000) found that students have difficulty understanding the mathematical implications of Coulomb's law pre-instruction. Post-instruction, they found that student gains in understanding were not as one would expect, for those which have completed exercises on Coulomb's law.

The application of the inverse square law to electrostatic forces was disputed for 40 years (Heering, 1992), it has since been proven and can now be demonstrated in a laboratory using charged domes and an electronic balance of precision to 4 decimal places (Cortel, 1999) or measuring the distance between charged and uncharged pith balls (Wiley and Stutzman, 1978). A simple approach for students to investigate the inverse square law is to use an electric field sensor held at various

distances from a charged body and plot the electric field against distance and analyse the data. This approach was taken for light intensity (Bohacek and Gobel, 2011). Other methods include presenting students with data and getting students to determine the relationship based on graphing activities (Hestenes and Wells, 2006). However, data alone does not enable students to develop a conceptual understanding of the inverse square law, nor provide a tangible context for them to relate the relationship to. In Conceptual Physics – 10^{th} Edition (Hewitt – 2009), the context of a spray paint can spraying drops of paint over different areas is used to illustrate the inverse square law. This model is easily understandable by students and adopted in the approach taken in this research.

Research literature also looks specifically at students understanding of the inverse square law, as applied to electric fields. Cao and Brizuela (2016) demonstrated that learners can qualitatively develop an appreciation that the closer charged particles are together, the stronger the force exerted between the charged particles. Conversely, Maloney, *et al.,* (2001) delivered a question involving Coulomb's law where the distance between a pair of charges was increased by a factor of three. It was seen after instruction, 54% and 32% of students struggled to apply the inverse square law to the increase in distance, with the most prevalent error submitted by the students was that the force would reduce by a factor of three. Marzec (2012) suggests that students misunderstanding of spherical scaling could lead to student to difficulties in this area. While there are many methods to experimentally demonstrate the inverse square law, research looking at students conceptual understanding of the law are scarce. Previous findings in this research, as part of an initial pilot study (Moynihan, *et al*., 2015) noted the that students have an over-reliance on the use of formulae and tend to unintentionally ignore the index value on the distance variable in the equation for Coulomb's law. These difficulties appear to be in line with Arons (1999), in which he notes that learners struggle understanding proportional scaling for area when individual dimensions of a geometric shape increase. For example, in the absence of formulae, learners struggle to explain how increasing the sides of a square by a factor of 3 results in an increase in the area by a factor of 9. When this reasoning is applied to a gaussian sphere to model an electric field, these struggles can indicate that scaling can cause difficulty in understanding the inverse square law.

2.3.1.3. Difficulties in understanding of field line concepts and their application to the electrostatic context

Törnkvist, *et al.,* (1993) paraphrased the work of Newton, Faraday and Maxwell in explaining that field lines presented an explanation of "action at a distance" for non-contact forces. For some, field lines presented a physical medium in which the force could act, where others saw them as representations. For a course that introduces the use of field lines, understanding the relationship between force, field and trajectory provides solid foundation for field theory. Greca and Moreira

(1999) showed that in a small group of undergraduate students taking an introductory course in electromagnetism, the students that could form working models of electromagnetic fields were comfortable exploring the concepts of a field mathematically or used models for field lines and solve problems using constructed images. Conversely, students who struggled to form working models of electric and magnetic fields appeared to overly rely on definitions without understanding their implications. They were unable visualise a field, using field lines or vectors, and in some cases, represented the fields incorrectly. Student's inability to apply the concept of field hinders their understanding and ability to progress to more advanced applications of electromagnetism in physics, electrical engineering and other relevant avenues of study.

As a diagrammatic representation of a vector field, a lot of information can be gleaned from a field pattern, that otherwise, would require complicated calculations to determine. Such examples are determining relative field strength at various positions, identifying relative magnitudes of charges / masses of bodies, identification of charge types, construction of reasonable paths taken by various charged and uncharged bodies in the fields and identification of charge on a particle moving in a field. While such a simple representation can be used to determine a vast amount of information, this also presents opportunities for students to develop confusion about the representation and errors in understanding (Maloney, *et al*., 2001).

Furio and Guisasola (1998) presented their student's difficulties in differentiating between field intensity, and the force acting on a particle in a field. One possible source of this difficulty is suggested by them that interpreting the definition of the magnitude of the electric field strength, $E = \frac{F}{q}$, the students interpret the proportionality between the electric field strength and force for an equivalence. Similar findings were presented by Cao and Brizuela (2016) who demonstrated that students can accurately produce canonical electric field lines, but struggle to attribute meaning such as force directions, velocity directions and trajectories to them. In some cases, students can attribute combinations of these meanings simultaneously to the field lines. These difficulties contrast the

convention that field intensity is based on the charge / mass generating the field and is independent of the test-charge / small-mass placed in the field. This is typically represented by the density of the field lines. Törnkvist, *et al.,* (1993) note that students struggle to transfer between field line representations and vector representations, in which some students produce curved vector force arrows to represent a curved field line, instead of producing a tangential arrow. They also showed that student errors include field lines overlapping, instead of use of the superposition principle to show a resultant field. Galili (1993) found that some students perceive field lines to be real tangible structures, which mediate the force acting on a particle in a field. Both Galili (1993) and Törnkvist, *et al.,* (1993) showed student represent the path taken by a body in a field to follow the curvature of a field line.

While the difficulties presented be used to inform development of instructional materials, Cao and Brizuela (2016) discuss how students do not always express in-depth reasoning initially but can develop sophisticated understandings whilst explaining and reinterpreting their work. They illustrate an example of a student making additions to their work during an interview to expand on their initial explanations and form an accurate model of force, motion trajectories using electric field line representations, but point out that they do not imply that students should be expected to develop sound reasoning without the aid of some manner of instruction.

2.3.1.4. Difficulties in understanding of work and potential difference

This section discusses literature related to student's understanding of work, potential and potential difference. Literature on these topics tends to focus on the work-energy theorem and how it applies to system interactions. Potential and potential difference are looked at from a calculus-based context, or their applications to electrical circuits. As this research focused on second level students, the literature reviewed focuses on students conceptual understanding of the mathematical implications of work, applying it to potential difference, and applying potential difference to systems, using an electrostatics context and reasoning, as opposed to current electricity.

Doughty (2013) conducted research with undergraduate students, in which she presented student difficulties in determining whether work done was positive, negative or zero when a test charge moves between various points in the electric field. Difficulties were observed for student determining the direction of the electric field, and force, relative to the displacement and assuming the direction of the force is collinear and in the same direction to the displacement. During student interviews, she noted students use energy conversion as a mechanism to explain the concept of positive work (increase in translational KE), negative work (decrease in translational KE) and zero work (no change to gravitational potential energy). This reasoning can be applied to systems, in which work can increase or decrease the energy in a system. Lindsey, *et al.*, (2009) conducted a study with over 4000 undergraduate students over numbers of years, where they developed tutorials focusing on systems involving mechanical work and springs. They identified numerous difficulties in student's understanding of work, such as belief that energy in any system is constant, thinking in terms of kinetic and potential energy and ignoring the cause and effect of the work-energy theorem, and associating work with change in kinetic or potential energy, as opposed to a system. They also identified specific difficulties related to work, such as treating the sign of work as dependant on a coordinate system and failing to consider the displacement of the point at which the force is applied. The approach taken in this research does not discuss systems, or internal energy, so it is expected to see students associate work with the change in kinetic and potential energy of charges in electric field, in the models presented to the students. Other approaches that were taken include a multi-

representation approach by van Heuvelen and Zou (2001), which focused on the use of verbal, pictorial, graphical and mathematical representations, and showed graphs and charts can be useful in helping student develop understanding of these processes.

Hazelton (2013) showed that students struggle to associate higher and lower potential, relative to the ground, to metal spheres with charged rods adjacent. While students could associate high and low potential to positive and negative charge, they incorrectly applied them to the metal spheres, both as the charged rod was initially placed over the sphere, and as time went on so charge was no longer moving between the ground and the rod. Student also demonstrated that they associate the potential with the charge on the body, but do not consider the overall system when multiple bodies of various charge are present. Maloney, *et al.*, (2003) showed that students struggle to understanding how a negative charge will move based being at a position of high potential, but not being aware of the potential in other regions, indicating they do not consider the movement of charged to be based on potential difference, as opposed to potential. Some of these difficulties can be explained by work completed by Guisasola, *et al*., (2002), in which they found that, when phenomena are related to the process of charge, students feel more comfortable when they talk in terms of charge, rather in terms of potential. This can lead to difficulties when students must consider the interactions with objects outside the system they are dealing with, and resort to imagining some form of contact or interaction with external bodies.

2.4. Implementation of lessons

The last section of this chapter discusses the overall teaching and learning approach adopted in this research. Section 2.3.1 highlighted that there are many issues and difficulties in the teaching and learning of Coulomb's law, electric field and work and potential difference. Difficulties and misconceptions were identified from literature and assessed using pre-tests to identify student's initial conceptions that can be targeted for conceptual change. Inquiry methods of learning were employed to direct the learning to promote conceptual change in the lessons (Tabak*, et al.*, 1995; Blanchard, *et al.,* 2010). The use of inquiry is employed in the research using tutorial lessons. The tutorial lesson structure is based on the lesson format as described in Tutorials in Introductory Physics (McDermott and Shaffer, 2003) and the design of some of the questions was influenced by Conceptual Physics (Hewitt, 2009).

Multiple external representations are utilised in these tutorial lessons, using various representational tools to enable the students to encounter information and display their understanding of the concepts underpinning Coulomb's law, the electric field and work and potential difference. It was aimed that the students would have been enabled to iso-morphically transfer the understanding

concepts in these topics between all the representations employed in the tutorial lessons, i.e., students can transfer proficiently between all representations without difficulty in any case. The tutorial lessons utilise representations to allow the students to (a) explore concepts and relationships in different manners, (b) glean different information from similar scenarios and/or (c) predict the behaviour of charged particles in different scenarios using the different representations. Homework and post-tests gauge the extent to which conceptual change occurred, and gather evidence for indications of conceptual extinction, exchange and/or extension. The details of how the tutorials lessons are implemented are discussed in detail in chapter 3.

Vygotsky’s (1978) zones of development were considered to determine the order of the lessons. Teaching Coulomb’s law, electric field, work and potential difference, without having covered any other underpinning topics would not be feasible, as without some foundational domain knowledge, these topics are in the zone of no development. In this research, vectors, the inverse square law and field lines are identified as element topics, as in they are elementary to the understanding the electrostatic topics. Once their understanding of these element topics is developed, the student’s zones of proximal development extends to encompass the electrostatic topics. The student’s development of the element topics is discussed in chapter 4, and the development of their understanding of the electrostatic topics is discussed in chapters 5 and 6.

Table 2.1 summarises this section in a pedagogical framework for the implementation of these lessons. It presents the several topics covered by the research and links them to the research questions that underpin this research. The target concepts within each topic are identified and the representations are presented. The later topics of Coulomb’s law, the electric field and work and potential difference display which element topics are employed, and the necessity for the students to have completed them prior to the electrostatic topics. The various teaching and learning (TandL) materials that were developed and utilised for this research are listed for each topic, with a reference to a copy of each set of the TandL materials listed in the Appendix column.

	Contextual / conceptual domain of physics	Research questions	Target concepts	Representations & elements used	Teaching & Learning (T&L) Materials	Appendix
Elements	Vectors	RQ 1	• Vector magnitude • Vector addition constructions • Horizontal / vertical component superposition.	Mathematical / algebraic. Vector diagrams.	Pre-test. Tutorial lesson. Homework. Post-test.	Appendix A
	Inverse square law	RQ 2	• Increase in x-variable reduces the y-variable • Change in y variable is invers square proportional to x-variable • Area and paint model to explain concepts.	Diagrammatic model. Tabular data. Graphs. Mathematical / algebraic.	Pre-test. Tutorial lesson. Post-test.	Appendix B
	Field lines	RQ 3	• Difference between force and field. • Field line density for strength. • Path taken not represented by field lines.	Field lines.	Pre-test. Tutorial lesson. Homework. Post-test.	Appendix C
Application of elements	Coulomb's law & Electric field	RQ 4	• $F \propto q_1q_2$ • $F \propto \frac{1}{d^2}$ • Vector addition of electrostatic forces. • Difference between force and field. • Field line density for strength. • Path taken not represented by field lines. • Negative charge moves against field lines. • Field patterns for attraction and repulsion. • Drawing vectors from field lines. • Drawing field lines from vectors.	Tabular data. Graphs. Mathematical / algebraic. Diagrams. Vectors. Field lines.	Pre-test. Tutorial lesson. Homework. Post-test. Pre-test. Tutorial lesson I. Homework I. Tutorial lesson II. Homework II. Homework III. Post-test.	Appendix D
	Work & potential difference	RQ 5	• Use of vectors to differentiative between displacement and distance. • Use of vectors and field lines to identify positive, negative and zero work. • Potential difference as a mathematical ratio.	Field lines. Vectors. Mathematical / algebraic.	Pre-test. Tutorial lesson I. Homework I. Tutorial lesson II. Homework II. Post-test.	Appendix E

Table 2.1, Pedagogical framework for the research studies.

Chapter 3. Research Design

3.1. Introduction

This chapter presents four sections: the research methodology, the implementation methodology, analysis methodology and description of participants. The research methodology overview focuses on the case study approach and outlines how and why it was adopted in this research. The propositions used to focus the research are outlined and the types of evidence collected for analysis to explore the propositions are detailed. The implementation methodology introduces the background to the tutorial lessons used in this project, outlines what occurs in a tutorial lesson, and justifies the use of structured-inquiry tutorials as an educational method. The ethical considerations for the students participating in the case for research is also discussed. The analysis methodology discusses the use of qualitative analysis in this project, whilst the description of participants illustrates the background of the fourteen students who took part in this project.

3.2. Research methodology

This section of this chapter discusses the research methodology employed in this research. It discusses published research on the use of case studies, primarily looking at the work of Yin (2009). The chapter then discusses how the case study methodology is adopted in this research, identifying the case central to the research and several propositions of the research. The final part of this section overviews the several types of evidence collected during this research and provides a descriptive commentary of each.

3.2.1. The use of case study in qualitative research

A case study is a study of something that occurs over time, with the subject case being a person, a group of people, an organization, other possible groups and even specific events. As empirical research into a chosen phenomenon within a context, a case study can gather both qualitative and quantitative data that can be analysed to construct conclusions (Given, 2008). Factors that affect this research include the sample size of the case being studied, and whether a single-case study or multiple case studies take place. As case studies look at individual contexts, it is not always possible to identify and control all variables that could affect the outcomes in the study, especially when research

involves multiple cases. While this can be considered a weakness of using case studies, they afford a unique opportunity to gather in-depth data that other research methods may not afford; moreover, it is often simply not possible to control all variables.

Yin (2009, p13) suggests a two-fold technical definition for a case study:

> "*A case study is an empirical inquiry that investigates a contemporary phenomenon in-depth and within its real-life context, especially when the boundaries between the phenomenon and context are not clearly evident*"

and

> "*The case study inquiry copes with the technically distinctive situation in which there will be many more variables of interest than data points and as one result, it relies on multiple sources of evidence, with the data needing to converge in a triangulating fashion, and as another result, benefits from the prior development of theoretical propositions to guide data collection and analysis*."

A case study is an appropriate methodology when the following considerations are applicable to the research:

1. A "how" or "why" question is being asked.
2. The events being researched are contemporary, i.e., they are events occurring in the present.
3. They are events in which the investigator is unable to control all the relevant variables that may affect the outcome. (Yin, 2009)

Other methods of qualitative research were considered for this research, such as social experiments, surveys, archival analysis and narrative historical accounts. However, due to the small sample size of the student cohort, the evidence collection employed in the research and the propositions use to guide the research questions discussed in section 1.3, a case study methodology was deemed appropriate.

When considering the rigor and reliability of using case study, the following advantages were identified: they

1. cope with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result,
2. rely on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result;
3. benefit from the prior development of theoretical propositions to guide data collection and analysis. (Yin, 2009)

3.2.2. Case study design

A case study is a research study of a person, group or situation over a period of time. Nisbet and Watt (1984) note that there are many strengths and benefits to the case study methodology. Case studies catch unique features that may otherwise be lost in large data-sets, and these features may hold the key to understanding the situation. As case studies tend to use relatively small sample sizes, numerous sources of data are recorded and analysed which can give a wide set of results that show the interaction of many factors being studied. They are strong on reality, in that they provide insight into an event being studied and seek to analyse the event. They can also provide insights into other, similar situations, thereby assisting interpretation of similar cases. Unlike alternative forms of research, case studies can be undertaken by a single researcher, instead of requiring a team of research for completion. Finally, they can embrace and build in unanticipated events and uncontrolled variables. When designing a case study, Yin (2009) suggests the following aspects of the research need to be considered, to implement the method efficiently:

- The research questions.
- The propositions of the research.
- The units of analysis.
- Linking of data and propositions.
- The criteria for interpreting the findings.

The research should be guided by an overall research question or set of research questions. This can guide the researcher to perform an effective literature review to inform and aid their research design. As case studies can produce a wide amount of varied data, when the researcher develops a research question, they should also identify propositions that guide the data analysis and provide a structured purpose to their research. As the sample size in this research is small, any hypothesis developed to guide the research would not be verifiable / rejectable when considering the general population. For this reason, the research is framed around propositions. Baxter and Jack (2008) explain that in a case study, propositions can be equated with hypotheses. This is justified as both have a predictable power to determine possible outcomes of the experiment/research study. Propositions are useful in that they allow the research to place limits on the scope of the study and increase the feasibility of completing the project, with specific propositions allowing for the construction of boundaries in a case study. Propositions may come from the literature, personal/professional experience, theories, and/or generalizations based on empirical data (Baxter and Jack, 2008). A case study may contain several propositions to guide the study. They are distinct from each-other and allow for a specific purpose when determine what data to collect and how to use that data to inform discussion. Each proposition serves to focus the data collection, determine

direction and scope of the study and together the propositions form the foundation for a conceptual structure/framework (Miles and Huberman, 1994).

The unit of analysis involves identifying the case that is to be researched and how it will be analysed. Over the course of research, the unit of analysis can change, due to improved designed research questions, or new opportunities for inquiry presenting themselves. When data is collected, the results can be linked back to the initial proposition stated in the research design. This allows for purposeful data analysis that links to the overall research question(s). The last consideration is criteria for interpretation, which allows the researcher to develop a manner for interpreting the analysed data to ensure it reflects what is being asked by the research. In the implementation of a case study, many sources of evidence can be collected, such as documentation, archival records, interviews, direct observation, participant-observation and physical artefacts (Yin, 2014).

Documentation involves the analysis of any documented material related to the case study. It can take the form of letters, memoranda, agendas, administrative documents such as reports, formal studies and/or evaluations. Archival records involve the analysis of archival data such as "public use files" (example, census data), service records and organisational records. Interviews allow for direct inquiry with the research participants and allow for a fluid generation of qualitative data from the interview participants. Direct observations involve directly observing the case participants, using data collection activities that range from casual to formal. In participant-observer data collection, the researcher assumes a role within the fieldwork situation and may participate in the actions being studied and record their own field notes as the research progresses. Finally, physical artefacts are the retrieval of physical evidence from the case study, with can be analysed upon completion of the evidence collection. The different strengths and weaknesses of each of these evidence collection types are presented in Table 3.1.

Using multiple sources of evidence allows for triangulation of data. The requirement for multiple evidence sources in case studies far exceeds that of other research methods (Yin, 2014), due to the method's limitations. Multiple evidence corroborating the same finding gives more weight to their validity. By triangulating data, the researcher constructs validity in their case study, and increases confidence that the research accurately renders the event being studied. Figure 3.1 illustrates the convergence of triangulated data to the same findings, and the non-convergence of un-triangulated data from separate sub-studies.

As every classroom is different and student's experiences and prior knowledge may be different, it is difficult to control all the variables that could affect student performance. To determine the progression of student's conceptual development, multiple sources of evidence are employed, discussed in-depth in section 3.4, such as pre-test – post-test comparisons, analysis of student's artefacts, teacher observations, teacher feedback, student feedback and teacher-student interviews.

This allows for the identification of patterns in the student's progression in their conceptual development and attribute this to the students completing the materials.

SOURCE OF EVIDENCE	Strengths	Weaknesses
Documentation	• Stable – can be reviewed repeatedly. • Unobtrusive – not created as a result of the case study. • Specific – can contain exact names, references, and details of event. • Broad – can cover a long span of time, many events and many settings.	• Retrievability – can be difficult to find. • Biased selectivity, if collect is incomplete. • Reporting bias – reflects (unknown) bias of any given document's author. • Access – may be deliberately withheld.
Archival records	• *[same as documentation]* • Precise and usually qualitative.	• *[same as documentation]* • Accessibility due to privacy reasons.
Interviews	• Targeted – focuses directly on case study topics. • Insightful – provides explanations as well Insightful into interactions and personal views (e.g., perceptions, attitudes and meanings)	• Bias due to poorly articulated questions. • Response bias. • Inaccuracies due to poor recall • Reflexivity – interviewee gives what interviewer wants to hear.
Direct observations	• Immediacy – covers actions in real time. • Contextual – can cover the case's context.	• Time – consuming. • Selectivity – broad coverage difficult without a team of observers. • Reflexivity – actions may proceed differently because they are being observed. • Cost – hours needed by human observers.
Participant – observation	• *[same as direct observations]* • Insightful into interpersonal behaviour and motives.	• *[same as direct observations]* • Bias due to participant-observer's manipulation of events.
Physical artefacts	• Insightful into cultural features. • Insightful into technical operations.	• Selectivity. • Availability.

Table 3.1. Six sources of evidence: Strengths and Weaknesses (Yin, 2014)

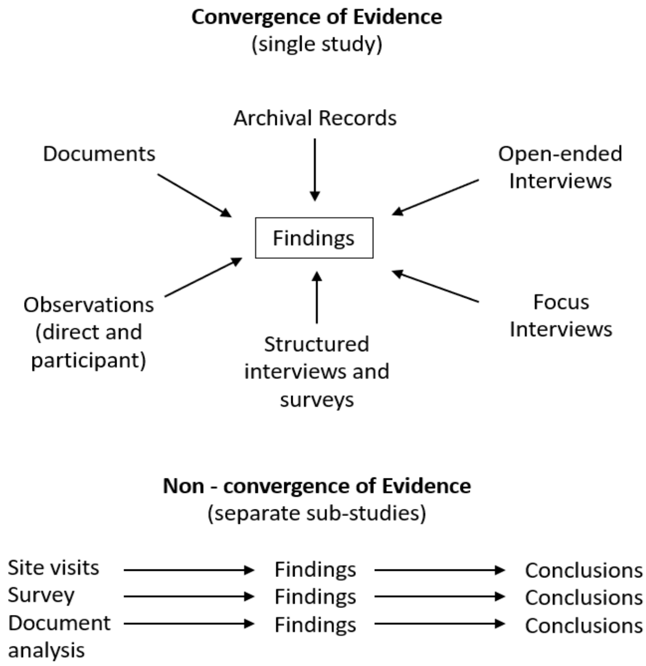


Figure 3.1. Convergence and non-convergence of evidence (Yin, 2014)

3.2.3. Case study limitations

Case studies produce unique research that can allow for understanding complex phenomena in particular contexts (Simons, 1996). The contexts tend to be unique and dynamic, allowing for investigation into complex and unfolding interactions of events, human relationships and influences and other unique factors (Cohen, *et al.*, 2000). However, the case study methodology is not without its limitations. Some researchers consider it to be a less than desirable form of research methodology, to which Yin (2009) suggests the following reasons:

- The lack of rigor of case study research, possibly due to a lack of literature on the subject when compared to other research methodologies.
- Single-case studies do not provide enough evidence for generalization.
- The time required to complete case studies, which can produce massive unreadable documents.
- Case studies do not provide enough evidence to determine "causal" relationships.

Nisbet and Watt (1984) note that results from this research (i) may not be generalizable to the wider population but may be useful to studies in which the contexts are similar, (ii) are not easily open to cross checking, and may be selective, biased, personal or subjective and (iii) prone to problems of observer bias. Simons (1996) notes that the first weakness is often cited as a case study

weakness, but this view assumes a polarity in research. However, an alternative perspective of this is that case studies allow for the development of descriptions and explanations to the case being studied, whilst acknowledging the numerous variables in the context that can affect the study. While it is outside the scope of this research to attempt to resolve these limitations of case study research, limitations of the case study methodology are acknowledged.

3.2.4. Applying a case study to the research

In this research, the case to be studied is the conceptual development and understanding of one class of upper secondary level physics students in the academic year 2016/17, in the topics of vectors, inverse square law and field lines, and how they transfer their understanding of these topics when learning about Coulomb’s law, electric fields and potential difference. As described in-depth in section 3.5., the group consists of mixed gendered, mixed ability students which varies from gifted to weak students.

A case study was chosen for this group for the following three reasons. The sample size of the case being studied consists of fourteen students. Any findings generated from data from a sample of this size are not reliable to make any generalisations that apply to the wider population of upper secondary physics students. Outside of instructional design, student abilities, student attitudes and previous learning experience from different teacher influences are factors that could influence the outcomes of this case study. A case study is an appropriate method to use when extra variables that influence the findings cannot be controlled. As the influence of these variables changes from school to school, the research can also define the case as a contemporary grouping of students, which is difficult to study in other forms of qualitative analysis.

As suggested by Yin (2014), numerous sources of evidence are collected for triangulation to determine conceptual development and link how the tutorials promoted the development. Using all the evidence collected, would produce a report of unreasonable length. Collating the data and providing a varied sample of student responses results in a concise overview of student’s progressions and difficulties encountered when completing the tutorial lessons. However, it is acknowledged that in some cases, the omission of some students work in favour of other’s, may not provide a complete picture of what occurred during the lessons, but it is endeavoured to keep this to a minimum. A final consideration is that while the tutorials are likely the only intervention the students use to explore concepts related to the research, it is acknowledged the evidence can only provide indications that attribute the developed materials to the student’s conceptual gains but cannot provide a causal link between the two.

In drafting the propositions that underpin this research, relevant sections of the thesis that cover the background used to inform them are referred to. The propositions used in the case study to frame the research and determine how to collect data are:

1. Tutorial lessons are a good teaching methodology to enable students to develop conceptual understanding in physics topics. (Section 3.1)
2. Students that learn a concept through, and develop the ability to transfer between, multiple representations to develop their understanding of that concept. (Section 2.2., section 2.3., section 3.3
3. Vectors, the inverse square law relationship and field line representations are tools, that when mastered by students, will enable them to describe electric fields, and the interactions of charges with electric fields. (Chapter 4)
4. Vectors and field line representations are tools, that when mastered by students, will enable them to describe the work done in moving a charge in an electric field, and conceptually explain the potential difference between two points in an electric field. (Chapter 5, chapter 6)

3.2.5. Evidence collection in this study

In this study, five types of evidence were collected. These evidence types were categorized as primary and secondary evidence. The primary evidence is used in all aspects of the data analysis through all concepts covered by the students and is used to glean the conceptual understanding developed by the students. The secondary evidence is used to support of the findings from analysing the primary data, but in and of itself, it not rigorous enough to construct meaningful findings on their own. The primary evidence types are used to pre-tests / post-tests, student artefacts and teacher – student interviews, while the secondary evidence are recordings of student dialogues and teacher reflections.

3.2.5.1. Pre-tests and post-tests

Pre-test / post-test comparisons are used in this research to gauge conceptual development of student's models over the course of the inquiry lessons. Each question on the pre-test is designed to elicit student's understanding of one aspect of a concept. An ideal pre-test question is structured in a manner that only correct understanding of the concept being asked about can bring about a correct answer. However, when asked to predict outcomes or produce rankings, it is possible for students to guess a correct answer, and so students are also required to display the reasoning they used to inform

their answer. The contexts of the questions are typically simple ones, so students are not influenced by the context itself. Additionally, by designing the pre-test questions to be as simple as possible, the students would be less likely to be put off answering the question by the perceived difficulty of the pre-test.

The post-test questions were designed in the same format as the pre-test questions. In some questions concepts were also asked in a manner that only one aspect of a topic was tested in any one question and correct understanding of the concept was required answer to the question correctly. The questions are not designed to guide students through the question, as they are in the tutorial. Additionally, there were questions asked that required the use of multiple concepts to answer correctly. By comparing student's answers between their pre-tests and post-tests, indications to the level of conceptual change that occurred could be observed. This provides evidence to judge the effectiveness of the instructional materials and the determination of what revisions need to be considered for redesigning future materials.

Students completed pre-tests in class, before completing the tutorial and after the teacher presentation of the topic. These tests were scanned and stored on an external hard-drive. Each pre-test was analysed, and student responses were categorized based on concepts / misconceptions used in answering the questions. These categories were used in the post-test questions, allowing for direct comparison between student answers in both tests. Further analysis of student's understanding was then gleaned from specifically looking at the student's articulation of their answers and their direct application of the concepts when answering questions. In this case, student's understanding must be interpreted from their written responses, especially in cases where the student used common colloquial language, instead of physics specific terminology and it is acknowledged that while this is a strong indication of student thinking, it cannot provide a complete representation of it.

3.2.5.2. Student artefacts

Samples of the student tutorial materials and the homework assignments were collected and scanned. The homework assignments were categorized in the same manner as the pre-tests and post-tests, while the tutorials were reviewed to identify where particular students encountered difficulties. When this occurred, students were requested to not destructively erase their earlier writing, and instead use extra space, to allow for the comparison of incorrect and correct reasoning produced by the student. This gives insight into student thinking, that is not afforded in the pre-test/post-test comparison. The use of the pre-test/post-test model can struggle to identify the source of difficulties in student reasoning. In analysing student artefacts, it is possible in some instances to pinpoint where student thinking diverged from what was the intended reasoning. In class, this was used by the teacher

to consider what line of questioning and prompts were required to enable the student to revise their thinking and develop news lines of thought to arrive at the correct conceptual understanding.

The homework assignments allowed students to review and extend their understanding of concepts covered in the tutorial. In some cases, questions from the pre-test appeared in the homework assignments. This gave the opportunity to observe how students answer the same question to determine if there is any change in their answers. When the homework assignment was designed to allow students to review the lesson material, it was possible to gauge whether there was divergence in student thinking when in a group setting in the tutorial or reaffirm correct reasoning between both the tutorial and the homework assignment.

3.2.5.3. Teacher-student interviews

The teacher-student interviews were conducted after the students completed the tutorial lessons. Students who completed the interviews were generally identified as having difficulties with concepts covered in the tutorials and post-tests. This allows for the use interviews as an extra mechanism for the student to help students alleviate their difficulties and provide insight to the source of their difficulties.

A small 20-minute tutorial worksheet is developed for these interviews, which explore the same concepts as those seen in the tutorials, but in an unseen context. As the interviewer, I provided prompts and scaffolds to the students, as they complete the worksheet. As students answered questions, I could ask the students to explain the reasoning used by the students to develop their answers. This gave insight into what the students were thinking and allowed them to articulate their reasoning more in depth than the reasoning some students submitted in the tutorials. I could also observe the students discussing the questions with each-other and take note of how the students developed and supported each-other's reasoning over the course of the teaching and learning interview. The interviews also allowed me to produce a record of what interventions and prompts I used that were effective in facilitating student conceptual development.

Engelhardt, *et al.*, (2004) highlight this as an advantage of using teaching-learning interviews. The teaching-learning interview also provides an opportunity to continually probe student's understanding in a manner not afforded when using post-tests. When students provide assertions, the interviewer can question the nature of these assertions. If the student's explanation is not complete, the interview can take time to explore their explanations, so the student clearly articulates their thinking, in a manner that they do not consider in the tutorial lesson. However, as this focus on student's specific explanations of their reasoning is atypical of the depth feedback generally provided in the tutorial lessons, the interview does not accurately reflect the learning experience the student

encounters in a classroom. Chini, *et al.*, (2009) note that this is a limitation of the interview, in that, what interventions work in this setting may not work in a classroom setting.

3.2.5.4. Recordings of student dialogues

Student conversations that took place during the lessons were recorded. As the students worked together to complete the materials, developments in student's conceptual understanding that were verbalised may not be recorded in the artefacts. Having recorded student's conversations, it is possible to analyse the reasoning used by students in their discussions to observe how student difficulties were overcome. Due to the massive amount of data generated, and the time required to analyse the data, the students were asked to record the time on the dicta-phones when they felt that other students helped them overcome difficulties they encountered during the lessons. The teacher also recorded the time in which they used questions and prompts to help students develop and consolidate their thinking.

3.2.5.5. Teacher reflections

Upon completing of the tutorial lessons, the teacher drafted reflections, based on his observations and feelings on what felt worked and did not work in the lesson. In a teacher reflection, the teacher engages in a cognitive process, which involves providing a commentary on difficulties encountered during the sequencing of the lesson. Reflective thinking generally addresses practical problems, allowing for doubt to be used as a mechanism to identify problems and shortcomings before solutions are reached (Hatton and Smith, 1995). By reviewing the teacher reflections, a comparison of the teacher's feedback and recordings of student dialogues can allow for the identification of student difficulties to take place. Specific difficulties that the teachers encountered during implementation can also be identified and open a conversation space to discuss ways to alleviate encountered difficulties.

3.3. Implementation

This section of the research discusses the implementation methodology for this research. It describes different influences used to develop the tutorial lessons implemented in this research. The structure and format of a tutorial lesson and details their implementation are then presented. The justifications of using tutorial lessons with the second level students is discussed. Additionally, as

the participants in this research were under 18 years of age, the ethical considerations that were made, and the approval by DCU's ethics committee is outlined in this section.

The approach adopted in this research is the use of structured inquiry. Instead of students being presented with the content and required to learn it off, learning occurs through students answering questions, solving problems and working through challenges (Lynn, *et al*., 2013). As seen in section 1.3, improving student's conceptual understanding is central to the research questions of this research, and inquiry has been shown to be effective in this pursuit (Tabak, *et al.*,1995; Blanchard, *et al.,* 2010). The tutorial lessons developed in this research as the teaching and learning materials, are based on the approach from Tutorials in Introductory Physics (McDermott and Shaffer, 2003). The use of tutorial lessons has been shown to promote gains in student's conceptual understanding of physics concepts (McDermott and Shaffer, 1992; Wosilait, *et al.*, 1998). As stated in section 2.2.2, exercises from the Conceptual Physics practice book, 10^{th} edition (Hewitt, 2009), were used to influence the design of the tutorials in this work, as the way Hewitt approaches contexts would be more accessible to the second level students. Sections 3.3.1 and 3.3.2 discuss how inquiry was employed in this research, by detailing what a tutorial lesson looks like, and justifying the use of inquiry in this research.

3.3.1. What does a tutorial lesson look like?

The following description of a tutorial has been patterned after Tutorials in Introductory Physics – Teachers Guide (McDermott and Shaffer, 2003). It summarizes what occurs when students complete a tutorial lesson, from start to finish. The logistics of implementing tutorial lessons into a second level classroom are also described.

A tutorial lesson is designed to supplement lectures and textbook materials in Physics courses. Students do not focus on reciting definitions, listing how to complete demonstration experiments, or completing quantitative problems. Instead, the focus is shifted on students developing their understanding of physical concepts and the use of their scientific reasoning skills. Tutorials provide a structured format for students to determine what they do and do not understand when learning physics. In small groups, students complete a series of questions in which they are guided through the reasoning necessary to construct scientifically valid models of a chosen topic in physics. They provide opportunity for students to interpret and represent concepts using a variety of representations, such as formulae, graphs, diagrams and verbal descriptions. Students take part in the tutorial lesson after they have been initially introduced to a concept through a lecture, presentation and/or laboratory, but can be used to introduce a concept or extend it.

As discussed in-depth in section 2.2.2., this research developed tutorial lessons to promote the student's conceptual understanding. The tutorial lessons comprise of a pre-test, worksheet assignment, homework assignment and post-test. The pre-test examines student's prior knowledge and understanding of a concept. This allows for the identification of difficulties to be targeted for conceptual change. The worksheet assignment is completed by the students in small groups. They work through the exercises together, engaging in dialogues between themselves or with the teacher when difficulties are encountered. The conditions of conceptual change (Posner, *et al.*, 1982) are generally encountered and met at this stage. The homework assignments are completed by the students, which gives them the opportunity to revisit the concepts from the worksheet or in some cases extend them. The context of the homework may or may not be altered from contexts in the worksheet exercises. The post-test is given at the end of the tutorial lesson. It is written to focus on the concepts and skills used in the tutorial lesson. Comparisons between the pre-test and post-test allow for the generation of evidence that can be used to identify and indicate the extent to which conceptual change occurred.

With the participants of this research, they completed a lesson in which concepts were introduced through power-point presentations, demonstrations and practicing qualitative problems and classroom discussion. These lessons occurred in either a single lesson period, (40 minutes in length) or a double lesson period (80 minutes in length). The students then completed tutorial lessons in a double lesson period. The materials themselves were written with the materials designed to take up to 50 minutes of class time, which affords time at the start of the lesson to be used to take the pre-test, and time at the end for a quick round up with the class in their entirety.

When covering inverse square law, vectors and field lines, the students completed one tutorial a week, and the pre-tests, homework assignments and post-tests are completed around the tutorial. During the section on electric field, a traditional lecture style class introduced concepts related to charge and Coulomb's law, and the pre-test and tutorial on Coulomb's law followed. Students were then given another lecture style lesson introducing the electric field and completed quantitative problems. Students then completed all the electric field tutorials in succession and a post-test based on Coulomb's law and the electric field was completed. When the students completed the tutorial lessons on work and potential difference, they completed the pre-test before completing the work tutorial, engaged in two tutorial lessons, one on work and one on potential difference, before completing the final post-test for these two topics together. An in-depth timeline for the implementation of the tutorials is presented in Chapters 4, 5 and 6.

3.3.2. Justifications for using inquiry tutorials

In section 2.1, the role of inquiry in science and physics education was discussed, and in the previous section, the application of inquiry in this research, using tutorial lessons, was outlined. This section provides justification for the use of structured inquiry in this research, in terms of the efficacy in promoting targeting and promoting conceptual understanding, how it aligns to some of the aims of the Leaving Certificate Physics syllabus (1999) and how structured inquiry allows for balance between content requirements and conceptual depth of treatment.

Structured inquiry was utilised in this research to promote conceptual change in the students understanding. Structured inquiry was chosen as it has been shown to be a favourable method to develop conceptual knowledge in students (Tabak, *et al.*, 1995; Blanchard, *et al.*, 2010). Ambrose (2004) notes that it is effective for targeting specific concepts for conceptual change. Section 2.1.3 illustrated the effectiveness of structured inquiry tutorials by detailing one example from research (Shaffer and McDermott, 2005) and referencing others (Close and Heron, 2010; Heron, *et al*., 2004; Hazelton, *et al.*, 2012). This research initially targets students conceptual understanding of vector concepts, the inverse square law and field line and then targets how they employ these topics to develop their understanding of Coulomb's law, the electric field, work and potential difference. Structured inquiry is employed in this research as evidence from the research literature suggests it is an effective method to achieve the teaching and learning outcomes that unpin this research.

Tutorial lessons involve learners working in small groups to discuss their thoughts, ideas and understanding of concepts that underpin the topics they are studying. This provides them the opportunity to articulate their thinking and communicate to each other to collate their understanding. This is in line with the aims of the Leaving Certificate Physics syllabus (NCCA, 1999, pg2) which states students should develop "an understanding of the fundamental principles of physics" and "develop the ability to observe, to think logically, and to communicate effectively." The tutorial lesson format provides students the opportunity to attain these aims.

Finally, as tutorial lessons allow for the targeting of specific concepts, the lessons were developed in a manner that the core learning outcomes for the Leaving Certificate Physics syllabus (1999) were met. The depth of understanding targeted by the research was balanced with the content the students were required to learn for completion of the course. This way, the students have completed the required content to complete questions on their terminal examination. Other forms of inquiry, such as guided and open, can result in students learning content not targeted by the teacher, and may miss specific content designated by the learning outcomes of the course they follow, but develop their competencies in their cognitive and procedural autonomy (Stefanou, *et al*., 2004). Guided and open inquiry was not employed in this research, as developing these competencies were not targets in this research.

However, there are notable difficulties in applying general inquiry methods to a classroom. These difficulties include, but are not exclusively, the workload involved, lack of resources and the effect the increased workload has on the teaching time required to finish a syllabus (Higgins, 2009). To address these difficulties, the materials are designed and prepared during school breaks, to not interfere with the time allocation to the teacher in preparing lessons. The topics in which the lessons are used, electric field and potential difference, take up a relatively small part of the overall physics course, and thus, the time allocated to completing the classes is minimal. Ambrose (2004) notes that structured tutorials allow for an inquiry implementation in a relative short duration of time. As the tutorials are mainly paper and pen focused, no special laboratory equipment needed to be procured to complete the material and any practical equipment the can be of use is generally readily accessible in the standard fit-out of a second level physics laboratory.

3.3.3. Ethical considerations for research involving second level students

To conduct this research, ethical approval was sought from the Ethics Committee of Dublin City University. As this project involved the analysis of data taken from students, it was important for the university to ensure that every effort was taken to ensure the educational experience provided to the students was well planned and would not hinder their academic progress in their physics course. The allotted time that could be spent on the research was also considered, to ensure they experienced minimal disruption to the lessons required to complete their physics course.

Other issues considered when seeking ethical approval for this project was honesty in reporting results, both positive research and negative results, identification of any potential bias, as the funding body are involved in the promotion of inquiry based learning, and human subjects protection, provided as an opt out of the research and communicating with the school care team regarding student welfare during the project, in the event that students were to find the teaching approach adding undue stress to their educational environment. Approval for this project was granted December 2013 (DCUREC/2013/197) for the pilot trail of the materials and amended September 2014, to include the additional schools for garnering data for external validation purposes.

All students were given two copies of an informed consent form, two plain language statements and a cover letter briefly outlining the aim and rationale of the project. As the students were under 18 years of age, their parents / legal guardians were required to sign the consent forms and return one copy to the teachers involved. Parents were informed of the aims of the project and given the opportunity over a two-month time frame to contact me with regards to any concerns regarding the project, prior to its commencement. Parents were also informed that their children could opt-out of the research at any time, in which any data recorded would be destroyed electronically, and paper copies of materials would be shredded.

Each student's name was known only to the teachers involved in the classrooms. This allowed for the comparison of pre-test data, post-test data, class worksheets and homework worksheets of individual students. Each student was given a unique reference code, which is used to identify him or her in this document. The reference code is made up of a number and a letter. The number referred to the year of the research and the letter referenced the student. In the case of the 2014/15 academic year, the second group to complete the material were given two letters. For example, student 2C took part in the 2014/15 editions of the materials, is from the second group of students to complete the materials and is the third person on the class roll as denoted by C being the third letter of the alphabet.

3.4. Analysis Methodology - Qualitative explanatory and qualitative descriptive

There are numerous types of case studies, as outlined by Baxter and Jack (2008), such as explanatory, exploratory, descriptive, intrinsic, instrumental and collective. The type of case study devised in this research contain elements of both an explanatory and a descriptive case study. Explanatory is when a case study would be used if you were seeking to answer a question that sought to explain the presumed causal links in real-life interventions that are too complex for the survey or experimental strategies. In evaluation language, the explanations link program implementation with program effects (Yin 2003). Descriptive is a type of case study is used to describe an intervention or phenomenon and the real-life context in which it occurred (Yin, 2003).

As the sample size of the students is small, quantitative data and quantitative data analysis tools cannot be used reliably. Instead descriptive qualitative data analysis is used, as it gives insight into understanding the context, participants and interventions encountered when the research was conducted. In this manner, a collection of data that allows for understanding the learning process that occurs during the tutorials lessons applied during the tutorial lessons could be generated. The data analysis allows for the drawing of patterns based on the progression of student conceptual development and provides illustrative explanations of their conceptions, based on their individual responses.

In the research, the educational instructional intervention used in the research is the developed inquiry materials used in a tutorial lesson. A case description (Yin 2009) is developed on the theoretical propositions discussed in section 3.2.2. Over the course of chapter 4, 5 and 6, the implementation of the tutorial lessons is described as an observational narrative. An analysis of the student's conceptual developments in vectors, inverse square law and field line representation is detailed in chapter 4 and provide a commentary on how these developments enable students to explore Coulomb's law, the electric field and potential difference.

Through this narrative, content analysis (Carley, 1993), is used to describe how the tutorial lessons promoted conceptual gains in the student models, using multiple forms of evidence gathered, as described in-depth in section 3.2.3. In the content analysis, the evidence was primarily used to populate matrices of student responses and concepts/misconceptions apparent in the data, to determine whether students are using key concepts in their correct contexts or overlapping their understanding (Yin, 2003). These matrices were analysed to produce the results tables shown in Chapter 4, 5 and 6. As a consideration to remove potential bias, initial blind marking was used when populating these matrices. This was done by removing the codes from the scanned student artefacts and initially recorded the frequency of the concepts used and difficulties observed in the student's responses. The artefacts were then analysed again, with the student codes intact, and were used to populate the student responses, as shown in the tables of results throughout chapters four, five and six.

An assumption of the research that students developing a functional conceptual understanding of vectors, inverse square law and field line representation will result in positive conceptual gains in electric field and potential difference, as explained in various sections of chapter 2. This is tested for using a type of pattern matching, determining whether the concepts used in the later topics are transferred from the former topics, and provide a narrative on how the students use these initial concepts to build their understanding of the electric field and potential difference. In the discussions of each section, instances of conceptual change will be discussed, collating evidence from as many of the various the evidence types as possible to identify conceptual extinction, exchange and extension where possible. If conceptual change did not occur, difficulties are identified and suggestions for redesign are given.

Four descriptors are selected in this research to indicate the extent of conceptual change that occurs over the course of this research. As shown in Figure 3.2, these descriptors are based on the total number of 14 students that were studied to measure their conceptual change for a given concept. Minimal conceptual change refers to instances in which between one and three students demonstrated that conceptual change occurred. Partial conceptual exchange refers to instance where between four and seven students were observed to have engaged in conceptual change. Moderate conceptual change is referred to when between eight and eleven students engage in conceptual change and ideal conceptual change is referred to when between twelve and fourteen students demonstrate conceptual change.

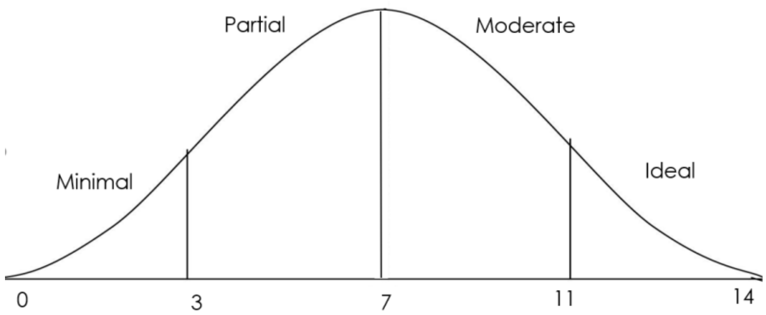


Figure 3.2. Illustration of descriptors used to describe extent of conceptual change within a group of 14 students.

3.5. Description of participants

The group of students that undertook this research was a group of 14 mixed ability students, aged 16 – 17 years old. The group is mixed gendered, but predominantly male (female = 4 students, male = 10). Three of the students speak English as a second language but are fluent in their use of the language. One of the students took part in the educational system of New South Wales, Australia for all the years they spent in formal education, before enrolling in the upper secondary Irish educational system in the academic year 2016/17. To illustrate the general ability of the students in science and mathematics, Table 3.2 shows the student's results from their lower secondary level final examinations in mathematics and science, which were produced by the State Examinations Commission. A copy of the results was obtained from the school where the research took place. These exams are unseen by students and teachers in Ireland, and all students partake in these so-called Junior Certificate examinations at the exact same time. The grades are presented as categorical ordinal data, with A being the highest grade and NG being the lowest grade. The table also includes their results from an in-house Christmas physics examination. This is an indication of the general ability of the students, having studied Leaving Certificate Physics for 4 months. This examination took place after the vector, inverse square law and field line tutorial lessons, but before the electrostatics tutorials. The grades for the in-house test are presented as categorical ordinal data, with H1 being the highest grade and H8 being the lowest grade, mirroring the grading system of the Leaving Certificate exam they undertake at the end of 6th Year.

Figure 3.3 presents the Junior Certificate exams results in bar charts, and the in-house Physics examinations results are shown in Figure 3.4. This allows for identification of the clusters of the results, to create a profile of the student's academic achievements in these exams. As both charts show the cluster of data centralized towards the higher grade, the student's results suggest that the students are a mix of high achieving and average achieving students.

	Junior Results		Senior Results
Student Code	**JC Science**	**JC Maths**	**5th Year Christmas**
4A	C	C	N/a
4B	B	B	H1
4C	B	B	H2
4D	B	D	H5
4E	A	B	H3
4F	N/a	N/a	N/a
4G	A	A	H1
4H	B	C	H4
4I	B	B	H1
4J	C	D	H2
4K	A	B	H3
4L	C	D	H6
4M	B	D	H6
4N	A	A	H2

Table 3.2. Summary of student's results pre-research.

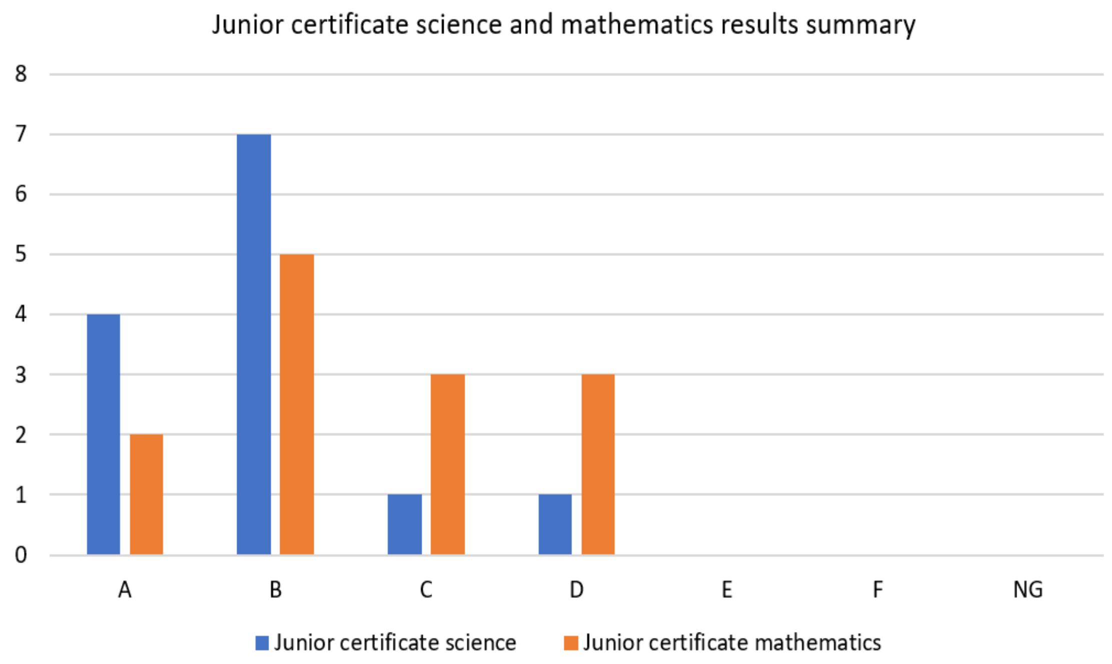


Figure 3.3. Student's results for Junior Certificate Science and mathematics examinations.

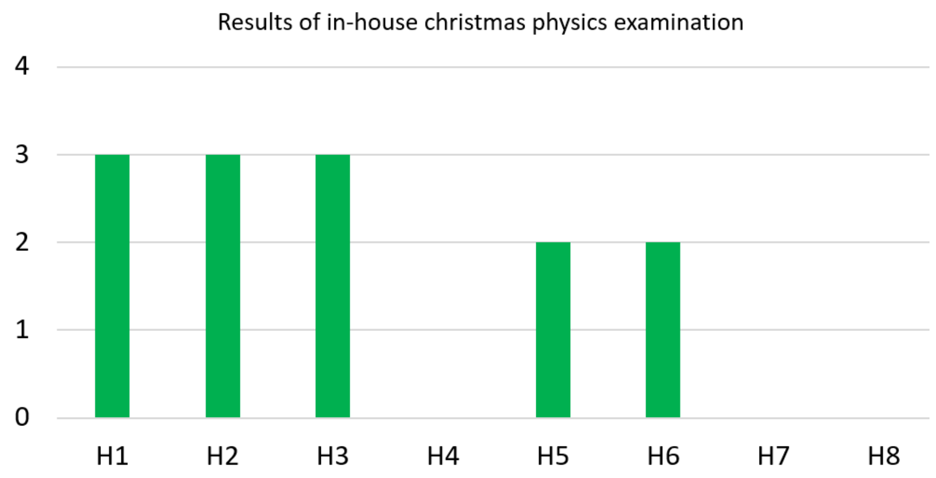


Figure 3.4. Student's results of in-house physics Christmas examination.

3.5.1. Participants' prior learning

This section discusses aspects of the student's prior learning in formal education, for lower secondary level science and mathematics. As the lower secondary level science and mathematics course are delivered for a duration of three years, this section will only reference the content covered in these subjects that are relevant for this research. As the students had different teachers for lower secondary science and mathematics, the aims and learning objectives from the syllabi for Junior

Certificate Science (NCCA, 2003) and Junior Certificate mathematics (NCCA 2012) will be referenced to illustrate the student's prior learning in these subjects.

According to the Junior Certificate Science syllabus (NCCA, 2003), the study of science contributes to a broad and balanced educational experience for students, extending their experiences at primary level. The course aims to promote student's development of scientific literacy, science process skills, and an appreciation of the contribution that science had on the humanity and the planet. The junior science course was designed to be investigative and activity based, and the courses' aims (NCCA, 2003) seek to encourage student development of manipulative, procedural, cognitive, affective and communication skills. Student are to be provided with opportunities for observing and evaluating phenomena and processes and for drawing valid deductions and conclusions, which aids to enable students to acquire a body of scientific knowledge appropriate to their age, and an understanding of the relevance and applications of science in their personal and social lives. The nature of the activities and investigations also aim to help students develop a sense of enjoyment in the learning of science. However, the extent to which teachers use lessons that allow student to engage in activity or investigative based exercises is not always in line with the extent intended by the syllabus (Weymms, 2008).

No matter the manner of instruction chosen by a lower secondary science teacher, the students must have completed the same learning objectives, as part of their lower secondary science course. The design of the tutorial materials identified the following objectives are relevant to the research, and considered the following to be required prior learning completed by the students:

- appreciate the concept of force; recall that the newton is the unit of force; describe forces and their effects.
- investigate examples of friction and the effect of lubrication.
- investigate the relationship between the extension of a spring and the applied force.
- recall that weight is the force due to gravity and that weight can vary with location; recall that mass in kilograms multiplied by 10 is approximately equal in magnitude to weight in newtons on the surface of the earth.
- carry out simple experiments to show attraction and repulsion between magnets and test a variety of materials for magnetism.
- plot the magnetic field of a bar magnet.
- demonstrate that the Earth has a magnetic field and locate north and south.
- use simple materials to generate static electricity; demonstrate the force between charged objects and the effect of earthing.

In the Junior Certificate mathematics syllabus, (NCCA 2012), it states that mathematics is the study of quantity, structure, space and change. In Mathematics, students develop skills in numeracy,

statistics, basic algebra, shape and space, and technology that have many uses in society. These skills can be utilized to allow students to make calculations and informed decisions based on information they come across in their everyday lives. Students also develop the skills to become good mathematicians, which allows them to compute and evaluate a calculation, follow logical arguments, generalize and justify conclusions, and apply mathematical concepts learned to real-life situations.

In the design of the tutorial materials, I took note of the learning objectives (NCCA, 2012) in the mathematics syllabus as relevant to the research. The mathematical aspects of the tutorial lessons were designed so they would align with the student's prior knowledge. This include students understanding co-ordinate geometry, the application of the rules of indices, the use of scientific notation. The algebraic understanding included the use of tables to represent a repeating-pattern situation, generalizing and explaining patterns and relationships in words and numbers. Writing arithmetic expressions for terms in a sequence and using tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations. When using tables, graphs and equations, the students would have learned to distinguish those features that are appear in the different representations, use the representations to reason about situation from which relationships can be derived and communicate their thinking to others and recognize problems involving direct proportion and identify the necessary information to solve them without formulae. Finally, the students have completed the using of graphs to represent various phenomena in different contexts including motion graphs, interpreted quantitative graphs, make connections between the shape of a graph and the story of a phenomenon and describe both quantity and change of quantity on a graph. In addition to considering the learning objectives, all the mathematical understanding and operations related to algebra (NCCA, 2012, pg. 28) is considered prior learning for this research.

This section of the description of participants illustrated the student cohort who agreed to undertake in this research. A description of the students was presented, results from both state examinations and an in-house exam were presented to help gauge the progress of the students in formal education in science, mathematics and physics thus far. An in-depth look at the specifics of the Junior Certificate Science and mathematics courses was presented, highlight the aims of both courses, and the objective completed by the students which are relevant to this research.

Chapter 4. Vectors, inverse square law and field lines

4.1. Introduction

This chapter addresses the development of student's understanding of vector concepts, the inverse square law, and field line representations as they completed the tutorial lessons. This chapter aims to address the first three of the research questions:

1. To what extent does the use of a structured inquiry approach develop student understanding of vector concepts?
2. To what extent does the use of a structured inquiry approach develop student understanding of the inverse square law relationship, by employing multiple representations
3. To what extent does the use of a structured inquiry approach develop student understanding of the field, when utilising field line representations?

These questions are important to the overall research as vectors, field lines and the inverse square law are the foundational topics for promoting conceptual understanding of Coulomb's law, the electric field and potential difference. We use structured inquiry tutorials to develop the student's understanding of three representations, as this allows us to (i) target specific concepts for the students to develop, (ii) design lessons that can easily be designed to be completed in a 60-minute timeframe and (iii) put the emphasis on the students to explore and develop their own understanding of vectors, field lines and the inverse square law. Having developed an understanding of these topics, the cognitive demand on the students should be lessened when they apply and utilise them in a electrostatics context, and instead they can focus on their understanding of the electrostatics concepts. In Vygotskian terms, when the students come to develop understanding of electrostatics, the required concepts for vector representation will have been moved from the student's zones of proximal development to actual development (Vygotsky, 1979). Multiple representations are employed, as Ainsworth (1999) shows that they can have many positive roles in the learning process, such as displaying difference processes and information, constraining the content and concepts learned by the nature what the representation can communicate, and constructing deeper understanding.

The tutorials, in the style of Tutorials in Introductory Physics (McDermott and Shaffer, 2003), utilise scaffolding questions to help students construct and develop understanding of a particular topic or concept. Breaking a topic down into small questions prevents the students from being overloaded by trying to assimilate too many new concepts at any one time. This approach also allows

for the observations of student difficulties as they progress through the tutorial, allowing for identification of the cause of the conceptual difficulty.

Figure 4.1 depicts where the topics fit into the student's broader understanding of Coulomb's law, the electric field and potential difference. Items presented in the left box are completed at Junior Certificate, items at the bottom are completed during Leaving Certificate Physics course, items on top are addressed specifically by this body of research and items on the right are the electrostatic topics covered by this research, discussed later in chapter 5.

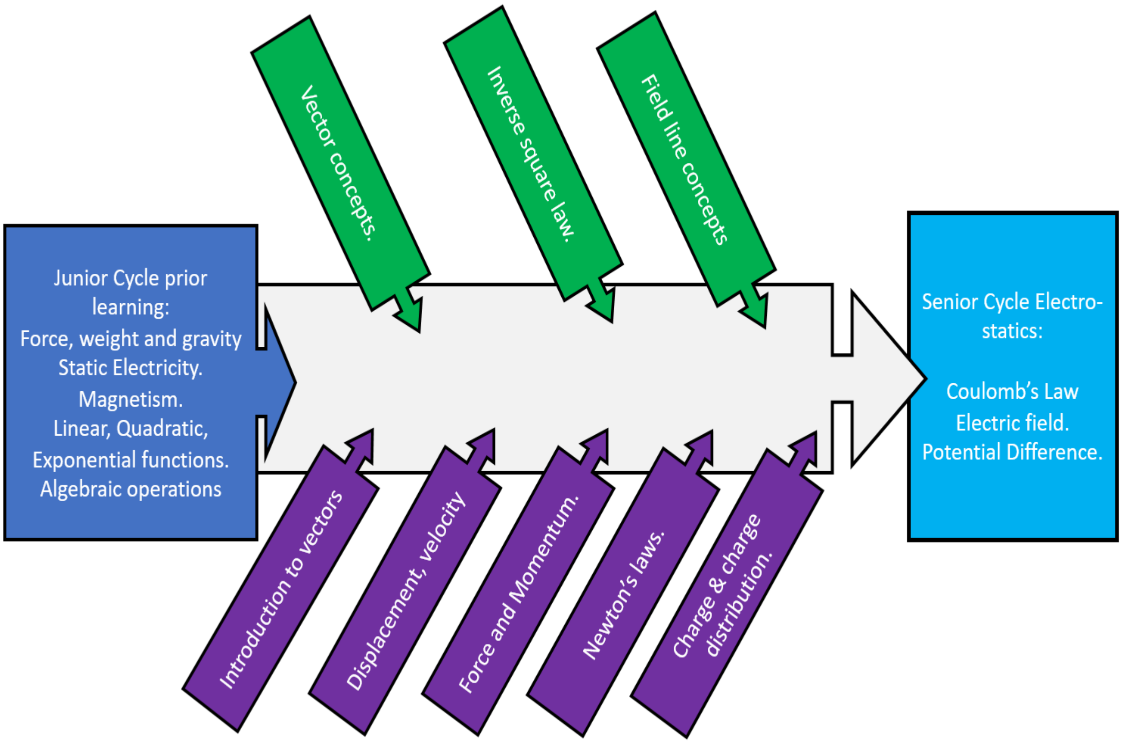


Figure 4.1. Flowchart depicting how the topics in this chapter contribute to electrostatics.

The timeline of this section of the project is shown in Table 4.1. Sections in bold refer to materials covered as they related to the research. Sections that are not presented in bold are required to be covered for completion of the required syllabus for Leaving Certificate Physics, but did not contribute to the collection of data used in this research. Week 1 denotes the beginning of the students studying mechanics, in their physics course. This coincided with the commencement of the first half of the research. These classes ran from the second last week in October to the second week in December 2016.

The order of the tutorials, (vectors, inverse square law and field lines) followed the order in which I normally teach the Leaving Certificate Physics course. This allowed for minimal disruption to the annual planning of topics. The vectors tutorial was implemented first, as it is one of the initial mechanics topics presenting in the Leaving Certificate Physics syllabus (NCCA, 1999). The inverse

square law and field lines tutorials followed later as Newton's gravitational law was contextually the first topic these representations could be applied to.

Week 1.	**Vectors Pre-test, vectors worksheet, vectors homework and vector constructions.**
Week 2.	Topics unrelated to project: Inclined planes, Momentum and Forces
Week 3.	**Vector Post-test.** Topic unrelated to project: Forces and Motion.
Week 4.	**Inverse square law pre-test**. Newton's law of gravitation. **Inverse square law tutorial.**
Week 5.	**Field lines pre-test. Field lines tutorial. Field lines homework. Further Newton's law of gravitation.**
Week 6.	**Inverse square law and field lines pre-test.**

Table 4.1. Timeline of the implementation of the first section of the research.

Section 4.2 covers the development of the student's understanding of vectors. Section 4.3 reports on the intervention used to promote the student's understanding of the inverse square law. Section 4.4 presents the development of the student's understanding of the use of field line presentation. In each of these three sections, a brief introduction presents common difficulties seen in literature by students used to form the learning objectives of the tutorial lessons. This is followed by a discussion of the student's initial understanding is elicited from the pre-test results, their development of the concepts is presented with a narrative of the tutorial lessons and homework assignments and their final understanding is delivered through an analysis of the post-test results. Each of the sections close with a discussion that compares the pre-test and post-test results. Difficulties targeted for conceptual change are identified and instances of Posner's conditions for conceptual change (1982) are referenced. Discussions of the post-test illustrate instances of conceptual change, when apparent. Finally, Section 4.5 presents the conclusions of this chapter, which discusses the progression of the student's understanding of the three different representations and implications for the use of the concepts in Coulomb's law, electric fields and potential difference.

4.2. Vector concepts

This section presents a narrative and analysis of the development of student's understanding of vectors. Section 2.1.3.1 detailed difficulties encountered by learners in understanding vector concepts

identified in the literature. The design of the vector tutorials focuses on student understanding of three introductory vector properties. Upon completion of the teaching and learning material, the students should be able to:

1. Differentiate between the magnitude and direction of a vector (Nguyen and Meltzer, 2003; Ivanov, 2011).
2. Demonstrate vector addition using the parallelogram and/or "tip to tail" constructions (Nguyen and Meltzer, 2003; Hewitt, 2009).
3. Consider vector components when adding vectors that are non-colinear nor perpendicular (Flores, *et al.*, 2008).

The inquiry approach consisted of a pre-test, a tutorial lesson, a homework and a post-test. This intervention ran over three weeks. A timeline for the implementation of this part of the study, including the target concepts for the intervention, is shown in Table 4.2. The vectors pre-test completed by the students was the first pre-test for the research, and the first experienced by the students. It was explained to them that the function of the pre-test was to allow for an indication of what they did and did not know about a given topic, that could later be used with the end of topic test (post-test) for comparative purposes.

Section 4.2.1 presents the pre-test results, looking at the difficulties the students have with representing vector magnitude, use of the parallelogram and "tip to tail" constructions and their prior understanding of horizontal and vertical components. Section 4.2.2 presents a narrative of the development of the student's understanding of vector concepts during the tutorial lesson. Section 4.2.3 presents an analysis of the homework assignment, which was developed to allow the students to practice the skills and apply the understanding they developed in the tutorial. Section 4.2.4 presents an analysis of the post-test results which, like the pre-test, focused on student's understanding of representing vector magnitude, the use of the parallelogram and "tip to tail" constructions and their understanding of vector components. Section 4.2.5 presents a comparison of the pre-test and post-test results, and a commentary of the student's progress during the tutorials. Examples of student progression and difficulties that persisted throughout the tutorial and post-test are discussed, and instances in which the conditions for conceptual change (Posner, *et al.*, 1982) were apparent during the tutorial lessons are discussed.

Time		Research Implementation	Target Concepts
Week 1	Class 1.	Pre-test	Relating magnitude to length of vector.
	Class 2.	Tutorial Lesson Homework	Parallelogram / Tip to tail construction of vector addition. Horizontal and Vertical components.
	Class 3		Lesson reserved for addressing difficulties observed during tutorial and homework, and extra time to practice using vector constructions.
Week 2		*N/a*	*Topics unrelated to project: Inclined planes, Momentum and Forces*
Week 3	Class 1	Post-test	Relating magnitude to length of vector. Parallelogram / Tip to tail construction of vector addition. Horizontal and Vertical components.

Table 4.2. Timeline of the implementation of the vector concepts study.

4.2.1. Pre-test: Vector Concepts

The pre-test on vector concepts was completed by the students before any instruction was delivered for vector concepts. As 12 of the 14 students study Applied Mathematics, they were familiar with some concepts related to vectors. Therefore, the pre-test could be used to identify what concepts the students understood before the tutorial, and what conceptual gains observed could be attributed to them completing the tutorial lesson. The students were given 15 minutes to complete the pre-test. The target concepts were magnitude of a vector and vector addition.

The first question, shown in Figure 4.2, looked at student's understanding of vector magnitude. The students were asked to rank the magnitude of five vectors represented by horizontal arrows from weakest to largest. Student's responses are presented in Table 4.3. Results highlighted in bold indicate a correct response or reasoning.

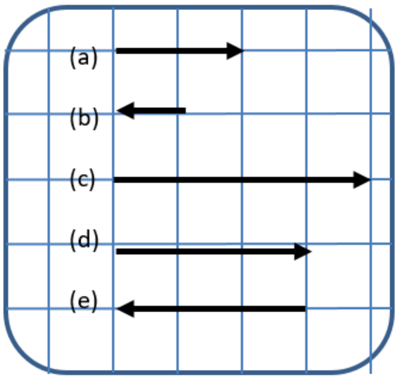


Figure 4.2. Pre-test vector magnitude ranking question.

A common difficulty (11/14) was that students thought the vectors that pointed to the left had a smaller magnitude than vectors pointing to the right. Students explain their reasoning as the vectors pointing to the left having a negative magnitude and the vectors pointing to the right having a positive magnitude. This was not addressed in their physics course, but introductory vectors using $\vec{\imath}$ and $\vec{\jmath}$ notation is covered in the Leaving Certificate Applied Mathematics course (NCCA, 2006), which all but students 4A and 4J completed. However, as Table 4.3 shows, student 4A produced the correct outcome but gave no reason to suggest they related the vector length to magnitude based on their understanding, and could have been the result of a reasonable guess. For the 11 students that did not produce the correct outcome, this indicates that students do not separate out the mathematical signs from the magnitude of the vectors, in this case, assigning negative to left and positive to right, and reasoning that negative integers have lesser value than positive integers. Nguyen and Meltzer (2003) also showed that when students were given scaled diagrams of various vector arrows, and asked to identify vectors of equal magnitude, their responses were influenced by not just the length of the arrows, but also the direction in which they pointed.

Responses	Students.
b < a < d = e < c	**4A**
c > e = d > a > b **(ranking from highest to lowest)**	**4C, 4M**
e < b < a < d < c	4B, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4N

Table 4.3. Summary of responses to the Vector magnitude ranking pre-test question.

In the second pre-test question students were asked to draw the resultant vector when two vectors at an acute angle are added (see Figure 4.3). Finding resultant vectors using the superposition principle is outlined as a mandatory skill in the Leaving Certificate Physics syllabus (NCCA, 1999). This pre-test question was designed only to determine if the students could combine vectors

diagrammatically, and if so, determine what vector construction they would use. The responses to this question are summarized in Table 4.4.

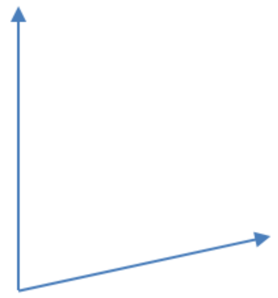


Figure 4.3. Pre-test vector construction of resultant question.

Construction Method Used	***Students***
"Tip to tail" construction (Nguyen and Meltzer, 2003)	**No responses**
"Parallelogram" construction (Hewitt, 2009)	**4B, 4H, 4I**
"Split the angle" construction	4C, 4D, 4G, 4M
Connects tails	4J
No attempt	4A, 4E, 4F, 4K, 4L, 4N

Table 4.4. Student's construction methods to find resultant of two vectors.

The results show us that only three of the students used a parallelogram construction, one of whom (4H) made a minor error in positioning the arrow head of the vector. Four of the students attempted to "split the difference" in which they bisected the angle but did not use any construction to determine the relevant magnitude (Nguygen and Meltzer, 2003). While these vectors may initially appear to be correct, there is no indication by the student's responses that they could determine any information about the resultant vector, other than its approximate direction. One student (4J) joined the resultant vectors with a curve but provided no reasoning, which indicates they made a guess to attempt to answer the question, without an understanding of what was being asked. The remainder of the students did not attempt the question. A sample of the student's constructions is presented in Figure 4.4.

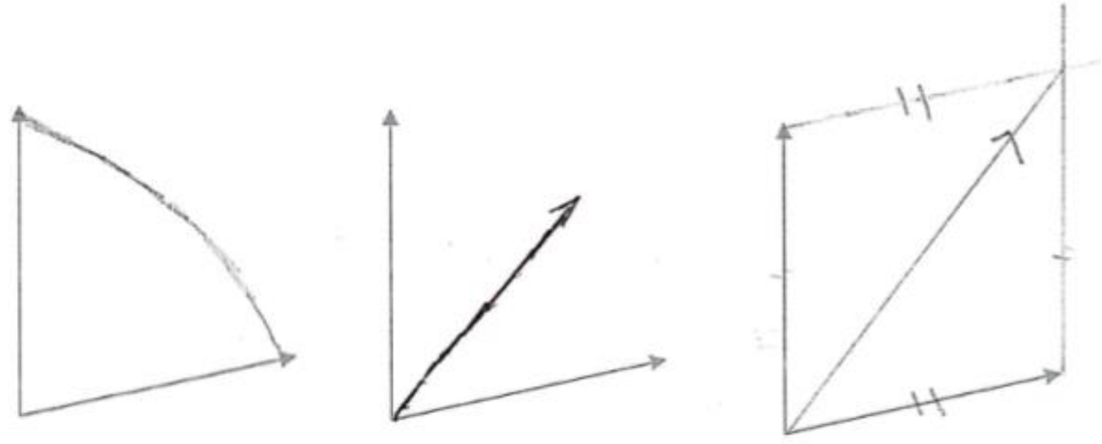


Figure 4.4. Examples of student responses, from student 4J, 4M and 4H respectively.

The last set of questions of the pre-test were used to determine student's understanding of vector addition in a conceptual vector question. Nguyen and Meltzer (2003) discussed common problems in vector addition, such as forcing results to align with the horizontal / vertical axis, or combining the magnitude of the horizontal components inaccurately, suggesting some attempt at the triangle / parallelogram rule. The students completed two questions looking to elicit their understanding of vectors. Both questions were published in Tutorials in Introductory Physics (McDermott and Shaffer, 2003). The first question, shown in Figure 4.5, asked students to determine which body will experience the highest net force, based on the vectors shown.

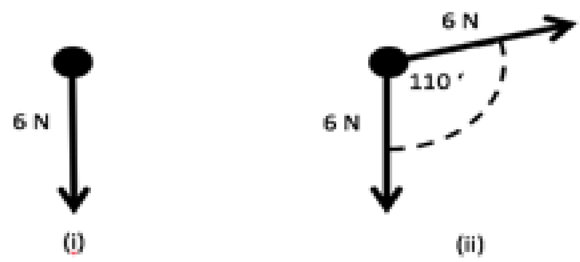


Figure 4.5. Vector pre-test question to elicit student understanding about vector addition.

The student's reasoning could be used to identity difficulties with vector addition. Table 4.5 shows the results from this question, with the students who used the correct concepts highlighted in bold. Where a student used the correct concept but made an error in applying it, their code is in bold and italics.

Concepts Used	Student Responses
Vector Addition	***4H,***
Parallelogram rule (Hewitt, 2009)	***4H***
Split the angle	4E
More forces lower the magnitude	4I
More forces increase magnitude	4D, 4G, 4K, 4M, 4N, 4L
Scalar addition	4C, 4B, 4F
No Reasoning Submitted	4A, 4E, 4J

Table 4.5. Student reasoning used in vector addition pre-test question.

While all the students identified that the net force for the vector diagram in Figure 4.5 (i) would have the highest resultant magnitude, only one gave an answer that explicitly referenced the use of vector addition. As the students were asked to submit their reasoning, the question allows for the identification of the reasoning used by the students. This enables and allows for differentiation between students that guessed the correct outcome, and those that determined the correct outcome. Table 4.5 shows a range of reasons submitted by the students; with the most common being that the more forces that act on the charge, the higher the magnitude of the resultant vector. In some cases, the reasoning given suggested vector addition, in other cases, it was suggestive of scalar addition, and in some cases the reasoning submitted was not explicit enough to include responses under either category, as seen in the following examples.

Student 4D: (ii) experiences the most force, because it experiences 2 forces in the diagram. (Not explicit to put into either category)

Student 4G: (ii) experiences the most force, because it experiences 2 forces in different directions, that when combined, are stronger than the force in (i). (Suggestive of vector addition)

In the case of the three students (4B, 4C and 4F) who used scalar addition, they explicitly stated the final vector would sum to 12 N, in effect disregarding the direction of the vectors. 4I stated that the angle between the vectors would reduce the resulting magnitude but gave no indication as to why. 4E gave a similar response, suggesting the resultant was "split between the angle" between the vector. When asked to explain what this reason meant, 4E volunteered that the combined force would spread across the 110°, so it'd be weaker as it went over a large angle, such as how intensity quantities get weaker as they are spread over larger areas. These responses show that the students not only used incorrect reasoning to attempt the question, but their reasoning was inconsistent with their choice of which of the two vector diagrams would result in the highest net force and did not recognise the inconsistency in their responses.

A fuller picture of student's understanding of 2D vector addition emerged from the second pre-test question This question specifically looks at their conceptual understanding of the addition of horizontal and vertical components. This question was adapted from Tutorials in Introductory

Physics (McDermott and Shaffer, 2003), in which the students were told the vectors represented forces acting on a body; they were asked to state which body experiences the most force, and to justify their choice. The vector diagrams are shown in Figure 4.6, and the student responses are summarised in Table 4.6.

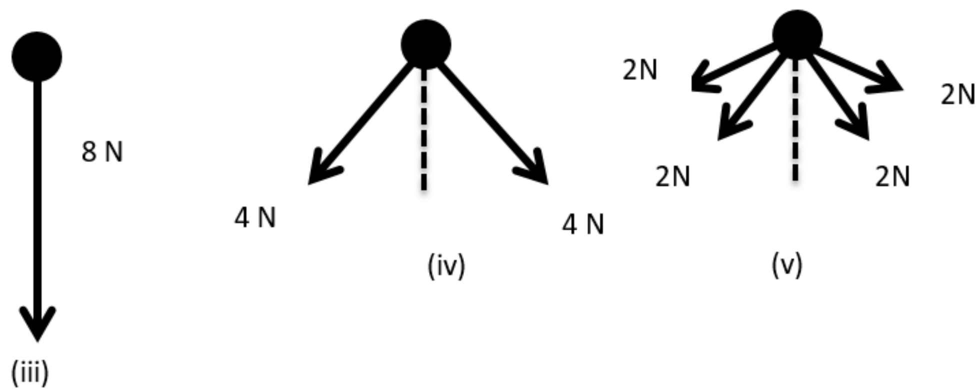


Figure 4.6. Vector pre-test question, to elicit student understanding of vector components

This question gave more clear results about student thinking. As can be seen in Table 4.6, eight of the 14 students added the vectors as if they were scalars, not accounting for the direction of vectors, nor the horizontal or vertical vector components. Only one student, 4H, attempted to use vector addition to complete this question, but errors in their application were apparent, as seen in Figure 4.7.

Concepts Used	Student Responses
Vector Addition	**4H**
Scalar Addition	4A, 4B, 4C, 4F, 4G, 4I, 4K, 4M
More angles mean more force	4D
No Reasoning Submitted / Reasoning unclear	4E, 4J, 4L, 4N
Correct Outcome for Setup	**4E**

Table 4.6. Student reasoning used in vector addition pre-test question, related to vector components.

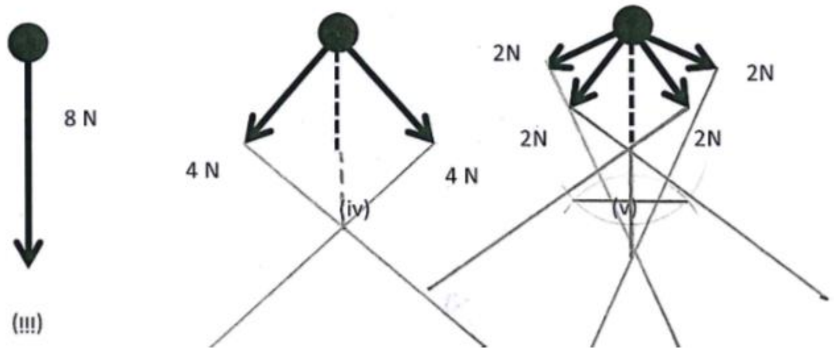


Figure 4.7. Student 4H's response to the vector component pre-test questions (iii)–(v).

Student 4H attempted to use the parallelogram rule to determine the resultant vectors, and in the case for (iv), their sketch is accurate. However, in (v), their attempt to resolve four vectors proved to be difficult and instead of using the parallelogram rule for the two most horizontal vectors, they translated them long an axis through the tips of the other two vectors, when attempting to find the resultant. This is a creative attempt for a student who never attempted a question with 4 vectors, but ultimately highlights an incomplete understanding of the contribution the horizontal and vertical components have in determining the net force.

Two students who used scalar addition were consistent in their responses, in which they found the sum of the magnitudes, as shown Table 4.7.

Student 4B:	Student 4C:
I think all the forces are the same because: *8 = 8* *4 + 4 = 8* *2 + 2 + 2 + 2 = 8.*	*They're all the same* *8N = 4N + 4N = 2N + 2N + 2N + 2N.*

Table 4.7. Student reasoning used in vector addition pre-test question, related to vector components.

Doughty (2013) showed students who completed introductory physics course in a third level setting struggled with vector addition, with vector components being a notable difficulty. It is not surprising to see the second level students in this study to have difficulties with vector addition questions, and due to their lack of experience with vector operations, it is not unreasonable to observe students disregarding the direction of vectors.

The pre-test results indicate that the student's understanding of vector concepts is undeveloped, even though 12 of the students were introduced to vector concepts in their Applied Maths course (NCCA, 2006). This is consistent with the work of Flores, *et al.*, (2008) and Nguyen and Meltzer (2003). The students did not consider direction and magnitude of a vector separately, had difficulty

in determining the superposition of two vectors, and appeared to lack conceptual understanding of the difference between adding vectors and scalars, and in their use and understanding of horizontal and vertical components to combine vectors.

4.2.2. Tutorial lesson: Vector Concepts

Vector concepts were introduced to the students during a lesson which consisted of a 15-minute teacher led class discussion and a 65-minute tutorial. The fourteen students were placed into two groups of four and two groups of three. During the class discussion, students were introduced to vectors by showing them a short clip of a plane flying in crosswinds, on a digital projector. A discussion between the students and teacher on the behaviour of the plane in the crosswinds led to informally discussing the difference between vector and scalar quantities. Formal definitions for each were then presented and examples of each were discussed on a PowerPoint presentation. On the presentation, students were presented with a grid with horizontal and vertical vectors. Students had to attribute positive and negative signs to represent the direction of a series of vector arrows (with reference to the right being positive, and up being positive). They then had to identify the longest vectors. This led to a discussion between students and teacher in which the magnitude of the vector was separated from its direction. An analogy of a strongman pushing a crate with 1000 N of force in various directions was used to aid the discussion. Students were also shown the use of the tip to tail method for vector addition and the parallelogram rule. Due to time constraints, the students were not afforded the opportunity to practice these constructions multiple times during the tutorial, and it was decided to complete them in the following class period.

The tutorial lesson was developed to allow the students to build up an understanding of how two-dimensional vector addition operates differently to scalar addition, explicitly exploring the use of vector components. Initially, students were guided through the labelling and drawing two vectors, $\vec{a}$ and $\vec{b}$, on a graph and sketching the vector components for each vector on the graph. Students were asked to match and draw vectors on graphs from coordinates. Upon sketching the components and producing two right angled triangles, the students had to apply Pythagoras' theorem using the components to determine the magnitude of the two vectors, $\vec{a}$ and $\vec{b}$. The students were then guided through the addition of vectors, using the tip to tail method, using the vectors from Figure 4.8, (i) and (ii).

The students showed an ability to apply the "tip to tail" method on the page to determine the resultant vector, $\vec{c} + \vec{d}$. However, all the students encountered difficulties when attempting to produce the horizontal and vertical components and use them to produce the resultant vector, $\vec{e} + \vec{f}$. Instead of adding the horizontal and vertical components, two groups of students attempted to add

the magnitudes of $\vec{e}$ and $\vec{f}$ directly to produce the resultant, while the other groups directly asked for assistance in completing this section. In all cases, the teacher requested the students sketch the horizontal and vertical components for $\vec{e}$, and then $\vec{f}$. Students were asked what the resulting horizontal magnitude would be if they were to combine the horizontal components only, and then apply the same reasoning to the vertical components. At this point, the student groups were comfortable to combine the resultant horizontal and vertical components to construct the vector $\vec{e} + \vec{f}$.

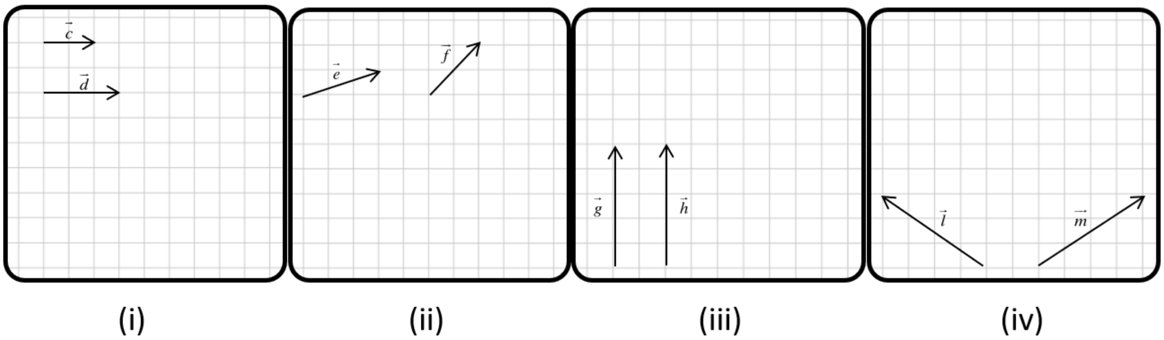


Figure 4.8. Vector addition diagrams from the vectors tutorial.

The students were then required to add the pairs of vectors shown in Figure 4.8 (iii) and (iv), using both the tip to tail method and by adding the horizontal and vertical coordinates. Upon completing this part of the exercise, students were asked to explain, in their own words, why the magnitudes in setup (iii) could be added directly and why they could not be added directly in (iv). In some cases, this proved to be the most challenging task on the whole worksheet, but ultimately, the only guidance needed for the students was to highlight the keywords required to give a full answer (horizontal and vertical components, magnitude, resultant), and they were able to construct their own valid meaning to this question. The following transcript, which was recorded on a dicta-phone during the tutorial lesson, illustrates this type of intervention.

Teacher — (reading student's work) "Because they are not going in the same direction so the"… ok, so the direction is important?... Ok I'm going to show you some keywords for your answer. (Teacher highlights the words horizontal and vertical components on the tutorial page).

Teacher allowed students to work on this section for 5 minutes before returning.

Student 4K: Is it cause the horizontal components are going in different directions?

Student 4L: *Yeah*

Student 4J: Well, I said, that way one goes up and then it returns, so then it is equal to zero. Like four plus six would be equal to ten, but we're taking them away, so I said it is zero here. This is why there is like, no change there.

Teacher: I didn't hear you, what were you saying?

Student 4J: Nothing (brief laughter)

Teacher:	No, go with it. It sounded like you were right, I just came in at the end.
Student 4K:	[4J] was saying that these parts are going in a different direction.
Student 4J:	And it is returning, so there is no change…
Teacher	There's no change in what?
Student 4L:	In the horizontal components.
Student 4J:	Yeah… (brief laughter) I came up with that!

This transcript of the dialogue between the teacher and the students showed that initially the students were not necessarily dissatisfied with their initial answer, but they were unable to reason through the task. Upon being prompted to consider the necessary keywords and being given the appropriate time, the students developed reasoning that was plausible and intelligible to them (Posner, *et al.,* 1982) and allowed them to obtain the correct answer of the task.

Upon reviewing the tutorial solutions, there is evidence that the basic skills required in understanding vectors, such as drawing vectors and combining them using the tip to tail method, were relatively straightforward for the students. The difficulties they encountered when using horizontal and vertical components to combine vectors indicates that this was a more challenging task for the students. The study showed that, unless guided, the students did not consider using the components to combine vectors and needed prompting questions to lead them through the process. At the end of the class the students demonstrated the ability to apply this process to vector pairs, as shown in Figure 4.8 (iii) and (iv) and explain how vectors of equal magnitude can produce resultants of various magnitude in terms of the combination of the vector components.

4.2.3. Homework: Vector Concepts

The homework assignment for vectors followed the format of the tutorial lesson. It allowed the students the opportunity to practice the skills developed in the tutorial and class discussion. The questions required students to (i) draw horizontal vectors of various magnitude (ii) combine non-collinear vectors arrows (iii) combine vectors in terms of their components mathematically and (iv) rank the net force acting on bodies with 2 forces acting on them, at different angles.

The first question on the pre-test, as shown in Figure 4.9, presented students with a vector, $\vec{b}$, and asked them to sketch the vectors defined as follows: $[2\vec{b}, 4\vec{b}, -\vec{b}, -3\vec{b}, 2\vec{b} - 3\vec{b}\ ]$. The students showed good ability to correctly represent the vectors, with only some minor errors shown by 4 students. Both student 4F and 4N included multiple arrow heads in their vectors, but otherwise produced valid answers to represent the required vectors. Students 4C and 4K did not use the scale

of $\vec{b}$ having a magnitude that corresponded to two boxes, as seen in Figure 4.9. Both student's vectors were relatively correct, for example, their representations of $4\vec{b}$ were longer than $2\vec{b}$, which were longer than $\vec{b}$, and $-\vec{b}$ pointed to the left with roughly the same magnitude as $\vec{b}$. These difficulties were briefly discussed with the whole class, where they commented on the errors produced by the four students and suggestions to how they may have represented the vectors more accurately were noted by the four students.

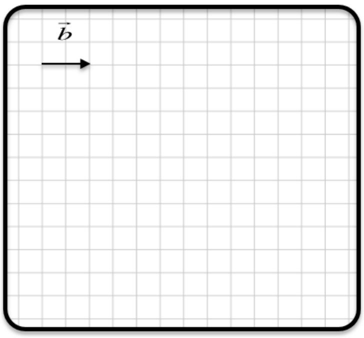


Figure 4.9. Homework question in which students sketch vectors.

In the second homework question, students were given 3 sets of vector pairs to add, as seen in Figure 4.10. In general, it was seen that students answered this section adequately; with all attempting to use the methods they learned in the tutorial, or class exercises, to find a resultant vector. Their results are summarized in Table 4.8.

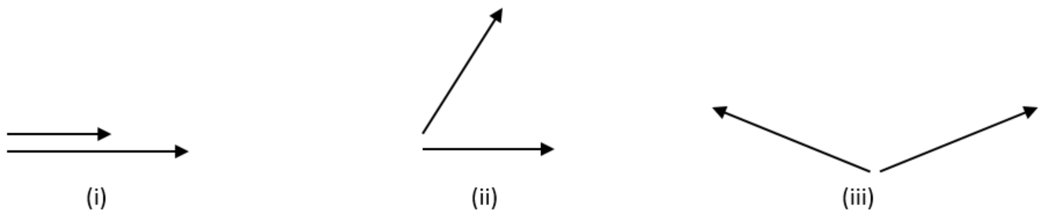


Figure 4.10. Homework question seeking to determine what construction students employ.

Concepts Used	Student Responses
Tip to Tail construction.	**4C, 4D, 4E, 4F, 4I, 4L, 4M, 4N**
Parallelogram construction.	**4A, 4B, 4G, 4H, 4K**
Errors in application of either construction	4E, 4F, 4I, 4K, 4L, 4N

Table 4.8. Constructions used by students to find the result of 2 vectors in homework exercise.

While all students attempted to use a vector addition construction to find the resultants, some did not draw their vectors to scale, and some did not complete arrow heads. In a same number of cases, the tip to tail / parallelogram was sketched but the resultant vector itself was not drawn, leading to an incomplete diagram. As shown in Table 4.2, the students were provided with an extra lesson to practise the vector constructions further. In this lesson, I set the students to work in pairs, to complete a series of exercises from their textbook over the course of 15 minutes, allowing them to attempt to overcome the issues observed in the homework assignment. Examples of student errors are presented in Figure 4.11.

(a) (b)

Figure 4.11. Examples of errors and incomplete diagrams from students 4L (a) and 4E (b).

The third homework question showed students the same vectors as seen in the second question but gave the students the vectors in terms of their horizontal and vertical components, as shown in Figure 4.12.

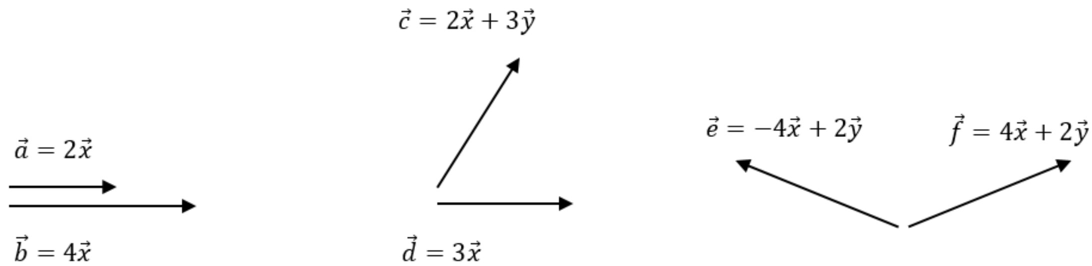


Figure 4.12. Homework question for students to add vectors using components.

The students were required to find the resultant vectors for $\vec{a} + \vec{b}$, $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$. All students demonstrated they could add the vector components in each combination, as this addition is similar to scalar addition in one dimension. Difficulties were observed in the addition of $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$, which were designed to resemble the vector addition pre-test questions, where the student had to consider the vector components in more depth. 8/14 of the students correctly used Pythagoras' theorem to calculate the magnitude of $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$. One student (4C) attempted to use the theorem but made errors in their work. For instance, instead of applying Pythagoras' theorem to $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$, they applied the theorem to $\vec{c}, \vec{d}$, $\vec{e}$ and $\vec{f}$ individually. The remaining students (4A, 4E, 4F, 4L and 4N) appeared unable to apply the theorem correctly. Difficulties seen in these students work showed error in picking appropriate values for the *x* and *y* component to use in Pythagoras' theorem (4A, 4F and 4L) or the student did not attempt the question (4E and 4N).

In the next section of the question, the students were asked the following question:

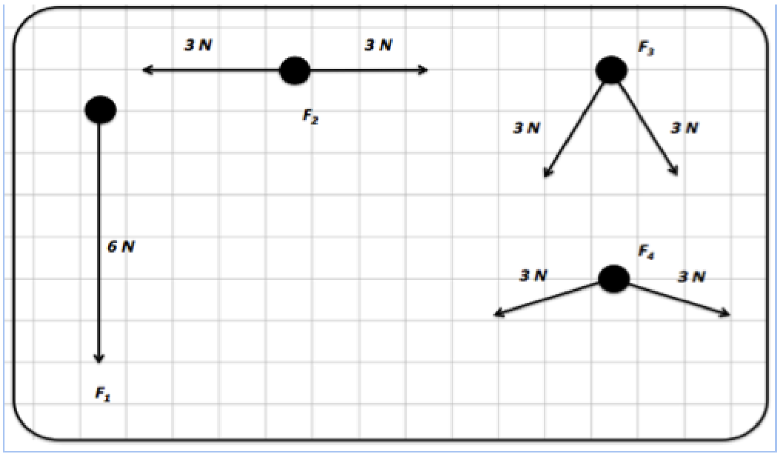
Explain, referring to the addition of horizontal and vertical components, explain why the magnitude of $\vec{c} + \vec{d}$ *is greater than* $\vec{c}$ *and* $\vec{d}$*, individually, but the magnitude of* $\vec{e} + \vec{f}$ *is less than* $\vec{e}$ *and* $\vec{f}$ *individually*

6/14 of the students referenced how the components of the vectors would affect the resultant, where horizontally cancelling components was observed in the $\vec{e} + \vec{f}$ combination. 3/14 of the students referenced the general direction of the vectors being opposite, and this would lead to a reduction in the overall magnitude but did not explicitly refer to the components. The remaining students did not attempt, and were likely unable to give any explanation. This highlights student difficulties to reason qualitatively, even when they are presented with information to help form their reasoning. This suggests the students would require more practise to develop the ability to reason scientifically, and as described by Hewitt (2011b), think in terms of concepts. As the tutorials progress, as shown in chapters 5 and 6, the students are given multiple opportunities to develop their qualitative reasoning skills and apply concepts qualitatively, as well as quantitatively.

In the last homework question, students were given a setup like the final pre-test question, with only a maximum of two vectors acting on a given body at any one time. The questions are shown in Figure 4.13, and the student's results are summarized in Table 4.9. 7/14 students gave the correct outcomes for the ranking. Two of them (4G and 4M) referenced the horizontal components cancelling out, while also using the parallelogram rule. A further three students relied solely on the parallelogram construction, without referring to the addition of either the vertical or horizontal components.

Concepts Used	Student Responses
Correct Outcome for Setup	**4B, 4E, 4G, 4H, 4J, 4K, 4M**
Horizontal / Vertical vectors referenced.	**4G, 4M**
Parallelogram / tip to tail	**4B, 4E, 4G, 4H, 4M**
Scalar Addition	4A, 4C, 4F, 4N
Incorrect ranking with no reasoning	4D
Not attempted	4I, 4L.

Table 4.9. Summary of student responses for homework vector addition conceptual question.



A small ball has a force or numerous forces acting on it as shown in the diagram to the right. The net force (resultant force) is labelled as F_1, F_2 and F_3.

(i) Rank the net forces acting on the ball, from highest to lowest. Explain your ranking.

Figure 4.13. Extract from vector homework.

Figure 4.14 presents a homework response from student 4E, which illustrates the use of the "tip to tail" construction to complete this task. The student applied the parallelogram rule to construct the resultant and presented 4 bullet points that commented on the resultant of the vectors. However, in the absence of a vector construction, there is no indication that the students could reason conceptually the correct ranking for this question. As discussed with the previous question, this illustrates the student's lack of practise in using qualitative reasoning in physics questions, or thinking in terms of concepts when approach qualitative exercises.

Students 4A, 4C, 4F and 4N submitted that all forces would be the same in all cases, contrary to the reasoning submitted in their tutorials. For all vector diagrams, they added the magnitudes of the vectors, ignoring the directions of the vectors. In a class review of the material, students 4F and 4N stated they did not think it would be the same as what they did in the tutorial, as in the tutorial they were dealing with magnitudes of vectors, while this was dealing with force. This can be explained by one any of the following three difficulties:

(i) The students could not connect the meaning of the term magnitude and its application to vector quantities.

(ii) The students could not connect the mathematical understanding to a physics concept.

(iii) The students did not consider force to be a vector quantity (4F and 4N explicitly stated this difficulty).

As part of the class discussion to review the homework, other students voiced their understanding of the terms magnitude, direction and vectors, and explained how the magnitude was represented by the length and how the length is a diagrammatic representation of the numerical measurement of the vector quantity. During this discussion section, students 4D, 4I and 4L were encouraged to communicate with the people beside them. Horizontal and vertical vectors were sketched on the board and these students were requested to use the diagrams to determine a ranking for the outcomes. Student 4D quickly realized that the horizontal vectors would sum to zero, whilst the other students came to this reasoning with the aid of their partners.

The homework assignment showed that the students were competent in representing vectors of various magnitudes and using the parallelogram and tip-to-tail constructions to combine vectors. Difficulties persisted in the conceptual understanding of combining vectors when considering the components of the vectors, and instead, students preferred to rely on the constructions to solve problems related to vector addition, for non-collinear vectors.

Student 4E: *F_1 – Longest vertical components*

F_3 – Second longest vertical components

F_2 – Third longest vertical components.

F_4 – That's just nil: +3N+(-)3N = 0N

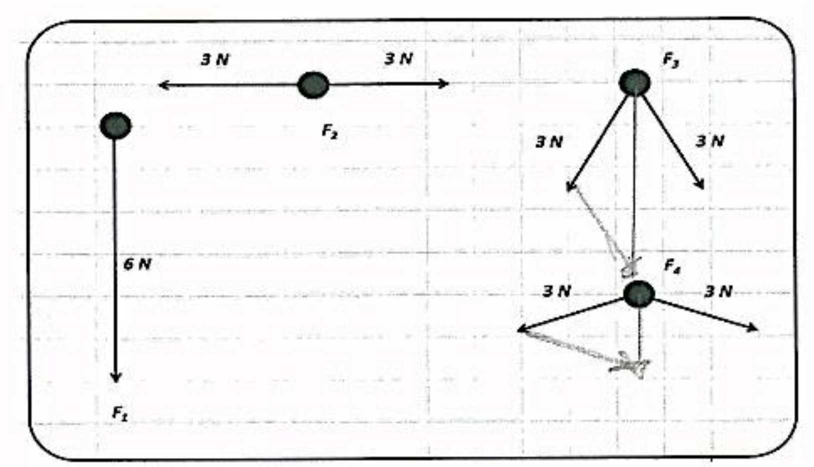


Figure 4.14. Student 4E's homework response, showing their work using the tip to tail to construct their ranking.

4.2.4. Post-test: Vector Concepts

The vectors post-test was written in a manner like both the tutorial lesson and the homework assignment. It tested the student's understanding of vector magnitude, by means of a ranking question, their ability to combine vectors using the "tip to tail," or parallelogram construction, and their understanding of vector components. The post-test took place approximately two weeks after the tutorial lesson. The large time duration was not due to research purpose, but due to a break in the tuition term.

In the first question on the post-test, the students were given a set of vector arrows and asked to rank them, from lowest to highest, based on their magnitude. This question was similar in nature to the first question they saw on their pre-test, to allow for a direct comparison, while also using result from their homework assignment to provide commentary on other difficulties that student may have, that were not seen in the post-test. The vectors are presented in Figure 4.15, and the results are shown Table 4.10.

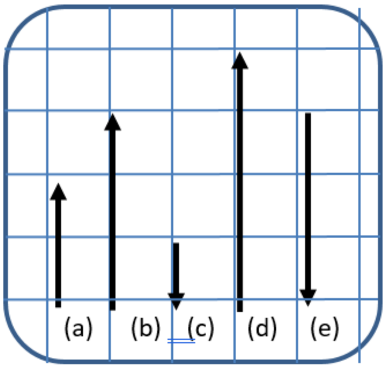


Figure 4.15. Post-test vector magnitude ranking question.

Vector Ranking	Student Responses.
C, A, B=E, D	**4A, 4B, 4E, 4F, 4G, 4H, 4J, 4K, 4L, 4M, 4N**
D, E=B, A, C	4C, 4D
E, C, A, B, D	4I

Table 4.10. Student responses from the post-test vector magnitude ranking question.

From Table 4.10, it is clear to see that all but one of the students ranked the vectors based on the lengths of the arrows, whilst acknowledging the direction of the arrow was not relevant to the magnitude. Students 4C and 4D may have submitted the incorrect ranking, but this is likely based on

misreading the question, as their submissions are correctly ranked from highest to lowest. Student 4I was the only student who based their ranking both the direction and the length of the vectors, as opposed to the length of the arrows alone when determining their magnitude. This shows evidence that most students overcame the difficulty that was apparent in the student's pre-test results.

In the second post-test question, students were required to construct resultant vectors by combining two vectors using the tip to tail, or parallelogram construction. The question is shown in Figure 4.16 and the student results are shown in Table 4.11.

All the students appropriately used one of the vector constructions to find the resultant vector. Some minor issues were observed in so far as some of the constructions were free hand sketches and slightly inaccurate in terms of scale (students 4E, 4F, 4I and 4J). Student 4A used the tip to tail method but extended the resultant beyond where it should have been. This was not expected as in the homework exercises, student 4A used the parallelogram construction and correctly represented the vectors using that representation.

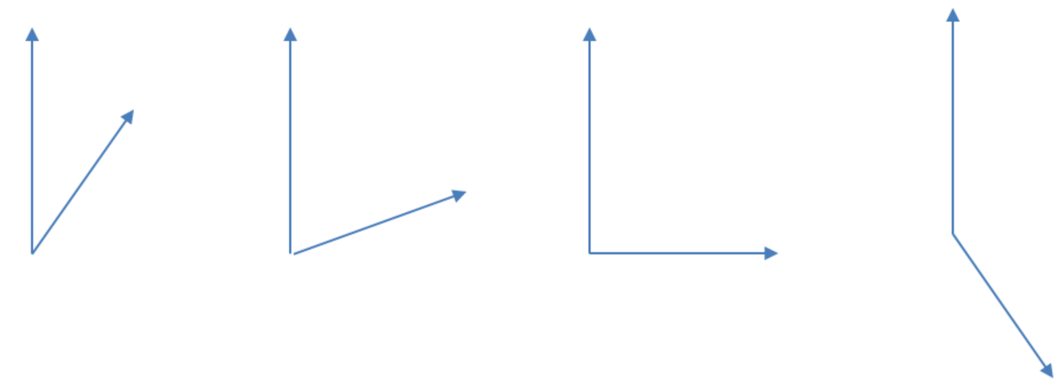


Figure 4.16. Post-test vector construction question

Construction Used	Student Responses.
Parallelogram construction	**4B 4I 4J 4K 4L**
Tip to tail construction.	**4A 4C 4D 4E 4F 4H 4M 4N**

Table 4.11. student's construction methods used in post-test to find resultant of two vectors.

Overall the responses indicate that students are aware of when to apply the vector constructions, but some students did not appear to consider the importance of using rulers to ensure they conserved the magnitude of their vectors, or the slopes of the vectors, when translating them to a new position, when completing the constructions. This was consistently seen in both the homework and post-test by students 4F and 4J, and in the post-test (but not the homework) by student 4E. All other students

consistently took care to draw the vectors to scale, using a ruler, in both the homework and post-test, or in the post-test only.

The final question on the post-test was developed to determine what conceptual understanding students had of adding vectors at angles. Students were given a scenario where an object was being pulled by two ropes, all with the same magnitude. Students were required to determine which of the three scenarios showed the strongest net force acting on the object and the weakest net force acting on the object. All the student's responses, except for 4N, showed they could determine the strongest and weakest force from the three diagrams. Student 4N reversed their answers, in which they incorrectly stated that the setup in which the force vectors were parallel would produce the weakest force, and the force vectors which were diverging with the largest angle would produce the strongest force. The questions are depicted in Figure 4.17, and the reasoning used by the students is presented in Table 4.12.

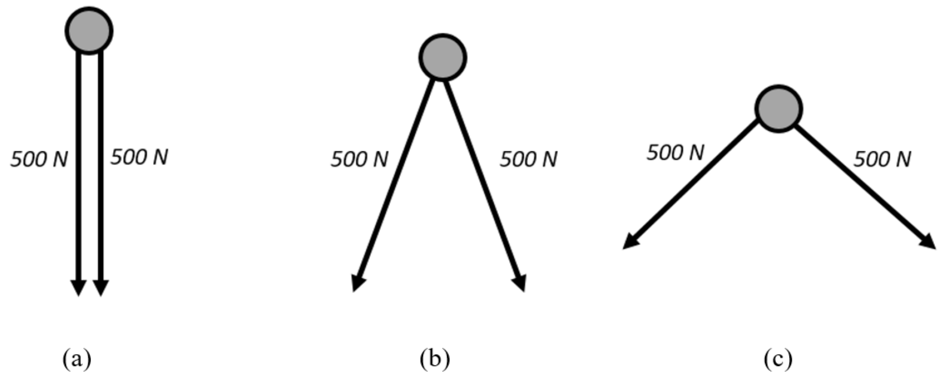


Figure 4.17. Vector post-test question to elicit student understanding of vector components.

Concepts Used	Student Responses
Parallelogram / tip to tail	**4E, 4F, 4G,**
Horizontal and Vertical components explicitly referenced.	**4B, 4C, 4D, 4E, 4G, 4I, 4J, 4K, 4M, 4N**
Horizontal and vertical components suggested / answer incomplete.	4A, 4F, 4L
Angle affect magnitude	4N

Table 4.12. Student reasoning used in vector addition post-test question, related to vector components.

The post-test results show that all but one of the students correctly determined the correct outcome. Of all the correct answers, the use of horizontal and vertical vectors was the primary reason chosen by the students, while a small number of students also included the use of the parallelogram / tip to tail constructions as evidence to support their answers, as shown in Figures 4.18 and 4.19.

Two of the students gave answers that did not fully articulate complete reasoning but were suggestive of the student's understanding about the horizontal components of the vectors cancelling out, while one student incorrectly related the magnitude to the angle and gave an incorrect ranking as a result.

In the cases where the students provided both justifications for their answers, it was seen that students tended to refer to both their diagrams and the components in their reasoning. This would suggest to us that students are not completing the constructions as a matter of rote – learned procedure but indicate that the students can determine the utility in both answering the question using vector constructions and in terms of the combination of vector components.

Student 4H:

The resultant for (i) is the strongest. Plus, it has no horizontal components. The resultant for (iii) is the weakest magnitude. The horizontal components cancel out.

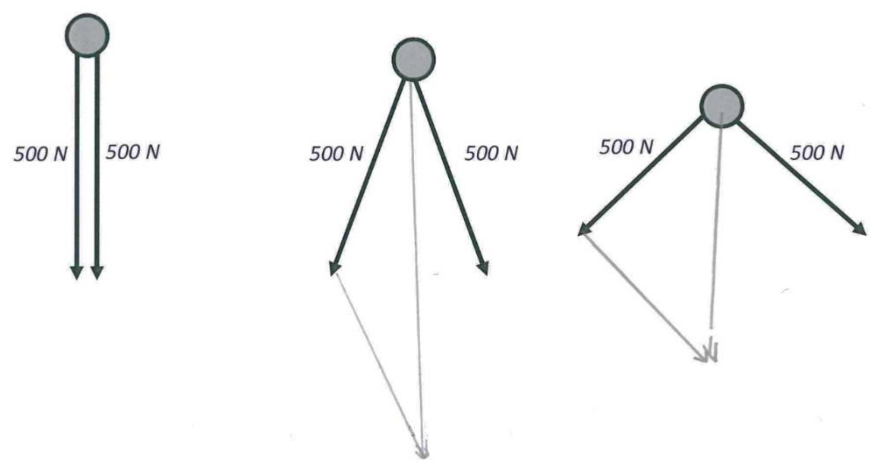


Figure 4.18. Student 4H's response for conceptual vector post-test question.

Other students chose to directly reference the horizontal vectors alone and did not attempt to generate evidence for their reasoning using the parallelogram / tip to tail method.

Student 4B: [(a)]. Both forces are acting the same direction, meaning there are no opposite forces cancelling out.

Diagram [(c)] shows the weakest force as there are large horizontal component forces cancelling out meaning there is less vertical component forces.

Student 4K: The third one, [(c)]. Even though the second one has a vertical and horizontal component, the third one has a wider horizontal component, so the force has to act on two difference horizontal.

Other students submitted the correct answer but gave incomplete reasoning that suggested the use of horizontal and vertical components, but lacked clarity and the use of keywords in the explanation:

Student 4E: A would allow a pulling power of 1000 N, as the combination of the two forces pulling in the same direction would be greater than that of two pulling in different directions, and cancelling each other out to a degree. [Diagram] C [is the strongest]. The forces are pulling in moderately different directions, which causes them to weaken the pulling power, by cancelling most of what the other is doing.

Student 4G:

1 only has vertical components and so is stronger than the others because the x components are 0, therefore all the force are acting the one direction making it the strongest. 3 has larger x values than 1 and 2, there the force acting on the x components are larger and cancel out. When they cancel out, the leave the vertical vector (the resultant vectors) to be weaker than 1 and 2.

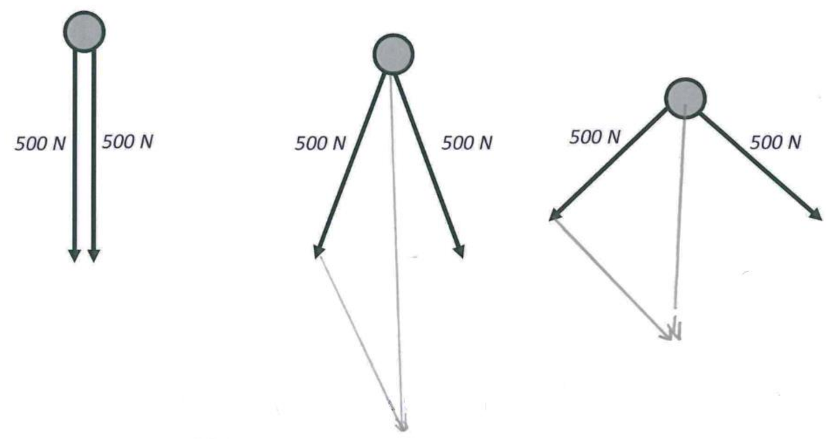


Figure 4.19. Student 4G's response for conceptual vector post-test question.

Student 4N, who did not give correct reasoning, associated the angle between the vectors with the magnitude of the resultant vector. Note that this reasoning was used by students 4D and 4J in the last two pre-test questions, as seen in section 4.2.2. Upon completion of the tutorial lesson, these students no longer produced work that indicated this type of reasoning. However, student 4N (who did not give a response to the pre-test question) was under the impression that a larger magnitude would result from a larger angle between the vectors. From discussion with the student, it was apparent that the student did not grasp the meaning of the term magnitude after completing both the tutorial and general class exercises. While they understood that numerically the magnitude was the number in any given quantity, they did not associate it with the length of the arrows explicitly and instead, associated it with any variable that could be quantified, such as in this case, the angle between

the vectors. Therefore, this reasoning produced a line of thought that a bigger angle had a bigger numerical measurement which gave the largest magnitude in the three setups.

The post-test results indicate that the students developed an understanding of vector magnitude and overcame the difficulty of not separating out the direction and magnitude of a vector (Nguyen and Meltzer, 2003). All the students also showed competency in combining vectors using a vector construction. The students were also able to apply horizontal and vertical component reasoning to the final conceptual question.

4.2.5. Discussion

By comparing the pre-test and post-test results, there is evidence to suggest that the students responded favourably to developing their understanding magnitude by using vector representations in a classroom discussion. Figure 4.20 presents a comparison of the pre-test and post-test results.

During the pre-test, it was observed the students had difficulties ranking vectors based on magnitude, a difficulty based on associating an influence of the direction of a vector on its magnitude (Nguyen and Meltzer, 2003). The occurrence of this difficulty in the class identifies the need for conceptual change during the lessons and tutorial (Hewson, 1992). During the class discussion, the students were presented with a similar ranking question as seen in the pre-test. The students submitted their rankings and reconsidered their rankings in terms of the definitions of vectors and scalar quantities. Through discussion with the teacher, it was seen that most of the students were easily able to identify that vectors associated with a direction that is denoted with a negative sign were equal in magnitude to vectors associated with a direction denoted with a positive sign, when the vectors are of equal length. The prompt to students to review their understanding of the term magnitude allowed become dissatisfied with ranking. They were then able to develop a correct ranking, that was in line with the correct understanding of the term magnitude. Students discussed their rankings in terms of their understanding of the word magnitude, and engaged in conceptual exchange, from which they ranked vector magnitude in terms of length and direction to just length in this task (Hewson, 1992). Based on the increase of correct reasoning used by the students in Figure 4.20, moderate conceptual change was recorded for this concept.

The gains in student understanding were apparent in both the homework and the post-test questions, in which it was seen that most of students submitted correct sketches and the correct rankings with valid reasoning. This classroom discussion appears to have been sufficient for students to engage with the visual representation and overcome the misconception that a negative sign on a vector is associated with a lower magnitude. The post-test results indicate that for most students there

are no persistent misconceptions for the student's understanding of how magnitude is represented by the length of a vector.

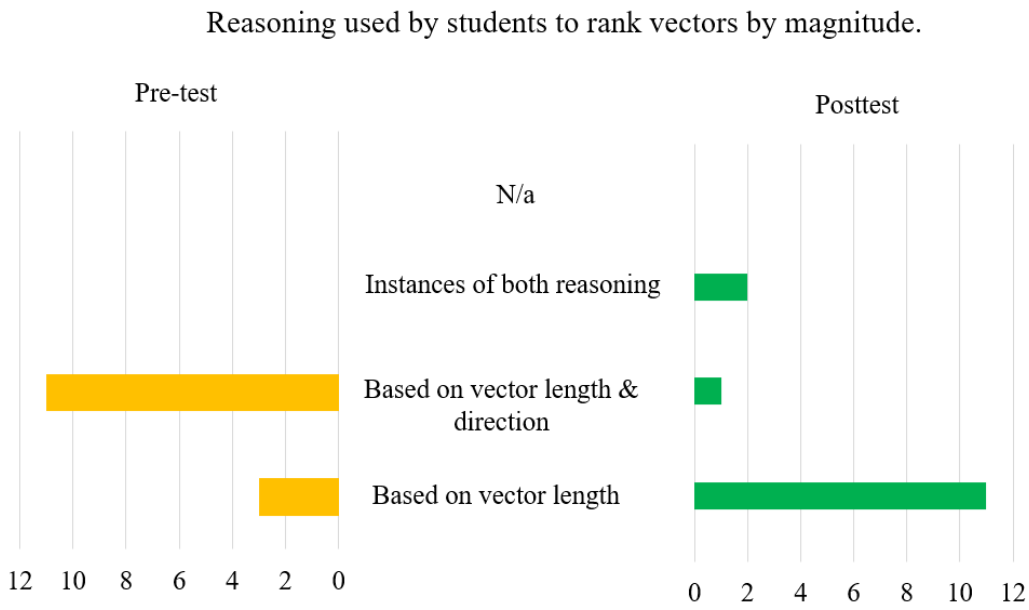


Figure 4.20. Comparison of reasoning used by students to rank vectors.

The next section of the discussion focuses on the students use of vector constructions to combine vectors graphically. Figure 4.21 presents a comparison of the constructions used by the students in both the pre-test and the post-test. During the pre-test, students had difficulties in combining vectors graphically. The most common error seen was students attempting to "split the angle" followed by connecting the tails of the vectors. These difficulties were encountered in literature (Nugyen and Meltzer, 2003) and were identified for conceptual change. In the homework assignment, discussed in section 4.2.3, that followed a tutorial lesson, all the students attempted to use either a tip-to-tail or a parallelogram construction to find the resultant between two vectors, as opposed to only3 students attempting the construction in the pre-test. However, numerous errors were observed in the student's application of the constructions in the homework. These were mainly, but not exclusively due to the lack of use of a scale or correct use of a ruler in completing the constructions. Some of the incorrect vectors also appeared to show some of the errors as seen in Nugyen and Meltzer (2003) such as "split the difference." As a response to the persistent difficulties that were observed in the homework assignment, extra time was additionally given to the students in a separate lesson. The students worked in pairs on completing more vector constructions and allowed for instances of peer tuition to occur. This allowed for an extended response to the student's initial difficulties to correctly implement the vector constructions.

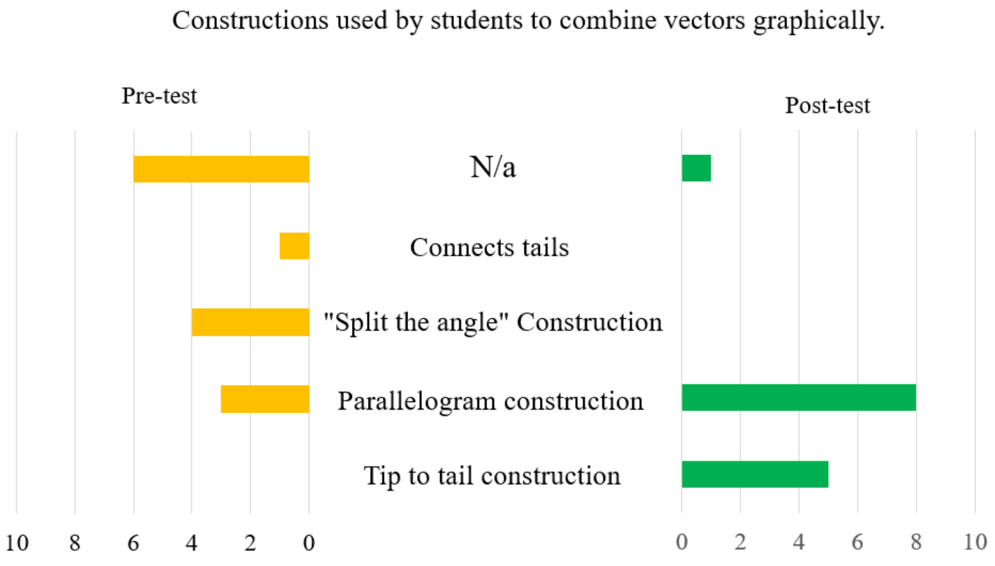


Figure 4.21. Comparison of vector constructions used by students

Upon completion of both the tutorial and additional set of exercises, the post-test showed all students using the constructions in their submissions, with most students having overcome the errors made in the homework. The post-test results showed that all, but one student effectively used one of the two constructions. This suggests that the students practise of the constructions during the tutorial and the extra 20 minutes after the homework activity allowed the students to see the constructions as intelligible and appropriate for adding vectors diagrammatically. The shift in results observed in Figure 4.21, indicates that conceptual exchange occurred, as the students were no longer producing the errors of connecting tail, or “splitting the angle” during the post-test. The shift in results also indicates that moderate conceptual change occurred over the course of this tutorial.

The last section of the discussion focuses on the student’s reasoning and understanding of horizontal and vertical components. Figure 4.22 presents a comparison of the reasoning used by the students in vector addition questions, in both the pre-test and post-test. The most prominent gains were seen in the student’s responses to ranking the net force acting on bodies when the forces acting are at obtuse angles to each other. Most students used scalar addition when completing the pre-test conceptual question, and this was targeted for conceptual exchange from their over-reliance on scalar addition (Hewson, 1992) to being able to identify when vector addition and scalar addition are appropriate.

In the tutorial lesson, it was observed that the students developed an understanding of components of the vectors, and they considered how these components affect the resultant of a vector. The discussion quoted in section 4.2.3 shows how the students required time to consider and discuss how all the components affected the resultant, and how the horizontal components summed to zero. In highlighting the reasoning used by the students wasn’t complete, the teacher provided a source of dissatisfaction in the initial reasoning (Posner, *et al.*, 1982), as students would realise that the

reasoning they provided would not provide an accurate outcome to a tutorial task. They then discussed alternatives amongst themselves. In a previous question, they had mathematically shown that the horizontal component vectors can sum to zero, leaving a vertical vector as the resultant, but the students did not initially consider this as evidence to support their reasoning in the final question of the tutorial. When students were prompted to review all their previous answers, and to consider what their mathematical answers could tell them about the resultant vectors, they developed reasoning that was plausible and intelligible (Posner, *et al.*, 1982) to provide an accurate and well thought out reason to the task.

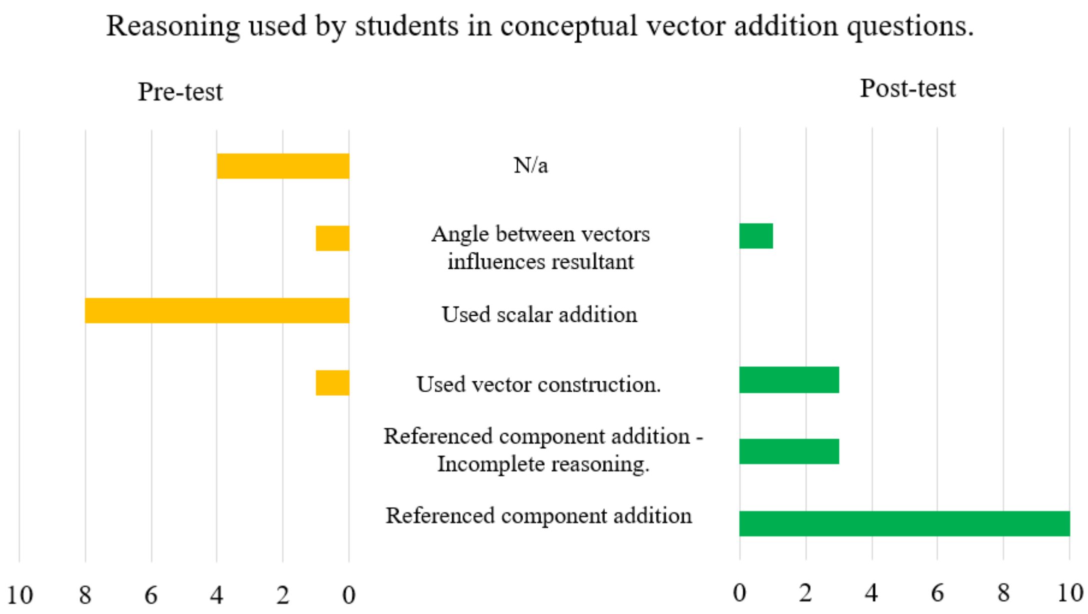


Figure 4.22. Comparison of reasoning used by student in conceptual vector questions.

In section 4.2.4, during a homework assignment it was seen that 7/14 of the students correctly ranked the net forcesacting on a body, at angles ranging from 0° to 180°. Students who submitted correct reasoning either referenced the vector components or used a vector construction. The remaining students reasoned incorrectly about vector components, or used scalar addition without applying the reasoning developed during the tutorial (see also Doughty, 2013). An open class discussion was held, in which the teacher sketched the horizontal and vertical component vectors, gave them the correct ranking and encouraged students to determine why the ranking was correct, referencing the vector components. As the students had the correct outcome presented, this allowed the student to focus on why the outcome was correct and review the reasoning they developed in groups in the previous tutorial lesson. The students worked in pairs in this discussion. As the groups were prompted to consider the components of the vectors, the groups were able to construct a comprehensive, concise and useful explanation that justified the correct ranking (Posner, *et al.*, 1982). The post-test results indicate this teaching approach was effective, as essentially all students gave the correct answers to the final conceptual vector question, with most of the students referencing the horizontal and vertical component summation in their answers. The increase in students

explaining vector addition in terms of vector components and reduction of students applying scalar addition, as seen in Figure 4.22, suggests the students engaged in conceptual exchange (Hewson, 1992) over the course of this tutorial. The increase in correct student responses and reasoning is indicative of moderate conceptual change having occurred. Sections 4.2.1 and 4.2.4 detailed the reasoning produced by the students and the jump from zero to ten students, in the pre-test and post-test, producing reasoning based on vector addition is evidence to support this.

In the homework assignment, the students who submitted a correct ranking tended to favour using the parallelogram or tip-to-tail method to produce their ranking, while only two students considered the vertical and horizontal components. After a discussion of the solution in the next lesson, which primarily used horizontal and vertical components, it was observed that most of the students acknowledged that considering the components provided an efficient method to solving the conceptual vector problem. This discussion could have heavily influenced the student's choice of reasoning and considering the students may have given both styles of reasoning in the post-test had the discussion being balanced to explore both methods to justify the correct ranking. Additionally, even though the students completed mathematical calculations relating to the summation of the horizontal and vertical vectors, in both the lesson and the homework, no students considered it's use in the conceptual questions at the end of either the homework or the post-test. This was unexpected, considering they would be familiar with the tools required to complete these calculations, and they draw a parallel to the reasoning the students developed and explored in their first and second year of second level mathematics, in which they covered the topic of coordinate geometry.

This chapter of the research presents evidence to suggest that conceptual change occurred to various extents between the three target concepts. On completion of the teaching sequence, most students showed they associated vector magnitude with the length of an arrow, regardless of direction. This can transfer to Coulomb's law, to represent the relative strengths of force acting on charged particles, and electric field to represent the field around a charge, and the superposition of two charges, at various points. The use of the constructions can also be applied to electric fields, in which students construct electric field of two, or more, charges and explain how the introducing more charges can increase / decrease the magnitude of the electric field at various points. The student's gains in understanding of components can be utilised by them to give a deeper explanation of the variation of the electric field strength at points between multiple charges, and develop an understanding of positive, negative and zero work and how it applied to potential difference in uniform electric fields.

4.3. Inverse square law

This section presents a narrative and analysis of the development of student's understanding of the inverse square law. The inverse square law applies to many contexts in physics, such as light intensity, sound intensity, Newton's gravitational law, Coulomb's law , electric fields and radiation. The participants in this study are only required to study three of these examples: Newton's gravitation, Coulomb's law and electric field and sound intensity (NCCA, 1999). Bardini, *et al.*, (2004) showed students learning to graph linear equations using contextual problems can help develop their understanding of the properties of an equation. There are numerous analogous practical activities that can be employed in teaching the inverse square law, subject to the time and equipment constraints of a secondary school environment. Hestenes and Wells (2006) showed that presenting the students with data can be sufficient. For the student to explore the inverse square law using a relatively non-abstract context, the tutorials employed a context of spray paint droplets spraying over various areas, which was adapted from Hewitt (2009).

Section 2.1.3.2 detailed difficulties encountered by learners in their understanding of the inverse square law. These difficulties informed the design of the tutorials, so students could recognise, explain and apply the inverse square law using multiple external representations. These are presented in the following learning objectives, as upon completion of the teaching and learning material, the students would be able to:

1. Accurately sketch and switch between graphical and algebraic representations of the inverse square law (Bardini, *et al.*, 2004; Hestenes and Wells, 2006; Bohacek and Gobel, 2011).
2. Apply a diagrammatic model utilising intensity to explain the behaviour of the inverse square law, and make predictions based on the model (Hewitt, 2009).
3. Demonstrate proportional reasoning using the inverse square law (Arons, 1999, Maloney, *et al.*, 2000; Marzec, 2012).

The inquiry approach developed for promoting student understanding of inverse square law consisted of a pre-test, a tutorial lesson and a post-test. This intervention ran over three weeks. The materials focused on student's ability to graph an inverse square law relationship, and modelling the uniform spreading of paint droplets over an increasing area to explain the behaviour of quantities that obey an inverse square law. A timeline for the implementation of the inverse square study, including the target concepts for the intervention, is shown in Table 4.13. As the field lines tutorial was completed in the same three weeks as the inverse square law tutorial, the field lines classes are also presented but bold font is applied to the classes which only applied to the inverse square law.

Section 4.3.1 presents the pre-test results, looking at the difficulties the students showed in representing an inverse square function on a graph, explaining the increase in the area covered by a

bulb illuminating a wall when the bulb is moved away from the wall, and a calculation based on the inverse square law. Section 4.2.2 presents a narrative of the development of the student's understanding of the inverse square law, by guiding them to develop an understanding of intensity applied to a context of using spray paint, model how the spray paint intensity varies as the distance from the source to the surface changes and present this model on a graph. Section 4.3.3 presents an analysis of the post-test results which, like the pre-test, focused on students representing an inverse square function on a graph, explaining the variation of area covered when a source is moved from a surface, and a calculation based on the inverse square law. Section 4.3.4 presents a comparison of the pre-test and post-test results, and a commentary of the student's progress during the tutorials. Examples of student progression and difficulties that persisted throughout the tutorial and post-test are discussed.

<table>
<tr><th colspan="2">Time</th><th>Research Implementation</th><th>Target Concepts</th></tr>
<tr><td rowspan="3">Week 4</td><td>Class 1.</td><td>Pre-test.</td><td>Representing an inverse square function on a graph.</td></tr>
<tr><td>Class 2.</td><td>Tutorial Lesson</td><td>Understanding of the increase / decrease in area model that follows an inverse square law.

Inverse square law ratios calculation.</td></tr>
<tr><td>Class 3.</td><td></td><td>Topics unrelated to project: Newton's gravitational law.</td></tr>
<tr><td rowspan="3">Week 5</td><td>Class 1.</td><td>Pre-test.</td><td>Topics unrelated to project: Newton's gravitational law. Field lines pre-test.</td></tr>
<tr><td>Class 2.</td><td>Tutorial Lesson
Homework.</td><td>Field lines tutorial
Field lines homework.</td></tr>
<tr><td>Class 3.</td><td></td><td>Topic summary.</td></tr>
<tr><td>Week 6.</td><td>Class 1.</td><td>Post-test</td><td>Representing an inverse square function on a graph.

Understanding of the increase / decrease in area model that follows an inverse square law.

Inverse square law ratios calculation.</td></tr>
</table>

Table 4.13. Timeline of the implementation of the vector concepts study.

4.3.1. Pre-test: Inverse Square Law

The inverse square law pre-test was designed to elicit student's understanding in various representations. The students were required to show their ability to (1) graph an equation of the form $y = k\frac{1}{x^2}$, (2) explain how the area on a wall, illuminated by a torch, varies as the torch is moved closer to or further from the wall, and (3) answer a mathematical question based on proportional reasoning involving the inverse square law. The use of different representations provides the opportunity to use many ways for students to show and for me to gauge their understanding.

The inverse square law pre-test took place after an introduction to Newton's universal law of gravitation. In a class discussion, which lasted 15 minutes, the students were guided through a series of slides outlining the Cavendish experiment, in which a diagram of the setup was shown, and a summary of the observations were presented. The slides also presented data they could use to qualitatively discuss, as a class group, the effect of varying the product of two masses, and the distance between the masses. The students determined the direct proportional relationship between the gravitational force and the product of the masses, but were unable to determine the relationship between the gravitational force and the distance. The students were informed that this was a relationship not covered in their mathematics courses and they would explore this relationship in the next lesson. The class discussion concluded with the students completing the pre-test, with a time limit of 15 minutes.

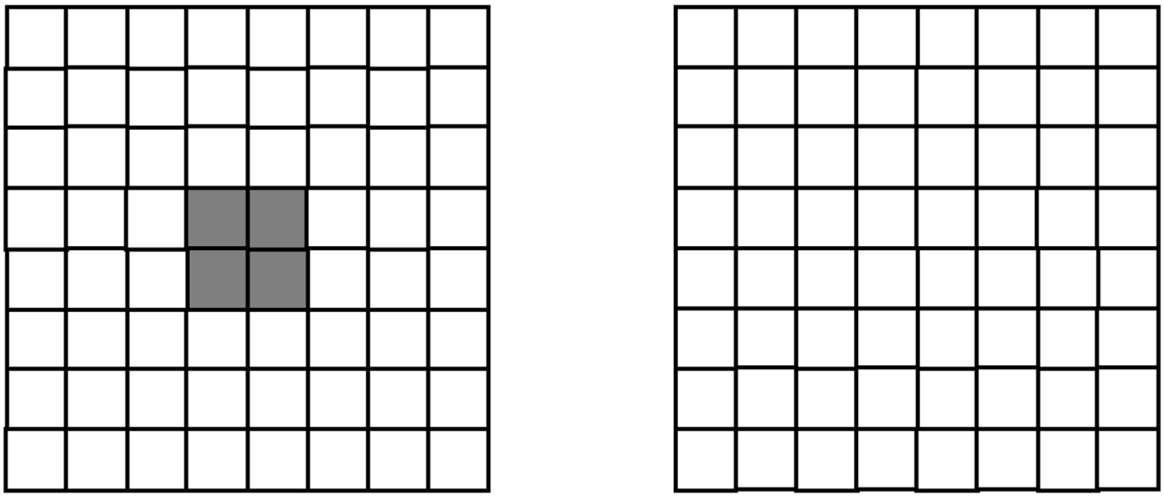
In the first question, the students were presented with blank *x*-*y* axes. They were asked to draw a pattern to match an equation of the general form $y = k\frac{1}{x^2}$. This allowed the students to demonstrate if they were aware how to transfer from algebraic to graphical representation, using the correct general characteristic curve for the function. A summary of the results is presented in Table 4.14.

Only three of the students correctly represented the relationship on a graph. The most common responses were quadratic curves. The responses from 4E, 4G and 4H were U-shaped curves, with an arbitrarily chosen intercept, while the responses from 4B, 4I and 4J were increasing quadratic curves with an intercept of zero. These students referenced the exponent on the *x*-variable in the equation to determine the function is quadratic. Student 4D presented a linear pattern with a positive slope, while student 4A sketched two linear patterns, one positive and one negative, overlapping. Both student's reasoning suggested they guessed the desired pattern. Of the correct graphs submitted, students 4C and 4M presented mathematics-based reasoning. Both students referenced that the function took the form of a fraction, and as the *x*-variable increased, the *y* value would approach zero, without reaching it, effectively defining an asymptotic function.

Response	***Students***
Decreasing asymptotic curve	**4C, 4F, 4M.**
Increasing curve	4B, 4I, 4J.
Quadratic curve	4E, 4G, 4H.
Decreasing straight line	4A.
Increasing straight line	4A, 4D.
N/a	4K, 4N.

Table 4.14. Student responses from the pre-test inverse square law graphing question.

The second pre-test question used a diagrammatic model for light intensity to probe student's understanding of the inverse square law. The students were presented with an 8 x 8 grid, with a torch illuminating 4 squares on the grid when it is 1 m away, as shown in Figure 4.23 (i). The students were asked to sketch, on the grid shown in Figure 4.23 (ii), the pattern observed if the torch was moved to 3 m away. This test was designed to elicit the student's understanding about scaling, as mentioned by Marzec (2012). The student's responses are summarized in Table 4.15.



(i) (ii)

Figure 4.23. Pre-test inverse area question involving scaling.

Response	Students
9 times bigger	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4N**
3 times bigger	N/a
4 times bigger	4M

Table 4.15. Responses for pre-test question seeking to elicit student's understanding of area scaling.

It was observed that all but one of the students were able to correctly represent the area covered when light source distance from the grid was tripled. However, not all student's reasoning directly

reflected this, and in some cases, student's reasoning contradicted their sketches. It was found that there were four lines of reasoning presented by the students:

- Students 4A and 4E submitted that they observed an increase in the "diameter" of the area by 3 times.
- Students 4B and 4H submitted that the area would increase 3 times, which is indicative of an inverse relationship. However, this is not reflected in their diagram, which shows an area that is 9 times bigger. Students 4D, 4F, 4G, 4I, 4J, 4K and 4N did not justify the quantity of their increase in area, but just qualitatively explained why the area of illumination increased, referring to how the light spreads across a larger area.
- Student 4C determined that for every metre the torch was moved back, the area given in Figure 4.23 (i) would increase by the power of the value of the distance from the wall: i.e., if the initial area covered is 4 units and the torch is moved to 3 meters from the wall, the final area would have a value of 4^3 squares on the grid. However, this student shaded 42 boxes, which does not correlate to the reasoning the submitted.
- Student 4M reasoned that in increasing from 1 m to 3 m, for every meter the torch was moved back, the area doubled. In going from 1 m to 2 m, the area increased from 4 boxes to 8 boxes, and in going from 2 m to 3 m, the area increased from 8 boxes to 16 boxes. This reasoning indicates a use of exponential proportional reasoning.

The last pre-test question looked at student's proportional reasoning, involving the inverse square law. Students were given a scenario in which a light sensor was placed 2 m from a bulb, and gave a reading 100 W m^{-2}, and were asked to determine what reading would be given on the sensor, if it was placed 4 m from the light bulb.

A summary of the results is provided in Table 4.16. 11/14 of the students determined that the light intensity would be half the original value, i.e., 50 W. The students did not submit reasoning, but instead produced a numerical value. These answers suggest that students did not consider the inverse square relationship, nor did they consider the patterns they drew in the second question. However, given that most of student's reasoning was incorrect for the second question, this is unsurprising. This indicates that the students had a mental model to help explain inverse relationships but had not developed an extension to the inverse square law.

The pre-test provides evidence to suggest that the students were generally unfamiliar with the inverse square law. 11/14 of the students did not relate the characteristic asymptotic curve associated with an equation of the form $y = k\frac{1}{x^2}$, instead producing a variety of linear and quadratic curves. 13/14 of the students were generally able to determine how the area covered by a source that follows an inverse square law changes, but showed reasoning inconsistent with their diagrams or no reasoning. This suggests some students may have guessed the correct outcomes, based on an

understanding that the area would increase in some manner. None of the students applied the necessary proportional reasoning to a mathematical exercise that required the understanding of the inverse square law. These difficulties in the pre-test agree with the findings of Marzec (2012).

Responses	Students
4 times smaller	**N/a**
2 times smaller	4A, 4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4M, 4N
Other reduction	4F, 4K
N/A	4L

Table 4.16. Responses for pre-test question probing student's proportional reasoning of intensity.

4.3.2. Tutorial lesson: Inverse square law

The tutorial lesson opened with a brief class discussion for 10 minutes, in which the previous presentation of the Cavendish experiment was reviewed. A formal definition for Newton's Gravitational Law was introduced and the discussion highlighted the similarities when comparing the equation $F = G\frac{m_1 m_2}{d^2}$ and functions of the form $y = k\frac{1}{x^2}$. Various physical phenomena that follow inverse square laws were listed by the teacher, such as light intensity, sound intensity, gravitational, static electrical force and the emission of particles from radioactive sources. This provided context for the importance of the mathematical relationship, which the students would explore in the tutorial lesson, which took up the remaining 70 minutes of the lesson.

The tutorial on the inverse square law was designed using an analogy of paint being sprayed over an increasing area, in which the students determined the number of particles of paint spraying over individual segments. In this way, spray paint "intensity" is used to conceptually model the inverse square law. This educational model was developed, and expanded upon, from Conceptual Physics, Practice Book (Hewitt, 2009).

Initially the students were presented with a scenario of a spray can emitting 100 drops of paint per second over a given area and were required to calculate the amount of paint droplets landing each second on a uniform area of 1 m^2. The students were then required to expand their model to the increase of area covered when distance from the can to the area is increased, as depicted in the Figure 4.24. This section of the tutorial was developed to promote the student's conceptual understanding of scaling (Arons, 1999), in which they were guided to reason why the area of the frames increases quadratically, instead of linearly, as the spray moves from left to right in the diagram.

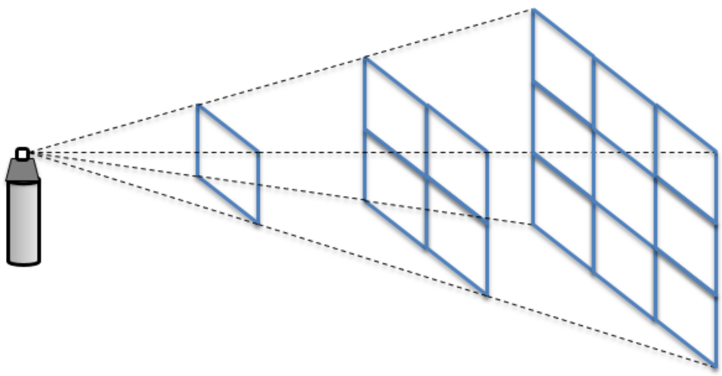


Figure 4.24. Diagram representing spray paint droplets passing through frames.

The students were required to explain, in the case of the second frame which was twice the distance from the can as the first, why the area of the second frame was four times the size of the area of the first frame. From this, they were required to determine how many droplets of paint pass through the individual smaller frames of the second setup, if 100 droplets passed through the first in 1 second. By determining that there were 25 drops passing per second in an individual frame in the second setup, the students were required to determine how this showed the spray paint "intensity" was following an inverse square law. Many discussions took place within groups to develop this reasoning, generally taking somewhere in the region of 15 minutes for the student groups to develop the reasoning to explain how the intensity drops. In the cases where students struggled to progress, the teacher asked the student groups to consider the increase in the length of the overall frame, the width of the overall frame, and discuss how both these increases affected the area of the overall frame.

Upon completion of this question, students had to consider the frame that was 3 times as far from the paint can as the first frame. Again, they were asked to determine why the area was 9 times the area of the first frame and use this to determine the spray paint "intensity" for one frame in one second, on the third setup. The students determined that there were 11 droplets per second, rounded to nearest whole number, and again used this to demonstrate the inverse square relationship. In completing this, the students demonstrated how the growing distance from the can decreases the number of droplets passing through an individual frame. To illustrate these points, the following quotes from student 4A, 4D and 4I were obtained by scanning the student artefacts.

Student 4A: Doubling the distance and the height, fits 4 plates in.
Distance triples and height triples, fits 9 plates in.
The drops are being divided (through the frames) as it grows

Student 4D: The farther away from the can, the bigger the area is because the lines are expanding, meaning more boxes (frames) can be filled in from each side. As the distance from the pain can increases the drops per second decreases because the same number of drops pass through each part but distributed equally into each frame.

Student 4I: Because you can fit 3 more square in horizontally and vertically, as it gets further away.
The droplets have to spread between the area. The more area there is, the less droplets of paint passing through 1 m^2.

This section of the tutorial gave the students a conceptual grounding in the inverse square law, using a tangible model which they could easily picture. The initial difficulty encountered by students suggested students recognized the limits of their understanding and became dissatisfied with it. Whilst some students required prompting to consider the length and width of the frames individually, the groups managed to discuss and construct sound reasoning that allowed them to develop explanations of what was described in the presented model.

The tutorial then turned to a graphical treatment, in which the students graphed the data for the paint "intensity" at various distances between the can and the frame, to show the inverse pattern. From this, they chose data points on the graph and use the data points to show the reduction ratio, when the distance from the can is increased by a factor of 2, and then again by a factor of 5. This enabled the students to confirm, using their graph, that an inverse square law is observed. To illustrate, work from student 4I is reproduced in Figure 4.25.

Student 4I: It is a quadratic graph, that is decreasing, a slope that gets smaller.
1 m, it is 100. At 2 m, it is 25.
If you move 2 m away, it will cause a decrease of 4 times the intensity.
5 m, it is 4.
If you move 5 m away, it will cause a decrease of 25 times the intensity.

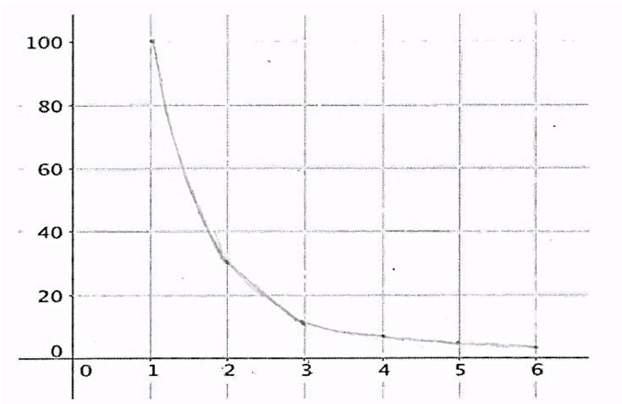


Figure 4.25. Student 4I's Graphical representation of the inverse square law.

There was not enough time for the tutorial to address student's complete quantitative problems the involved the inverse square law. However, in completing quantitative problems in Newton's Gravitational law in subsequent lessons unrelated to this project, the students were afforded the opportunity to practice the mathematical operations involving the inverse square law.

4.3.3. Post-test: Inverse square law

The inverse square law post-test was completed by the students two weeks after completing the tutorial lesson on the inverse square law. It consisted of questions that probed student's understanding of the inverse square law using graphical, diagrammatic and mathematical means. All questions revolved around the context of a spray-paint can spraying on a wall with a grid, which was used to model intensity.

In the first question, the students were presented with a formula for spray paint intensity and asked to produce the shape of an intensity vs distance graph and suggest reasoning as to why they chose the shape that they did. This question was similar in nature to the first pre-test question and would allow for direct comparison. It would allow us to determine if students learned to recognize and transfer the algebraic characteristics of an inverse square function to a graphical one and articulate their justifications. The question is shown in Figure 4.26, and the student's responses are presented in Table 4.17.

While 9/14 of the students answered correctly, only two (4H and 4I) of these students referenced the mathematical relationship between the variables referenced in the questions. These students directly referenced the exponent of the distance variable given in the provided equation, justifying not only the shape of their graph, but the type of inverse relationship in the equation.

Student 4H: As the distance from the nozzle increases, the intensity decreases by a square of the distance.

Student 4I: The intensity decreases by a square factor.

One of the students (4B) referenced the area model used in the tutorial to justify the shape of their graph but stopped short of relating this to the equation given. Another two students (4C and 4E) briefly explained the shape of the graph and how it relates the intensity to the distance from the can. This suggests memorization of the graph shape, but an inability to interpret it correctly in the real-world context.

Student 4B: As the distance from the nozzle increases, the paint is spread over a wider area, meaning the intensity decreases proportionally.

Student 4C: It shows that intensity is affect by distance.

Student 4E: As the distance from the nozzle increases, the intensity decreases.

Furthermore, one student (4K) produced a linearly increasing graph but referenced the inverse square law for the intensity of the spray paint. This indicates a lack of understanding and an inability to link the graphical representation with the relationship they encountered in the inverse square law.

This suggests the student resorted to the use of rote memorization of the law but employed a familiar graph shape that they do not realise does not represent the law they stated. The remaining students explained that a linear pattern produces a line on a graph or submitted no reasoning at all.

A can of spray paint emits 200 droplets of paint per second from the nozzle. The amount of droplet from a can of spray paint that fall on a given area (intensity – I) is given by the formula: $I = \frac{200}{0.125\pi r^2}$

Draw a sketch of the graph to show the relationship between the spray paint intensity (I) and the distance from the nozzle (r) and explain how it shows the relationship.

How it shows the relationship:

Figure 4.26. Post-test question asking students to represent inverse square equation on a graph.

Responses	Students
Decreasing asymptotic graph	**4B, 4C, 4E, 4G, 4H, 4I, 4J, 4M, 4N.**
Decreasing linear graph	4L.
Increasing linear graph	4A, 4D, 4F, 4K.

Table 4.17. Student responses from the post-test inverse square law graphing question.

The second question referred to viewing the wall itself, in which they were presented with the grid in which the paint was only landing a small section of it, when placed two meters from the grid. The question layout was identical in nature to the second pre-test question, as discussed in section 4.3.2, but with a different number of boxes shaded in, and different distances referenced in the question. The students told the spray paint was 2 m from the wall, and the paint spray covered 4 boxes. They were asked to sketch the shape of the painted sections on the grid if the distance were to be increased to four meters. This gave us an opportunity to determine if they could apply the

inverse square law to the area covered by the paint. The diagrams from this question are presented in Figure 4.27 and the student results are shown in Table 4.18.

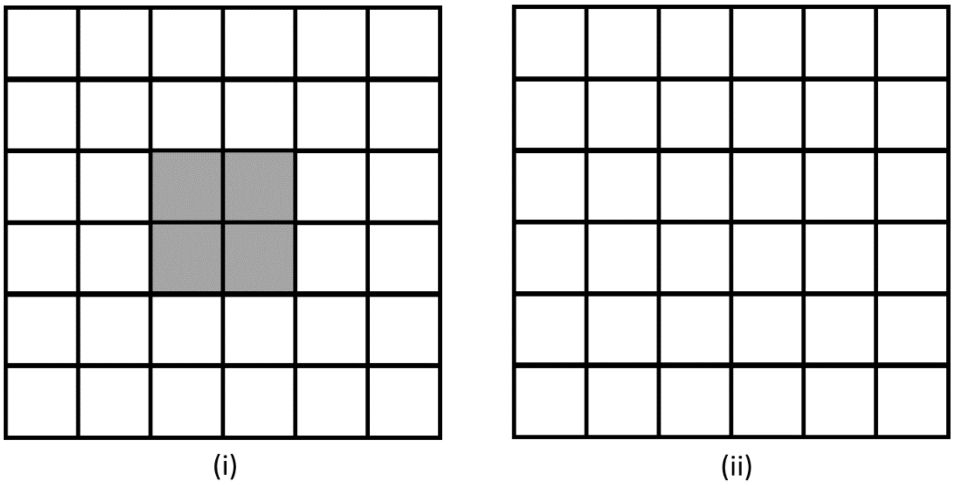


Figure 4.27. Area covered by the spray paint when (i) held 2 m from the wall and (ii) the blank grid.

Responses	Students
Doubling the distance, quadruples the area.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**

Table 4.18. Responses for post-test question seeking to elicit student's understanding of area scaling.

As can be seen from the responses, all students showed that the increasing the distance from the wall increases the area of the spray of paint quadruples. While it its noted that doubling the distance and squaring the distance can result in the same result, if students were to apply this to the grid, it would give an incorrect response (doubling 4 square results in 8 squares being shaded, as opposed to squaring 4 to result in 16 squares being shaded). The students were also asked to determine what the distance the paint spray would be if 36 squares were covered. The diagram presented in the post-test is shown in Figure 4.28, and the student's results are summarized as shown in Table 4.19.

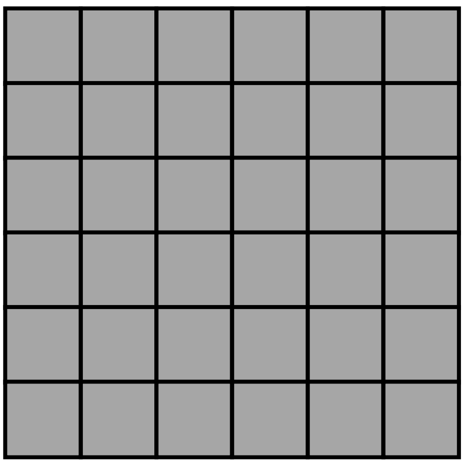


Figure 4.28. Post-test question where students apply proportional reasoning to scaling.

Responses	Students
36 boxes are produced by a radius of 6 m.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4M, 4N**
N/a	4L

Table 4.19. Responses for post-test question in which students determine the distance from the spray paint can to the wall.

Only one student was unable determine the distance required to produce 36 boxes covered in paint. The remaining students produced four lines of reasoning to determine the distance from the can to the wall.

- Mathematical: Students calculated the square root of the area of the shaded boxes to find the length / width of the square. They reasoned, and generalised, in this question that the length / width of the shaded area was equal to the distance from the paint can to the wall. This was valid for this particular question, and is a limitation of it is design. The question was intentionally designed as such, so the ratios would not be difficult for the students to work through when exploring this concept initially. A similar question was also completed by the students during the electric field post-test, in which the numbers are more difficult, and this manner does not directly produce the correct answer. The comparison of these two questions is discussed later in section 6.3.3. As an example, student 4B's reasoning is shown in Figure. 4.29.

Student 4B

$$x^2 = 36$$

$$\sqrt{x^2} = \sqrt{36}$$

$$x = 6$$

Figure 4.29. Sample of reasoning presented by Student 4B.

- Graphical: Students overlaid the paint patterns from the previous two questions. By superimposing these diagrams for 2 m and 4 m, it is easy for the students to extrapolate a pattern across the diagram to determine the distance from the can to the wall. Figure 4.30 shows this solution used by student 4D

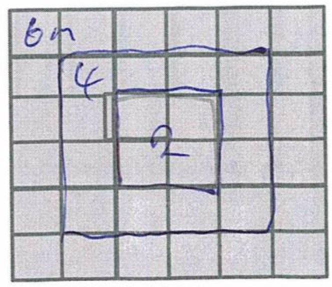


Figure 4.30. Student 4D's graphical reduction used to determine distance.

- Tabular: One of the students, 4E, used a table to determine how many boxes would be shaded at different distances. This acts as a combination of the previous two methods. The student could probably determine how many boxes would be covered for any value of *r*. Note that this student did not employ this method when looking at the electric field, as discussed in section 6.3.3.

Student 4E:

r	*1*	*2*	*3*	*4*	*5*	*6*
r^2	*1*	*4*	*9*	*16*	*25*	*36*

Table 4.20. Data produced by student 4E to demonstrate quadratic change.

- Changing length and width: Only one student (4H) determined the area of the wall when the can was 6 m from the wall using reasoning related to the change in the lengths / widths of the boxes. As the area of 36 boxes has a length of 6 boxes, and the initial diagram showed an area of 4 boxes with a length of 2 boxes, the student divided the lengths to determine that each side of the area grew by a factor of 3. The student then used this factor to multiply the original distance for the area of 4 boxes (2 m) by 3 to determine the correct radius. In their submitted post-test, it was evident that the student struggled to clearly articulate this in their answer.

Student 4H: *If you divide 6* [width and length of shade in question] *by 2* [the original length and width of the shade], *you get 3. So 2 m x 3 = 6 m.* [2m is the original distance]

In the final question presented on the post-test, the students were asked to use the formula presented on the first question to determine the spray paint intensity at both five meters and ten meters from the can. This would afford them the opportunity to use the data to verify that spray paint intensity followed an inverse square law, as developed from their comments in the previous questions in the post-test and use a method that would confirm their reasoning mathematically. The results of this section are summarized in Table 4.21 and Table 4.22.

Responses	Students
Correctly determined both intensity values.	**4B, 4C, 4E, 4F, 4G, 4H, 4J, 4K, 4M**
Correctly determined one value	4A, 4I, 4D
Inverted the distance only.	4N
N/a	4L

Table 4.21. responses for post-test question probing student's mathematical proportional reasoning of intensity.

Values used to verify inverse square law	**4C, 4E, 4G, 4H, 4K**
Incomplete reasoning	4B, 4J, 4M, 4N
Misconception not relating intensity to area	4A, 4D, 4F,
No reasoning given	4I,
N/a	4L

Table 4.22. Students that calculated values to verify the intensity as an inverse square law.

It was seen that 9/14 of the students completed the substitution and evaluation required to determine the spray paint intensity, but only 5 of these used the values to show an inverse square relationship was observed. They used the formula, developed a ratio and commented on how the change in distance from the can to the wall affects the intensity. They could all calculate the intensities and demonstrate that the doubling the distance produces one quarter the intensity.

Student 4C:	Student 4E:
$\frac{200}{0.125\pi(5)^2} = 20.37$	$I = \frac{200}{0.125\pi(5)^2} = 20.37\frac{W}{m^2}$
$\frac{200}{0.125\pi(10)^2} = 5.09$	$I = \frac{200}{0.125\pi(10)^2} = 5.092958$
Distance x 2 = 2^2 = 4.	
20.37 ÷ 4 = 5.09	$\frac{20.37\ W/m^2}{4} = 5.092958$
The intensity is dependent on the distance. If the distance is doubled, it is 4 times less intense.	

Table 4.23. Post-test calculations presented by students 4C and 4E.

Four students could qualitatively explain the effect of increasing distance on intensity but struggled to completely justify their understanding quantitatively, as seen in the following submissions:

Student 4B:	As the distance increases, the paint intensity decreases proportionally (exponentially).
Student 4J:	It is an example of an inverse square law because 5.0929 is more than half the intensity for another 5 m away.
Student 4N:	The intensity decreases the further out you go.

A misunderstanding that arose in the reasoning in the post-test was seen in the submissions from students 4A, 4D and 4F, in which they appeared to indicate their understanding of intensity was the amount of paint drops being emitted from the can, as opposed to the paint droplets passing through a defined area

Student 4A:	The intensity is the same, only dispersed over a larger area.
Student 4D:	It is the same, just displaced over a larger area.
Student 4F:	The intensity is spread over a larger area.

However, even accounting for this misunderstanding, these students did not demonstrate an inverse square law mathematically. The remaining students who completed the post-test did not submit any reasoning for their answers or did not attempt this question.

The post-test results indicate that the tutorial lesson had a positive effect on aspects of the student's understanding of the inverse. 9/12 of the students correctly represented a pattern that follows the inverse square law on a graph with an asymptotic curve. Some students demonstrated they could correctly determine the increase in area covered by the spray paint droplets and used reasoning indicative of the increase of the lengths and widths, either graphically or in written form, or analysed the pattern to extrapolate the correct answer. In some cases, the students showed they were applying inverse square proportional reasoning in mathematical questions, although these were not addressed in the tutorial lesson. This can be attributed a combination of the students applying the reasoning developed in the tutorial to mathematical questions and applying the skills they developed in solving problems involving Newton's gravitational law.

4.3.4. Discussion

This section discusses the student's understanding of the inverse square law, by comparing the pre-test and post-test results and referencing development shown by the students during the tutorial lessons. It will then discuss the student's conceptual understanding of the inverse square law using graphical representations, diagrammatic representations and algebraic representations.

Initially, the pre-test results indicated that the students were unaware of the shape of the graph and numerical patterns of an inverse square law. This immediately highlighted an issue for conceptual change to be addressed. During the tutorial lesson, the students were guided in mapping an inverse square law graphically. The post-test results show an increase in the number of students who could transfer the mathematical formula to a graph, represent the function using the correct shape, and provide justifications for the graph choice. This is presented in the Figure 4.31, showing the frequency of different responses in the pre-test and post-test.

The gains seen in the student's responses are in line with the findings of Bardini, *et al.,* (2004), in which guiding students through a function in context can help them develop an understanding of the equation and its transfer to a graph. Several difficulties were seen in the student's pre-test submissions, there were no difficulties that trumped any others, and thus, all difficulties were considered for conceptual change. A lack of clear concise reasoning for the graphs drawn in the pre-test would indicate that the students were not satisfied with the reasoning they were using to construct their graphs (Posner, *et al.,* 1982). In the tutorial, it was observed that the students could be guided to represent data that follows an inverse square law on a graph. They then used the data from the graph to develop ratios to determine that it shows an inverse square law.

They also clearly demonstrated that a decreasing asymptotic graph can be used to represent how the intensity decreases as the distance from the can to the wall increased. This evidence indicates that conceptual exchange and extension occurred, as the students demonstrated they could transfer the inverse square law from one representation to another, and then extended their understanding to develop an intelligible method to analyse the data on the graph (Posner, *et al.,* 1982; Hewson, 1992). Comparing the pre-test and post-test results directly from Figure 4.31, the results indicate that partial conceptual change occurred over the course of this tutorial.

Upon completing the lesson centred on the inverse square law, it was observed that students began to associate the general shape of an inverse square graph to any inverse pattern. Some difficulties persisted as five of the students demonstrated that they were still unaware how to either recognise or transfer an inverse square relationship from a mathematical symbolic representation to a graphical representation, in which four of the students sketched a linearly decreasing pattern, and one student sketched a linearly increasing pattern. This difficulty was later re-addressed in the Coulomb's law tutorial lesson, in which the inverse square is highlighted to the students in the

mathematical notation and the student then completing a graphing exercise with force and distance data, to display and analyse the graphical pattern. This is discussed in section 5.5.2.

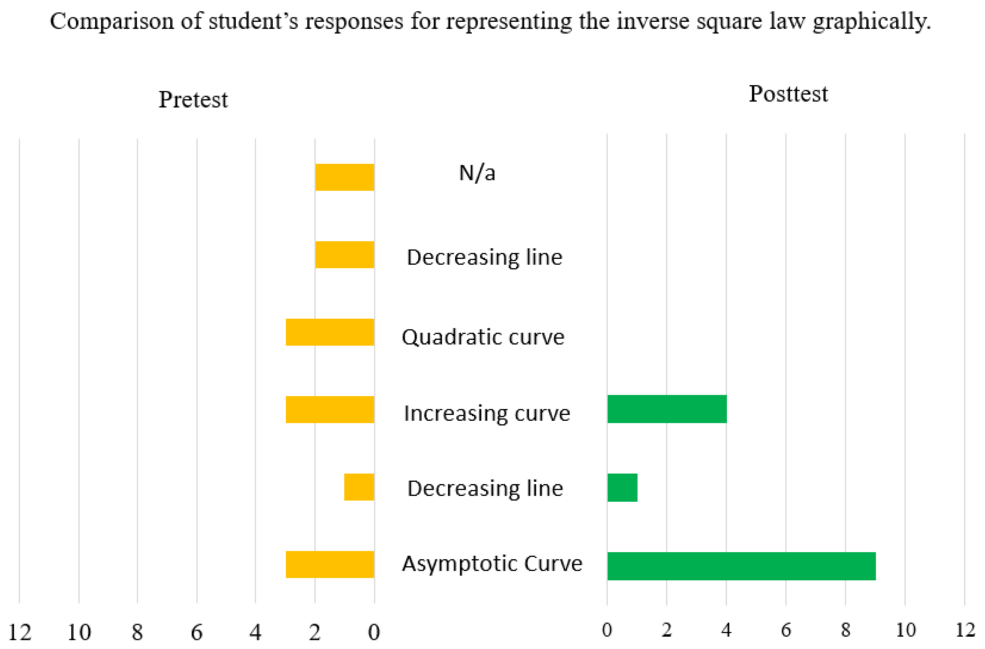


Figure 4.31. Comparison showing for student's graphs of inverse square law.

The development of student's understanding of the area change due to scaling is presented in Figure 4.32. In both pre-test and post-test, it was observed that students could correctly determine the increase in the area illuminated when a light is moved back from a wall. While it was a positive outcome where it was observed that the students could predict the change of the area in the given exercise, it was later observed that this increase did not correlate to student's understanding of a concept like intensity, in which a quantity is spread out evenly over this area. This would indicate that the difficulties to be targeted for conceptual change was not just dimensional scaling, but also applying the scaling area to other quantities and explaining how it applying to concepts like intensity, in which paint / energy is "spread evenly" over the increasing / decreasing area.

During the tutorial lesson, it was found that the model adopted (Hewitt, 2009) of using spray paint passing through square frames helps students visualise the inverse square law, in a relatively simple tangible context. When developing the student's understanding of scaling in the inverse square law they struggled to articulate why the area of the frames grew quadratically with the increase in distance between the nozzle of the paint can and the frame. The students were asked to consider both the increase in the width and height of the frames, and to consider how both these increases could explain the quadratic growth observed in the diagram. Difficulties were also encountered when the students needed to determine the amount of paint droplets passing through the frames when they were presented with 4 and 9 frames. The students tended to consider the total area of all the frames, instead of looking at them individually. The teacher was required to provide initial dissatisfaction to

reasons that focused on the overall area of the frames only. The students were required to extend their thinking to the amount of paint passing through all the frames, and then using their answer to determine how much paint was passing through the individual frame and provide reasoning to scale the “paint intensity” from the larger frames to the smaller individual ones. In the post-test, a small number of the students still considered the overall area, while the remaining students to focused on the change in dimensions of the shape, indicating that conceptual exchange occurred the understanding of these students (Hewson, 1992). This would indicate that minimal conceptual exchange occurred, but an increase in the students focusing on the dimensional scaling is an optimistic finding.

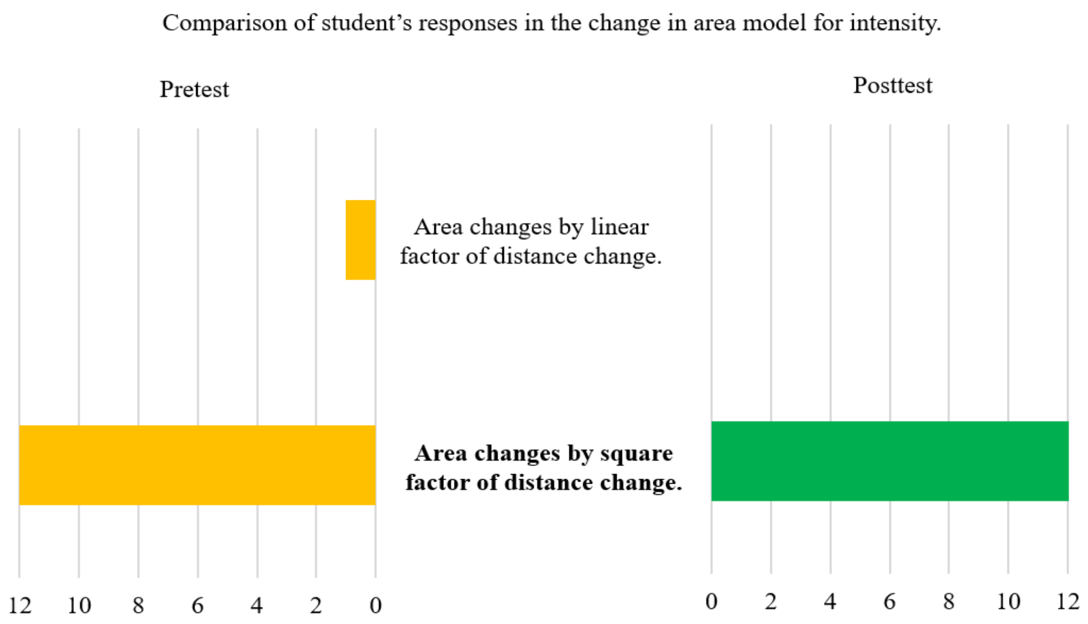


Figure 4.32. Comparison for student’s responses using area model.

The last section of this discussion looks at the student’s mathematical understanding of the inverse square law. A comparison of the pre-test and post-test results is presented in Figure 4.33. The mathematical pre-test question, in which students needed to apply the appropriate proportional reasoning consistent with the inverse square law, they used either proportional reasoning consistent with a linearly inverse law or qualitatively stated that a reduction would occur. The overwhelming occurrence of students applying linear proportionality indicated a difficulty to be targeted for conceptual extension (Hewson, 1992), as the students need to be aware when to use linear proportionality, and non-linear proportionality. In the post-test, the students were presented with a formula for intensity and were asked to prove that intensity followed an inverse square law, using the same skills developed in using ratios as completed in the tutorial when completing the graphing exercise.

It was observed that 12/14 of the students used the formula to produce at least one correct set of results, but only 5/14 students calculated a ratio, as they were directly instructed to do. These five

students demonstrated transfer of understanding between representations, and their consideration of the overall inverse square law in a task unseen from the tutorial. This suggests conceptual exchange (Hewson, 1992) occurred as the students demonstrated conceptual understanding in an unknown context (Konicek – Moran and Keeley, 2015), and that the students have develop intelligible reasoning that is applicable to further contexts (Posner, *et al.*, 1982). Based on these results, the extent of conceptual change observed was partial. The results also suggests that the difficulty for the remaining students was not the mechanics of using the mathematical operations, but how to apply their calculations to demonstrate an inverse square law as it was observed that nine students could produce values using an equation that involved an inverse square law, but only five could use their calculations to demonstrate the relationship. This is indicative of a gap between their mathematical ability and their ability to understand and apply it in a physics context. The Coulomb's law tutorial was augmented to account for this, as discussed in section 5.4.2.

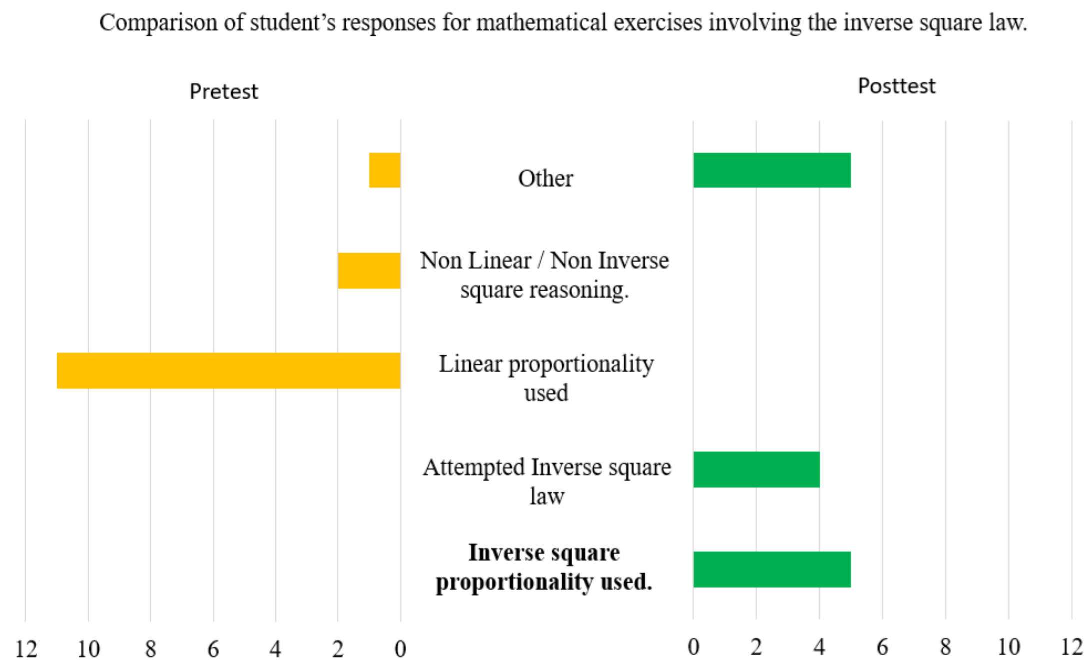


Figure 4.33. Comparison of student's responses for mathematical exercises using the inverse square law.

As discussed in section 4.3.2, the students plotted a graph of data using the inverse square law and completed mathematical calculations based on their graph to demonstrate an inverse square relationship, which they demonstrated. The tutorial itself did not address exploring the inverse square law mathematically. However, the students practised qualitative problems involving calculations involving Newton's gravitational law between the tutorial lesson and post-test. Therefore, we can attribute the increase in understanding of the inverse square law demonstrated mathematically to be a combination of representational transfer utilised in parallel to solving qualitative problems.

This section has shown evidence of gains in the student's understanding of the inverse square law. The discussions show indications that conceptual change occurred, but some difficulties persist with the students. This was to be expected as Arons (1997) and Marzec (2012) showed difficulties can persist beyond initial instruction. The approach adopted allowed for students to progress their understanding of the inverse square law. The reasoning developed by the student can be transferred to Coulomb's law, to explain the variation of the force felt between two charges as the distance between them is varied. This reasoning can also be utilised with students developing their understanding of electric fields, in which the students can explain the variation in the electric field strength at varying distances from a single charge. As discussed in section 5.5.3, one of the electric field homework assignments is written in a format similar in nature to the inverse square law tutorial, where field lines are substituted in lieu of paint droplets, so the students could use the frame model to explain the behaviour of electric field lines, and thus model the variation of field strength with increasing distance.

4.4. Field line concepts

This section presents a narrative of the teaching sequence the students experienced to develop their understanding of field line representations. Field is a key concept in physics, providing a model to explain "action at a distance" for non-contact forces, such as gravitational, electric and magnetism. Greca and Moreira (2006) noted that field theory is rarely covered in high school physics and students are mainly introduced to the theory in the study of electromagnetism third level. When students apply field theory to electromagnetism, the emphasis is placed on mathematical representations. This can lead to student difficulties in their understanding, application and interpretation of field lines as discussed in section 2.1.3.3.

The difficulties identified were used to construct the following learning objectives. Upon completion of the field line tutorial lessons, the students would be able to:

1. Distinguish between force and field (Furio and Guisasola, 1998).
2. Sketch field lines diagrams from vector fields, and vice-versa. (Törnkvist, *et al.*, 1993).
3. Recognise field lines are representational tools and not tangible objects (Galili, 1993).
4. Reasonably determine the trajectory of a body under the influence of a field. (Galili, 1993; Törnkvist, *et al.*, 1993).

The inquiry approach developed for promoting student understanding of field lines consisted of a pre-test, a tutorial lesson, homework, and a post-test. This intervention ran over three weeks. The intervention aimed to promote student's understanding of (1) field strength using field line density,

(2) the direction of force acting on a body in a field and (3) the path taken by a body moving through a field. A timeline for the implementation of the inverse square study, including the target concepts for the intervention, are shown in Table 4.24. As mentioned in section 4.3, this intervention took place in the same period as the inverse square law materials. The sections relevant to field lines are presented in bold.

Section 4.4.1 presents the pre-test results on how students represent field strength, sketch vectors to show the direction of force at different points in a field, and show the path taken by a body in a field. Section 4.4.2 presents a narrative of the development of the student's understanding of the field line conventions, initially looking at uniform fields, then varying fields and finally looking at bodies interacting with a field. Section 4.4.3 presents an analysis of the homework assignment, which was developed to allow the students to practice the skills and apply the understanding they developed in the tutorial. Section 4.4.4 presents an analysis of the post-test results which, like the pre-test, focused on student's understanding of field strength using field line representations, sketching the direction of force at different points in a field and showing the path taken by a body in a field. Section 4.4.6 presents a comparison of the pre-test and post-test results, and a commentary of the student's progress during the tutorials. Examples of student progression and difficulties that persisted throughout the tutorial and post-test are discussed.

Time		Research Implementation	Target Concepts.
Week 4	Class 1.	Pre-test.	Inverse square law pre-test.
	Class 2.	Tutorial Lesson	Inverse square law tutorial lesson.
	Class 3.		*Topics unrelated to project: Newton's gravitational law.*
Week 5	Class 1.	**Pre-test.**	**Field strength.** **Direction of force related to field line.**
	Class 2.	**Tutorial Lesson** **Homework.**	**Path taken by body in field.** **Topic summary.**
	Class 3.		
Week 6.	Class 1.	**Post-test**	**Field strength.** **Direction of force related to field line.** **Path taken by body in field.** **Inverse square law post-test.**

Table 4.24. Timeline of the implementation of the field lines study.

4.4.1. Pre-test: Field line Concepts

The pre-test for field lines was based on gravitational fields, as this is the first topic in Leaving Certificate Physics that students experience in which field line representation can be used. At Junior Certificate level, the would have used field lines to represent the magnetic field of a bar magnet. During the initial presentation of Newton's universal law of gravitation, the students were introduced to field lines as an alternative manner to visually represent the gravitational field of a large mass, as opposed to using vectors.

The students then completed the pre-test, in which the results could be used to determine how much understanding the students develop at Junior Certificate level about field line convention. The results were also used for comparison with the post-test results, to determine any conceptual gains that can be attributed to the students completing the tutorial lessons. The students were given 15 minutes to complete the pre-test and it was administered before the class discussion to introduce the topic. The tutorial mainly revolves around scenarios of bodies moving through space and feeling the gravitational force between themselves and planets. The pre-test probes student's understanding of the following concepts:

- The field strength is represented by the field line density. (Furio and Guisasola, 1998)
- The direction of the field is tangential to the field lines. (Törnkvist, *et al.,* 1993)
- The field lines represent the direction of the force acting on a body, not the path taken by a body. (Galili, 1993; Törnkvist, *et al.*, 1993)

The field line patterns used in probing the student's understanding of both field strength and the direction of the field at various points are shown in Figure 4.34.

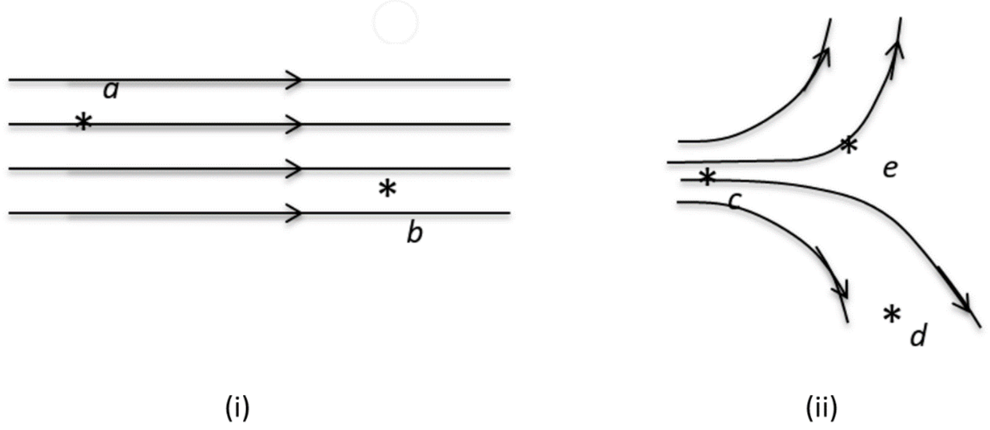


Figure 4.34. Pre-test field line question.

The student's rankings of field strength, from highest to lowest, is shown in Table 4.25. The correct ranking is denoted in bold. The pre-test results indicate that no student had a full understanding that the field line density represented the relative field strengths of the field. The most prominent rankings submitted by students were "d > e> c > a = b" and the converse ranking, "a = b > c > e > d." These rankings could suggest some indication that the field line density represents strength, but some of the reasoning provided by these students indicate otherwise. Reasoning provided included that the straighter the lines, the stronger the force (4B); the more the lines turn, the stronger the force (4H); the more the direction faced downwards aligned with gravity, the stronger the force (4J). The remaining students who chose these ranking did not submit reasoning.

Ranking	Students
c > a = b > e > d	n/a
d > e> c > a = b	4A, 4E, 4F, 4H, 4J.
a = b > c > e > d	4B, 4I, 4N.
e > d > a = b = c	4C
d > e > a > b > c	4G, 4L
d > a > b > c > e	4K
a > c > e > d > b	4M
N/a	4D

Table 4.25. Student's pre-test rankings of field strength, from highest to lowest

The other students who completed the pre-test and gave alternative rankings submitted reasoning such as the angle of the lines determines strength (4K), the apparent (or lack of) movement of the lines as you travel from left to right (4C) and the distance of the points to the field lines themselves (4M). Overviewing these results, it shows that the students were familiar with field line representation, and that when faced with a question on this, approximately half of the students were unable to correctly interpret how field lines representations represent field strength.

The second pre-test question asked the students to determine the direction of the field at the points marked with an asterisk (*) in the gravitational field, and to use vector arrows to represent them. The results are summarized in Table 4.26, with the correct response denoted in bold writing.

Response	Students
Force is tangential	**4M, 4N**
Force follows field line	4B
Force from (a) to (e)	4C, 4G
N/A	4A, 4D, 4E, 4F, 4H, 4I, 4J, 4K, 4L

Table 4.26. Student pre-test responses to representing the field using vector arrows.

Only two students correctly determined the direction of the force at the points labelled, 4M and 4N. Their results showed the force acting in the direction of the uniform field, tangential to the curved field lines. One student, 4B, showed the force following the field lines, not only showing a misunderstanding about the force direction, but also that the vector arrows curve, as opposed to show directions at a single point. Two students appeared to misread the question and drew a vector arrow from the point (a) directly to the point (e), 4C and 4G. The remaining students did not answer the question, suggesting they likely did not possess the understanding required to complete the question. Overall, this suggests that students were unaware of how field lines represent the direction of the force

The final question on the pre-test determined whether students can predict a reasonable path taken by a stationary body when it moves under the influence of a gravitational field of two nearby planets. The field is shown in Figure 4.35, and a summary of the student's responses is shown in Table 4.27.

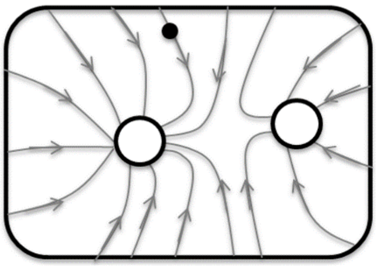


Figure 4.35. Pre-test question in which students were required to draw the path taken by a stationary body under the influence of the gravitational field of two nearby planets

Responses	Students
Path does not follow field line	**N/A**
Path follows field line	4B, 4E, 4G, 4M, 4N.
Directly to planet	4C
N/A	4A, 4D, 4F, 4H, 4I, 4J, 4K, 4L.

Table 4.27. Student's pre-test paths taken by small body in a gravitational field.

Students typically did not understand what information can be gleaned from the field line pattern. No students determined how the field lines represent the direction of the force at the point. Five students appeared to think that the field line represents the path. One student, 4C, indicated that the body would fall directly towards the leftmost planet, ignoring the effect of the gravitational field generated by the rightmost planet, and any acceleration it may have experienced due to it. The remaining students were unable to formulate any reasoning to allow them to attempt this question, and left it blank on the pre-test.

The pre-test results present evidence that students were unaware of the conventions of using field line representations. When presented with a field line pattern, the students were unable to accurately determine the field line strength, using the field line density as a guide (Furio and Guisasola, 1998). 12/14 students showed difficulties in using vectors to represent the direction of the force at various points in the field (Törnkvist, *et al.,* 1993). When required to draw the path taken by a body in a gravitational field, 6/14 students thought the field lines present the path taken by the body, or ignored the patterns of the field lines and directed the body directly to the nearest mass (Galili, 1993; Törnkvist, *et al.,* 1993)

4.4.2. Tutorial lesson: Field line Concepts

The students were briefly introduced to field lines in a class discussion, looking at the use of vectors to demonstrate the inverse square law. They were presented with a planet and vectors shown at points around the planet, getting shorter as the points were further from the planet. The students were asked to explain how the vectors represent that the field was getting weaker as the points were getting further from the planet. In pairs, they discussed this, and all pairs volunteered that the shorter arrows demonstrated a weaker field. They were then informed that drawing vectors in the manner shown can be cumbersome and they were presented with the same planet with eight field lines converging towards the planet. I explained, using diagrams to demonstrate, how field lines and vector arrows could be used to demonstrate the same thing. The conventions of relative field strength being represented by field lines being close together was discussed, and highlighted that they represented the direction of force, as opposed to the trajectory of bodies under the influence of the field lines. I presented three diagrams of field line patterns and the students in pairs practised interpreting the field line patterns, in light of the conventions shown, and verbally gave feedback. Eight of the students initially confused the field strength convention, in which they reasoned field lines further apart were stronger but over the three, they appeared to rectify this error. A depiction of a satellite in orbit was used to illustrate how field lines represent the direction of force experience by a mass at different points, but not the path taken by the satellite, as they clearly were different in the diagram. The students were then informed in the tutorial lesson, the use of field lines would be explored, which they then commenced.

In the first section of the tutorial, the students were presented with a body falling off a cliff, in the path taken as shown in Figure 4.36. The students were guided through a series of questions asking about the force acting on the body as it fell, to guide them to understand that the ball was under a constant force due to gravitational attraction. It was expected that the students would identify that the acceleration due to gravity on the ball was constant and could represent it as such using vector arrows.

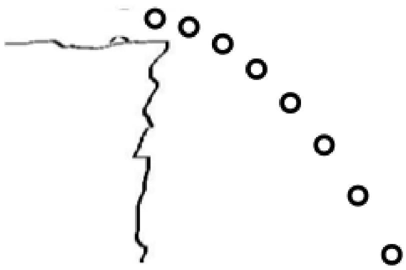


Figure 4.36. Motion diagram of a body falling from a cliff, from the field lines tutorial.

Some of the students highlighted the negligible change in acceleration due to gravity due to the ball being closer to the centre of gravity of the earth as it falls. This was discussed, and they concluded that the change would be extremely small, and practically immeasurable.

Student 4B:	It (gravitational force) changes very slightly. No (acceleration is not constant), as it changes as it gets closer to the centre of gravity.
Student 4D:	Yes (gravitational force is constant), gravity doesn’t change. Yes (acceleration due to gravity is constant), as the force of gravity doesn’t change.
Student 4K:	It (gravitational force) remains constant. Yes (acceleration remains constant) because the acceleration due to gravity will be 9.8 m/s^2.

All students presented identical vector arrows to show the acceleration to due to gravity was constant, including students who mentioned the negligible change due to the decreased distance as the body fell. They were then introduced to field lines, and explicitly told that the field line density showed relative strength and the direction of field was in a direction tangential to the field lines. They were asked to apply these conventions to the field line patterns they were shown in the pre-test, as previously shown in Figure 4.34.

As the convention was explicitly presented to the students, there was little difficulty encountered by them to apply the convention correctly to determine the correct ranking and the direction of force acting on a body placed at the points shown. The students were then invited to represent the gravitational field on the body falling from the cliff. It was observed that when representing the uniform field, the following two difficulties were shown by the students, as seen in the sample of their work presented in Figure 4.37.

- These were that the field lines began at the ball when it was falling.
- The field lines terminated before running down beyond the cliff.

These errors were seen in approximately half of the student's responses. This error was mentioned to students, and is explicitly addressed in a later tutorial involving electric fields.

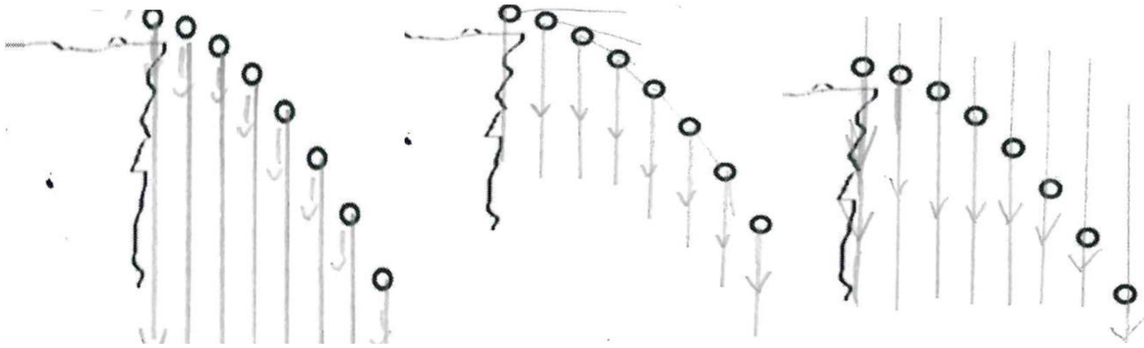


Figure 4.37. Examples of responses, in which field lines begin in body, field lines begin in body and terminate, and an accurate depiction of field lines.

The students were then presented with the diagram shown in Figure 4.38, showing a small meteor moving with an initial velocity and the earth. They were required to draw in the path taken by the meteor and explain how the field shows the variation in field strength, with a suggestion to compare it to the uniform field they previously encountered.

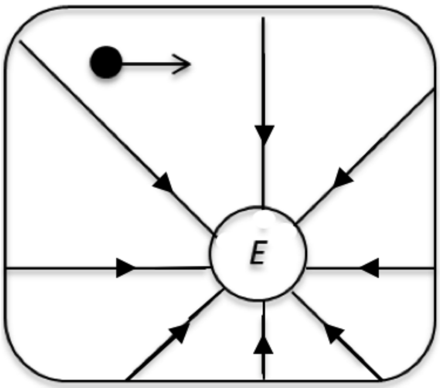


Figure 4.38. Tutorial diagram for difference between the direction of a field line and the path taken by a body.

All the students identified that the field gets stronger at points closer to Earth. Six students referenced that the strength decreases with the square of the distance from the planet referencing the inverse square relationship they encountered in the previous tutorial lesson. However, they could not explain how the field line pattern itself could be used to attributed this relationship, and were instead considering Newton's universal law of gravitation in justifying their reasoning. The remaining students used the field line representation for their justification, as shown in the following quotes taken from scans of student's tutorial worksheets.

Student 4A: No. The field lines are not all equal… the further away they are, the weaker the field.

Student 4E:	The lines are far apart at a and b, but c and d are close, so the force is stronger. (Points drawn on diagram by student)
Student 4M:	No. The strength decreases. The further from earth you go because the distance between the lines increases, weakening the strength.

The students then had to determine the path taken by the meteor under the influence of the gravitational field. It was expected that students draw their paths so that they would follow one of the field lines, as expected from errors seen in literature where leaners consider the field lines to represent the path taken by a body (Galili, 1993; Törnkvist, *et al.,* 1993). There errors were also observed in earlier versions of this tutorial trailed with pilot groups. However, during the tutorial lesson, the students drew individual paths, and then in their groups, all the students engaged in discussions to determine which of their paths they considered to be valid. While no group said that the meteor would follow a line, there were differences of opinion as to whether the path would be a linear path, a circular path or a curved path due to the initial velocity. When students suggested a deviated linear path for the meteor, the teacher suggested that their chosen path may not be accurate. This allowed other members of their group to explain why a circular or curved path would be an appropriate choice. Although the most accurate paths to represent the paths would have been hyperbolic or elliptical, these types of paths would require a level of depth of understanding the students would not have developed at this point in their education of Leaving Certificate Physics or Leaving Certificate Applied Mathematics. Therefore, both a circular and curved path were considered valid, given the nature of the question, the concept being taught, the lack of numerical details for the necessary calculations to determine the exact path and the prior learning of the students. Each group was asked to explain this in detail to the teacher, to which all the explanations are summarized in the following bullet points:

- The meteor will try to move in the direction it is going with the initial velocity, but the gravitational attraction between the meteor and the earth will cause it to turn.
- This force will cause the meteor to deviate from its original path.
- The field lines will show the direction the meteor will attempt to turn instead of the path.

In the last section of the tutorial, the students were once again presented with the scenario depicted in Figure 4.35. In this case, the meteor has no initial velocity. First, to deepen their understanding of the field line representation, the students were asked to identify which of the two planets had a larger mass, based on the field line density around the planets presented. The students were then required to determine what path would be taken by the meteor. To help them with this process, the tutorial provided a dialogue between two hypothetical students discussing the path taken:

S_1: The field lines indicated the direction of the force, so the meteor will be forced along the line until it hits the left planet

S_2: As the small meteor begins to accelerate, its gained velocity will make it move away from the field line that it was on originally, so we can be sure it'll hit either planet.

Two groups of students inquired into the meaning of the term "gained velocity," as seen in the reasoning seen by S_2. It was explained that this was shorthand for "the meteor would experience a force and accelerate". The students were asked to determine if the velocity would be zero or non-zero, due to the acceleration, after a small amount of time passed.

Initially, students 4F, 4H and 4M agreed with S_1, explaining that the meteor has no initial velocity in this case, leading to the force felt by the particle at all points to turn in tandem with the field line. The teacher then suggested they consider the velocity a small instant after the particle starts to accelerate. The students acknowledged that there would be an non-zero velocity after a small instant in time. The students were then directed to review their reasoning from the previous section covered and discuss with their groups.

Upon review, the students agreed with S_2, and sketched a path consistent with this reasoning. All other students initially considered that the force would cause an acceleration that would take the path off the field line, into a curved path towards the negative charge. Two examples of student work are presented in Figure 4.39.

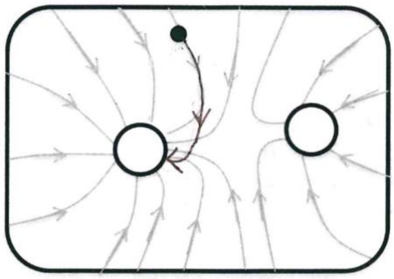
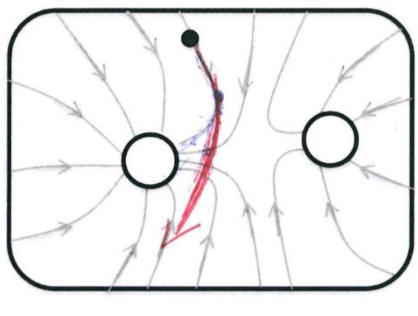
Student 4C: S_2, because it has gained velocity, it will move off the field line, to hit the planet. 	Student 4H: I agree with S_2, because as the meteor begins to accelerate, the gained velocity will make it move away from the field line that it was on originally. 

Figure 4.39. Paths depicted by students 4C and 4H.

In summary, the tutorial discussion illustrates how the students developed their understanding of field line conventions. The students were guided to represent a uniform field using vectors and then

transferred the vectors into field line representation. The students were presented with both uniform and non – uniform fields and guided through the reasoning to accurately determine the relative field strength using field line density as an indicator. With assistance from the teacher, the students developed reasoning to explain why a mass does not follow a field line, referencing force, acceleration, velocity and time, to construct a path taken by a body in a field, under the influence of that field.

4.4.3. Homework: Field line Concepts

The homework assignment was developed to reinforce the target concepts developed in the lesson. The homework assignment questioned student's ability to draw field lines, determine field strength based on field line density, and depict the path taken by bodies in various gravitational field.

In the first question, the students again used the context of meteor travelling past planets, in slightly different scenarios to those shown in the tutorial. The first scenario is presented in Figure 4.40.

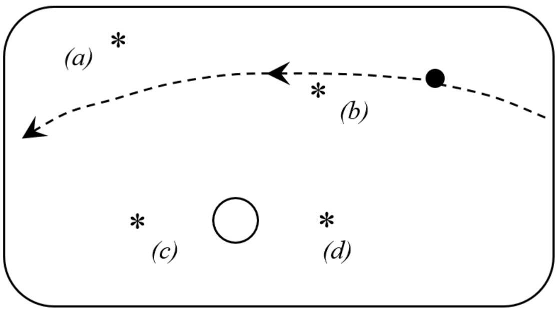


Figure 4.40. Homework question of comet passing planet.

The students asked two questions, in which they were required to draw the gravitational field lines caused by the planet and the second question required them to rank the field strength, from lowest to highest, at the points labelled (a) to (d). The latter question explicitly required the students to reference the field line pattern they sketched for the former question. Their results are summarized in Table 4.28.

Eight of the 14 students sketched a correct field line pattern for the planet's gravitational field, in which straight lines extended from the surface of the planet to the boundary of the diagram, as depicted in Figure 4.38 in the tutorial lesson. Student 4D sketched field lines that terminated in space, before the boundary of the diagram, unlikely realising this depiction suggests the gravitational field terminates at a distance away from the planet. Students 4E and 4F made a similar error, but also sketched their field lines with minor curves, likely not realising that this suggests other masses would be required to be nearby to cause the minor curves they sketched. However, outside of these error in

understanding by the students, they provided an accurate depiction of the field. One student, 4N drew a magnetic field of the earth, but did not provide any reasoning as to why they did this. However, magnetic field patterns are the first field line pattern the students are exposed to (at Junior Certificate level) and it is possible that the student was recalling a pattern they came across two years previously.

Responses	Students
Correct Field line pattern	**4A, 4B, 4G, 4H, 4I, 4J, 4K, 4M**
Used vectors instead of field lines	4C
Field lines terminate	4D
Field lines curve and terminate	4E, 4F
Sketched Earth's B-Field.	4N

Table 4.28. Student's representations of the gravitational field of the planet.

The students were also required to rank the field strength, from highest to lowest, at points marked (a) – (d). Their rankings are summarized in Table 4.29. Seven students provided an accurate ranking. Six students did not explicitly define the ranking between (c) and (d). It is reasonable suspect that they considered the field strength at (c) is greater than (d) in these cases. This would be consistent with reasoning submitted by these students.

Student 4F: C is closest, so it gets affect by the gravitational field the most.

Student 4H: Because as you increase the distance from a planet, you decrease the gravity.

Responses	Students
c = d > b > a	**4A, 4C, 4D, 4E, 4J, 4K, 4M**
c, d, b, a	4B, 4F, 4G, 4H, 4I, 4N
N/A	4L

Table 4.29. Student's rankings of the gravitational field of the planet.

This reasoning was also seen in some students who provided the correct ranking, whilst one student used the field line density to justify their choice.

In the final question, students were asked to sketch the field line pattern of two planets of equal mass and from this, determine a reasonable path the meteor would take, when starting from rest, as depicted in Figure 4.41. There were some difficulties observed in the student's representation of the field lines, such as terminating field lines (4C and 4D), the use of vectors instead of lines (4E) and failing to show the superposition of the two fields (4A, 4B, 4G, 4J and 4K), where lines overlapped instead and two magnetic fields with no superposition (4N). Despite these errors, most of the students

produced a reasonable path to be taken by the meteor, where the path did not follow either the field lines or vectors when used by the students. However, this may have been due to recall of the paths taken in the tutorial questions and applied to this question.

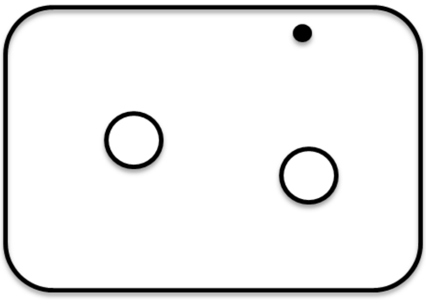


Figure 4.41. Homework question of comet with no initial velocity near two planets.

4D, 4J and 4N submitted paths that were considered unreasonable due to the meteor moving in a direction which does not align with the direction of the net force acting on it (4D), associating a higher gravitational pull to one of the planets at a point equidistant between them (4J) and a path that ignores one of the planets and collides into the other planet (4N).

The homework assignment indicated that the tutorial produced positive gains in some student's understanding. 8/14 students could correctly represent a gravitational field of a planet, with persistent difficulties present such as using vectors instead of field lines to represent the field, field lines terminating inconsistently, field lines being curved when unnecessary or representing the field as the pattern for a bar magnet. 7/14 students could correctly rank the field strength at various points, but in this case, typically used the distance of the points from the planet to justify their ranking. A further 6/14 students submitted answers indicating the correct ranking, also typically referencing the distance from the planet to the points. When representing the field of two planets, student difficulties were more commonly observe, such as representing terminating field lines, drawing vector instead of field lines, not applying the principle of superposition, and drawing bar magnet patterns. Despite these errors, only three students drew paths considered to be unreasonable between the two planets. This indicates that the they were considering the influence of the force of gravity from both planets, the acceleration of the mass, the changing velocity and how these affect the trajectory of the mass.

4.4.4. Post-test: Field line Concepts

The post-test for field lines was undertaken by the students approximately one week after the they completed the field lines tutorial, along with the post-test for the inverse square law. The post-test was designed to elicit student understanding the following three conventions:

- The field strength is represented by the field line density. (Furio and Guisasola, 1998)
- The direction of the field is tangential to the field lines. (Törnkvist, *et al.,* 1993)
- The field lines represent the direction of the force acting on a body, not the path taken by a body. (Galili, 1993; Törnkvist, *et al.,* 1993)

In the first section of the first question, the students were presented with a section of a gravitational field, as shown in Figure 4.42. The students were asked to trace their finger along the lines, starting from where the lines are closest together, so that it travels against the direction of field line, and explain how the field strength varied as they did so.

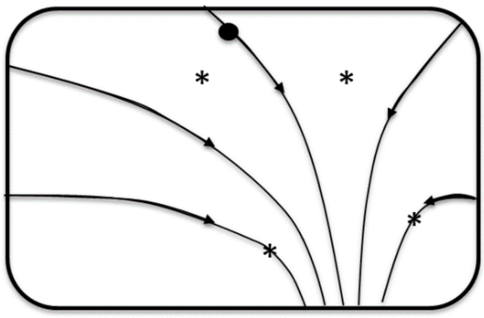


Figure 4.42. Post-test question field lines question.

All the students, except 4I and 4L, explained that the field strength decreases, and this was represented as the field line spreading further apart. Student 4I did articulate that the lines spread out as the field lines were traced but did not reference how this affects the strength of the field itself. Student 4L was not present on the day to complete the post-test.

In the second section of the first question, the students were required to determine the direction of force at the points. The results are summarized in Table 4.30.

Most of the students appeared to have grasped how the direction of the force is represented by field lines. Students 4I and 4N represented the force as acting vertically downwards, which is symptomatic of students who do not consider the field line as a representation of force, and instead assume beyond the bottom of the diagram is a planet in which the gravity uniformly is directed down towards. Student 4G's diagram also suggests a similar reasoning, albeit with the planet just below where the lines appear to converge and the gravitational field acting towards it in a radial pattern.

In the last section of the first post-test question, the students were asked to determine what path a small body would take when it was released from the position marked with a black dot on the diagram. The student's responses are presented in Table 4.31.

Most of the students explained that the meteor's trajectory would not follow the field line, as the bodies inertia would prevent it from directly following the line. They explained that the body would move in the direction of the force acting upon on it and produce a path that is not represented by the

pattern of the field lines in Figure 4.43. The students articulated this by using simpler terms reasoning. The students themselves articulated this as velocity gained by the meteor would cause that path that would carry it from the sketch of the field line. The response summary presented in Table 4.31 indicates that eleven of the students did not think of field lines as a path, and ten of these students could interpret the diagram to draw a reasoning path taken in which the trajectory was influence by, but not identical to, any of the field lines shown.

Responses	Students
Force is tangential to the field lines.	**4A, 4B, 4C, 4D, 4E, 4F, 4H, 4J, 4K, 4M**
All forces point downwards.	4I, 4N
All vectors point to where the line appear to converge	4G
N/A	4L

Table 4.30. Student post-test responses to representing a field using vector arrows.

Responses	Students
Path trajectory sketch diverges from field line pattern in a reasonable path	**4A, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4K, 4M.**
Path trajectory sketch diverges from field line pattern but is an unreasonable path.	4B
Path taken follows the field line.	4N
No path was determined.	4J

Table 4.31. Student's post-test paths drawn taken by a body under the influence of a gravitational field.

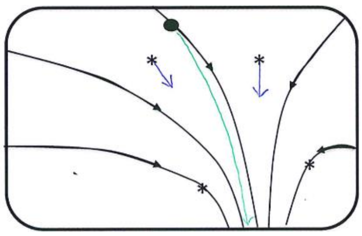
However, some students gave additional information about the interaction between the body and the field lines that showed misconceptions. Student 4F correctly reasoned the path and referenced the acceleration, but then also volunteered reasoning that the mass of the body must compete with, and overcome, the force generated by the field lines. Another student, 4H, also gave the field lines a tangible property, in which case the path was a result of the acceleration off the line, the gravitational pull towards the bottom of the diagram and the other field lines pushing the body away to stop the path from intersecting with other field line. This tangible property of field lines has been observed in other research (Galili, 1993)

Student 4F: As the body accelerates, it moves off the field line towards the gravity centre, because its mass is greater that the force of the gravity field.

Student 4H: The body will go off the field line as it accelerates and gains its own velocity. However, the other field lines will push it downwards.

Student 4B reasoned that the area where the gravitational field was strongest would have the most pull, therefore it would pull the body off the field line as shown in Figure 4.43. This suggests confusion to attributing a gravitational force to the field line themselves, as opposed to a representation of which way a body would experience a force. Student 4N acknowledged the body would accelerate, and as it got closer to the bottom, this acceleration would increase. They did not however consider that this acceleration would generate a velocity that would carry it from the field line.

<table>
<tr>
<td>Student 4B:

The small body is pulled towards the field lines that are closer together, because the strength of the gravity is stronger there.
</td>
<td>Student 4N:

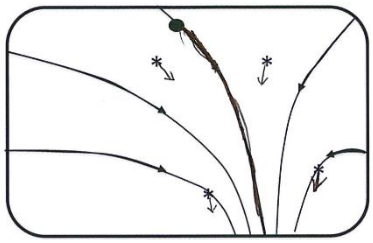
The body accelerates faster when the lines are closer together and the body is being pulled down by the force of gravity.
</td>
</tr>
</table>

Figure 4.43. Path taken by the body from rest from student 4B and 4N.

The post-test showed that student gains in understanding of how the electric field lines represent the relative strength of the field. 12/14 of the students correctly explained the variation of the field line patterns, with another student providing reasoning alluding to the convention. 10/14 students accurately represented the force vectors as tangential to the field lines, which persistent difficulties in the remaining students observed such as directing the vectors to the point where the field line converge or ignoring the field pattern entirely. The most notable student difficulties were students representing the path taken by a stationary body in a gravitational field. 10/14 of the students sketch the path taken as a pattern that did not follow the field lines, but there were examples of students associating a tangible nature to the field lines themselves, in which they described the field lines are having a gravitational pull of their own. This reasoning appears to treat gravity as a contact force, instead of a non-contact force, and shows the approach did not address the difficulty for these students.

4.4.5. Discussion

From comparing the pre-tests, lessons, homework and post-tests, it can be determined that most of the students made progress in their understanding of field line conventions. The first concept addressed in this discussion is the student understanding of field line density and its representation of relative field strength. Figure 4.44 compares the pre-test and post-test results for this concept.

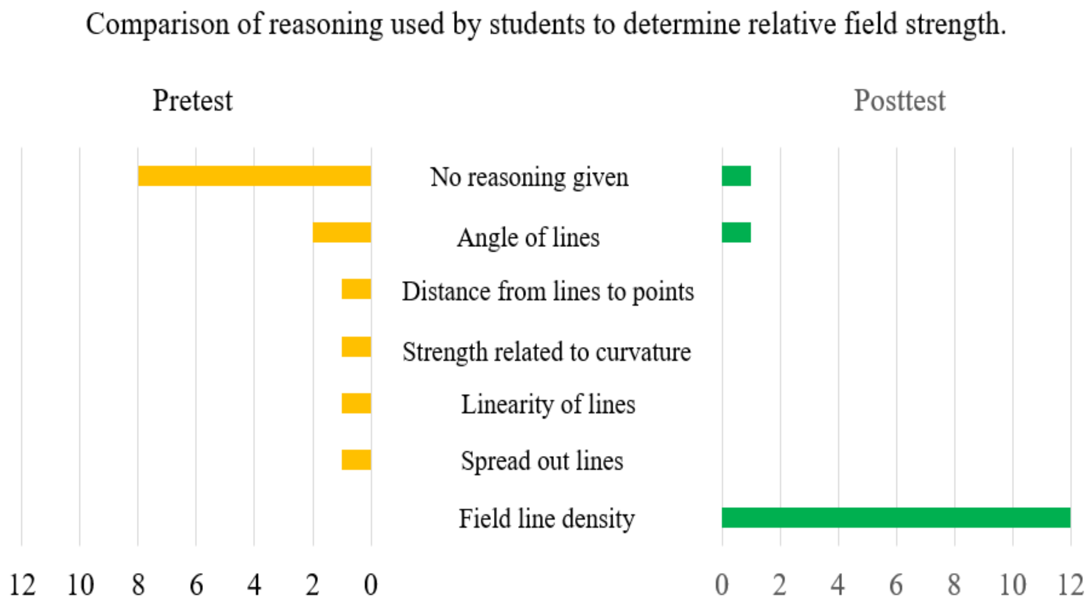


Figure 4.44. Comparison of reasoning used by students to determine relative field strength.

As seen in Figure 4.44, a shift in student reasoning occurred from the pre-test to post-test. In the pre-test, six of the students used incorrect reasoning such for stronger field strength, such as (i) the further spread the field lines, the stronger the field strength, (ii) straight lines being stronger, (iii) the more pronounced the curvature of the line, the stronger the field (iv) the further a point is from a line, the weaker it is and (v) the higher the angle the lines make with the horizontal / vertical, the stronger the field. Eight of them submitted no reasoning for their rankings. The eight submissions with no responses suggest the students had no explanations they were satisfied with to justify their rankings (Posner, *et al.*, 1982), while difficulties the remaining students had were identified. The students were introduced to the convention for field line density representing strength during the tutorial lesson, and section 4.4.3 outlines multiple instances that were sufficient for the students to explore and adopt the convention of field line density to represent relative field strength.

Figure 4.44 indicates that the tutorial lesson was effective in promoting the student's conceptual development, as a clear shift in reasoning submitted by the students was observed in the post-test results. All but two of the students correctly used the field line density to produce the correct rankings, and except for one student, there were no references to the misconceptions observed in the

pre-test. This indicates that conceptual extinction occurred (Hewson, 1992), with the shift in the pre-test and post-test results demonstrating that ideal conceptual change occurred.

The next section of this discussion presents a comparison of the student's transfer from field line representation to vector arrow representation. Figure 4.45 presents a comparison of the pre-test and post-test results for this concept.

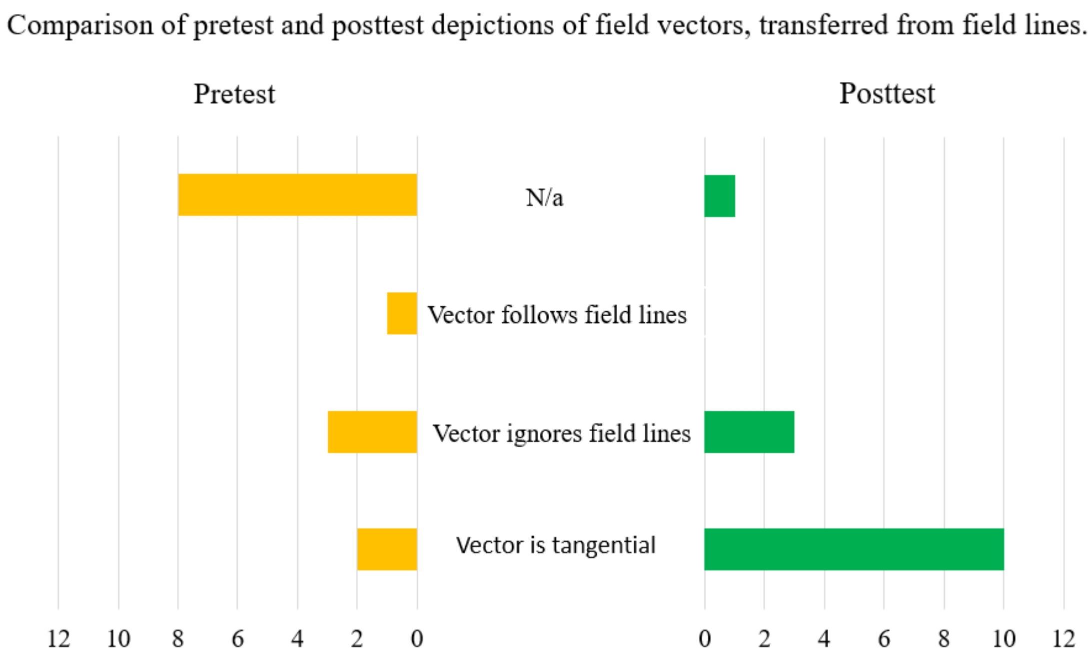


Figure 4.45. Comparison of depictions of field vectors, transferred from field lines.

Figure 4.45 shows that the tutorial lesson also produced gains in student's understanding that the electric field vector at a point is tangential to the field lines. Initially, most of the students were unable to attempt this question in the pre-test and did not have enough understanding to approach the question. The student's inability to reason this question suggests dissatisfaction with their prior understanding (Posner, *et al.*, 1982), while the difficulties observed by the three students who ignored or followed the field lines were identified for conceptual extinction. The students were guided to transfer from field lines to vectors during the tutorial lesson, provided with opportunities to practise the transfer between the representation in the tutorial and homework assignment. During the tutorial lesson, the students used rulers to practise drawing the arrows and repeated this for the homework activity. Figure 4.45 indicates that there was a shift in the number of students that could accurately apply vector diagrams to an electric field context, from the pre-test to the post-test. As the participants demonstrated proficiency of using vectors in section 4.2, this shift indicates conceptual extension occurred (Hewson, 1992), with the shift from two to ten students producing tangential vectors indicating that moderate conceptual change occurred. However, no students attempted an accurate scale, in which the vectors were longer where the field strength was greater, ignoring the magnitude

of the vectors, and instead, focusing on the direction only. This indicates conceptual change occurred, but issues of transfer between the vector representation and the field line representation persisted.

Additionally, in the tutorial lesson, the students built up a model of the gravitational field of the earth and transitioned it to a field line model, but half of the students demonstrated errors such as field lines beginning at the object in the field and terminating before reaching the planet generating the gravitational field. These errors are further discussed in sections 5.6 and 5.7 where the students were given the opportunity to address them.

The last section of this discussion focuses on the student's predictions of the path taken by a body under the influence of a field. Figure 4.46 presents a comparison of the pre-test and post-test responses from the students for this concept. As discussed in section 4.4.1 and 4.4.4, these results are for scenario's in which the body under the influence of the field has an initial velocity of zero.

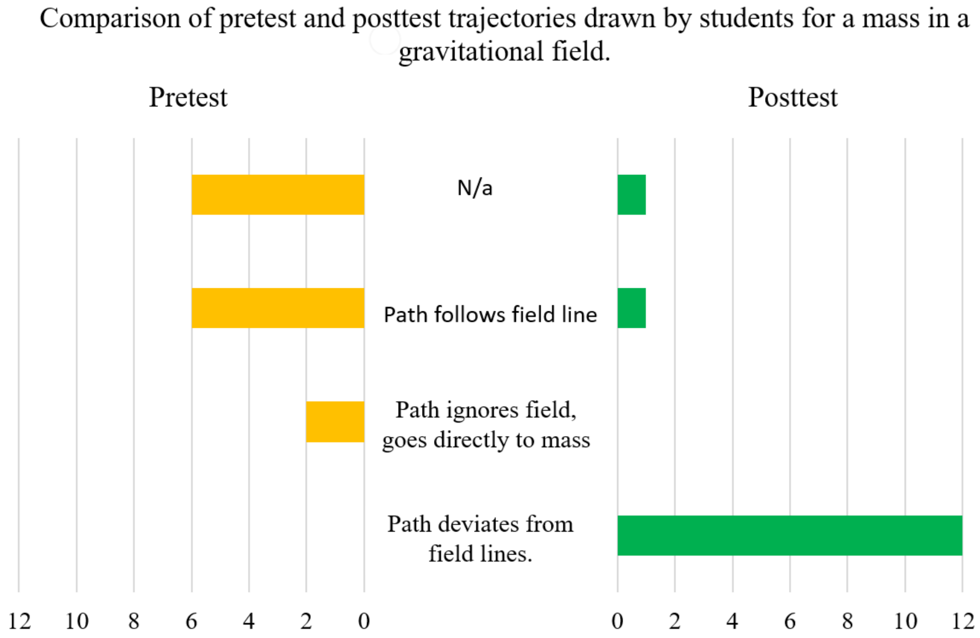


Figure 4.46. Comparison of trajectories drawn by students for a mass in a gravitational field.

The most prominent error by students in the pre-test was that the object would follow the field lines or ignore the field lines and go directly to the mass generating the gravitational field. Both difficulties were predicted from literature (Galili, 1993; Törnkvist, *et al.*, 1993). As the reasoning that produces these difficulties is erroneous, these difficulties were targeted for conceptual exchange (Hewson, 1982). Section 4.4.2 illustrated many examples of the students developing their reasoning for this concept. The tutorial was written with three examples of a body moving with a trajectory influenced by, but not identical to, a gravitational field. The students explicitly looked at this concept on two questions during the tutorial and referenced the first question during discussions with the teacher. As the students were familiar with in the initial scenarios presented in the tutorial, they became dissatisfied with erroneous reasoning that did not explain the observations accurately (Posner

et al., 1982). The students were given ample opportunity to develop and apply the field line representation to explain the observations in the tutorial, and they used it to develop intelligible reasoning to predict the behaviour of objects under the influence of a field, both in contexts they were familiar with and in contexts unseen to them (Posner *et al.*, 1982). In the post-test, all but two of the students depicted a path that diverges from field lines. This shift in student responses from the pre-test to post-test indicates that the tutorial lesson was effective in promoting conceptual exchange in the student's understanding (Hewson, 1982), with the results indicating that ideal conceptual change occurred.

In the absence of presenting students with a field line pattern, such as those seen in the homework activity, it was observed that the students used different reasoning to justify their claims. In the first homework activity given to the students, they were not explicitly presented with a field line pattern, as previously seen in Figure 4.42, and were asked to rank the field strengths. In this case, the students justified the ranking in various manners. 8/14 students based their ranking on using the distance to indicate the gravitational field strength, and all but one of the students ignored their sketches of the field lines and used these to justify their ranking. The students could have also referenced the inverse square law for gravitation to justify their rankings, but this was not seen in any student's responses. Only one student (4B) explicitly based their ranking on the field line density. However, when shown a field line pattern and asked to explain how the field strength varies, as seen in the post-test, 12/14 of the students used the convention for field strength with the representation. This indicates that while the students were comfortable with the representation, they did not always consider it a valuable tool to use, unless directly required to do so.

In the final homework question, the students attempted to use the reasoning they developed in the tutorial lesson, but many errors were observed in the field lines patterns they produced. One tool not observed in the homework responses by the students was the additional use of vectors or motion diagrams to justify the path chosen. When students ran into difficulties in the tutorial lesson, the teacher would ask the students in which direction the body would move and represent this with a vector and sketch the body again at the end of this vector, indicating that this was the body's new position after a short interval of time. The teacher would repeat this process two more times, using the student's answers to define the direction of the movement of the body and its new position after a short interval of time, thus generating a motion diagram with the aid of the teacher. This line of reasoning was not considered by the students in their homework activity. An exercise designed to look which representations students prefer to use is discussed in chapter 5 in which students plot the path taken by a negative charge when placed between two positive charges. In this question, the students are not asked to represent the field in any way, so it will give us an insight as to which representation, or combination they choose to use.

This section has shown evidence of student's gains in understanding of field line conventions. While there are still some difficulties that persist with the students, they appeared in a small number

of the student's responses. Instances of Posner, *et al.*, (1982) conditions for conceptual change were indicated and when possible, the manner of conceptual change that occurred was identified (Hewson, 1992). The reasoning developed by the student can be transferred to electrostatics, to represent the fields of one or two charges. They can also use the representation to help explain the behaviour charged particles in an electric field. Field lines are also utilised in an electric field tutorial to aid students in associating the inverse square law to electric field, using a model of field line passing through a unit frame, similar in nature to the tutorial discussed in section 4.3.3. Students will also employ the use of electric field lines to develop an understanding of positive, negative and zero work in an electric field, to develop their understanding of potential difference.

4.5. Conclusions

The results of the students presented in this chapter show that the student's understanding of vectors, inverse square law and field lines improved by the employing tutorial lesson. Evidence provided supports that conceptual change occurred in some of the student's understanding of these topics. The tutorials both introduced students to the topics, and specifically addressed difficulties typically encountered by students, as seen in literature. This approach has been shown to be effective to address student difficulties, over using traditional instructional methods. (Dykstra, *et al*., 1992; McDermott and Shaffer, 1992).

The results indicate that the approach adopted promotes conceptual understanding of vector magnitude, vector addition and the implications of adding horizontal and vertical vector components. While some of the students preferred to use vector constructions over reasoning based on vector components, it was observed that students engaged with, and overcame, difficulties in the vector concepts such as linking the magnitude of a vector to its direction (Nguyen and Meltzer, 2003), incorrectly combining vector arrows (Nguyen and Meltzer, 2003) treating vector addition as scalar addition, with no consideration for either the directions of the vectors or the summation of the vector components (Doughty, 2013). Regarding these difficulties, the evidence presented in this chapter indicates conceptual exchange (Hewson, 1992) occurred in the student's models, although some difficulties persisted. As the students became more proficient in these vector concepts, the representation could then be utilised by the students to explain the direction and variation of electric field strength between two charged sources, and the forces acting on the charges.

When the students completed the inverse square law tutorial, it was observed that the students could recognize the general shape of a graphical pattern that follows an inverse square function. However, it was noted that students upon completion of this topic, the students could not differentiate between data that followed an inverse square pattern, and an inverse pattern when they complete

investigations into Boyles' law and the focal length of a concave lens. This indicates that students had an over-reliance on the general shape of the pattern and did not consider to mathematically analyse the data presented on a graph. Mathematically, it was seen that the students could evaluate an equation for intensity, but also struggled to analyse their produced values to show an inverse square law. This lack of consideration to find meaning in the numbers produced echoes student's experience of solving quantitative problems that is typically found in traditional methods of instruction in physics education. The students were given more opportunity to consider the inverse square law and practice their ability to analyse the data, both on a graph and using algebraic evaluation, when they completed the Coulomb's law tutorial lesson, as these skills are employed in that lesson, as discussed in section 5.5.2.

Conceptually, it was observed that students could explain the variation of area covered in intensity contexts, when the distance between an object and the frames were varied. Section 4.3.3 narrates how students could reason that area change of frame was a quadratic factor of the change in the distance between the objects due to a change in bother the length and width of the frame, demonstrating their understanding of quadratic pattern observed when scaling up the area of a surface. Difficulties were seen when applying this to intensity when considering how much of a given phenomenon, in this case droplets of paint, passing through the frame. This highlights confusion between their understanding of intensity of a phenomenon and the phenomenon itself. This can also lead to confusion about which factors follow a quadratic increase, in the case of the tutorial; area, and which follow an inverse square law, in the case of the tutorial; paint intensity.

From the student's exploration of field lines, it was shown that the students are reasonably proficient in the use of the representation and can interpret information from a field lines pattern. It was demonstrated, that upon completion of the materials, they could determine the field strength based on field line density, determine the direction of force acting on a body at a point in a field and reasonably plot a path taken by a body in a field, based on this information. However, in the absence of a field line pattern, when asked to construct the field for two masses, the students produced errors in their field lines patterns. The most persistent difficulty seen was the students not applying the principle of superposition of the two fields, field lines overlapping and using vector arrows in lieu of field lines. These persistent difficulties are addressed in a later tutorial, discussed in sections 5.6 and section 5.7.

In the cases of all three of the topics, there is evidence that conceptual change occurred. The pre-test, tutorials and post-test, student - student discussions and teacher-student discussions presented in this chapter provided many instances of the 4 conditions presented by Posner, *et al.*, (1982) required for conceptual change and when the evidence was presented, the type of conceptual change; extinction, exchange and/or extension (Hewson, 1992) was identified. The extent of conceptual change that occurred varied from minimal to ideal, and the instance of each descriptor is presented a

line plot, shown in Figure 4.47. A legend of the codes used in Figure 4.47 can be found in Appendix F.

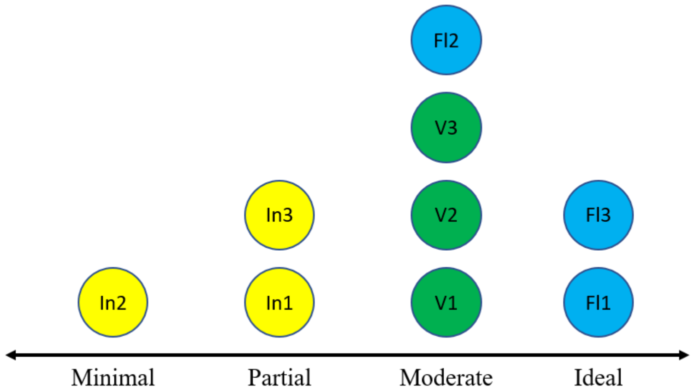


Figure 4.47. Line plot of extent of conceptual change for vectors, inverse square law and field lines.

The extent to which the student's engaged in conceptual change over the course of the tutorials varies depending on which topic the approach addressed. The tutorials that were most effective in promoting conceptual change for the concepts related to the field lines, as moderate and ideal instances of conceptual change were recorded. There were moderate instances of conceptual change for the vectors concepts and minimal and partial instances of conceptual change for the student's conceptual understanding of the inverse square law. The vectors and field line tutorials generally omitted the requirement for students to use mathematical reasoning, while the inverse square law required both mathematical and scientific reasoning to be employed by the students. This requirement to employ dual reasoning could have overloaded the number of items being processed in the students working memory (Reid, 2009) and halted their ability to fully develop their understanding.

By developing the student's understanding of vectors, the inverse square law and field lines, this sets a foundation for the students to build their understanding of Coulomb's law, electric field and potential difference. The students will be able to apply their understanding to these topics to help develop their understanding of electrostatic force, and field. Vectors and the inverse square law are central concepts underpinning Coulomb's law, in determining the forces acting on charged particles, in collinear and non-collinear settings. Vectors and field lines can be used to represent electric field and the behaviour of charged particles in these field, whilst the inverse square law can be used to mathematically quantify the variation in the electric field strength of a charge. When a charged particle moves in an electric field, the use of vectors and field lines can be used to identify whether positive, negative or zero work occurs, which can be used to discuss the variation of potential or define the potential difference between two points in an electric field.

Chapter 5. Coulomb's law and electric fields.

5.1. Introduction

This chapter discusses the development of the student's understanding of Coulomb's law and electric fields, by using inquiry tutorials that employ a multi-representational approach. The tutorials embed vector concepts, exploration of the inverse square law and field line representations in electrostatics. Combined with their prior learning of mechanics, forces and charge, this allows for the students to generate links and their own understanding of the topics and develop the ability to transfer between all representations. This chapter identifies instances in which the students (a) used their understanding of the three element concepts to develop their understanding of Coulomb's law and electric fields, (b) used their experience in developing their understanding of Coulomb's law and electric fields to better their understanding of the target elements, or both.

The following research question is addressed in this chapter:

- To what extent does the use of a multi-representational structured inquiry approach develop student understanding of electric fields?

The following points were considered when addressing this research question:

1. The student's ability to demonstrate that Coulomb's law is an example of an inverse square law.
2. To what extent the students demonstrate their understanding of electric fields and the interaction of charged objects with fields using vector representations.
3. To what extent the students demonstrate their understanding of electric fields and the interaction of charged objects with fields using the field line representation.
4. To what extent the students demonstrate their ability to transfer a depiction of an electric field from one representation to another representation.

Figure 5.1 depicts how the concepts discussed in chapter 4 and the concepts covered external to the project are prerequisites to learning electrostatics. As before, the colour purple denotes topics completed, at this time, during the project; it shows that the students have studied all prerequisite concepts.

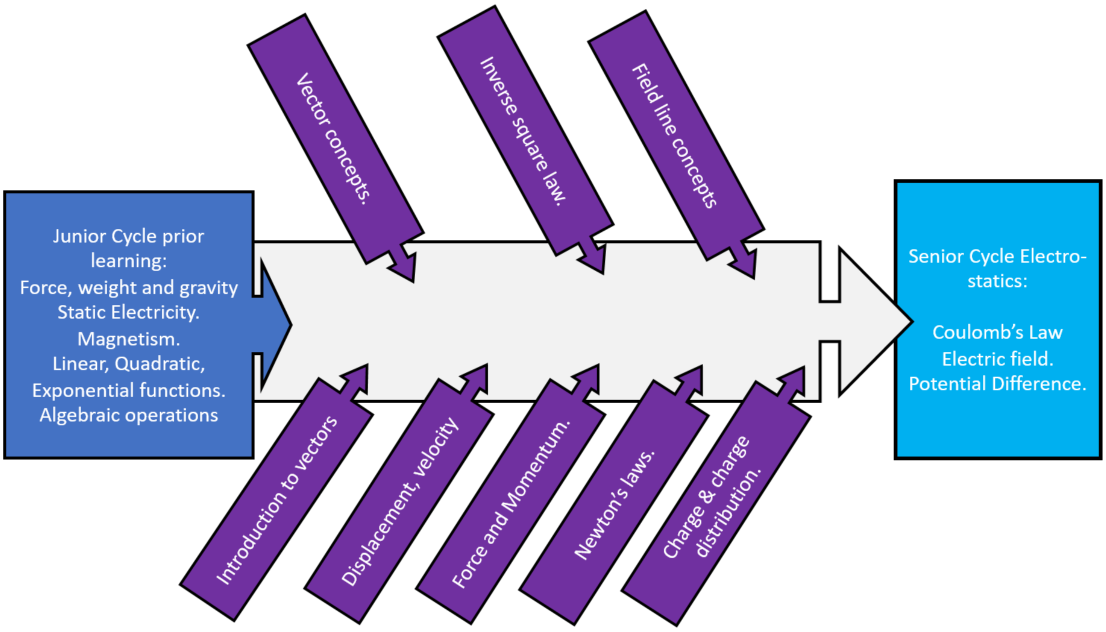


Figure 5.1. Flowchart depicting the topics completed by the students, prior to developing their understanding of Leaving Certificate electrostatics.

The timeline of this section is shown in Table 5.1. Sections in bold refer to materials covered are directly related to the research.

The chapter presents a narrative of the student's use of vectors concepts, the inverse square law and field lines, as they were applied to Coulomb's law and the electric field concept. The student's conceptual development is presented by comparing pre-test and post-test results for the different topics, as well as snapshots of their tutorial worksheets and excerpts of recordings of their conversations during the tutorial sessions.

Section 5.2 reviews difficulties encountered by students in their understanding of vectors, the inverse square relationship and field lines, and their application to Coulomb's law and electric fields, as described in section 2.3.1. Section 5.3 discusses the lessons learned from the research presented in chapter 4, in terms of the difficulties observed in the students understanding after the tutorial lessons, and the implementation of the tutorial lesson format. Section 5.4 discusses how students apply their understanding of vector concepts to the electric field in the context of Coulomb's law. This section focuses on student's use of vector constructions, representing the relationship between magnitude of field strength and distance, and consideration of horizontal and vertical components in vector addition. Section 5.5 looks at the student's application of the inverse square law, and how they apply it to Coulomb's law and electric fields. Students apply the law using tables, graphs, algebra and a scale model, in which they discuss field lines passing through various frames. This approach adapts the model used in Conceptual Physics (Hewitt, 2009). Section 5.6 looks at the student's understanding of the field line representation in electric fields. Here they apply the convections of field line density to represent relative strength, the direction of a field being tangential to a field line. They should understand that the path taken by a body in a field does not follow a field line but the

trajectory of the body is influenced by the field, and the field lines give an indication of the direction and strength of force acting on a body in the field. The students are further introduced to the conventions that field lines do not overlap, and only start and end on electric charges. In the discussions, the results are reviewed to present the impact of the approach on student learning, particularly their ability to transfer representational tools to the electric field context and their reasoning skills.

Week 1, Class 1 (35 mins)	**Presentation to introduce to Coulomb's law.** **Pre-test**
Week 1, Class 2 (80 mins)	**Practice class: Qualitative problems involving Coulomb's Law.**
Week 1, Class 3 (76 mins)	**Research lesson: Coulomb's Law worksheet. Homework assignment given.**
Week 2, Class 1 (35 mins)	**Presentation to introduce to Electric field.** **Pre-test**
Week 2, Class 2 (80 mins)	**Practice class: Qualitative problems involving Electric field.**
Week 2, Class 3 (76 mins)	**Research lesson: Electric field worksheet. Homework assignment given.**
Week 3, Class 1 (35 mins)	**Practice class: More difficult qualitative problems involving Electric field. Homework assignment given**
Week 3, Class 2 (80 mins)	Presentations and demonstrations: Charge distributions and charging by induction.
Week 3, Class 3 (76 mins)	Practice of Leaving Certificate past paper questions.
Week 4, Class 1 (35 mins)	Review of Topics with students.
Week 4, Class 2 (80 mins)	**Post-test.**

Table 5.1. Timeline of the Coulomb's law and electric field tutorial lessons.

5.2. Vectors, inverse square law and field lines in electric fields

This chapter presents a narrative and analysis of the development of the student's understanding of Coulomb's law and the electric field, in terms of their ability to apply vectors, the inverse square law and field lines to this domain. As the students have developed an understanding of these topics, as discussed in chapter 4, the tutorial lessons for this part of the research embed these topics throughout. This gives the students the opportunity to review the material covered in the electric field context, and develop a deeper understanding of the electric field through using multiple representations (Ainsworth, 2006).

The Coulomb's law and Electric field tutorials were designed to provide the students with opportunities to address the difficulties encountered by learners in their understanding of vector concepts, the inverse square law and field line representations detailed in Section 2.1.3. detailed difficulties. The following learning objectives for this section of the research ensued: upon completion of the teaching and learning material, the students would be able to:

- Construct and interpret a uniform and/or non-uniform electric field represented by vector arrows, consider both vector magnitude and direction (Maloney, *et al.*, 2001)
- Apply the principle of superposition to two vector field representations (Maloney, *et al.*, 2001; Nugyen and Meltzer, 2003).
- Students can discuss electric force and field superposition in terms of vector component addition (Furio and Guisasola, 1998; Cao and Brizuela, 2016).
- Demonstrate that Coulomb's law and the electric field follow an inverse square law using a variety of representations (Maloney, *et al.*, 2001; Hewitt, 2009; Moynihan, *et al.*, 2015)
- Apply proportional reasoning involving scaling to inverse square law problems (Arons, 1999, Marzec, 2012).
- Construct and interpret a uniform and / or non – uniform field using field line representations (Törnkvist, *et al.*, 1993; and Galili, 1993; Cao and Brizuela, 2016).
- Apply the principle of superposition to field line diagrams (Törnkvist, *et al.*, 1993; Galili, 1993).
- Accurately predict the behaviour of charged particles under the influence of an electric field (Cao and Brizuela, 2016).

5.3. Lessons learned from previous research

In chapter 4, it was seen that the use of tutorial lessons promoted the development of student understanding of vectors, the inverse square law and field line representations. This approach, patterned after Tutorials in Introductory Physics (McDermott and Shaffer, 2003), breaks concepts

and topics down into a series of lower and higher order questions, designed to elicit student thinking, identify difficulties and provide opportunities to help students overcome difficulties. The emphasis is on students working in groups to think and reason their way through the worksheet. This allows them to apply what they know, organise their thoughts and make judgements and evaluations about physical phenomena. The approach adopted produced various degrees of gains in understanding of vector concepts, though some conceptual difficulties with vector addition remained as it was observed that numerous students did not consider vector addition in terms of component addition. On completion of the inverse square law tutorial most students could recognise and represent an inverse square proportional relationship graphically, explain the variation in the area model using scaling and could apply inverse square proportional reasoning to mathematical questions. Finally, students made some gains in their understanding of field line representations such as relative field strength being represented by field line density, field vectors pointing tangentially to field lines at a point and field lines representing the direction of force experienced by a body, and not the path taken by a body under the influence of a field. However, difficulties related to the superposition of field lines persisted. These difficulties are addressed in the electric field tutorial, discussed in section 5.6.2.

These representations can be used as an aid for students developing conceptually accurate understanding of the electric field, at both second level and during further study at third level. Field theory replaced the preceding model of action at a distance and employs the use of both vector mathematics and mathematics involving the use of calculus. While the application of calculus to electric fields would be beyond the capabilities to the average second level student, an understanding the basic and slightly sophisticated vector concepts can underpin the foundations for future development of further advanced studies into electromagnetism. The inverse square law links to the model of a Gaussian sphere and electric flux density. In Ireland, students typically encounter this model until they have completed second level physics education. If they have not developed a complete understanding of the inverse square law, any developed difficulties would need to be overcome at third level. Field lines are a simple model than can be used to represent a field, and when interpreted correctly, can display a lot of information about a field that would be cumbersome to represent in another format. In developing understanding of these three “pillar” concepts, students can be guided to transfer their reasoning to representations they can struggle with, such as the mathematical formula for Coulomb’s law, or problems that employ the use of unfamiliar contexts.

5.4. Student’s use of vectors in electric fields

This section presents the student’s use of vectors in Coulomb’s law, and electric fields. Section 5.4.1 details the Electric field pre-test results, in which it was observed that students struggled to transfer and apply the vector concepts they developed in chapter 4, such as vector magnitude and

superposition. Section 5.4.2 narrates the use of the electric field tutorial lesson, in which the students review their understanding of magnitude, presented as uniform and varying electric fields. The students then apply the concept of superposition to finding the net electric field at various points using vector addition, and discuss the magnitude of the resultant vector in terms of its horizontal and vertical components. Section 5.4.3 discusses the post-test results, in which positive gains were seen in the student's understanding of vector magnitude. Sections 5.4.4 and 5.4.5 detail research on the student's understanding of horizontal and vertical components through the analysis of a student homework assignment, and a teaching and learning interview involving the concept. Section 5.4.6 presents a comparison of the pre-test/post-test results and discusses student gains observed during the tutorial lesson and teaching and learning interview, as well as illustrating persistent difficulties in student understanding of vector concepts and their transfer to the Coulomb's law and electric fields.

5.4.1. Pre-test: Student's use of vectors in electric fields

In the Electric field pre-test, the students were given a question where they were asked to construct an electric field using vector arrows at various points, based on their relative position around (a) a positive charge, (b) a negative charge, and (c) and positive and negative charge. In this way it was determined to what extent students could depict the direction of the field of the charge correctly, display the relative magnitude at different distances from the charges, and apply vector addition for the fields produced by a positive and negative charge at given points. The diagrams are presented in Figure 5.2, and a summary of student responses is presented in Table 5.2.

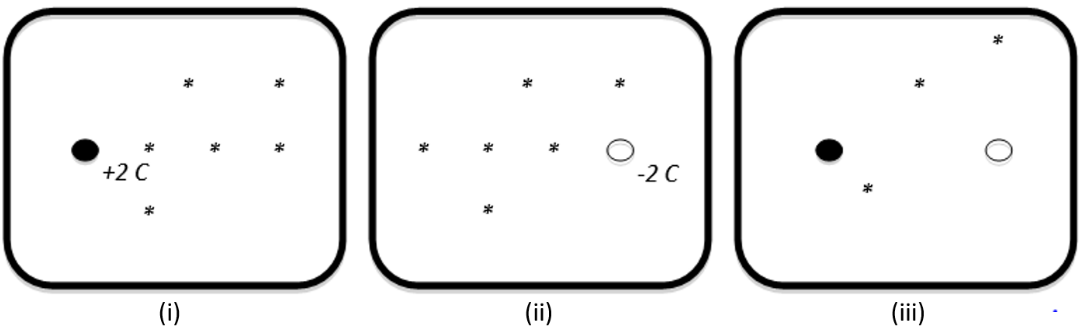


Figure 5.2. Pre-test question applying vectors to electric field context.

The pre-test result shows that a common difficulty was a reversal of the direction in which the vectors should point, (towards positive charge / away from negative charge) to represent the electric field. The students were familiar with using vectors to represent the gravitational field of a planet, so for them to apply the same directions for the vectors in the diagram for the first question was not

unreasonable. It followed that the next question, which asks them to use vectors to represent the field of a charge of an opposing sign, which resulted in the opposite direction being plotted.

Responses	Students
Direction point away from positively charged particle.	**4L.**
Vectors point towards positively charged particle.	4A, 4B, 4D, 4E, 4H.
Attempted field lines.	4F, 4G.
Non-discernable.	4C, 4J.
One vector, originating from charge.	4K, 4M, 4N.
Direction points towards negatively charged particle.	**4A, 4H, 4L, 4G**
Vectors point away from negatively charged particle.	4B. 4C. 4D, 4E. 4F, 4K, 4M, 4N
Vectors originate from charge.	4B, 4C, 4K, 4M, 4N
Attempted field lines.	4D, 4E. 4F, 4G
Non-discernable	4J

Table 5.2. Student responses to vectors and electric field pre-test question.

It was also observed that four students attempted to use field lines instead of vectors to represent the field, and a further five students drew their vectors originating from the charge. The latter suggests that the students have combined elements of the two representations or have confused the field vector for a displacement vector, representing the perceived path to be taken by a charge at a given point.

Table 5.3. presents summary of the vector representational errors made by the students in representing the magnitude of the electric field at the various points. The pre-test showed the students also had difficulties in representing magnitude of the electric field at various points. Only two students drew vectors in which the strength decreased as the distance from the charge increased. Two other students drew vectors that suggested the field strength increased at distances further from the charge, and while one student did not show any variation in field strength. The remaining students results showed difficulties that were not typical of those found in literature, such as vectors of varying magnitudes with no discernible patterns, or displacement vectors between randomly chosen points.

Responses	Students
Magnitude of vectors decreases with distance from charge.	**4H, 4L**
Magnitude of vectors increases with distance from charge.	4A, 4G
No variation in vector magnitude	4M
Magnitude variation follows no discernable pattern.	4C, 4D, 4E, 4F
Vectors magnitude is relative to distance to next point.	4B, 4J
N/a	4K, 4N

Table 5.3. Students responses to variation of field strength with distance.

In the final diagram (Figure 5.2, (iii)), students were required to draw the resultant electric field at three points. A summary of the results to this question is presented in Table 5.4.

Reasonable superposition of vectors demonstrated.	**N/a**
No superposition of vector demonstrated.	4A, 4B, 4C, 4D, 4F, 4H, 4J, 4L, 4M, 4N.
Superposition of field lines attempted.	4E, 4G,
Not attempted	4K

Table 5.4. Students use of superposition with electric fields.

This pre-test question showed that many students did not consider the superposition of their vector arrows from the previous two parts of the question to be appropriate, when drawing the electric field at the points highlighted. Figure 5.3 illustrates some of these difficulties. Ten of the students represented the field but showed no indication of vector addition. In some cases, students drew two vectors acting at the points (4A – Figure 5.3, i), displacement vectors from both charges (4C, 4F), vectors pointing to one charge only (4H, 4J, 4L – Figure 5.3, ii) or force vectors representing attraction between the charged bodies (4B, 4D, 4M, 4N – Figure 5.3, iii).

Two of the students attempted the use of field line representations instead and produced patterns consistent with their observations of the field between two planets (4E), as seen in the field lines tutorial lesson or a pattern consistent with the field lines of a bar magnet (4G).

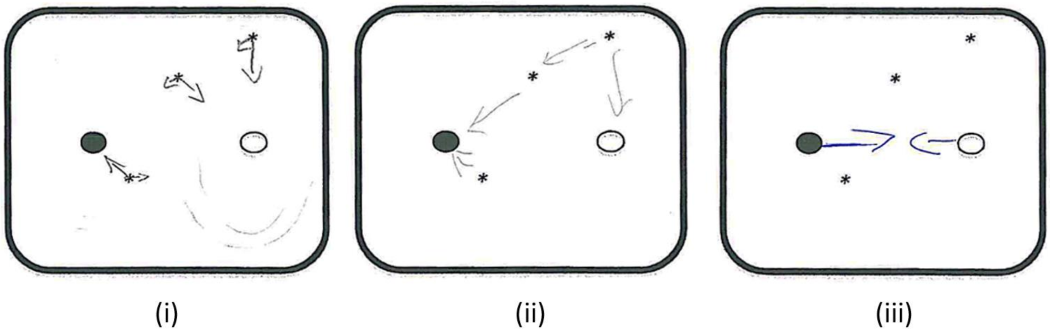


Figure 5.3. Student responses to applying the superposition of vectors to an electric field.

This pre-test showed poor performances by the students to apply their understanding to the electrostatic field context despite previous gains in their understanding of vector magnitude and vector addition. This inability to extend their understanding of vectors to apply it to an electrostatic context indicates that the approach in the tutorial lesson wherein the students are guided through the use of vectors to describe electric fields is justified. This can help promote their ability to transfer their understanding between these two domains.

5.4.2. Tutorial lesson: Student's use of vectors in electric fields

The electric field tutorial lesson was designed for students to transfer the concepts they developed in the tutorials outlined in chapter 4 to electrostatic fields, and further their understanding of electric fields. The students were presented with opportunities to represent electric field of 2 charge systems using both methods. This section will discuss the first half of the tutorial lesson, in which the students explored the use of vectors to show the superposition of an electric field, for a two charged-particle system. The latter half of the tutorial lesson, which looks at the use of field lines, is discussed in section 5.4.3.

The tutorial introduced students to a uniform electric field represented as vectors. The tutorial worksheet defined a uniform field as a field the is equal in magnitude and direction at all points. The students were then required to observe a uniform field and were asked to explain how the vector representation indicated the presence of a uniform field, as shown in Figure 5.4.

The students were required to justify that the vectors are of equal length, and all point in the same direction. Additionally, they were required to rank the electric field strength from strongest to weakest at various given points. While an understanding of vector magnitude should produce a ranking of A=B=C=D, there were discussions within group as to whether D>C>B>A was correct. The reasoning initially used to justify this was that as the vectors move from left to right, more vectors pass through each point. When asked to clarify what was moving, the students would reply that the

field was moving, as indicated by the direction of the arrows. However, when asked to recall a demonstration experiment for lower second level, in which they sprinkled iron filings over a bar magnet to show the field. They recalled that the field pattern they demonstrated did not move from North to South and determined the field. These students were then invited to revisit their notes from the presentation about how the electric field represented the force at an individual point, and with the aid of other students in their groups, they generally volunteered that neither the field or vectors move. The students generally came to the correct ranking during these discussions. In the questions of this initial section, the students were required to represent an increasing electric field, and decreasing electric field, which all students successfully completed.

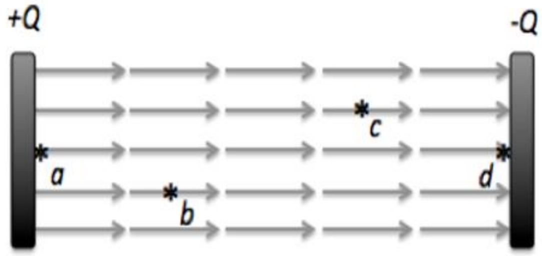


Figure 5.4. Uniform electric represented using vector arrows.

The tutorial then presented a two-dimensional vector addition exercise, in which the students were presented with the steps required to determine the combined field at a point, by utilising a vector construction to determine the superposition of two electric field vectors at a point, as shown in Figure 5.5.

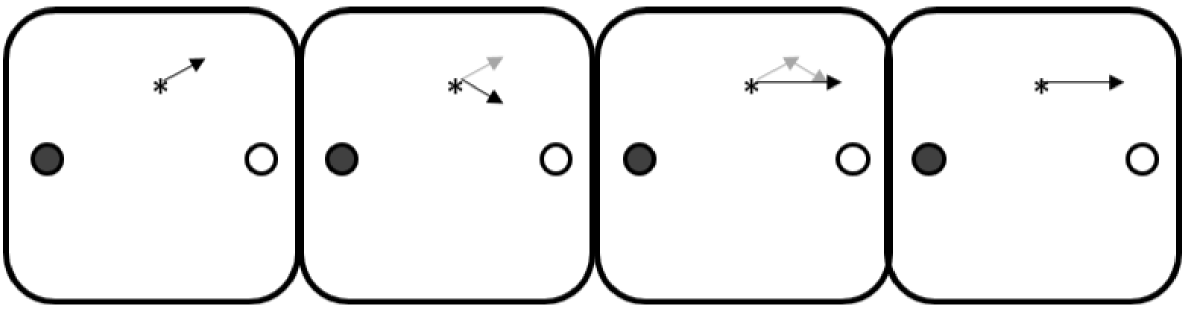


Figure 5.5. Demonstration of the superposition of two vectors representing an electric field.

The students were required to explain each of the steps, demonstrating and applying their understanding of adding vector components. By presenting the steps, and getting them to justify them, it was expected that the students could explain why the electric field is horizontal, and be able to explain the cause of the change in the resultant electric field magnitude, compared to the individual electric field vectors initially presented.

Student 4D: The vertical component cancel out, leaving only horizontal vectors. The vertical components cancel out, because they go in opposite directions, leaving only horizontal components to add to get the magnitude.

The students then were presented with another diagram like that shown in Figure 5.5, in this case where the two particles were positively charged, as opposed to positively and negatively charged. The students were required to use the either the “tip to tail” or parallelogram construction to represent resultant vector. The students were also asked to explain why and how the direction and magnitude of the resultant vector was affected in terms of the horizontal and vertical components. Again, the student groups produced reasoning in which the horizontal vectors cancelled each-other out, leaving only the vertical vector components to combine to produce the resultant electric field vector in a vertical direction.

In the last section of the tutorial lesson that addressed student’s understanding of vectors in an electric field, students were presented with the a positively and negatively charged pair of particles, and various positions were highlight for the students, in which they were required to construct the superposition of the electric field at various points. Errors were caused by students initially not considering the magnitude of their vectors to be different, regardless of their distance from the charges. When prompted to consider the field strength based on their distances, the students were quick to realise their errors, and redrew their vectors accordingly. An example of student 4L applying the parallelogram rule to the electric field is presented in Figure 5.6.

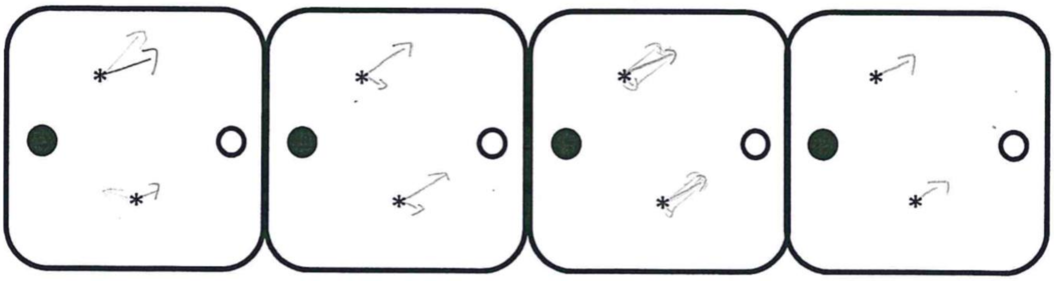


Figure 5.6. Student 4L applying the principle of superposition to represent the electric field.

This section narrated the tutorial guidance used to encourage the students to transfer their understanding of vector concepts to representing electric fields. The student’s ability to consider direction and magnitude of vectors was discussed in the initial section of the tutorial, student’s understanding of vector addition and how component addition affects the direction and magnitude is illustrated and the student’s ability to combine vectors using vector constructions was shown.

5.4.3. Post-test: Student's use of vectors in electric fields

In an Electric field post-test question, the students were presented with a diagram in which they were given the electric field surrounding a charged particle of unknown sign, as shown in Figure 5.7. The students were asked to use the vector representation to identify the sign of the charge and explain why the length of the vectors decreases as the distance from the charge increases. A summary of the student responses is shown in Table 5.5.

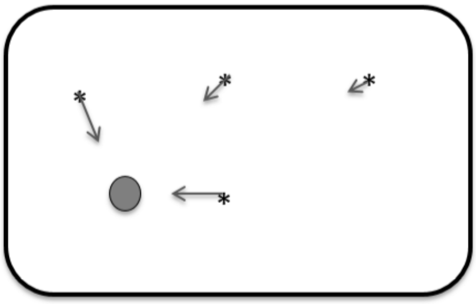


Figure 5.7. Diagram used in Electric field vector post-test question.

Response	Students
Charge is negative	**4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4L, 4M, 4N.**
Charge is positive	4A, 4F, 4K.
Associates direction with negative charge	**4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4L, 4M, 4N.**
Associates vectors with electrons.	4A.
Positive pulls in all arrows	4F.
Associates colour with charge	4K.

Table 5.5. Student responses to Electric field vector post-test question.

Upon completing the post-test, it was clear that most of the students could identify the sign of the charge based on the information in the diagram. Students 4A and 4F associated a negative charge to the arrows and treated them as electrons. This was a persistent difficulty for 4F, as discussed in section 5.4.5, in which they associated a gravitation force to field lines. This would suggest they consider the field to be a tangible construct (Galili, 1993), regardless of whether it is represented with field lines or vectors. Another student, 4K, associated a positive charge to diagrams in which the body is represented as a dark circle, as was done in the tutorial. In the tutorial, this was done to

help students differentiate the two charges with ease but was never intended to set a convention to indicate charge.

The students were then asked to construct the resultant vectors when particle with opposite charge was placed in close vicinity to the original charge, as shown in Figure 5.8.

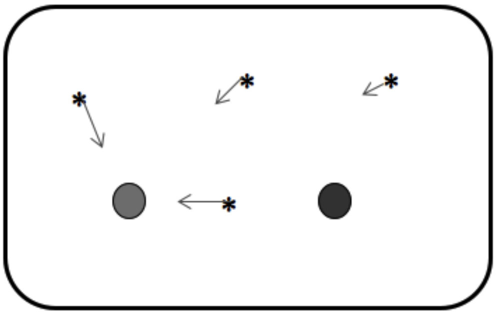


Figure 5.8. Post-test electric field question in which student sketch arrows to represent field components due to positive charge

This would allow me to determine if the students could represent the vectors using the correct direction, display reasonable relative magnitudes and use of the principle of superposition to determine the resultant field vectors. The results are presented in Table 5.6.

All students drew a second set of vectors representing the electric field due to the second charge. Twelve students drew these vectors with a length that varied reasonably with distance from the charge; ten of these correctly drew vector pointing away from the second charge, while two drew vectors pointing towards it, .

The answers of students 4J and 4M are shown in Figure 5.9. Student 4J drew vectors of equal magnitude perpendicular to the original vectors Student 4M drew field lines resembling a dipole field. This strategy could have worked, but the student used superposition to add a vector derived from the dipole field line pattern they drew (i.e., the correct answer) to the original vectors.

Seven students correctly used either the superposition principle, including two students (4A and 4F) who had difficulties in identifying and representing the direction for electric field for the different charges. Additionally, three students (4B, 4C and 4D) correctly applied the principle but for reasons unclear did not apply it to all the points. The remaining students did not apply the superposition principle at all.

Across the three questions in the post-test, it is observed that only two students, 4G and 4I, produced answers which showed they could completely transfer their understanding of drawing and adding vectors to the domain of electric fields. The other students showed errors in some aspect in representing the resultant electric field at the various points, in which case eight students only made one error (4B, 4C, 4D, 4E, 4F, 4H, 4K and 4N) in using vector representation for the electric field of the two charges. The remaining students made two or more errors in the representational transfer.

Responses	Students
Second vector arrow point away from positive charge.	**4B, 4C, 4D, 4E, 4G, 4H, 4I, 4K, 4L, 4N.**
Second vectors point towards positive charge.	4A, 4F.
Vectors are forced to be 90° to previous field vectors.	4J.
No pattern to the vectors	4M.
Reasonable variation of vector length with distance	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4K, 4L, 4N.**
Unreasonable variation of vector length with distance.	4J.
Field line representation used	4M.
Appropriate use of superposition	**4A, 4F, 4G, 4I, 4J, 4L, 4N**
Appropriate use of superposition for some / one of the positions.	4B, 4C, 4D
No superposition of vectors found.	4E, 4H, 4K, 4M.

Table 5.6. Student's application of vector concepts to electric field context.

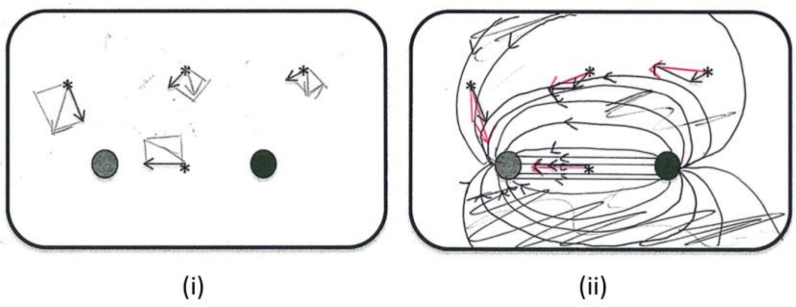


Figure 5.9. Errors in electric field vectors by (i) student 4J and (ii) student 4M.

5.4.4. Homework: Student's use of vectors in Coulomb's law

The students also completed a homework question, in which they could apply their understanding of vector components to a conceptual force question, presented in Figure 5.10. The students were asked to compare the net force acting on the -1 C charged body in (a) with that of (b), and then with that of (c). The question invited the students to use whatever reasoning they deemed appropriate and suggested vector reasoning, calculations or any other reasoning deemed fit by the students. While the vector nature of Coulomb's law was discussed in the class discussion before the tutorial, the tutorial itself did not directly look at the vector nature of electrostatic forces. This question tested if students could transfer their reasoning of vector component directly to the electrostatic context without explicitly exploring it in the tutorial. A summary of the student's responses is shown in Table 5.7.

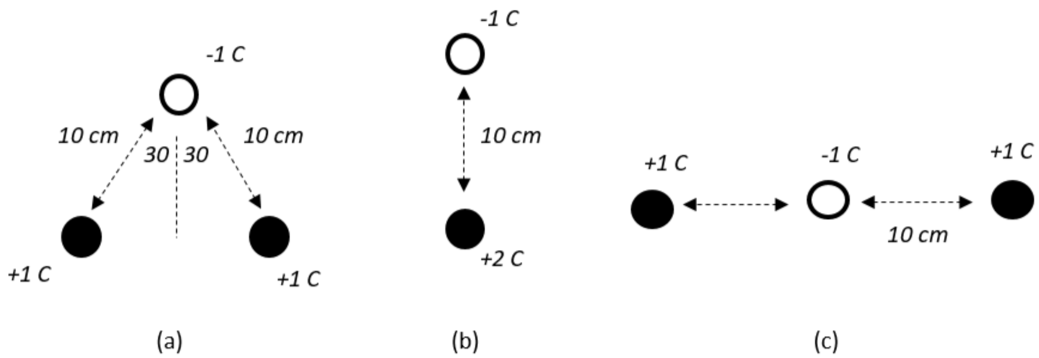


Figure 5.10. Coulomb's law vector concept question

Student Reasoning	Students.
Applied vector component reasoning	**N/a.**
Applied incomplete vector component reasoning	4K
Applied scalar reasoning.	4G, 4M
Described forces in terms of attraction and repulsion.	4C
N/A	4A, 4B, 4D, 4E, 4F, 4H, 4I, 4J, 4L, 4N

Table 5.7. Student's application of vector components to electric field context.

None of the students gave a complete answer: the horizontal components in (a) cancel, resulting in a net magnitude less than (b), and that the horizontal forces in (c) would sum to zero. Student 4K, when describing the forces in diagram (a), acknowledged that there is a combination of horizontal

and vertical vector components acting on the negatively charged particle, while only vertical and horizontal vectors act on negatively charged particle in (b) and (c). However, they stated that the net force is stronger in both cases (b) and (c), and did not consider that the horizontal net force is zero in (c).

Both students 4G and 4M used the equation $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$ in all the setups, and in setup (c) treated the two positively charged bodies as one charged body with a magnitude of +2 C. Student 4M also mentioned that the force would be slightly reduced due to the repulsion between both positive charges, but did not explain this in any detail. Student 4C interpreted the question to explain whether attraction or repulsion existed between the different combinations of charged particles. The remaining students, bar three who were absent and unable to complete the homework, did not make headway with the question. They stated that they were unaware how to approach the question. This indicated that the suggestion to use vector reasoning or calculations did not prompt them to use the understanding they had previously developed in the context of electric fields.

5.4.5. Interview: Student's use of vector components in Coulomb's law

There was no post-test question developed to elicit student's thinking about vector components in Coulomb's law, or electric fields. Instead three of the students were interviewed. They were asked to revisit the homework question discussed in section 5.4.4. The students were told they were permitted to ask questions during the interview to help them along, but they were not permitted to ask directly for the solution.

At the beginning of the interview, students 4A, 4B and 4H stated that the forces would be equal in all cases as the -1 C charged particle was being attracted by a net charge of +2 C, in all cases. However, upon being informed that their reasoning was incorrect, and the net force on the particles was not equal in all cases, they considered the use of vectors to analyse the question. The following interview extract illustrates the student's reasoning.

Student 4H:	The distance is there [a] cause that one will be pulled down the centre line. That is just as strong as charge [b], but it is is the most [strongest force], cause it is direct. And that one will cancel out [c], so it'll be zero. That one [b] will be twice as much if that was one [c].
Teacher:	So, C = zero. Why did you say that?
Student 4B:	Cause it cancels out.
Teacher:	What cancels out?
Student 4H:	The horizontals.

Teacher:	Ok… so now we have horizontal vectors. What type of vectors do we have acting here [b]?
Student 4A:	Vertical vectors.
Teacher:	And everything is vertical? [Students nod in agreement] Ok, so let's just say here [one horizontal vector is sketched on c] is 10 N, and this [vector sketched acting in opposite direction] is 10 N, now what's the force acting on this [b]?
Student 4H:	20 N.
Teacher:	Ok, so look here [a] and ignore this [right positive charge]. What force acts on the negative charge?
Student 4B:	10 N.
Teacher:	Now ignore this [left positive charge] What's the force?
Student 4B:	10 N.
Student 4H:	And we can add them tip to tail now.
Teacher:	We can, but also, consider, you mentioned horizontal and vertical components earlier. Keep the idea of components in your head. Do you think the 10 N and 10 N will sum to 20 N?
Student 4B:	The horizontal components in that one [a] will cancel out.
Teacher:	So, you're only left with what?
Student 4H:	Just vertical components.

When the students had to consider alternative reasoning they resorted to vector reasoning without much prompting. The extract shows the teacher did not volunteer any reasoning the students did not mention themselves but guided them to use the reasoning in the three different layouts. The student's reasoning was based on how the component vectors combined. Based on this, they produced an accurate ranking of B > A > C = 0, to represent the net force on the negative charge in each layout.

5.4.6. Discussion

This section presents a discussion of the student's use of vectors in Coulomb's law and the electric field. It presents a comparison of the student's ability to represent the variation in field strength using vectors, and their ability to apply the principle of superposition of electric fields using vectors. Figure 5.11 presents a comparison of the student's responses in the pre-test and post-test, looking at student's representations of the electric field strength at various points in an electric field.

In the pre-test, it was observed that two students accurately used vectors to represent the magnitude of the electric field at points at various distances around a charged particle. As the students

showed good understanding of vector magnitude, as discussed in section 4.2.5, the following reasons could explain the student's difficulties:

- The students consider that a form of proportional relationship exists between electric field strength and the distance from the charged particle.
- They do not consider electric field strength to vary with distance at all.
- They do not consider electric field strength to be a vector quantity.
- The students were unable to differentiate between electric field and other vector quantities, such as displacement.

These reasons are suggested as they are based on the interpretations of conversations with the students during the tutorial lessons and interpretations of discussions overheard between the students combined with interpretations of the student artefacts which were scanned.

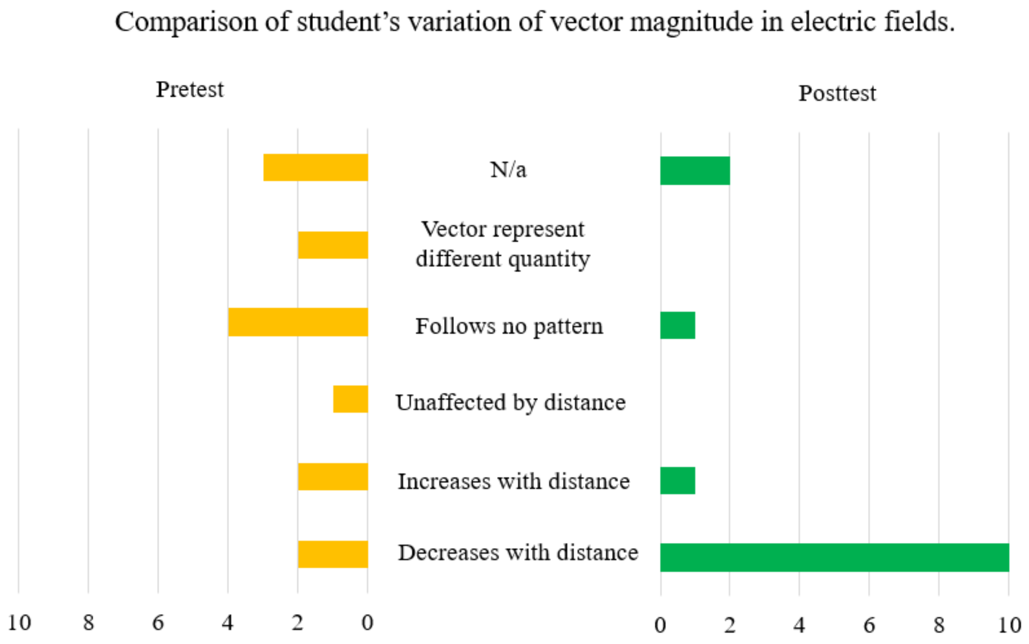


Figure 5.11. Comparison of student's representations of vector magnitude for an electric field.

The post-test showed considerable gains by the students. Most of the students (10/14) correctly represented the change in field strength with distance using vectors of different lengths. The shift in student's abilities to correctly represent the field lines indicates that conceptual exchange occurred (Hewson, 1992), with the comparison of the pre-test and post-test indicating that moderate conceptual change occurred. Ten of the students applied the magnitude convention to their vectors in the post-test, and no longer showed the difficulties seen in the pre-test. Through completing the tutorial lessons, the students applied the reasoning they developed in section 4.2 to a new context, utilising the representation as a useful tool to represent and verbally explain a vector field pattern in an unseen context (Posner, *et al.*, 1982). Not all the difficulties were overcome however, as two of

the student's presented difficulties in which they used field lines and then drew vectors to match the field line pattern, or used vectors previously presented on the diagram as a scale to determine the magnitude of the vectors.

Figure 5.12 presents a comparison of the pre-test and post-test results for the student's use of the superposition principle. In the pre-test, none of the student's applied it to electric fields. This suggests difficulty in transfer to this context, as all the students demonstrated the ability to construct resultant vectors in section 4.3.4. The difficulties reflect those seen in literature when learners struggle to apply vector concepts to the electric field context (Maloney, *et al.*, 2001).

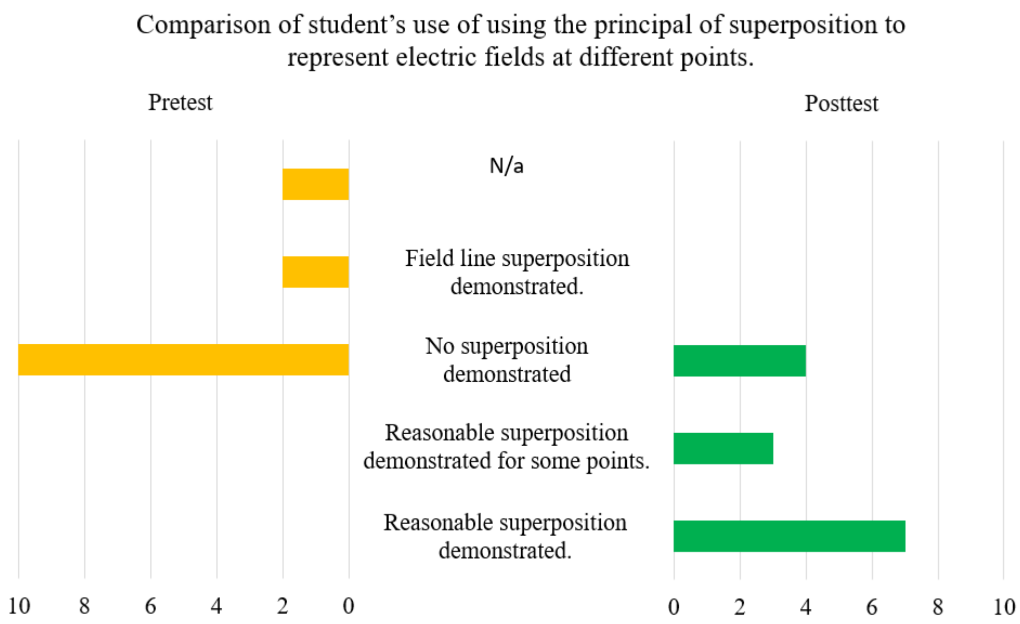


Figure 5.12. Comparison of student's use of superposition to draw an electric field using vectors.

After the tutorial lesson, 10/14 students applied the principle of superposition to the electric field in the post-test. This indicates that students did not initially transfer the skills they developed, as discussed in section 4.2, to the context of electric fields. The students demonstrated this skill in the vectors tutorial but did not transfer this skill to the electric field context until after they had completed the electric field tutorial. This suggests that conceptual extension occurred from completing the tutorial lesson (Hewson, 1992), with a moderate conceptual change observed from comparing the pre-test and post-test results. As discussed in section 5.4.2, the tutorial gave the students the opportunity to practice the vector constructions they previously developed in the context of electric fields, furthering their overall conceptual understanding by applying it to new contexts (Konicek-Moran and Keeley 2015).

Marzec (2012) suggested that learners require multiple opportunities to develop understanding of the inverse square law. The pre-test/post-test comparison and discussion of the tutorial lesson suggest that this could also apply to using vectors in multiple contexts. The students did not

experience any significant difficulties in applying the vector constructions in the tutorial lesson, which indicates that the central difficulty in the pre-test was students not realising the application of their vector understanding. This was also observed in the Coulomb's law homework, in which the students were unable to apply the reasoning they developed in the vectors tutorial question described in section 5.2.4. Apart from one student, the students did not recognise that vector components were applicable to the question given. However, during the electric field tutorial, the students had little difficulty in applying vector component reasoning when drawing the electric field at various points around two charged bodies. The student interview, in section 5.2.5, also indicates that with minimal guidance, the students applied vector reasoning to the homework question. Initially the students based their equal ranking on the net positive charge in each case, instead of the positioning on the charges. This indicates that students value magnitude as the most important aspect of force. This, in turn, suggests that for the transfer of vector concepts to electric fields to be complete, students require the opportunity to develop their reasoning in this context in a classroom setting, such as a tutorial and/or a class discussion, as was the case in this research.

5.5. The inverse square law applied to electric fields and Coulomb's law

The following sections discuss student's understanding of the inverse square law and how they applied it to Coulomb's law and electric fields. Section 5.5.1 discusses results based on student's answering a mathematical problem in a pre-test question, and a problem involving the use of vector representation to show their understanding of the inverse square law. Section 5.5.2 discusses the Coulomb's law tutorial lesson, which focused on the student's understanding of proportionality and the use of tabular data, graphs and mathematical methods to explore the inverse square law. Section 5.5.3 discusses a homework assignment applied the scale model, adapted from Conceptual Physics (Hewitt, 2009) to the electric field. A difficulty in student's understanding of the scaling of area is identified, and a section of a teaching and learning interview is presented to show how students overcame the difficulty. Section 5.5.4 presents the results of a post-test that looks at the student's ability to represent the inverse square law on a graph, how students differentiate between an inverse and inverse square law graphically, student's use of the inverse square law in a mathematical question, and their understanding in change of an area based on a scale model.

5.5.1. Pretest: Coulomb's law and inverse square law

This section discusses the results from the Coulomb's law pre-test, in which the students had to use vector arrows to demonstrate their understanding of a directly proportional relationship, and a

relationship that follows the inverse square law. The latter part of this section presents students graphical representations of a directly proportional relationship, and the inverse square law.

In the first question, the students were asked to state the relationship, between the magnitude of the force, and the distance between them, when presented with the Coulomb's law equation. This allows for gauging the student's ability to recognise relationships in algebraic form and transfer them between tabular, graphical, diagrammatic and mathematical symbolic representations. The students were familiar with the general structure of the Coulomb's law equation, as they had studied Newton's gravitational law and they were formally introduced to Coulomb's law in a presentation and class discussion preceding the tutorial lesson. The equation was presented to the students in the form $F = k\frac{q_1 q_2}{d^2}$. This form presents Coulomb's law as a scalar equation, as the Leaving Certificate Physics course does not employ the use of vector algebra. The students were made aware that this equation can only be used to determine the magnitude of the force between two charges, and when the students are required to determine the directions of the forces acting on the charges, other appropriate methods are employed. A summary of the student's results are presented in Table 5.8.

Inverse square relationship	**4H, 4K, 4M**
Inverse relationship	4G, 4E
Increase distance, decrease the force.	4A, 4C, 4I, 4J
Directly proportional relationship	4B, 4D
N/a	4F, 4L, 4N

Table 5.8. Student pre-test responses to transferring from equation to verbal relationship.

The results show that three of the students could glean the formal relationship between both quantities from the law equation. Two students stated that the force was inversely proportional to the distance, suggesting the students did not observe the index of distance variable, or did not consider its relevance in defining or naming the relationship. A further four students could relate the position of the distance variable as a denominator to determine that increasing the distance from the charges reduces the magnitude of the force between them. Two students were unable to relate the positions of the variables in the equation and determined the variables were directly proportional to each other, while three students did not give an answer.

The students were then presented with a mathematical question, in which they had to apply the inverse square law. The students were not provided with a value for *k* in the Coulomb's law equation, so they would have to employ proportional reasoning to answer the question. The question is provided in Figure 5.13. The student's results are presented in Table 5.9.

Two +8 C charges are held a distance of 10 cm from each other, and the force acting on both charges is 90 N. The charges are moved so the distance between then is now 30 cm. What is the new force acting between the charges? Explain how you know what the change in force is.

Figure 5.13 Pre-test question seeking to elicit student's ability to mathematically apply inverse square law

As the distance between the charges was increased by a factor of 3, the magnitude of the force was reduced by a factor of 9. Only one student, 4C, completed this. It is interesting to note that 4C responded to the previous question by stating the force would decrease without quantifying this decrease, but provided the correct outcome in this question. Three students (4D, 4G and 4K) reduced the force by a factor of 3. Two of these responses were surprising considering student 4D previously stated there was directly proportional relationship and student 4K previously stated there was an inverse square relationship between the variables. This indicates the students did not understand the nature of the relationships or did not apply them in this context. Four students attempted to use the formula to attempt the question, but were unable to complete their calculations since they did not know a value for *k* and could not determine how to tackle the question. This difficulty echoes that shown by Arons (1999). Two students (4E and 4M) respectively stated that the force would increase and decrease, but they did not quantify their answers. This answer was not consistent with 4E's previous question, in which they stated there was an inverse relationship, and 4M did not apply the inverse square relationship they stated in the previous question. The remaining students did not answer this question.

Responses	Students
Reduces force to one ninth original	**4C**
Reduces force to one third original	4D, 4G, 4K
Attempts calculation	4B, 4F, 4L, 4N
Increases force	4E
Decreases force	4M
N/a	4A, 4H, 4I, 4J

Table 5.9. Student's pre-test responses to applying the inverse square law mathematically.

In the final question of this pre-test, the students were asked to use vector arrows to show the change in magnitude of the force with distance. The students were presented with two charges, with vector arrows showing an attractive force between the two charges. They were asked to determine

the effect of doubling the distance between the charges and representing this using vector arrows. The question is presented in Figure 5.14, and the student's results are presented in Table 5.10.

As can be seen in Table 5.10, none of the students correctly represented the vectors by reducing the magnitude by a factor of four. The two more prevalent vector representations showed that the force would reduce by a factor of two or would not be affected at all. This highlights inconsistencies from the four students that previously defined or applied the inverse square law, and the students who applied an inverse relationship, or explained the increasing the distance would decrease the force felt by the charges.

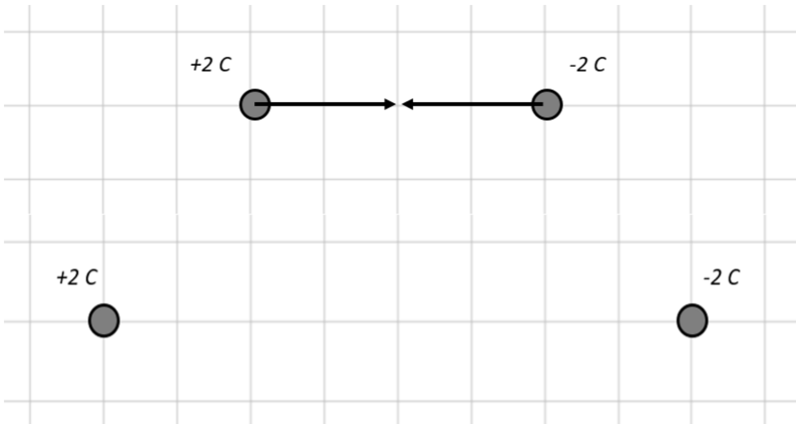


Figure 5.14. Pre-test question utilising the inverse square law and vector representations

Responses	Students
Quarters both vectors.	N/a.
Doubles both vectors	4B
Halves both vectors	4A, 4C, 4D, 4M, 4N,
Increases vectors	4J
No change to vectors.	4E, 4F, 4G, 4H, 4I, 4K, 4L

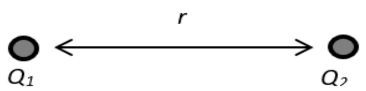
Table 5.10. Student responses to pre-test question that looked at student's application of the inverse square law and vector representations.

These pre-test results indicate that the students encountered difficulties in transferring their understanding of the inverse square law to Coulomb's law. Only a small number of students recognised the inverse square law in the Coulomb's law formula, and only a single student applied the law mathematically. None of the students presented the inverse square law using vector representation, and most of the students displayed reasoning consistent with an inversely proportional relationship.

5.5.2. Tutorial lesson: Coulomb's law and inverse square law

During the Coulomb's law tutorial lesson, the students were introduced to a formal definition of Coulomb's law, in which the magnitude of the force between two point-charges is directly proportional to the product of the magnitude of the charges, and inversely proportional to the square of the distance between them. The lesson aimed to allow students to verify these relationships themselves, using tabulated data, graphs and calculations based on the formula. In the first half of the tutorial, the students were guided through this process for a directly proportional relationship, and in the latter half, they had to apply the same skills to show an inverse square proportional relationship.

The students were presented with tabulated data for the force between two charges, and 3 columns with values for the magnitude of the first charge only, the magnitude second charge only and the values for the product of magnitudes of the two charges, as shown in Figure 5.15.



q_1 (C)	q_2 (C)	$q_1 q_2$ (C^2)	F (N)
1	1	1	2
1	2	2	4
2	1.5	3	6
4	1	4	8
1	5	5	10
2	3	6	12

Figure 5.15. Data set from Coulomb's law tutorial, relating force to product of charges.

The students were to identify which column, when coupled with the force values, provided a familiar pattern, being linear, quadratic, exponential, inverse or inverse square. The students then rewrote these two columns in a separate table and were guided to show that the ratio of force to the product of the charges was constant, by dividing the values for F by the values for q_1q_2. This would allow the students to demonstrate that the pattern followed the general form $y = mx$. The students were then asked to graph their data and explain how the shape of the graph showed a directly proportional relationship. They were required to use their graph to determine the magnitude of the force when the product of the charges was 2 C^2 and 6 C^2. From this, they explained that tripling the product of the charges has the effect of tripling the magnitude of the force.

The students were then required to complete calculations to show the effect of tripling the produce of the charges. They were provided with a sample calculation between a 6 μC charged

particle and a 3 µC charged particle that were placed a distance 1 cm apart, as shown in Figure 5.16. The mathematical operations were completed in the sample and they were required to identify which operations took place. This ensured they were familiar with the nuances of completing the calculations and aimed for students to avoid errors in performing the calculations. They then completed similar calculations, between two charges of magnitudes 3 µC and 9 µC.

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

A

$$F = \frac{1}{4(3.14)(8.9\,x\,10^{-12})}\frac{(6\,x\,10^{-6})(3\,x\,10^{-6})}{(1\,x\,10^{-2})^2}$$

B

$$F = \frac{1}{1.12\,x\,10^{-10}}\frac{(1.8\,x\,10^{-11})}{(1\,x\,10^{-4})}$$

C

$$F = (8.9\,x\,10^{9})(1.8\,x\,10^{-7})$$

D

$$F = 1{,}602N$$

Figure 5.16. Coulomb's law tutorial extract, in which students identify the operations in the calculation.

In the second section of the tutorial lesson, the students were asked to develop the reasoning to show the relationship between the force between two charges and the distance between them was an inverse square relationship. Students were given the table of data shown in Figure 5.17, and they were required to either use the data directly to show it followed an inverse square relationship or place it on a graph and use the graph to do so. During the inverse square law tutorial, as discussed in section 4.3.2, the students can pick x-values from the domain of the graph and read the corresponding y-values for these, they can calculate what factor the values change by and determine if an inverse square law was observed. For instance, choosing the values 1 and 3 from the x-axis would produce a decrease by a factor of 9 in the two y-axis values, if it followed an inverse square law.

To complete the tabular section of the tutorial lesson, the students were presented with the general forms of linear equations, quadratic equations, inverse equations and inverse square equations, in which they were shown what operations between the variables produced a constant. The general forms of $y = \frac{k}{x}$ and $y = \frac{k}{x^2}$ were less familiar to the students, as they generally picked either the quadratic or the inverse relationship initially to use to manipulate the data. However, it was clear that the ratios of force to the square of the distance or the products of force and distance values

did not produce a constant. When the students picked the inverse square proportional equation, the students determined that the product of the square of distance and the force produced a constant. Overall, this section proved to be the most challenging for the students, and took all groups approximately twenty minutes to complete, as they picked the different relationships and explored them.

The students were also required to show the inverse square law on a graph. It was observed that the students plotted the data on the graph, which produced the characteristic curve typical of an inverse square law. However, the students were satisfied that this graph demonstrated an inverse square law, based on its shape, even though it was similar in shape to an inverse graph of the form $y = \frac{k}{x}$. As the students had data points to follow, they did not repeat the errors of drawing quadratic patterns, or linearly decreasing patterns, discussed in section 4.3.4. All groups had to be prompted to pick two points on the graph and show, in this case limited by the domain and range of the data, that double the distance would result in one quarter the force.

Directly proportional general equation: $y = mx,\quad \frac{y}{x} = m,\quad \frac{y}{x} = constant$

Directly proportional to square equation: $y = ax^2,\quad \frac{y}{x^2} = a,\quad \frac{y}{x^2} = constant$

Inverse proportional general equation: $y = \frac{k}{x},\quad xy = k,\quad xy = constant$

Inverse square proportional equation: $y = \frac{k}{x^2},\quad x^2y = k,\quad x^2y = constant$

y	F (N)	100	25	11.11	6.25	4	2.78
x	d (m)	1	2	3	4	5	6

Figure 5.17. Coulomb's law tutorial extract, using data to demonstrate the inverse square law.

In the last section of the tutorial, the students were presented with the Coulomb's law equation, with a set of data, and asked to determine the relationship, as shown in Figure 5.18. While this section took time for the students to complete due to the many steps involved in the calculations, no groups found it overly challenging. They did need to be prompted use the values of the two forces to make a ratio at the end of the calculations, in which they could determine the factor decrease observed when the distance between the two charges was doubled. This method is similar in nature to that described on the previous page, involving the use of graphs.

At the end, the students were asked which method they found to be the most effective and simplest to use. Six of the students mentioned that they preferred to use the equation method, as they stated there was a structured approach to follow, in which the ratio was easy to determine. Four students preferred the graphical method, in which the students read the values for the forces directly from the graph and developed the inverse square relationship from their values. They found the method easier than the others, and the values can be easily obtained. Teacher observations from this lesson stated that initially, the students only used the shape of the graph as the reference to the inverse square relationship, and the values were initially ignored. This observation is considered in the post-test question utilising a graph, see section 5.5.4.

This section illustrated that students encounter difficulties in analysing tabular data and graphical representations, when seeking to determine the mathematical relationship displayed. Difficulties seen in using tabular data was determine the steps required to manipulate the data to show that $x^2y = k$, even though they were guided through the process for a linear pattern. This indicates difficulty in both their mathematical ability and the physics context in which the pattern is applied. While it may have been preferable to do this mathematical work during the tutorials completed in chapter 4, time constraints did not allow for this and instead, it was integrated into the electric field tutorial lesson. The student's unfamiliarity with the approach presented in the tutorial is likely to contribute to the difficulty observed, as the method used for analysing data in a table, for graphical analysis in their math course, is not applied to this function (LC Project Maths syllabus, NCCA, 2013). The prevalent difficulty involved in the student's use of analysing the graph was an over-emphasis on the shape of the graph itself. Students tended to ignore the values from the graph and did not analyse the values to show an inverse square pattern, suggesting they would be unable to differentiate between an inverse graph, and an inverse square graph.

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

$\varepsilon = 8.85 \times 10^{-12}\ F\ m^{-1}$, $q_1 = 6 \times 10^{-6}\ C$, $q_2 = 4 \times 10^{-6}\ C$.

$d_1 = 4\ cm$, $d_2 = 8\ cm$

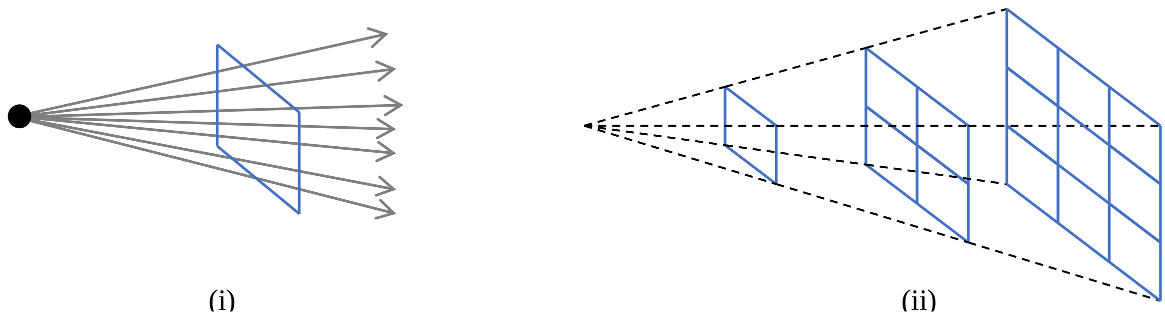
Figure 5.18. Coulomb's law tutorial extract, to demonstrate inverse square relationship mathematically.

5.5.3. Homework: Electric field and inverse square law

This section presents an analysis of the electric field homework tutorial. This was developed to apply the scale model adapted from Conceptual Physics (Hewitt, 2009) and the use of field lines in lieu of spray paint droplets, as discussed in the previous chapter, in section 4.3.2.

The students were presented with the diagrams shown in Figure 5.19. They were told that 100 field lines diverging from a charged particle placed at the left of the diagram pass through the first frame, as shown in Figure 5.19 (i). The model was designed to give students an appreciation of concept of an electric field integrated over the surface area of a sphere, without formally introducing electric flux or Gauss' Law for closed surface contexts. The students were required to explain why the area increased in the pattern shown in the diagram, and to explain what effect this had on the lines passing through each frame as they moved from left to right, as shown in Figure 5.19 (ii).

Five students qualitatively described the increase in area as the frames moved from left to right. This reasoning revolved around the distance from the charge being greater results in a bigger area, without articulating that field lines diverge as the distance increases. One student described the field lines diverging further apart as the distance increase as a reason for the increased area. Three students explained that increasing the distance from the charge increases both the length and width of the frame, increasing the area as shown in the diagram.



(i) (ii)

Figure 5.19. Electric field line homework extract, applying the scale model to electric field and field lines.

The remaining students used mathematical calculations, in which they squared the values of two to get four. It is unknown if this represented squaring the length of the square frames, squaring the factor in which the distance from the charge to the frame, or both. The students consistently repeated applied their reasoning to the 9 frames, and all students determined that the number of field lines passing through each frame would be 25 lines and 11.1 lines respectively. When asked to explain the behaviour of the field lines passing through the frames, eight of the students qualitatively explained that the field lines were spreading out more the distance increased, resulting in a lower number of

field lines passing through the individual frames. Only four of these students added that this models an inverse square law. The remaining students did not answer this question.

Explaining the change in area proved to be difficult for some of the students. During an interview with students 4C and 4E they resorted to relating the change in area to the distance from the charge, whilst not considering the change in the lengths and widths of the frames.

Student 4C: If that distance is 1, [distance to first frame] 1^2 is 1, which is 1 frame. Then this distance is 2 [distance to second frame], so 2^2 is 4, which is four frames.

It was clear that the students could use the model presented to describe the inverse square law but did not consider the source of the change in the area, in terms of scaling up the dimensions of the frames. The teacher engaged the students in a discussion to focus on the length and width of the frames, and to link these back to the increase in distance, to show the overall increase in the area.

Student 4E: The length of the second frame is 2, and the width is also 2.

Student 4C: The distance increases by 2, and so does the length and width.

Student 4E: And therefore, the area by 4.

This suggests that students need to be guided to focus how the change in distance affects the length of the frame and width of the frame separately. They can use the increases in these dimensions to explain the overall increase in the area of the frames.

This section has illustrated the student's application of scale model adapted from Hewitt (2009) to electric field lines. The context used displayed field lines passing through various frames and the students analysed this context to explain the relationship between the distance from a point to a charged body and the magnitude of the electric field strength at the point. Using the scaling model, they demonstrated the electric field follows an inverse square law mathematically. However, some students still struggled to articulate why the area increased in a quadratic pattern, and their final explanations did not reference the inverse square but did reference the divergence of field lines through different frames, showing a conceptual understanding of the behaviour of the model. The interview with two students illustrated the need to be guided to focus how the change in distance effect the length of the frame and width of the frame separately. They can use the increases in these dimensions to explain the overall increase of the area of the frames.

5.5.4. Post-test: Coulomb's law, electric fields and the inverse square law

The first question presented in this section was completed by the students not immediately after completing the Coulomb's law tutorial lesson (which focused on the inverse square law) but as part of the electric field pre-test. As this was given after the Coulomb's law tutorial lesson, the question presented can pragmatically be considered a post-test question.

In the question, the students were presented with the formula that relates the magnitude of the electric field strength at a point to the magnitude of the charge, and the distance of the point to the charged particle; $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2}$. The students were asked to explain the relationship between the electric field strength and distance based on the equation, and to graph the pattern that represents the mathematical relationship. The question is presented in Figure 5.20, and the results are summarised in Table 5.11 and Table 5.12.

The results indicate that 10 of the students associate the asymptotic pattern based on the relative position of the electric field strength and the distance variables in the equation. However, only two students, 4E and 4G, could determine the relationship from the equation. Five students stated an inverse relationship existed between the variables, which would also account for general shape of the graph. Student 4D was consistent in their explanation that doubling the distance would result in one quarter the electric field strength. Student 4F was consistent in graphing a directly proportional relationship, indicating they were unable to determine the relationship from the equation. Both 4J and 4N stated that increasing the distance would increase the field strength, but drew graphs contrary to this reasoning, also indicating they were unable to determine relationships based on the equations. The results suggest that students require more attention in overcoming difficulties in recognising and articulating relationships presented in equations, but progress was made in student's ability to transfer relationships from one representation to another.

4.. The electric field strength around a charge is given by the formula $E = \frac{1}{4\pi\varepsilon}\frac{Q}{d^2}$.

What is the relationship between the electric field strength and (i) the magnitude of the charge causing it, and (ii) the distance from the charge.

(i)

(ii)

Draw these relationships on the graphs below.

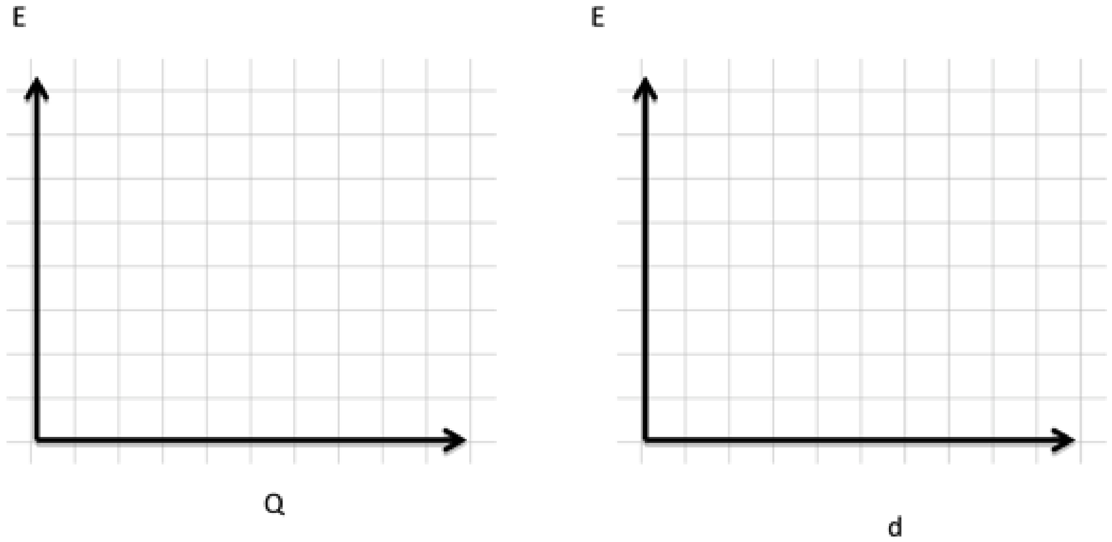


Figure 5.20. Electric field question in which students transfer inverse square law from symbolic to word and graphical representations.

Responses	Students.
Inverse square proportionality.	**4E, 4G,**
Inverse proportionality	4A, 4B, 4C, 4H, 4M
Illustrates inverse square relationship with examples.	4D
Directly proportional	4F, 4N.
Qualitative answer only.	4J, 4L, 4K
N/A	4I

Table 5.11. Student's responses from electric field pre-test, determining student's ability to transfer from equation to verbal relationship.

Responses	Students.
Asymptotic decreasing graph.	**4A, 4B, 4C, 4D, 4E, 4G, 4H, 4J, 4M, 4N.**
Directly proportional relationship.	4F.
Increasing quadratic curve.	4L
N/a.	4I, 4K.

Table 5.12. Student's responses from electric field pre-test, determining student's ability to transfer from equation to graphical representation.

The previous questions indicated that the students related an asymptotic pattern to an inverse and inverse square relationship. However, it did not determine if the students could recognise the difference between the patterns on the graph, or if they had just memorised the general shape of the graph. The first question presented in the Coulomb's law and Electric field post-test tested for this. The students were asked to pick out and justify the correct shape of both a directly proportional relationship and inverse square relationship in the graph shown in Figure 5.21. The students were also asked to determine which patterns showed the relationship between the force between two charges and (i) the product of their magnitude and (ii) the distance between the charges, to see if they could transfer the formal definition to a graphical representation. A summary of the student's responses is presented in Table 5.13.

The following graph shows 3 patterns for different types of functions, *A*, *B* and *C*.

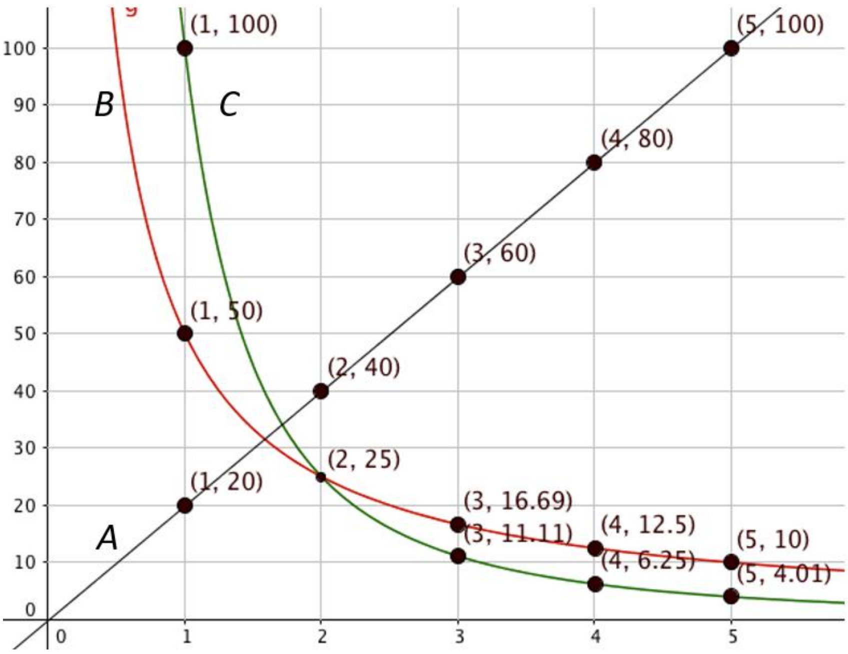


Figure 5.21. Graphical representations of a directly proportional, an inverse and an inverse square relationship.

Responses	Students.
Picks A as directly proportional.	**4A, 4B, 4C, 4E, 4F, 4G, 4H, 4I, 4J, 4L, 4M.**
Picks B as directly proportional.	4N
Picks C as directly proportional.	4D
N/a	4K
Picks C as inverse square.	**4B, 4E, 4G, 4H, 4J, 4I, 4L, 4M**
Picks B as inverse square.	4D, 4N
Picks B and C as inverse square.	4C, 4F
N/a	4A, 4K

Table 5.13. Student's post-test responses in determining relationships based on graphical data.

For both relationships, the results show that eleven and eight students respectively determined the correct graphical pattern for the directly proportional and inverse square relationships in Coulomb's law. The students primarily referenced the shape of the graph, being linear with a y-intercept of zero, to explain the direct proportion relationship. One student, 4E, also employed the use of a table to demonstrate the relationship, although they had apparent issues articulating the final reasoning.

Student 4A: They form a straight line… product (of charges) are proportional.

Student 4E: $y = mx$, is a straight linear function. y is directly proportional to x, it has to get bigger.

Student 4K: A. Because $y = mx$ is directly proportional, which tells us it is a straight line.

However, only 8 students correctly determined which of the graphs represented the inverse square relationship Typically students substituted values from the graphs into the two equations and saw which set of results fit the equation. Some students determined a value for "k" in the manner they were guided to use in the Coulomb law tutorial lesson (section 5.5.2). Mistakes commonly made by the students were that they looked at the pattern and recognised an inverse relationship, but could not differentiate between the inverse and inverse square pattern shown on the graph, in which they picked either B or both B and C.

Student 4B: (1, 50)
$50 = 100\left(\frac{1}{1^2}\right)$
Not curve B.
(2, 25)
$25 = 100\left(\frac{1}{2^2}\right)$
Curve C is of the form $y = k\frac{1}{x^2}$. As the distance increases, the force decreases

Student 4F: C is $y = k\frac{1}{x^2}$, as it does not decrease at a constant rate.

B. It is inversely proportional to the distance squared.

Student 4G: Coulomb's law states that the force between 2 point-charges is directly proportional to the product of the magnitude of their charges and inversely proportional to the distance squared. If x = 1, and y = 100, when the distance is doubled, it is 2 (x) and $100\left(\frac{1}{2^2}\right) = 25$ (y), and that point is on C.

Students 4N and 4D consistently picked B as the graph which represented both $y = k\frac{1}{x^2}$, and the magnitude of the force between two point charges as a function of the distance between them. Student 4C picked both B and C to represent the mathematical equation and the relationship in Coulomb's law, suggesting that they focus on the shape, but not the values produced by the pattern. Student 4F correctly determined that C represented $y = k\frac{1}{x^2}$, but chose pattern B as the representation of the inverse square relationship in Coulomb's law. As they quoted the inverse square law in their response, this would also indicate they focused on the shape over the values produced by the pattern. Students 4A and 4K did not complete this question, which suggests the tutorial was ineffective for these students in aiding their development of transferring the inverse square law to this context.

The results of the last two questions would indicate that 8 of the students were able to graphically represent the inverse square law on a graph and differentiate it from a pattern that represents an inverse relationship. Some students had persistent difficulties in which they ignored the data and based their responses on the recollection of the shape of the graph, focusing on the mathematical implication that as one variable increased, the other decreased. Additionally, while the students were able to apply their understanding of the inverse square law in this graphical form, they struggled to transfer the relationship from an equation into written form.

A later question on the post-test was designed to determine the student's ability to apply the inverse square law in an electric field context. The students were presented with a positive charge held in a fixed position, with 3 points around the charge, as shown in Figure 5.22. The students were required to rank the electric field strength around the electric field, and determine the ratio of the electric field between points A and C. The student's responses are summarised in Table 5.14.

The results from this question suggest that the students are clearly aware that as distance from the charge increases, the electric field strength decreases. Nine of the students showed that the electric field strength at *c* was one quarter the strength at *a*, whilst another two students (4E and 4J) produced a ratio that had a value of 1:4, but not in its simplest form.

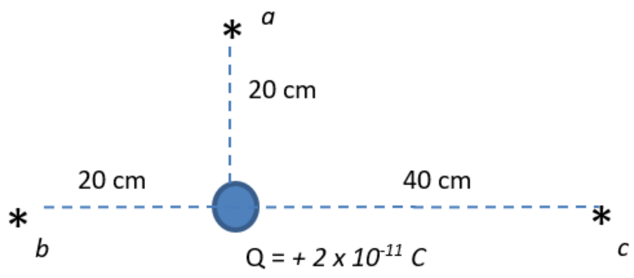


Figure 5.22. Post-test electric field question, testing understanding of inverse square law.

Student's reasoning was predominantly based on calculations (seven students) using $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2}$ or developing reasoning that doubling the distance would produce an electric field strength that was reduced by a factor of four (six students). All students who used the latter reasoning produced the correct ratio. In some cases, they substituted values for the charge and distance variables but ignored the $\frac{1}{4\pi\varepsilon}$ term in the equation. Two of the students made calculator errors, but they did not attempt to revise their work to develop alternative values for the magnitude of the charges. Two of the students simply halved the magnitude of the field strength when comparing *a* to *c*, as the distance was doubled.

Responses	Students
c < a = b	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M.**
Ratio of c to a is 1/4	**4B, 4C, 4D, 4G, 4H, 4I, 4K, 4L, 4M.**
Ratio of c to a is 1/4, not in simplest form	4E, 4J
Incomplete due to error in using calculations to determine ratio.	4F.
Ratio of c to a is 1/2	4A, 4N

Table 5.14. Student's post-test responses applying the inverse square law mathematically.

From looking at the examples of students work in Table 5.15, there were numerous errors in the student's calculations, which could still lead the students to develop the correct ratio. Difficulties that stand out are the use of scientific notation and prefixes, use of the $\frac{1}{4\pi\varepsilon}$ terms in the electric field equation, and misinterpreting electrostatic force for electric field strength.

<table>
<tr><td colspan="3">Student 4L:
The distance between the charge and particle is doubled. This means that the magnitude of “c” is ¼ that of “a”.</td></tr>
<tr>
<td>Student 4C:
$\frac{Q}{d^2} : \frac{Q}{d^2}$
$\frac{2\,x\,10^{-12}}{0.2^2} : \frac{2\,x\,10^{-12}}{0.4^2}$
$5\,x\,10^{-11}$
$:\ 1.25\,x\,10^{-11}$
$4 : 1$</td>
<td>Student 4J:
$E = \frac{Q}{d^2} \qquad E = \frac{Q}{d^2}$
$E = \frac{2\,x\,10^{-6}}{0.2^2} E = \frac{2\,x\,10^{-6}}{0.4^2}$
$E = 1.25\,x\,10^{-5}\ E = 5\,x\,10^{-5}$
$1.25\,x\,10^{-5}\ :\ 5\,x\,10^{-5}$</td>
<td>Student 4A:
$\frac{1}{4\pi\varepsilon}\,x\,\frac{q}{d^2}$
$\frac{1}{4(3.14)(8.89\,x\,10^{-12})}\,x\,\frac{2\,x\,10^{-11}}{0.2^2}$
$1.118\,x\,10^{-12}\,x\,\frac{2\,x\,10^{-11}}{0.04}$
5.59
$5.59\ \div 2 = 2.80\,N$</td>
</tr>
</table>

Table 5.15. Examples of responses from student 4L, 4C, 4J and 4A.

This reliance on calculations, while useful for students to generate their own evidence / data to interpret, can lead to a habitual approach where the students unknowingly make careless mistakes, but as they were familiar with what the answer should be, from their tutorial exercises, they did not revise their calculations and are satisfied to submit their answer. For example, in Table 5.15, it was seen that student 4J made errors in their calculation using the electric field formula and also mathematically demonstrated the 1:4 ratio, with non-simplified values. Conversely, six of the students demonstrated the ability to correctly develop the ratio using no formulae, overcoming difficulties presented by Arons (1997) and Maloney, *et al.*, (2000).

In the final question, the students were presented with a shaded grid, similar to the one used in the inverse square law paint can “intensity” homework. They were presented with the diagram in Figure 5.23 (i) and told that 100 field lines pass through the shaded region when the charge is held 10m from the charge. They were then asked to determine at what distance the 100 lines would only pass through the shaded region, as seen in Figure 5.23 (ii).

This question was designed as a manner to engage their conceptual understanding of the area model employed in the homework, as discussed in section 5.5.3. A similar question was presented in section 4.3.3, which the students did well on. However, here the distance values do no related to the area covered in the diagram, as like previous questions, this allowed me to determine to what extent the students were focusing on the area of the shaded squares and/or the distance from the charge. A summary of the student results is presented in Table 5.16.

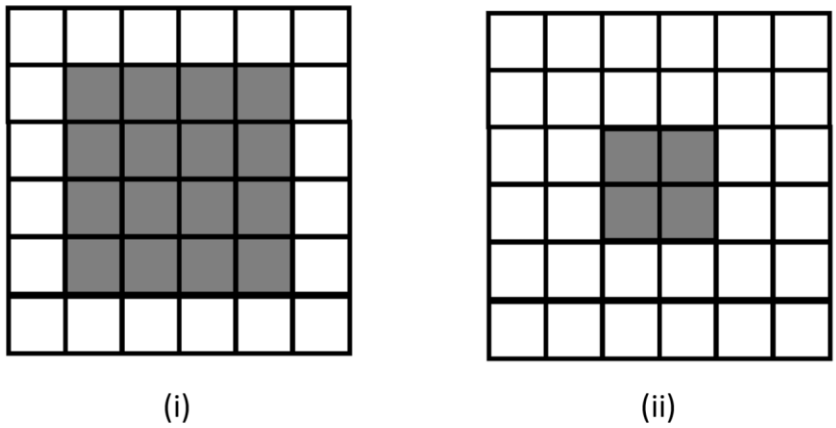


Figure 5.23. Post-test electric field question, utilising the area / scale model.

The results showed that five of the students could correctly identify the distance from which the charge would produce one quarter the field line intensity. There was a variety in responses from students who determined the correct distance that the distance was *5 m*, in which the they sketched an isometric view of the charge and the frames, demonstrated partial or complete reasoning based on reducing the length / widths of the frames, and qualitatively referencing the related increase and decrease of the variables involved. Some examples of this reasoning are shown in Figure 5.24.

It was also seen that students submitted the correct answer but give the wrong reasoning, based on an inversely proportional relationship, such as 4A and 4F. Both these students stated that as the intensity is doubled, the distance is halved. It is not clear how these students determined the intensity doubled, when considering going from sixteen frames to four frames. However, this error allowed them to produce the correct final distance from the charge to the frame.

Responses	Students.
r = 5 m	**4A, 4D, 4E, 4F, 4G, 4L.**
r = 2.5 m	4C, 4K, 4N.
r = 2 m	4B, 4I
r = 20 m	4H, 4M
r = 40 m	4J
References quadratic link between distance and area – Scaling	**4E, 4G, 4H, 4I, 4M.**
References inverse relationship	4A, 4C, 4F, 4K, 4N.
Closer to charge puts lines through less boxes.	4B, 4D
Uses alternative mathematical ratio.	4L.

Table 5.16. Student responses to scaling model, relating distance to area covered by spray can.

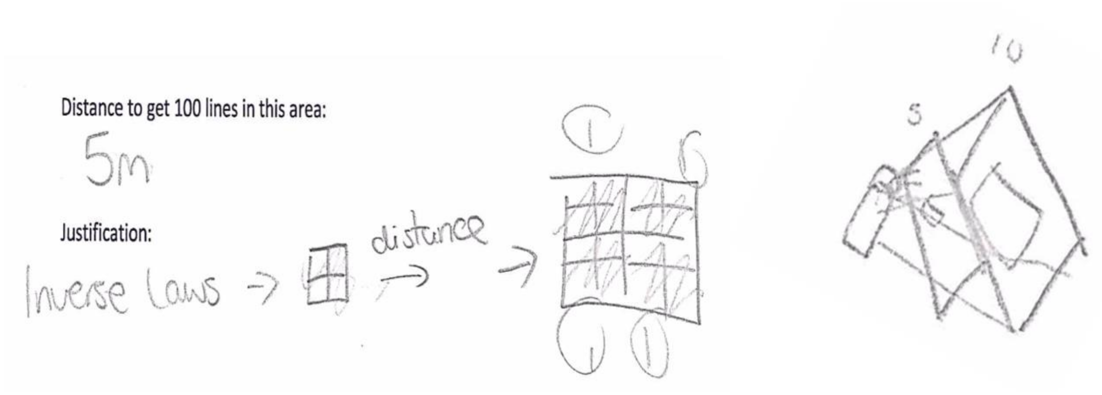
<table>
<tr><td colspan="3">Student 4E:

</td></tr>
<tr><td>Student 4D:
The closer to the charge, the less spread out the field lines are.</td><td>Student 4G:
The length of the width / length is half the original $(4 \div 2 = 2)$. This means the distance from the charge must also be half the original $(10\ m \div 2 = 5\ m)$.</td><td>Student 4L:
If $4\ x\ 4\ \rightarrow 10\ m$
Then $2\ x\ 2\ \rightarrow 5\ m$</td></tr>
</table>

Figure 5.24. Inverse square law reasoning provided by students 4E, 4D, 4G and 4L.

Another difficulty was presented by some students determined that the area of the second frame was one quarter the area of the first, and that the distance reduced by the same ratio. Other students determined that the distance reduced by a factor of 2, but instead of charging the 10 m distance by this factor, they suggested new distance was 2 m. One student attempted to use an inverse square law, but instead increased the distance from the charge to the frames, instead of decreasing it, suggesting they did not consider the effect of diverging field lines passing through multiple frames, as displayed in Table 5.17.

The post-test results show that students gained understanding of how the inverse square law applies to Coulomb's law and the electric field. Most students could draw the shape of an inverse graph from the electric field formula. The post-test question showed that eight of the students could differentiate between an inverse and inverse square pattern, primarily using a form of data analysis of values obtained from the graph to verify their choices. Prevalent difficulties in the remaining students involved interpreting Coulomb's law as an example of an inverse law. When analysing graphs, it was observed that without intervention, the students focused solely on the shape of the graph, and did not consider obtaining values from the graph and analyse them, as discussed in section 5.5.2.

Student 4B:	Student 4M:
$\frac{4}{16} = \frac{1}{4}$ $\frac{16}{4} = 4$ $\sqrt{4} = 2\,m$ *If I move the particles closer, the number of field lines that pushes through each box increases. To have 100 field lines pass through the grid, the number of boxes must decrease.*	$16\ x\ \frac{1}{4} = 4$ $\frac{2}{1} \to \frac{1}{2^2} = \frac{1}{4}$ *Double the distance. – 20m.*

Table 5.17. Erroneous reasoning produced by students 4B and 4M.

When completing the ranking question, and asked to develop a ratio, it was seen that six of the students applied the inverse square law while seven attempted calculations. Some errors in mathematical operations caused difficulties for some of these students, preventing them from showing the ratio. In two cases, the students produced an accurate ratio, but did not reduce it to its simplest form. A small number of students demonstrated that they still applied inverse proportional reasoning in this question. The final question indicates that just under half of the students could applied the inverse square law correctly using a scale model to determine the distance from a charged particle to the frames presented. Prevalent difficulties presented themselves as students applying inverse proportional reasoning, using qualitative reasoning instead of quantitative, or applying the inverse square law incorrectly.

5.5.5. Discussion

This section presents the discussion on the student's application of the inverse square law to Coulomb's law and the electric field. The student's pre-test and post-test results are compared to indicate instances of conceptual change that occurred and references to the tutorial lesson and homework discussion (section 5.5.2 and 5.5.3) are highlighted as examples of evidence that conceptual change occurred. The discussion mainly focuses on the student's application of the inverse square law mathematically, while issues and examples of understanding of scaling are referenced (Arons, 1999; Marzec, 2012).

The pre-test indicated that students encountered difficulties when required to transfer their understanding of the inverse square law to the Coulomb's law and electric field contexts. As seen in Figure 5.25, in the Coulomb's law pre-test question presented, it was observed that only three students could recognise that Coulomb's law followed contained an inverse proportional relationship. Additionally, only one student could successfully mathematically apply the inverse

square law to the pre-test. The remaining students either qualitatively, randomly attempted some manner of calculations or were unable to attempt the question. This suggests that the students required instruction that would result in conceptual extension, to promote the students to apply the reasoning and understanding they demonstrated in section 4.3, to an electrostatics context (Hewson, 1992; Konicek-Moran and Keeley, 2011).

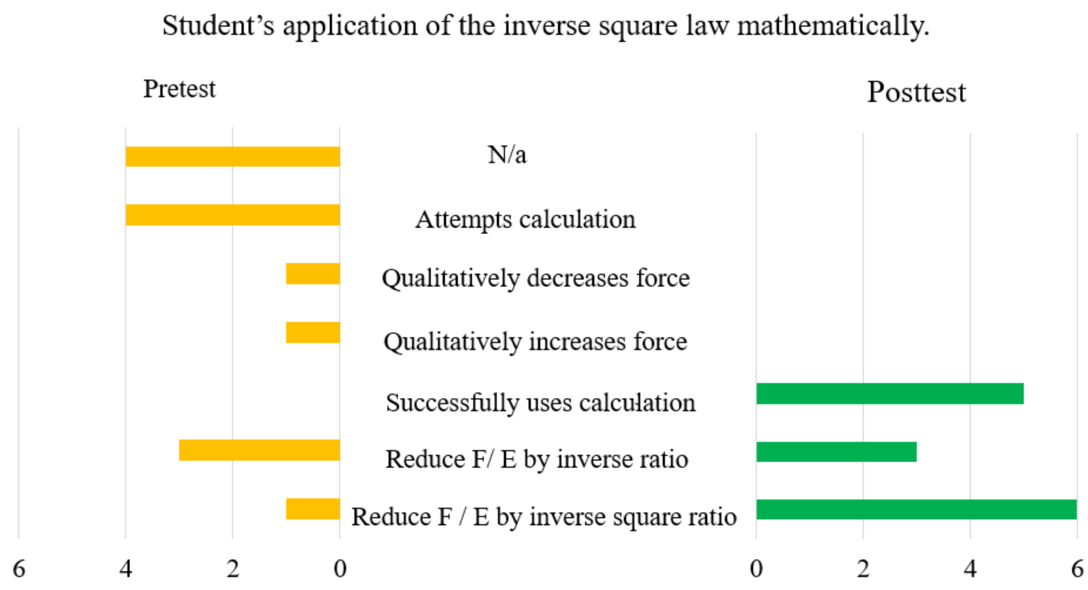


Figure 5.25. Comparison of students use of inverse square law in mathematical problems.

Gains were observed in the student's understanding and application of the inverse square law in the post-test results, as discussed in section 5.5.3. In the post-test six students applied inverse square law reasoning, without relying on formulaic substitution and evaluation, whilst a further five students did rely on the formula. Two students relied on use of the formula but used inverse proportional reasoning, while one student produced a ratio based on the distances presented in the question. This suggests that half of the students are reliant on the use of formulae, whilst the other half consider their understanding of the proportional nature of Coulomb's law and the electric field sufficient to approach the problem. The shift in results from the pre-test to post-test indicate that conceptual exchange (Hewson, 1992) occurred, as more students are observed correctly applying the inverse square law to the post-test contexts than was observed in the pre-test, and the students with persistent errors made more progress before encountering difficulty in the post-test, whereas they were unable to attempt the questions in the pre-test. Based on the shift in student responses shown in Figure 5.25, partial conceptual change was observed over through employing the use of the tutorials in the Coulomb's law and electric field contexts.

The student's reliance on formulae in the post-test is unsurprising. In tutorial the use of formulae was the most preferred method to explore the inverse square law. The students used a multiple representational approach to explore the inverse square law mathematically, and modelled the inverse

square behaviour of electric fields in the homework exercise. This homework activity was intentionally written in the same style as the spray paint model, discussed in section 4.3, as the students were familiar with the model, to allow for ease of transfer. The homework responses indicated that the students were proficient in the use of the frame model to explain the behaviour of the electric field lines. The interview indicated that the approach was unsuccessful in promoting student's consideration of mathematical area scaling, which underpins learner difficulty (Arons, 1999; Marzec, 2012). During the interview, the students demonstrated that they did not think in terms of the concept (Konicek-Moran and Keeley, 2011) of varying dimensions as they used the frame model to explore the variation of intensity through the unit areas at different distances from the charge, as discussed in section 5.5.3. However, the presentation of the frame model itself may reduce the requirement to rely on the understanding of the dimensional scaling, as students can consider the area of the frame directly.

Difficulties were also observed in the final question of the post-test using the area model, as depicted in Figure 5.23. Some students struggled to correctly show that reducing the distance from the can to the frames by a factor of a half would decrease the area by a factor of four. Students referenced the inverse square law in this question, suggesting they did not consider the quadratic nature that links the area of the spray to the distance from the can. Whilst the students attempted to approach the problem without using formulae and utilise their understanding, their application of the inverse square law directly instead of considering the dimensional scaling of the area. It suggests that they incorrectly apply the wrong relationship when considering the scaling itself but can determine the distance related to the given area, as both distance and area are familiar quantities to the students.

The electric field graphing pre-test question asked students to represent the relationship between the magnitude of the electric field at a point, and the distance from the charge. Ten of the students correctly represented the relationship using a characteristic decreasing asymptotic curve, while the remaining students erroneously produced linear of quadratic graphs or did not answer the question. However, in the post-test question, in which the students were presented with both an inverse and inverse square pattern, it was observed that difficulties were prevalent in six student's abilities to determine which pattern followed an inverse square law. Student difficulties related to the shape of the graph, not analysing the data on the graph by mapping the data into equations of the form $y = k\frac{1}{x}$ and $y = k\frac{1}{x^2}$, or analysing the data in terms of the behaviour of the reduction factors. This indicates the difficulties presented in section 4.3.3 were persistent at the end of this study, and that conceptual extinction of them did not occur.

5.6. Student's use of field lines to represent electric fields

Literature informs us that there are many representational difficulties that students can have when using field lines to represent an electric field (Galili, 1993; Törnkvist, *et al.*,1993; Maloney, *et al*., 2001). The following representational conventions for field lines were identified as target areas for the students to learn during the tutorial lessons, as they are accessible to the student's level of cognitive capability and could be used in application for other parts of the course, such as explaining the photoelectric effect, thermionic emission, production of x-rays and the use of particles accelerators.

- The closer field lines are, the stronger the field.
- When a field line curves, the direction of the force is tangential to the field lines.
- The field line represents the direction of force acting on a body, not the path taken by a body in the field.
- Field line patterns do not overlap or intersect. Two bodies that cause a field both contribute to one overall field surrounding both objects.
- Field lines only terminate on electric charges. They should extend to infinity / off the page / to the end of the diagram boundary.

Section 5.6.1 discusses the pre-test results, illustrating the student's understanding of the direction of force on a charged particle in a field, student's representations of the path taken by a body in a field, student's determination of relative field strength, and using vectors to represent the field at various points. Section 5.6.2 details the tutorial lesson in which students applied field line conventions to electric fields. Section 5.6.3 reviews the student's responses from the post-test, in which the student's gains in their understanding of field line conventions are illustrated.

5.6.1. Pre-test: Electric field

In the pre-test, the students were presented with the field pattern presented in Figure 5.26. While this field pattern shows some errors, due to poor design, from the learning outcomes for fields described in section 5.6, the questions asked of the students generally do not require the reasoning that field lines extent to infinity to correctly answer the pre-test questions. This field pattern is also not representative for a single point charge, before the students commenced the pre-test, that were informed that this diagram was a snapshot of a bigger field diagram with other charges elsewhere affecting the shape of the field lines. They were to only focus on the information they could use from the pattern they observed in the pre-test question.

They were asked a series of questions to determine the student's transfer of conceptual understanding from the field lines tutorial, and gauge what new concepts would be developed when transferring from gravitational fields to electrostatic fields. The students were asked the following questions:

- To draw in the path taken by an electron placed at the white circle.
- To rank the field strength of the point *a, b* and *c*.
- Draw vectors to represent field at the points *a, b* and *c*.

The first two of these three questions are discussed in this section. The last question, which involves the transfer of field lines to vector representation, is discussed in section 5.7.

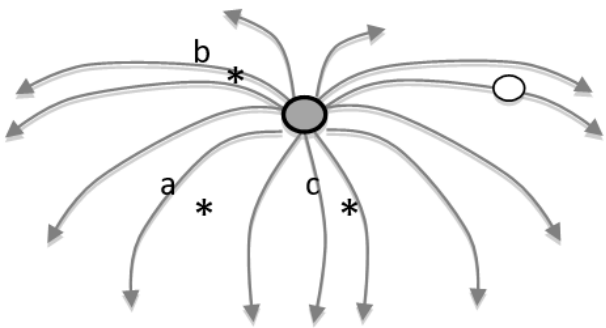


Figure 5.26. Pre-test question electric field pattern.

When representing the path taken by a negatively charged particle in this question, the students needed to consider the direction of the force acting on the charged particle, and how to represent the path under the influence of the field. Expected errors would be that the force on the negatively charged particle would be in the direction of the field line and that trajectory of the particle would follow the field line. A summary of the student's results for this question is given in Table 5.18.

The results show that three of the students determined that the direction the charge would follow was against the field line, while seven of the students determine it would go with the field line. In the class presentation to the students, the direction of the field line indicating the direction of force acting on a positive test charge was emphasized, and the opposite direction for a negatively charged particle was referenced. However, only three students recalled and applied this to their pre-test responses. This is not unreasonable as, except for the class discussion, the students did not spend any time exploring this concept.

Responses	Students
Electron moves against field lines.	***4A, 4J, 4M.***
Electron moves with field lines.	4B, 4C, 4E, 4F, 4G, 4K,4L.

Electron ignores field lines.	4D, 4H, 4N
N/a	
Electron diverges from path.	***4A, 4B, 4D.***
Electron takes linear path	4H.
Electron take unrealistic divergent path.	4E, 4K.
Electron sticks to field line.	4C, 4F, 4G, 4M.
Circular path	4N.
Electron ignores field and moves towards charge directly.	4D, 4J.

Table 5.18. Students pre-test results in determining the path taken by a negatively charged particle in an electric field.

Two of the remaining students (4D and 4J) drew paths in which the particle ignored the field and moved directly to the positively charged body. This is not considered a valid answer for this concept, even though the direction of the force acts against the direction of the field, due to the interpretation that the students did not consider the shape of the field lines when determining the path taken by the negatively charged particle. The final student (4N) ignored the field lines and drew a circular path for the electron around the positively charged particle. As there was no initial velocity for the charged particle, it is reasonable to speculate that student 4N is recalling aspects of the field line tutorial involving gravity. The production of a circular path would reflect the curved orbital paths referenced in the earlier tutorial.

Table 5.18 also shows the students results for using the field line as a guide for the path taken by the electron. Four of the students drew paths that reasonable diverged from the field lines. Errors were seen in one student (4H) drawing a path in the correct direction, but linear instead of curved. This indicates the student was not considering the force acting on the charge as it moves, and instead considered it a “once only” interaction between the field and the charged particle. Four students (4C, 4F, 4J and 4M) draw paths along the field line, indicating that the field line represented not on the force acting on charged particle, but the trajectory taken by the path. Two students (4E and 4K) drew an unreasonable diverging path, in which the electron moved away from field line, in a direction that does not correspond to direction of the force at its initial point. This indicates that while they are aware that field lines do not represent path taken, they are unclear how to apply the concepts of force, acceleration and velocity to determine the path taken. A sample of these paths are depicted in Figure 5.27.

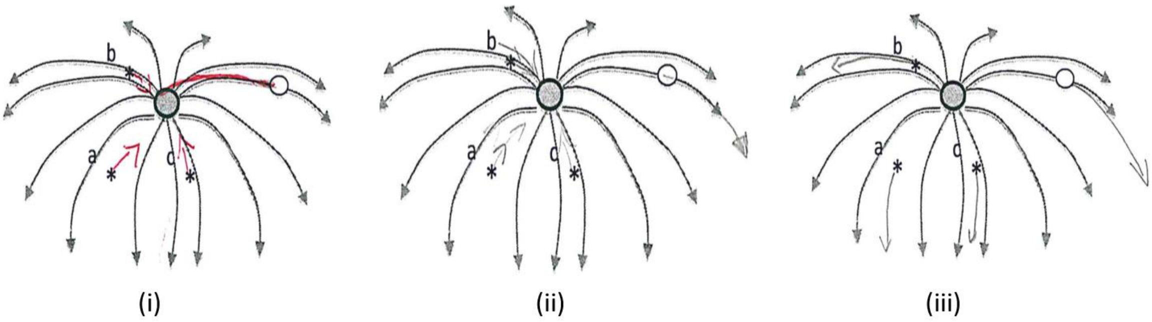


Figure 5.27. Student depictions of path of charged body which reasonably diverges from field lines (i), follows field line (ii) and diverges unreasonably (iii).

The next question on the pre-test looked at student's ability to rank the field strength at various points in a field, based on the diagram they were given. It was envisaged that successful students would use the field line density to determine the relative field strength, with a small number of students relying on the distance from the positively charged particle. A summary of the student's results is shown in Table 5.19.

Responses	Students
B > C > A.	**4A, 4B, 4D, 4E, 4F, 4G, 4H, 4J, 4K, 4L, 4M, 4N.**
B > A > C.	4C.
Field line density reasoning.	4E, 4G.
Distance from charge reasoning.	4A, 4B, 4C, 4D, 4E, 4F, 4H, 4L, 4N
Distance from field lines	4J, 4K, 4M.

Table 5.19. Summary of student's pre-test ranking of electric field strength and reasons used.

Many students were able rank the field strength surrounding the group of charges accurately. However, the reasoning utilised by nine of the students was based on the distance from the charge, whilst only three of the students attempted to use field line density as the evidence to justify their rankings. Additionally, three students used erroneous reasoning, utilising the distance from the points to the field line to represent the strength, i.e., the closer to a field line, the stronger the field. This suggests that most of students did not transfer this skill from the field line tutorial, or they did not warrant it as important over the reasoning based on distance from the charge. This could be highly likely, as this pre-test followed the tutorial on Coulomb's law, in which the inverse square relationship between force and distance was stressed as an important relationship.

Student 4G: b > c > a.
b is between 2 lines that are closest together and c is between 2 that are slightly further apart and a is between 2 lines that are very far apart.

Student 4D: b, c, a.
Because the closer you are [to the charge], the stronger the strength.

Student 4M: b, c, a.
B is closed to the long field line, c is slightly further away and a is much further away.

In summary, the pre-test results show that the student gains discussed in section 5.4 did not transfer to electric field as much as one would have hoped. A notable number of thought the field lines represent the trajectory of a body under the influence of a field. Most of the students assumed a negatively charged particle would follow a field line, although this is reasonable for the students at this point, as they had not yet explored the behaviour of a negatively charged body in a field. Moreover, in gravitational fields, there is no equivalence for this behaviour. In ranking electric field strength, most of the students relied on distance from the charge to determine strength, as opposed to using the field line density to justify their rankings.

5.6.2. Tutorial lesson: Electric field

As mentioned in section 5.4.2, the electric field tutorial lesson addressed both vector representations and field lines representations. This section looks at the latter half of the tutorial lesson. In this section, the students recapped previously covered concepts in their study of field lines, such as (a) the closer field lines are, the stronger the field, (b) when a field line curves, the direction of the force is tangential to the field lines, and (c) the field line represents the direction of force acting on a body, not the path taken by a body in the field. From the previous tutorial on field lines, as mentioned in section 4.4.5, it was observed that students made errors that the electric field line tutorial addresses. Those errors were as follows:

- Field line patterns do not overlap or intersect. Two bodies that cause a field both contribute to one overall field surrounding both objects.
- Field lines only terminate on charges. They should extend to infinity / off the page / to the end of the diagram boundary.

These rules were presented to students in an introductory paragraph on the tutorial sheet, and students were prompted to refer to them during course of the tutorial lesson. The students utilised these rules to explain the variation of an electric field of a positive charge and construct the electric field for a negative charge, whilst explaining the similarities and differences of both patterns.

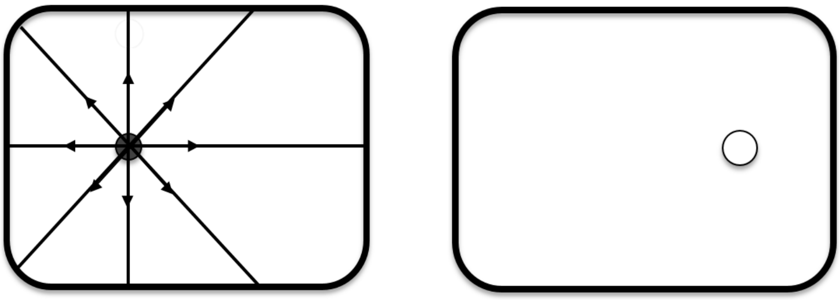


Figure 5.28. Tutorial setting where students represent electric fields using lines.

The students were required to use the field lines seen in Figure 5.28 to identify the sign of the charge as positive, and explain how the variation of field strength was represented. All students were consistent in referring to the direction of the field line pointing away from the charge as the indicator for the positive charge, in some cases referring to the fields moving to infinity. The students were also consistent in using the field line density as an indicator for the field strength. All students successful represented a negatively charged particle using the same field line pattern, with the directions placed towards the negatively charged body, as opposed to away from it, highlighting the differences between the two patterns.

Student 4E: Positively charged because field line are pointing outwards to infinity. [Field strength] decreases, field lines are further apart from each-other. [Differences] D = Arrow changes direction because negatively charged. [Same] S = field line still stretch to infinity.

The next section of the tutorial required students to apply the superposition principle to construct field lines for a system of two charges. In the first case, the students were required to represent the field of two dissimilar charges, and in the second case, the field of similar charges. In both cases, the students were required to draw the path taken by a positively charged particle under the influence of the field at a point. There were no errors shown in the student's representation of field lines for these cases. Some groups represented the path taken by the charge along the field lines. Questioning from the teacher or discussions with other students about the initial acceleration of the particle helped these students confront the errors in their initial responses. The prompt questions asked students to consider the direction of the force at the initial moment and consider the position of the particle at the next moment, having moved under the influence of the force. Student's revised their paths taken, using this approach, but still produced reasoning that was indicative of some confusion about the field line representation. For instance, one group of students, consisting of students 4D, 4E, 4F and 4N submitted the reasoning shown in Figure 5.29.

The use of the term "off track" in this case is still indicative of the error that the field lines represent a path or trajectory of some type, but the forces acting on the charge have caused it to move away from the track, akin to a car sliding off the road on a turn. Students 4G, 4H and 4I also submitted

the correct path, but gave reasoning based on the force interaction of the charges, as opposed to the field. This indicates the students were considering the interaction of the forces between the charges over the interaction of the positive particle and the field. Using this force orientated reasoning, the students appear to use the field to guide the trajectory, that's influenced by the charges generating the field.

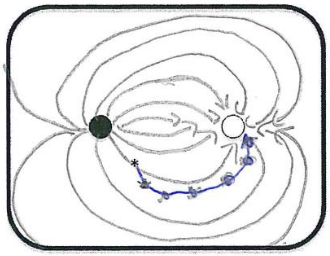
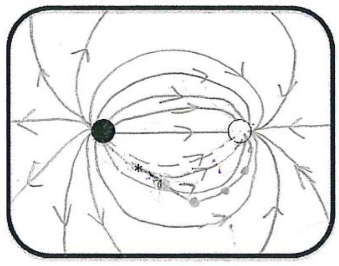
Student 4D:	Student 4G:
No velocity at the start. It accelerates a little and begins to move off track.	*As the particle is repelled by the positive charge, it will gradually be more affected by the negative charge than the positive charge, as it moves further away from the positive charge.*
	

Figure 5.29. Students 4D and 4G's depiction of path taken by charged particle in an electric field.

This tutorial narrative presented evidence that the students applied their understanding of field lines to electric fields, to some degree of success. The students identified and represented the electric fields for single charges and use the field line density to identify the variation of electric field strength for the charges. The students could also represent the superposition of similar and dissimilar charges, using field lines. While further work appears to be required to further develop the student's reasoning for the interactions of charged particles in an electric field, the students responses showed, with prompting, they accurately predicted the behaviour of a charged particle in an electric field and draw its trajectory accurately.

5.6.3. Post-test: electric field

In the post-test, the students were presented with the electric field of two positive charges shown in Figure 5.30. The students were asked a series of questions to determine their understanding of the information provided in the diagram. They were required to:

1. determine the charges on P and Q,
2. determine which of the two charges had the greatest magnitude,

3. explain the variation of field strength as a field line is followed from P to the boundary of the diagram,
4. draw the path a negative charge would take, if placed at R,
5. represent this electric field using vector arrows at different points. The results are shown in the following table.

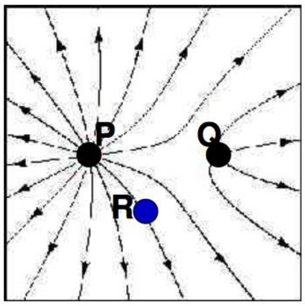


Figure 5.30. Diagram from the electric fields lines post-test question.

The first four of these questions are presented in this section, while the fifth question is addressed in section 5.7, as it specifically deals with transfer between representations. Table 5.20 presents a summary of the student's responses for the first two questions, including the reasoning provided by the students.

The post-test results indicate that the students were successful in applying their understanding of field line representation for positive and negative charges to this question. Except for one student (4F), all the students correctly identified that both bodes were positively charged, using the field line direction as justification for their responses. All the students recognised that the number of field lines coming out of the bodies could be used to determine the relative magnitude of their strength, which is an application of the field line density concept.

Student's 4A and 4G also incorporated the field line density into their responses. Student 4N referred to the number of field lines, but showed an error in their reasoning, alluding to the field lines of P pushing the field lines of Q away, indicating the field lines themselves exhibit their own tangible charge like property, or force-like behaviour.

Student 4E:	[The magnitude of P is] greater. The electric field lines are closer together.
Student 4G:	[The magnitude of P is] greater than [Q], because there are more field lines that are closer together from P, meaning that it is stronger.
Student 4N:	[The magnitude of P is] greater, because it has more field lines, and it pushes the field lines of Q away.

Responses	Students
P and Q are positive	**4A, 4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**
P and Q are negative	4F
Based on field lines leaving the charge.	**4A, 4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**
No reasoning.	4F
Magnitude of P is greater than Q.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**
Answer recognized the number of field lines is greater for P over Q.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**

Table 5.20. Student responses of the charges on P and Q, and relative charge magnitude between the bodies.

The third question asked students to place their finger on the charge P and follow one line to the boundary of the diagram. They were required to explain the variation of field strength as their finger moved and justify their explanation. Student responses are shown in Table 5.21.

The results clearly indicate that the students correctly identified the variation of the field strength using the field line density as justification. Only one student, 4M, used the distance from the charge as an indicator for field strength. Neither of these examples of reasoning are incorrect, but the results show a shift by the class to using the representational conventions of field line density, from relying on the relationship between the field strength and distance from the charge.

Student 4J: It decreases cause the field lines are more separated.

Student 4M: It gets weaker because the further you go from a charged particle, the weaker the electric field strength.

The fourth question required students to draw the path taken by a negatively charge particle in the field, when placed at the point R, depicted in Figure 5.29. This question was like the pre-test question, looking to determine if the students would draw a path that reasonably diverted from the field line, and whether the force acting on the negatively charged particle would go against the direction of the field. A summary of the student's responses is presented in Table 5.22.

While most students were able to show a reasonable path diverging from the field lines in a direction that moved against the field lines, minor errors were still observed in some student responses. Students 4L and 4M both submitted the correct path, but used naïve reasoning based on the attraction of the positive charge P, and the negatively charged particles placed down. These students failed to explicitly articulate their reasoning for the shape of the path. By presenting a reasonable curved path that diverges from the field line, they indicated that they understood the

velocity (or inertia) of the particle moves it from the field line. Student 4B drew a path in which the initial force was in the correct direction and was tangential to the field line. However, as the particle moved, the direction of the net force on the charge would have changed, but 4B did not take this into account and drew a linear path that followed the direction of the initial force. Student 4C also drew a linear path, but in this case, they ignored the shape of the electric field, and draw a path directly to the charge, basing their path on the attraction between the charges.

Responses.	Students.
Field strength decreases.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N**
Field strength increases.	N/a
Field strength remains unchanged.	N/a
Justified by using field line density.	4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4N
Justified by distance from charge.	4M

Table 5.21. Student's post-test responses for the variation of field strength, and their justifications.

Responses.	Students.
Negative charge moves against field line.	**4A, 4B, 4C, 4D, 4G, 4H, 4I, 4K, 4L, 4M, 4N**
Negative charge moves with field lines.	4F
Reasoning based on attraction	4L, 4M
N/a	4E, 4J
Reasonable path off the field lines produced.	**4A, 4D, 4F, 4G, 4H, 4I, 4K, 4L, 4M, 4N**
Paths was linear	4B
Path ignore field lines, direct to P.	4C
N/a	4E, 4J

Table 5.22. Summary of student's post-test responses to drawing a negatively charged particle moving in an electric field.

Student 4F had incorrectly identified that P and Q were negative charges instead of positive charges and drew their path accordingly. While the path did leave the field line, in what is considered a reasonable path, the student did not use information based on the field line direction to determine what direction the force on the charge would act. They used the interaction between the negative charge placed down at R and the force it would experience based on its repulsion from P and Q. Student 4F did however explain that the increasing velocity would move it from the field line, and

drew a reasonable curved path that indicated repulsion on the mobile charged particle from both P and Q. A sample of the students correct, and incorrect paths are displayed in Figure 5.31.

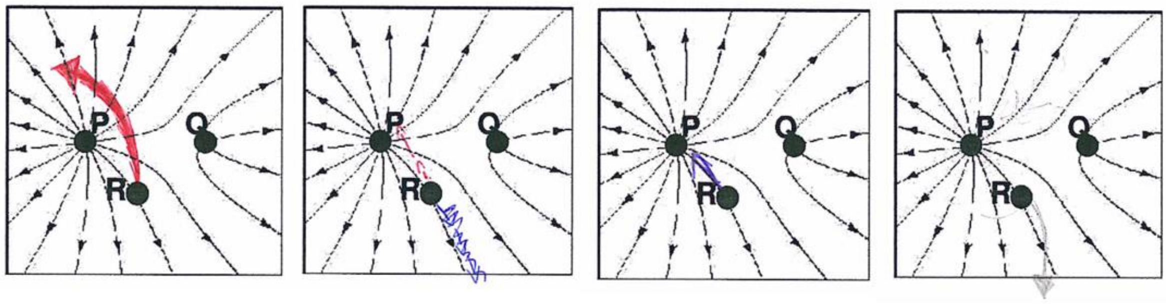


Figure 5.31. Paths taken by negative charge in field, from students 4H, 4B, 4C and 4F.

This section discussed the post-test results. The results indicate that, across the whole group of 14 students, gains occurred in the student's understanding and application of field line concepts. The students demonstrated they could identify a charge, based on the field line pattern, and determine the relative magnitude of a charged body based on the number of field lines going into or out of it. All but one of the students used the field line density to explain the variation in field strength as an indicator for relative field strength. The final question discussed in this section showed that the majority of students could accurately draw the path taken by a negatively charged particle in a field accurately but errors were persistent for a small number of students, such as the direction of the force acting on the negative charge in a field, the continuous nature of the force acting on the negative charge, and students ignoring the field and drawing the path based on attraction interaction between the negative charge and P or Q.

5.6.4. Discussion

This section discusses the student's pre-test and post-test results, focusing on their understanding of the direction of force acting on a charged object, using the field lines as a guide to draw a reasonable trajectory, and the used of field line density as an indicator for field strength. When appropriate, the student's ability to transfer their reasoning from the tutorials in Chapter 4 is also discussed in each section. The first part of this discussion focuses on the student's understanding of the direction of force acting on a charged particle, under the influence of an electric field. Figure 5.32 shows the pre-test – post-test comparison for the student's understanding that the force on a negative charge acts in a direction that goes against the field.

The pre-test results indicate that over half of the students were unaware of how a field line can be used to determine the direction of force acting on a charged negatively charged particle or ignored the field itself and focused on the interaction of charges. Both difficulties are commonly observed in

learners understanding of electric fields (Furio and Guisasola, 1998; Cao and Brizuela, 2016), and both were identified as targets for conceptual extension (Hewson, 1992).

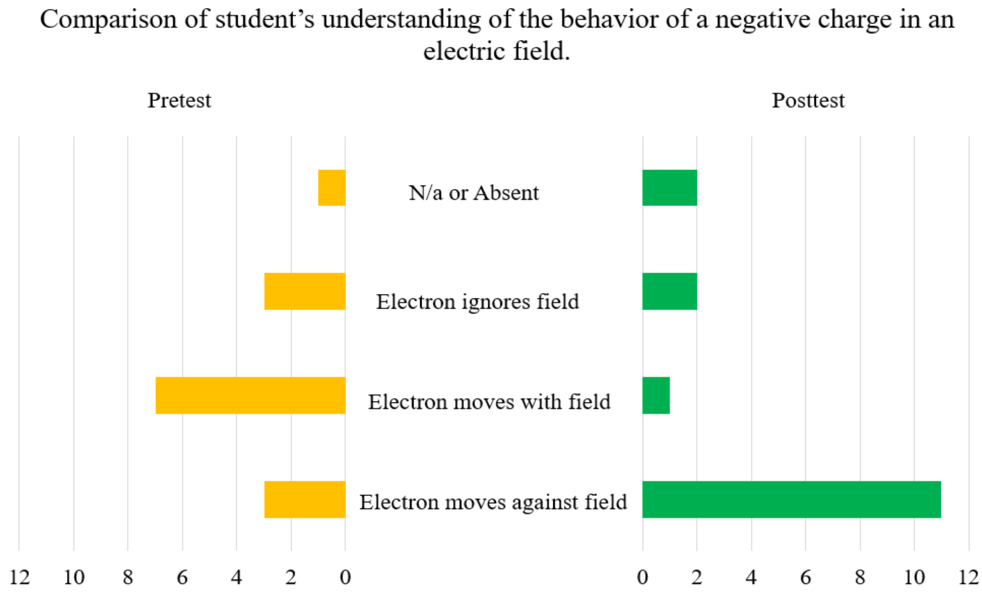


Figure 5.32. Comparison of results regarding the force on a negatively charged particle in an electric field.

Section 5.6.2 narrated the student's development during the tutorial. The latter part indicated that when both a field and charges are presented to the students, the students reasoning is based on the interaction of charges, as opposed to the interaction of the field with the charged particle. However, this was not observed with high frequency in the post-test. The post-test showed a shift in the number of students that determined an electron would be influenced by a force to move against the direction of the field using reasoning based on the charged particle and the field. Figure 5.32 displays the student's responses to this question and indicates that conceptual extension occurred, with a jump of 3 students to 11 students using field-based reasoning. Reasoning based on the interaction between charges was rarer in the post-test with only two students (4L and 4M) using the reasoning. The paths drawn by these two students were correct and suggest that they were aware of how the electron would behave, but they appeared to value the interaction between the charges as a stronger indicator to determine the path than the interaction with the electron and the field. Both types of reasoning, charge interactions and field interactions, were observed in the tutorial and post-test, and this suggests that the students consider the scenarios in terms of both styles of reasoning and applied them as they saw fit to the task presented to them. This suggests conceptual exchange did occur in the student's models, but conceptual extinction did not occur. As the comparison of the results demonstrates an increase in eight students utilising the correct reasoning, the extent to which conceptual change was observed was moderate.

The next section compares the paths drawn by a charged particle under the influence of an electric field. Figure 5.33 shows the pre-test – post-test comparison, in which it is clearly observed that there was a reduction in the number of difficulties encountered by the students.

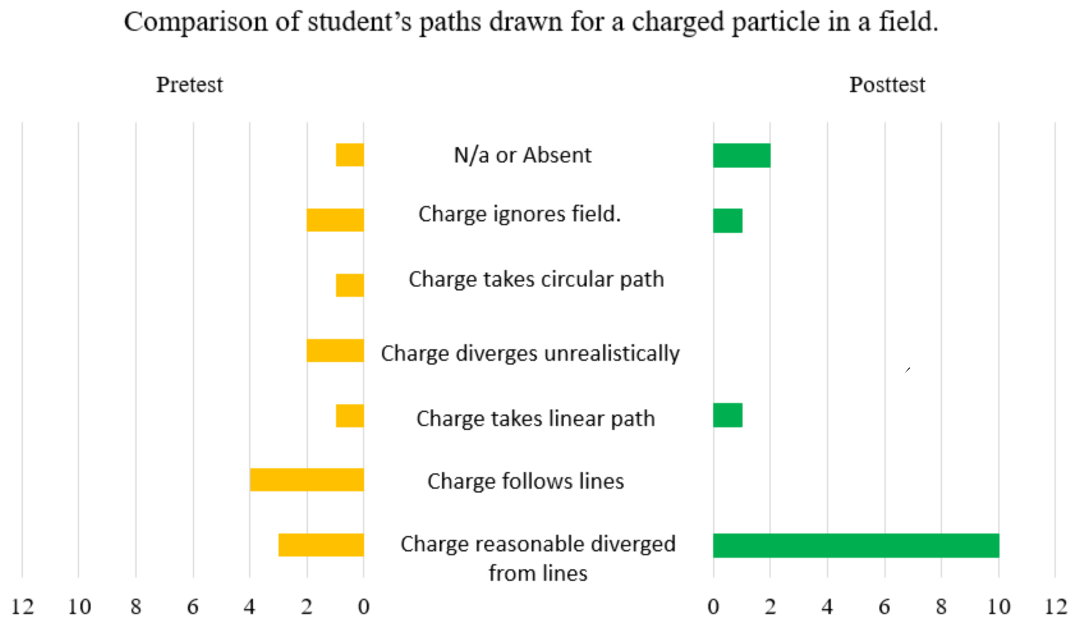


Figure 5.33. Comparison of results regarding the path taken by a charged body in an electric field.

While only a small number of students showed the path following the field line in the pre-test, which was expected to be the most prominent error, the results from Figure 5.33 suggest that students struggled to construct a reasonable path during the pre-test (Törnkvist, *et al.*, 1993; and Galili, 1993). In section 4.4.4, it was seen that 12 of the students constructed a reasonable path taken by a body under the influence of a gravitational field. When this is compared to the electric field pre-test, it is clear the students did not transfer their understanding to the electric field context. An inability to transfer a concept to multiple contexts is an indicator of the limit of the student's conceptual understanding (Konicek-Moran and Keeley, 2011) and the tutorial was developed to aid student extend their understanding to the electric field (Hewson, 1992), to which partial conceptual change was observed.

In the electric field tutorial lesson, it was seen that students required time and guidance to consider how the force acting on charged particles affects their acceleration and velocity to construct a reasonable path. When difficulties occurred, the motion diagrams were utilised as prompts by the teacher. This allowed the students to consider how the charged particles moved from moment to moment, whilst the students explained how the force at every moment would cause the particle to change its motion. The teacher provided the dissatisfaction in the explanatory power of considering the field lines as indicative of the path taken by a charged body and presented a motion diagram which allowed the students to develop intelligible reasoning based on their understanding of forces, acceleration and velocity (Posner, *et al.*, 1982). As the students to focus on applying the outcome of

the interaction and the concept involved, as opposed to focusing solely on the outcome in the electric field context (Konicek-Moran and Keeley, 2011). The post-test results showed that 10 students constructed reasonable field line trajectories, overcoming most misconceptions presented in the pre-test. This suggests an extension in the student's understanding to generate a reasonable path in a gravitational field to an electric field context (Hewson, 1992)

Figure 5.34 illustrates the pre-test post-test comparison for the student's justifications in determining relative field strength in an electric field.

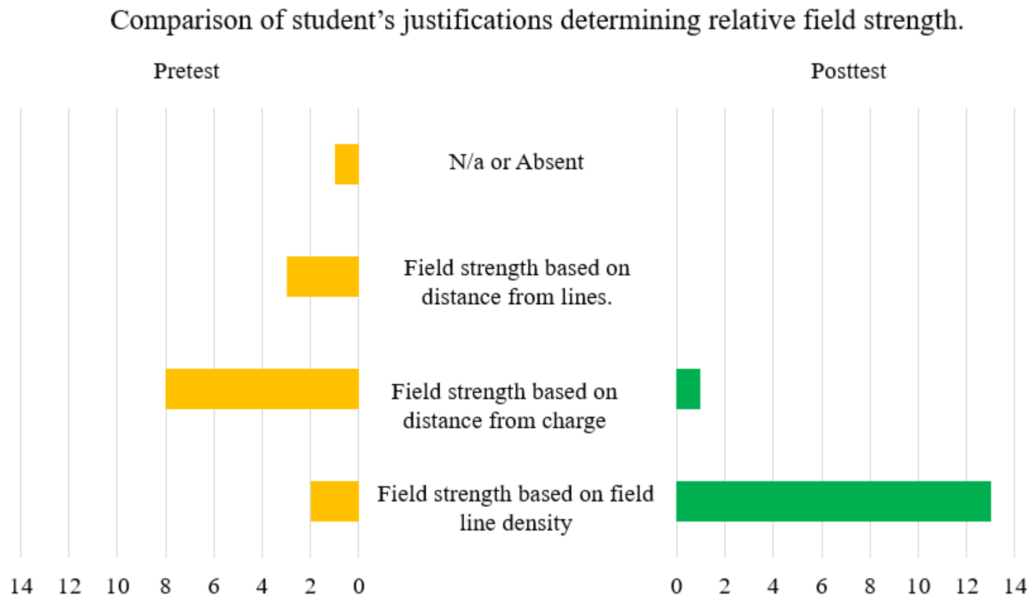


Figure 5.34. Comparison of results regarding the relative field strength in an electric field.

In chapter 4, section 4.4.5, it was observed that 10 of the students could rank field strength based on field line density. In the electric field pre-test, it was seen that only two students extended their understanding field density to determine the relative field strength of different points to the electric field context (Hewson, 1992; Törnkvist, *et al.*, 1993; and Galili, 1993), whilst eight students relied on distance from the charge. It is unclear whether the students were unable to extend and transfer their understanding to from gravitational lines, or whether they did not consider this concept to be more appropriate to employ over using distance from charge to justify their ranking. As the students previously completed the tutorial on the inverse square law and Coulomb's law, it is reasonable they considered the qualitative relationship first as it was recently used in their memory. During the tutorial lesson, the students demonstrated that there was no difficulty in apply the convention and apply it in the electric field context (Konicek-Moran and Keeley, 2011), and in the post-test, it was observed that all but one of the students applied it in the post-test. As this question could be answered using both distance reasoning and field line density reasoning, with both concepts recently in their memory, most students opted to use field line density as their justification. This indicates that the students were able to extend the concept of field line density to represent the relative field strength

between points, but they may opt to using alternative, but valid, reasoning when the task they were completing allowed them to do so. Therefore, the level of conceptual change observed in the student's understanding of relating relative field strength to field line density was moderate.

This section discussed the student's understanding of the force acting on a negative charge and showed the tutorial lesson allowed for conceptual change to occur. Post-instruction, some minor errors were persistent such as students ignoring the field focusing on the interaction between the charged particles, or directing the force acting on negatively charge particle following the field line direction. Ten of the students showed that the path taken by a charge would reasonably diverge from field lines, with only one student considering the force to act on the charged particle at a single instant, and one student ignoring the field. Finally, the students demonstrated that they could utilise the field line density to rank the field strength at different points, instead of relying on the relationship between electric field strength and distance, when the concept was recently employed in their classwork.

5.7. Student's use of vector and field lines representations in electrostatics

Thus far, this chapter has illustrated and discussed the student's use of vectors, the inverse square law and field lines, as they applied them to Coulomb's law and the electric field. This section details how students transferred between field line representations and vector representations (Törnkvist, *et al.*, 1993; and Galili, 1993). The ability to transfer isomorphically, that is transfer between both directions without difficulty either way, is demonstrated as a trait of expert problem solvers (Kozma, 2003). The first half of this section discusses an example of students representing vector arrows from a given electric field, and the latter half discusses an example of students representing a vector field using field lines.

5.7.1. Student transfer from field lines to vectors

In the electric field post-test, the students were presented with an opportunity to construct a vector field plot from a field line diagram. The field line diagram was shown in Figure 5.30, at the beginning of section 5.6.3. The students were presented with a diagram that presented the charges P and Q and six points indicated with asterisks shown in Figure 5.35.

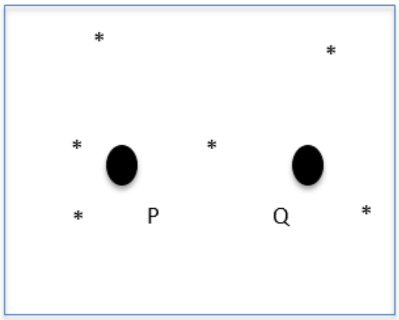


Figure 5.35. Points to represent vector arrows from field lines diagram.

When asked to represent the electric field using vector arrows at the points shown, most of the students encountered difficulties in transferring between the two representations. The student's results are summarized in Table 5.23.

Responses	Students.
Vectors are correct direction and correct relative magnitude	**4B, 4D, 4E, 4G**
Magnitudes are not relatively in scale.	4C, 4H, 4I, 4J, 4L
Directions of vectors are mildly non-tangential.	4M
Tangential, but points towards P and Q.	4C, 4J, 4K
Vectors from both charges shown, no super-position of vectors attempted.	4A, 4F
Attempted superposition shown	4N

Table 5.23. Student's attempts to transfer from field line to vector representation.

Only four of the students could correctly represent the electric field at various points around the two charges using vectors. The most prominent error shown by the students was sketching vectors of the correct orientation but incorrect relative magnitudes. Section 5.6.3 showed that the students were aware of how the field line density represented the field strength, and during section 5.4.2, it was noted in the tutorial lessons that the students tended to forget to represent the magnitude of the field correctly before applying the superposition principle. Although the students were prompted to the error and they rectified it in the tutorial, it was clearly not sufficient to produce a long-term change in their vector representation, when constructing vector fields.

Three students, 4C, 4J and 4K, reversed the direction of the vectors relative to the direction of the field at the point. All these students previously identified the sign of the charge on P and Q, based on the direction of the field lines, so these vectors are not consistent with their previous reasoning. It is possible that these students drew these vectors to represent the force acting on a negative charge at the given points, as the previous question required them to draw the path acting on a negative

charge. The relative magnitudes of these student's vectors were reasonable and accurately represented the field strength at the points shown.

Another difficulty in transferring to vector representation was encountered by students 4A and 4F. This their case, they drew vectors surrounding each individual charge, but attempted no superposition. This is not considered an attempt at transfer between the representations, but an attempt at construction of the vectors from scratch, suggesting that the student's do not comprehend the link between the two representations.

Student 4N had similar difficulties in which they attempted to construct the vector field based on the positions of the charges, without using the direction of the field lines as a guide for the direction and relative magnitudes of the vectors. Student 4N also overlaid the electric field on their diagram and produced reasonable resultant vectors. Whilst their vectors were drawn reasonably, it is clear they did not ground their construction in transferring between field lines, but instead relied on the principle of superposition. Examples of the student's constructions are presented in Figure 5.36.

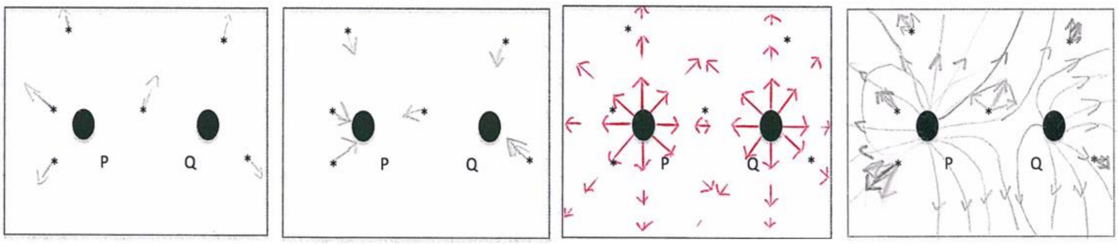


Figure 5.36. Vector fields transferred from field line representations, from students 4D, 4C, 4A and 4N.

This section showed that the approach undertaken in this study did address the student's ability to transfer from field lines to vector representations. Difficulties about representing vector magnitude qualitatively using the length of the arrows were the most persistent difficulty presented by the students, whilst most students appeared to grasp that the direction of the field was tangential to the field lines at the points shown. In some cases, students did not transfer the direction of the arrows correctly, even though they were depicted on the original field lines diagram. This difficulty may have been influenced by the previous question in which the students was asked to consider a negative charge, indicating they do not consider the field independent of the charges it interacts with. Other students resorted to rote learned vector patterns without drawing the superposition and one student was able to represent the field drawing vectors but relied using both superposition between the charges and overlaying the field line pattern to scaffold their construction.

5.7.2. Student transfer from vectors to field lines

In a separate question in the post-test, the students were presented with a vector field, as shown in Figure 5.37 (i), and asked to identify the sign of the charge on the particle. They were told an oppositely charged body was nailed down next to the first charge and they were asked to construct the superposition of the field at the points shown in Figure 5.37 (ii). The student's responses to these questions were discussed in section 5.4.3. As an extension to these questions, the students were required to draw electric field lines to match the vector field they constructed, as seen in Figure 5.37 (iii). A summary of the student's responses is presented in the Table 5.24.

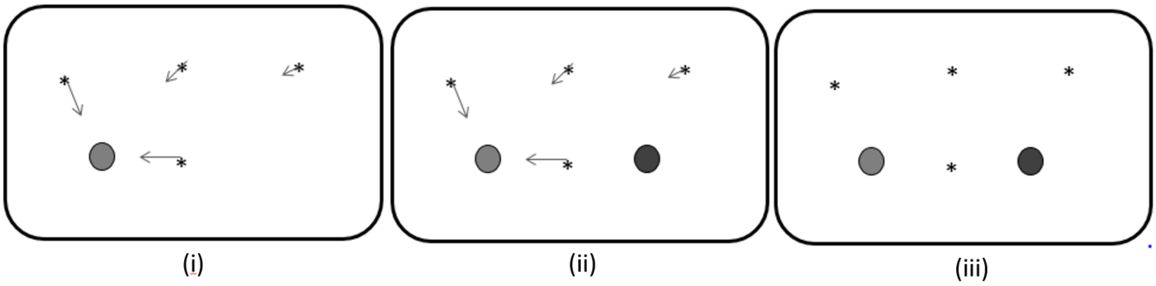


Figure 5.37. Post-test question in in which students apply vectors to an electric field.

Responses.	Students.
Field lines consistently represent vectors	**4D, 4E, 4H, 4J, 4L, 4M.**
Field lines not consistent with vector arrows	4A, 4B, 4F, 4G
Incorrect superposition – Not all vectors consistent with lines.	4C, 4I, 4K,
No transfer demonstrated – used both representations	4N

Table 5.24. Student's attempts to transfer from field line to vector representation.

The results indicate that half of the students could construct a field line representation that was consistent with their vectors, while the other half of the students could not. The six students who correctly transferred their vectors to field lines aligned field lines reasonably well with the directions and magnitudes of the vectors, as shown in the example of student 4H's response in Figure 5.38. In all cases, minor errors were seen, and it was reasonable if the student's diagrams accurately represented 3 of the vectors, and reasonably represented the fourth.

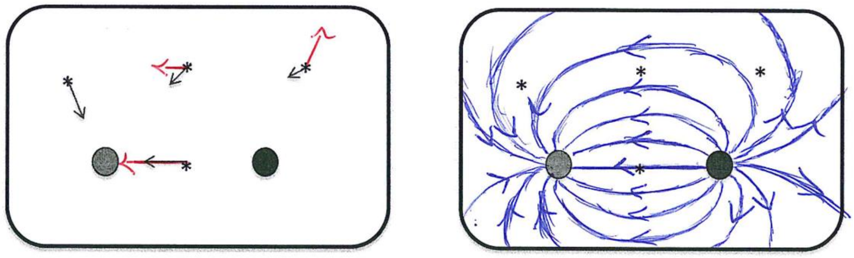


Figure 5.38. Example of Student 4H representing vectors using field line representation.

As both charges were equal in magnitude, the field line pattern should have been symmetrical, but in cases where the student's vectors were inaccurate with magnitude, this error was transferred to their field line diagrams. These errors are considered to be reasonable as the diagrams are qualitative in nature, and the consistency in the error suggests the students were employing representational transfer. For instance, student 4E did not find the superposition of the vectors, and when representing the field lines, skewed the symmetry of the diagram to produce the shape of the field in line with the strongest vectors in their field, except for the top right point, as seen in Figure 5.39.

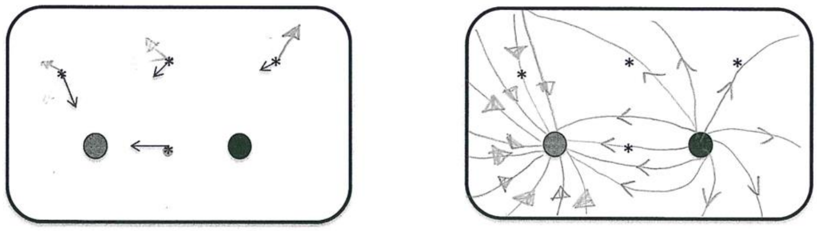


Figure 5.39. Example of Student 4E transferring error consistently to field line representation.

Four students drew field patterns which were correct and symmetrical but did not represent the vectors. This indicates they may have produced a rote learned model but did not consider the direction of the field at all points. For instance, Figure 5.40 shows student 4G showed the superposition at various points, but when drawing the field lines, three of the points did not align to their vectors. Furthermore, their field line diagram suggests only the field in the space between the charges is where the superposition principle applies, ignoring the areas on the left and right of the diagram. This contradicts the vector superposition they represented in the vector field diagram.

This error was also seen in the responses of students 4A and 4B. Student 4F gave incorrect vector directions. They interpreted the question to be two negatively charged particles. However, the field line pattern they drew was indicative of oppositely charged particles. This indicates the student was resorting to rote learned patterns, as their diagrams are inconsistent.

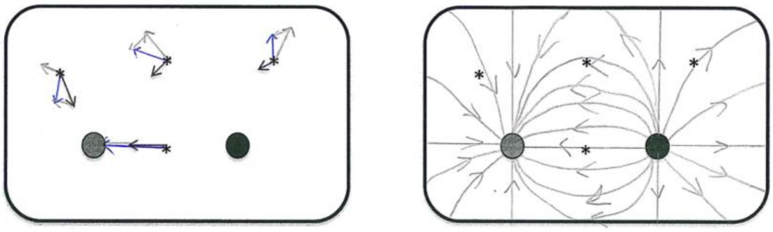


Figure 5.40. Inconsistencies in the vector transfer to field line representation, from student 4G.

Students 4C, 4I and 4K drew patterns that also did not transfer from their vector arrows. As seen in Figure 5.41, the two points in the centre of the diagram are not represented for their field. They drew a pattern that indicates repulsion and is inconsistent with their vectors. However, an example of transfer can be seen on the points shown to the top right and top left of their pattern, in which their patter follows the direction of the vector of highest magnitude, an error also seen in student 4E's submission. Student 4K also displayed this error. Student 4I drew overlapping lines of the field lines pattern for both attraction and repulsion and display the field line direction to be in the opposite direction of that which they drew directions of the vector arrows. This is depicted in Figure 5.42.

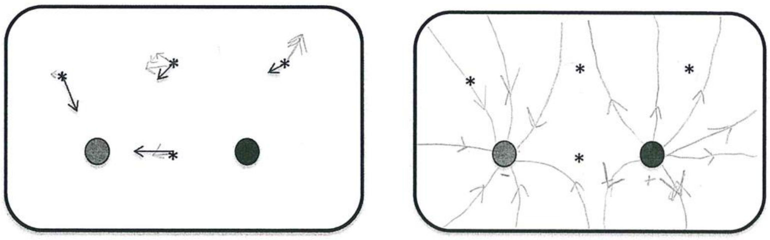


Figure 5.41. Inconsistencies in the vector transfer to field line representation, from student 4C.

This section showed that only a small number of the students represented a field line pattern using field lines. The most persistent difficulty in students was maintaining consistency in the direction in the diagrams drawn from vector arrows to field lines. A likely issue in the responses was students resorting to rote learned patterns for field lines of oppositely charged particles. The students appeared to demonstrate an overreliance on position of the charges to produce their field line pattern, and not on the vector arrows they drew.

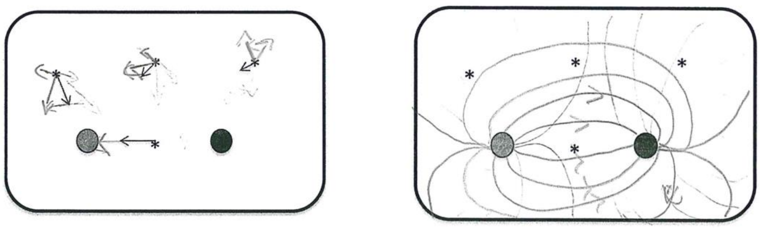


Figure 5.42. Examples of errors in the vector diagram transfer to field line representation, from student 4I.

5.7.3. Discussion

This discussion reviews the student's understanding of concepts related to vector and field lines representations and provides a commentary on student gains observed in the previous two sections. Section 5.4 illustrated the student's progress in extending their understanding of vectors to applying it in an electric field diagrams, when required to represent the field using arrows at various points around a charged particle (Maloney, *et al.*, 2001). Section 5.6 illustrated the students understanding of the relative field strength and the direction of force acting on a charged body at various points in an electric field (Galili, 1993; Cao and Brizuela, 2016). As seen in section 5.7.1, the gains that the students displayed in the post-test for these representational contexts individually were not as frequent when the students were required to transfer between the two representations.

Section 5.7.1 showed that the prevalent difficulty in student's ability to transfer between representations was representing vector magnitude that was consistent with a given electric field line diagram (Törnkvist, *et al.*, 1993). This difficulty was not observed to the same degree in the post-test question, seen in section 5.4, as in this section where the students were required to transfer between field line representation to vector representation. As section 5.4 indicates that the students could apply the vector convention to the electric field context, this suggests that conceptual extension did not occur effective (Hewson, 1992), to enable the students to apply the concept in a task that involved transfer between the two representations. As nine of the students were able to correctly represent the directions of the vectors, this indicates that some level of transfer occurred, in which the students could recognise the tangential nature of the electric field at positions represented along a field line.

Section 5.7.2 detailed the post-test question, which elicited the student's ability to transfer from vector representation to field line representation. It was observed that half of the students were able to construct a field line pattern that was consistent with their own vector constructions (Törnkvist, *et al.*, 1993). The most common difficulties in the post-test were that students drew vector fields that were suggestive of both charges having the same charge, but the fields indicated their charges were opposite or vice-versa. This suggests these students did not extend their understanding to interpret their vector and apply their field line reasoning but constructed their field line patterns from patterns that were memorised, or results to guessing.

From reviewing both section 5.7.1 and 5.7.2, it was observed that only students 4D and 4E consistently answered the post-test questions without error and demonstrated the understanding to allow them to transfer between the representations isomorphically. Students 4B, 4G, 4H, 4L and 4M answered one of the two questions without error, and tended to make minor errors in the other question. This suggests that conceptual extension (Hewson, 1992) occurred for these students during the tutorial, as these students displayed proficiency in the vector and field lines tutorials, as seen in chapter 4, and showed progress in transferring their understanding to the electric field context. The

remaining students demonstrated errors in both transfer questions. The errors displayed by these students are reflective of the errors seen in chapter 4 and/or the difficulties observed when applying these concepts to the electric field, as seen in section 5.4 and 5.6, such as terminating field lines, or lines overlapping. As these students displayed errors in their understanding of both representations in chapter 4, it is reasonable that difficulties in transfer between the representation occurred.

5.8. Conclusions

This section of the chapter presents the conclusions based on the studies presented in this chapter in two separate subsections. They address the impact on student learning, and the student's ability to transfer the representational tool to the electric field context. The conclusion addressed the overall research question that was introduced at the beginning of this chapter:

- To what extent does the use of a multi-representational structured inquiry approach develop student understanding of electric fields?

This question is addressed in the impact of student learning, which addresses the following considerations related to the research question:

1. The student's ability to demonstrate that Coulomb's law, and the electric field, is an example of an inverse square law?
2. To what extent the students demonstrate their understanding electric fields and the interaction of charged objects with fields using vector representation.
3. To what extent the students demonstrate their understanding electric fields and the interaction of charged objects with fields using field line representation.

The student's ability to transfer representational tools to the electric field context addressed the following consideration related to the research question:

4. To what extent the students demonstrate their ability to transfer a depiction of an electric field from one representation to another representation?

5.8.1. Impact on student learning

The approach adopted in this study focused on developing student's ability to transfer and their understanding of vectors, the inverse square law and field lines to Coulomb's law and the electric

field. The vector nature of forces is a fundamental pillar of electrostatics, and further electromagnetism, in which vector mathematics is utilised, along with calculus. The inverse square law is one of the fundamental relationships studied by students, which is seen in optics, sound, gravitational and electrostatic forces, and radiation. The concept of a field underpins the interactions of particles and field lines are an efficient manner to represent them, as they can convey various information about a field, in a simple manner.

The first of the research considerations looks at student's application of the inverse square law to Coulomb's law and electric fields. The approach adopted utilised two methods for students to apply their understanding of the inverse square law to Coulomb's law and the electric field. The first method involved mathematical data analysis in various forms, as suggested by Hestenes and Wells (2006), and the second method was adapted from the spray paint model, presented by Hewitt (2009). The pre-test results for the student's application of the inverse square law to Coulomb's law and the electric field suggested that the students did not incorporate the ability to recognise Coulomb's law as an equation of the form $y = k\frac{1}{x^2}$, and apply mental mathematical ratios to determine a reduction in force, or did not remember how to apply inverse square proportional reasoning. This suggests the students did not transfer what they learned during the inverse square law tutorial lessons, as discussed in section 4.3, where nine students successfully used the law or made reasonable attempts in applying it. However, in the Coulomb's law pre-test, this number dropped to four across different manners of representing the inverse square law. Upon completing the Coulomb's law tutorial and electric field homework exercise, it was seen that approximately half of the students still require formulae to apply the inverse square law, and struggle to fully conceptually grasp the mathematical scaling that applies to the inverse square law. However, they were able to apply the scaling to diverging field lines. The tutorial also showed the students prefer to utilise algebraic substitution and evaluation to apply the inverse square law, due to their perceived ease of use. This is also an interesting observation to note, as there are less mathematical operations involved in using the graphical or tabular analysis that the students completed. This may be due to the student's familiarity with solving quantitative problems, which rely on algorithmic problem-solving strategies. The post-test results showed that repeated exposure to the inverse square law promoted student's ability to perform proportional reasoning operations to show the ratio of reduction for 2 points, but over half the students relied on the use of mathematical formulae with substitution and evaluation to complete the task.

Additionally, the electric field homework interview and post-test scaling model question results indicate that the students do not consider the dimensional change in width and height of the scale model when using area problems, and do not link this to the change in distance. This error in student thinking can lead to them apply directly proportional thinking to the model, which in turn leads to inverse proportionality when applying concepts such as intensity and field strength to the model. Further development of these concepts, in a mathematical or physics context, could employ active learning strategies focusing on constructions of geometrical regular shapes such as squares and

rectangles which involve increases in dimensions of lengths and width, and recording and analysing the patterns between the variables could help students with these concepts.

The second research consideration addresses the student's ability to transfer their understanding of vectors to the electric field context. Section 5.2 detailed the student's progression in learning vector concepts in isolation of any physical context, and the discussions showed that progress was made by the students with regards to representing vector magnitude using the lengths of the arrows, accurately using vector constructions to show the superposition of two vectors and consideration of horizontal and vertical components of vectors. The electric field pre-test suggests that the students struggled to recognise and transfer this representation to the electric field context. Errors were seen in which students did not demonstrate the reduction in magnitude with distance and could not consistently apply the principle of superposition. It is unlikely they were unaware of the relationship between the distance from a point to electric field strength as the preceding Coulomb's law tutorial and class discussion introducing electric field reviewed the relationship. The electric field tutorial lesson afforded students the opportunity to apply vector concepts and vector addition to an electric field context. This explicit application was justified as the students required prompting to consider both the magnitude and direction of the field at different points, and then apply the principle of superposition. The student gains in transfer manifested in the post-test results, in which most of the students demonstrated the relationship between distance and electric field strength using vector arrows, and reasonable apply the vector constructions to find resultant vectors. This indicates that students learning vector concepts in isolation in not effective for contextual transfer, and the concepts require explicit application in a variety of contexts for students to utilise vectors as a tool to explain and represent those contexts.

The third research consideration addresses student's use of field lines in representing the electric field. Section 5.4 showed progression in student's understanding of field line density representative of relative field strength, the force on a body acting tangentially to a point on a field line and reasonable deviations of a body from a field line. The students did not demonstrate transfer of these concepts as much as was expected to the electric field pre-test. Few students could reasonably draw the path taken by a charged body in a field and field strength was ranked by students using qualitative reasoning based on distance instead of the field line density. The tutorial lesson allowed students to apply the field line conventions to field line contexts and gains were seen in the post-test results. As was seen with the vector concepts, explicit lessons in which field lines are applied to electrostatic contexts are required for them to correctly apply it to the context. This is justified as the results indicate the field line tutorial using a gravitational context alone were not sufficient in helping students develop and transfer the understanding of basic field lines to electrostatics. Additionally, the behaviour of negatively charged particles in field lines does not have an equivalent concept in gravitational field lines. Further explicit use of vectors and field lines in context is appropriate in magnetostatics and electromagnetism, to explain the variation of magnetic field strengths, application

and understanding of the right-hand grip rule and Fleming's left-hand rule and determining the direction of induced current in electromagnetic induction.

The extent to which the student's developed their understanding of Coulomb's law and the electric field varied during the tutorials. Figure 5.43 presents a line plot to illustrate and compare the extent of the conceptual change recorded in these studies. A legend of the codes used in Figure 5.43 can be found in Appendix F.

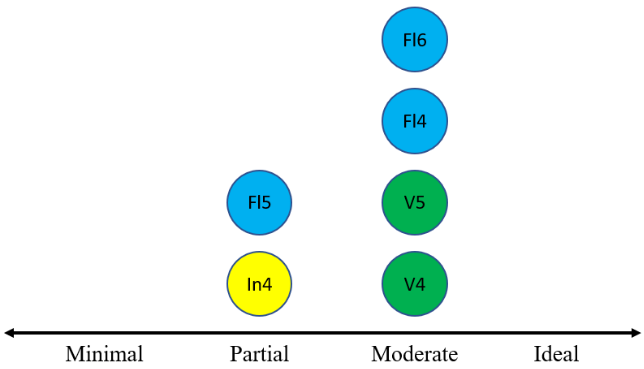


Figure 5.43. Line plot of extent of conceptual change in student's understanding of Coulomb's law and the electric field.

Unlike the results shown in section 4.5, the range to which the extent of conceptual change occurred was not as large between the student's transfer of vectors, the inverse square law and field lines in this section of the research. The most gains were observed in field line and vector concepts, but this is unsurprising as the most gains were observed in with these representations in chapter 4. The student's development and application of the inverse square law in an electro-static context was the most challenging, as seen by only one instance of partial conceptual change observed in Figure 5.43. As discussed in section 4.5, the tutorials in the electrostatic context also employed both mathematical reasoning and scientific reasoning when the students were applying the inverse square law to Coulomb's law, and when using the scale model applied to field lines, as opposed to field lines and vector concepts, when the students were primarily engaging with scientific reasoning. This requirement to employ dual reasoning may once again have overloaded student's working memory resources (Reid, 2009) and impede the student's progress.

5.8.2. Student's transfer between representations in electrostatics

This section addresses the last of the research considerations, in which present the conclusions for the student's ability to transfer. Section 5.7 outlines the student's ability to transfer between vector and field line representations. In transferring from field lines to vectors, persistent difficulties generally related to either students ignoring the magnitude of the vectors, relative to the field strength,

or drawing the direction of the arrows opposing the field lines. The former was a difficulty persistent seen in the students use of vectors, but suggests these students only considered the direction of the field lines when transferring between the representations. The latter difficultly it likely due to a previous question which involved the presence of a negatively charged particles influencing their rationale. This also indicates the students consider the field to be influenced by a charge that interacts with it, instead of being independent of the field.

In transferring from vectors to field lines, it was observed that just under half of the students drew the correct field diagram consistent to their vector representation. The most common difficulty observed was the student's reliance on drawing a field based on the position and types of charged bodies present, instead of being consistent with their vector diagrams. One student utilised the superposition principle to find the resultant vectors are various points, but also sketched field lines, in which their resultant vectors were consistent with their field lines, suggesting the student did not transfer directly from one representation to another, but utilised both representations to accurately show the field.

In comparing both transfers, it was seen that only two students iso-morphically transferred between vectors and field lines, i.e., they could transfer from field lines to vectors accurately, whilst transferring from vectors to field lines. The results suggest that the students were more comfortable with transferring from field line representation to vectors, with minor errors in the transfer present. When transferring from vector to field line representations, slightly more students completed this task effectively, but the errors shown by the remaining students suggested they did not consider the vectors but the setup of the charged particles, which is a major hindrance in their ability to transfer between representations.

One aspect of the difficulties observed in the student's transfer could relate to the mathematical underpinnings required to fully construct a mental model of electric fields. The mathematical courses completed by the students do not contain vector mathematics, nor have the students completed the calculus section of the mathematics course required to completed build a more accurate model of electric fields, in line with the scientific consensus. Issues of transfer between the representation were presented as rules, as opposed to delving into the reasoning underpinning them. For instance, the equation $\vec{E} = \frac{\vec{F}}{q}$ is treated as an algebraic equation as opposed to a vector equation. Reference to the direction of the force being tangential to the field is referenced by the sign of the variable is the same when the charge used is positive, and they're opposite when they're negative. However, without a deeper understanding of vector mathematics, the students unable form coherent models and instances of the students operating with incomplete models were observed in the tutorials sections of this chapter. These issues like have contributed to the relatively low numbers of students being able to transfer between vector and field line representations iso-morphically.

Chapter 6. Work and potential difference

6.1. Introduction

This chapter discusses the development of the student's understanding of the concepts of work and potential difference, as applied to electric fields. The Work tutorial employs the use of vector concepts and field lines to get students thinking about positive, negative and zero work. The potential difference tutorial employed verbal, mathematical and graphical representations, to promote student understanding of the concept, and ability to predict the behaviour of charges acting under the influence of a potential difference. The students also apply their understanding of work and potential difference, along with concepts covered in chapter 5, to explain the behaviour of current and potential difference in a simple circuit.

The following research question is addressed in this chapter:

- To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

The following points are considered when addressing this research question:

- To what extent does the use of vectors and field lines, representing force and displacement, enable students to identify positive, negative and zero work?
- What affect does the use of graphs and diagrams have on students understanding of potential difference?
- What difficulties are encountered by the students during this transfer to a potential difference context?

The timeline of this section of the project is shown in Table 6.1. Sections in bold refer to materials covered as they related to the research. Sections that are not presented in both are required to be covered for completion of the required syllabus for Leaving Certificate Physics. As the Coulomb's law, electric field, work and potential difference tutorial lessons were run concurrently, Table 6.1 starts on Week 5, which chronologically followed Week 4 of the Coulomb's law and electric field tutorial lessons, as was shown in Table 5.1.

The last two subsections of section 2.1.3 detailed difficulties encountered by learners in their understanding of work and potential difference. Based on the difficulties typically encountered by learners, as discussed in these sections, the Work and Potential Difference tutorials were designed

provide the students with opportunities to overcome these common difficulties. These difficulties influence the drafting of the learning objectives for this section, as upon completion of the teaching and learning material, the students would be able to:

1. Identify and explain instances of positive, negative and zero work (Lindsey, *et al.*, 2009; Doughty, 2013)
2. Identify work and displacement, based on electric field line diagrams (Doughy, 2013)
3. Associate relatively high and low potential to positively and negatively charged particles respectively (Hazelton, 2013).
4. Explain the behaviour of charged particles, under the influence of a potential difference (Guisasola, *et al.*, 2002; Maloney, *et al.*, 2003; Hazelton, 2013).

Week 5, Class 1 (35 mins)	Presentation to introduce to review work **Pre-test**
Week 5, Class 2 (80 mins)	**Research lesson: Work worksheet.**
Week 5, Class 3 (76 mins)	Presentation to introduce potential difference. Practise class: qualitative problems.
Week 6, Class 1 (35 mins)	Further qualitative problems involving potential difference, work, potential energy and kinetic energy.
Week 6, Class 2 (80 mins)	Practice class: Qualitative problems involving Electric field.
Week 6, Class 3 (76 mins)	**Research lesson: Potential difference. Homework assignment given.**
Week 7, Class 1 (35 mins)	Review of topics.
Week 7, Class 2 (80 mins)	**Post-test.**

Table 6.1. Timeline of implementation of work and potential difference tutorial lessons.

This chapter presents a narrative of the students use of various representations as they were applied to the context of work and potential difference, whilst targeting the 4 learning objectives. The student's development is presented by comparing pre-test and post-test results for the different topics, as well as display the development of students understanding during the tutorial lessons, with both snapshots of their tutorial worksheets and extracts of recordings of the student's conversations that occurred during the tutorial sessions. Section 6.2 discusses the use of student's application of their understanding of vector and field concepts to develop their knowledge of work, and the use of various representations to develop their understanding of potential difference as a mathematical ratio of work done per charge, their association of relative high and low potential to charged bodies and

how charges move under the influence of a potential difference. Section 6.3 compares student difficulties before and after instruction, and comments on expanded contexts that were used in the post-test, that draw on concepts seen in chapter 5, the pre-test and the tutorial materials. The chapter closes with conclusions, which address the research question and considerations under the headings of the tutorial's impact on student learning, and how the use of various representations helped develop student understanding.

6.2. Work and potential difference tutorials

This section of the chapter discusses the implementation of the work and potential difference tutorials. Section 6.2.1 discusses the pre-test, which focused on student's understanding of work done as a charge is moved from in an electric field, how force and displacement vectors are used to determine several types of work, the behaviour of charged objects in the presence of a potential difference. It also focuses on the association of relative high and low potential to regions surrounding positively and negatively charged particles. This concept is illustrated in Figure 6.1 using a pHet simulation, in which the positive and negative charge are held fixed and equipotential lines are displayed. Any positively charged mobile test charge placed in the space between the charges held fixed would be influenced to move from left to right in this figure, while a negatively charged mobile test charge would move in the opposite manner. This concept would be more commonly applied to the relative potentials of the terminals of a battery, or the plates of a capacitor.

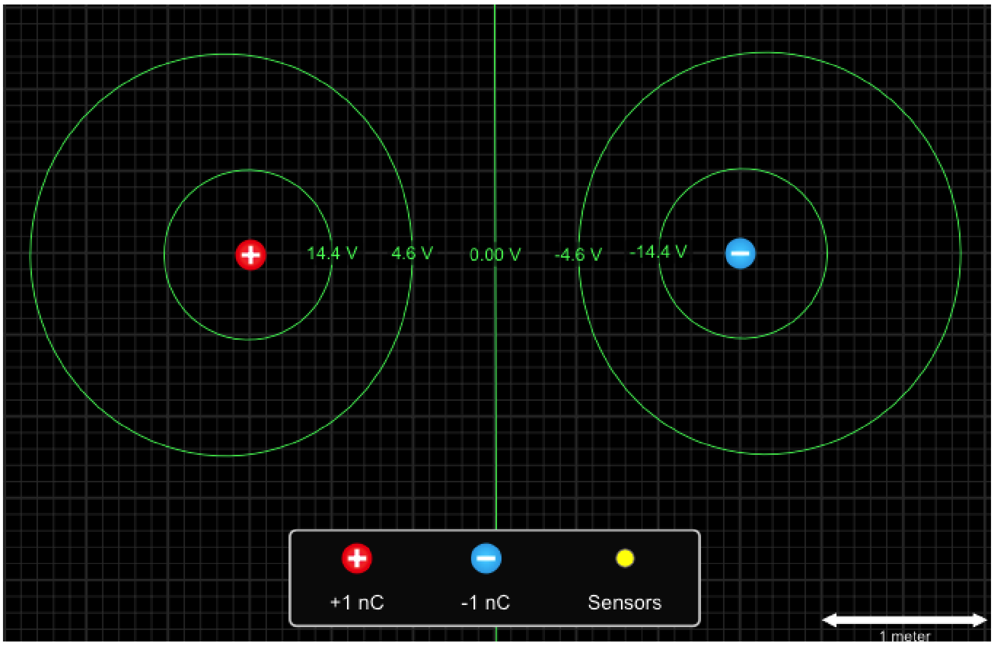


Figure 6.1. pHet simulation displaying the relative high and low equipotential lines due to the presence of positive and negative charges.

Section 6.2.2 and 6.2.3 narrate the Work and Potential Difference tutorial lessons, which identify instances of difficulties encountered by the students. Section 6.2.4 discusses the homework, which advances the students understanding of potential and applies a graphical approach to thinking about the variation of potential due to positive and negative charges. Section 6.2.5 analyses question given to students in the post-test, that either directly relate to questions asked in the pre-test, or present contexts that are extensions from questions seen in the pre-test, tutorials and homework. The tutorials were written in the same style as those seen in Tutorials in Introductory Physics, with some ideas and contexts taken from Conceptual Physics (Hewitt, 2009).

6.2.1. Pre-test: Work and Potential difference

The pre-test question was designed to test student's understanding of work, focusing on their understanding of displacement vs distance travelled when a body moves from one point to another in a field. Figure 6.2 shows the diagram, in which the students were to rank the work done in moving from A to B, along the three paths. Correct reasoning would show that the work done is the same in all cases. This could be argued by the displacement in all cases being the same, or discussing the contribution of positive, negative and zero work along all the paths taken. Expected incorrect reasoning would be the student's relying on the distance travelled along the different paths, instead of displacement. A summary of the student's responses is presented in Table 6.2.

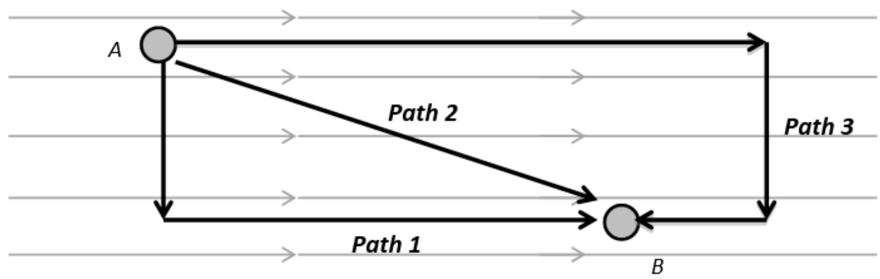


Figure 6.2.Extract from pre-test question in which student's rank work done in 3 paths.

The pre-test results indicate that the students did not consider positive, negative and zero work reasoning to the problem, and predominantly focused on the distance travelled along the paths as the justification of their ranking. Furthermore, five of the students referenced that in path 3, the path moves against the field which leads to more work, with two of the student referencing the ease taken along the other paths and the resistance encountered in going against the field. This suggests these students still conceptualise the field with tangible properties, in this case, as contributing to a resistance that must be overcome, similar in nature to swimming against the current in a river. Similar reasoning was also observed in student 4H's responses, in which they said the path was deflected by the field. They reasoned that path 2 had only one deflection at an acute angle, while path 1 deflected

at ninety degrees leading to more work involved in this path and as there was two perpendicular turns in path 3, there was more work in that path. Another student, 4C, referenced both the distance taken in the paths, but also noted that path 2 was the resultant of the components vectors for path 1, and equated this as contributing to more work in path 2.

Responses.	Students.
$\boldsymbol{W_1 = W_2 = W_3}$	**N/a.**
$W_2 < W_1 < W_3$	4A, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M.
$W_2 < W_1 = W_3$	4B.
$W_1 < W_2 < W_3$	4C.
$W_2 < W_3 < W_1$	4N.
Displacement / Positive, negative, zero work reasoning.	**N/a**
Reasoning based on distance travelled.	4B, 4G, 4I, 4J, 4K, 4L, 4M.
Reasoning based on “resistance” encountered.	4A, 4C, 4F, 4G, 4I.
Reasoning based on vector resultant.	4C.
Reasoning based on field “deflection” of paths	4H.
No clear reasoning	4D, 4E, 4N.

Table 6.2. Student’s responses and reasoning to pre-test work ranking question.

In the second pre-test question, the students were required to rank the magnitude of the work done, for various pairs of force – displacement vector pairs. Christensen, *et al.*, (2004) noted ~75% of undergraduate students in their study could mentally apply the dot product to rank vector pairs when the angles were in the range $0^{\circ} \leq \theta \leq 90^{\circ}$, but when angles greater than 90° were introduced, the percentage dropped to ~60%, noting students had difficulties in recognising vector layouts that would produce negative work. Their research specifically looked at vector mathematics, whilst the pre-test question uses a work context and as the magnitude of negative and positive work are equal in this question, it was decided that the sign of the work would not have to be observed to consider a correct answer, if the students referred to the scalar nature of work. Figure 6.3 shows the question presented in the pre-test, and Table 6.3 summarises the student responses.

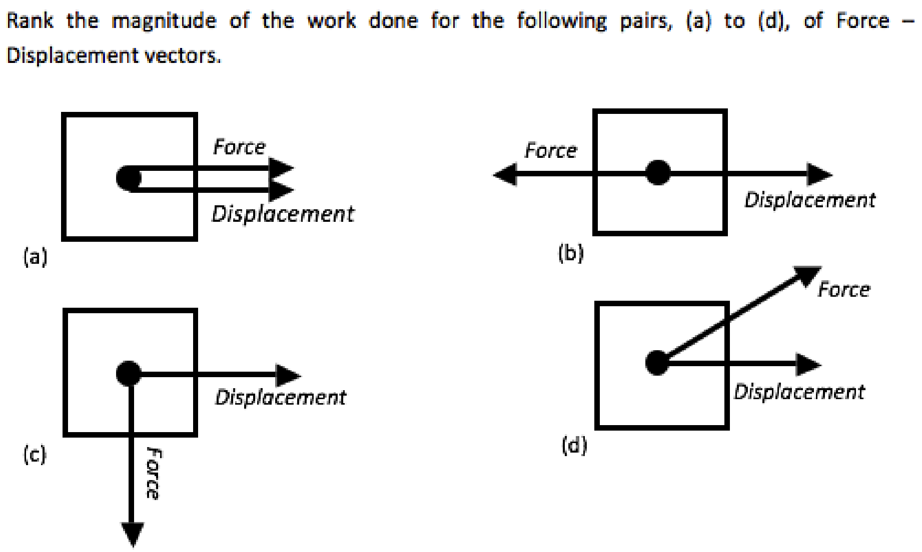


Figure 6.3 Pre-test question in which students use vectors to rank work done.

The results show that none of the students ranked the magnitude of the work done correctly in any of the setups shown. The most common incorrect ranking, produced by 9 of the students, was $W_A > W_D > W_C > W_B$, and the reasoning produced by these students varied, with three students (4C, 4D and 4E) using vector addition reasoning with the force and displacement. Two students (4F and 4M) referenced the force as adding resistance to the displacement when they are anti-parallel. These students may think force adds a thrust to the displacement when they are parallel, and does not affect displacement when perpendicular. Thus, the angle between the force and displacement would determine the work done (student 4G) and the relative displacement between the arrowheads of the force vector and displacement vector indicates the amount of work done in the system. Of the other rankings, only student 4H provided reasoning, suggesting as the displacement in all cases did not change, the force was indicative of the work and used this to produce their ranking.

The next question was designed to determine the student's understanding of the behaviour of charged objects in a potential difference. They were required to predict the movement of the two boxes, shown in Figure 6.4, and justify their predictions. A summary of their responses is shown in Table 6.4.

Responses.	**Students.**
$\boldsymbol{W_A = W_D > W_C = 0 > W_B}$	**N/a**
$\boldsymbol{W_A = W_B = W_D > W_C = 0}$	**N/a.**
$W_A > W_D > W_C > W_B$	4A, 4C, 4D, 4E, 4F, 4G, 4K, 4M, 4N.
$W_A > W_B > W_C > W_D$	4B.
$W_D > W_A = W_B = W_C$	4H.

$W_C > W_B > W_D > W_A$	4I, 4J.
$W_D > W_C > W_A > W_B$	4L.
Force and displacement cancel out.	4C, 4D.
Force decreases with distance.	4A.
Determines resultant of force and displacement.	4E.
References resistance.	4F, 4M.
Wider angle results in more work.	4G.
Force magnitude determines work.	4H.
Distance between arrowheads / vectors indicates work.	4K, 4N.
N/a.	4B, 4I, 4J, 4L.

Table 6.3. Student's responses and reasoning to pre-test work questions, based on force and displacement components.

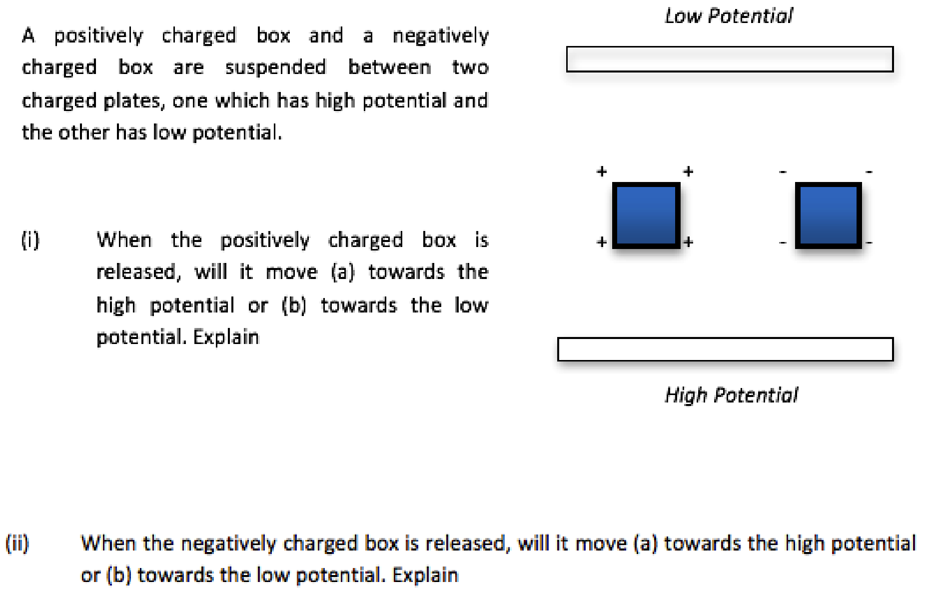


Figure 6.4. Pre-test question about charged objects under the influence of a potential difference.

While the students correctly predicted the behaviour of the charged boxes in a potential difference, they did not apply correct reasoning to do so, based on the information they had. The most common reason used to justify the student's predictions was to associate a positive charge with the plate labelled high potential and negative charge to the plate labelled low potential. Once they had determined the charges on both of the plates, they predicted the motion of the boxes based on the force interactions between the boxes and the charge of the plates. While such reasoning is valid,

the students did not come across this in the tutorials nor the class discussions and may have (a) guessed this was how the potential of the plates can be used as a reference to determine the charge on them or (b) assumed that positive indicates high and negative indicates low due to positive integers having higher values than negative integers, which are relatively lower in value. Two students (4J and 4K) referenced the effect of gravity on the boxes, although student 4K's responses were not consistent with their predictions, in which the bodies would move in anti-parallel directions. Students 4C and 4D referenced that the boxes had less and more protons for their respective charges and moved to the low potential and high potential because of this, without clarifying why they referenced the protons as they did or explain how the behaviour was dictated by protons. Student 4B referenced that the bodies become unstable due to having potential energy. As they stated that both boxes would move from high to low, this may be a reference to gravitational potential energy.

In the final question on the pre-test, the students were given a graphical representation of a profile for potential difference and asked to sketch the positions of charges to produce the low – high – low potential variance, as shown in Figure 6.5. This question was designed to elicit student's thinking about the relative potential associated with positive and negative charges, positive being high potential and negative being low potential. A summary of the student's responses is found in Table 6.5.

Responses.	Students.
Positive moves from high to low.	**4B, 4C, 4D, 4E, 4F, 4G, 4M, 4N**
Positive moves from low to high.	4A, 4K, 4L 4J
N/a	4I, 4J
Negative moves from low to high.	**4A, 4C, 4D, 4E, 4F, 4G, 4H, 4J, 4L, 4M**
Negative moves from high to low.	4B, 4K
N/a	4I
References behaviour due to potential difference.	**N/a.**
Assumes charges on plates.	4E, 4F, 4G, 4M, 4N.
Use gravitational field as for reasoning.	4J, 4K
References protons.	4C, 4D
Instability due to potential energy.	4B
Reasoning unclear / no reasoning submitted.	4A, 4I, 4L

Table 6.4. Responses and reasoning to pre-test question involving the movement of charges bodies acting under the influence of a potential difference.

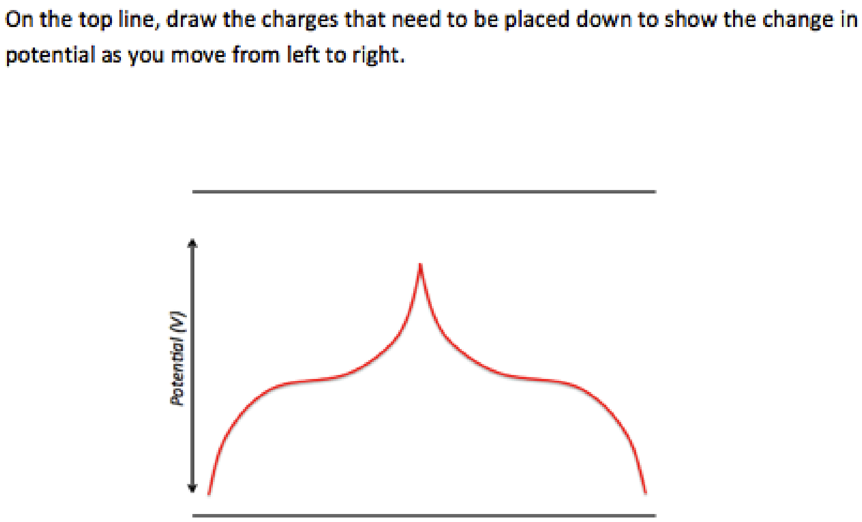


Figure 6.5. Pre-test question eliciting student's association of high and low potential to charges.

Responses	*Students.*
Associate high potential with positive charge.	**4B, 4G, 4I.**
Associate low potential with positive charge.	4E, 4F, 4L, 4M.
Does not associate charge with potential.	4A, 4C, 4D.
N/a.	4H, 4J, 4K, 4N.
Associate low potential with negative charge.	**4G, 4I.**
Associate high potential with negative charge.	4E, 4F, 4L, 4M
Does not associate charge with potential.	4C, 4D.
N/a.	4A, 4B, 4H, 4J, 4K, 4N.

Table 6.5. Responses to pre-test question involving the association of high and low potential of charged bodies.

A correct response to this question would place negatively charged particles at the two positions where the graphs are at the minimal values. A positively charged particle would also be expected to be placed at the position where the graph was at the maximum value. This would show the student applying the association of relative high and low potential to the areas around positively and negative charged particles, as described in section 6.2 and illustrated in Figure 6.1. The pre-test results show that most of the students did not associate relative high potential to regions with positively charged bodies and regions negatively charged bodies with relative low potential. Only two students, 4G and

4I, submitted the correct charges for the high and low potentials, with only student 4G submitting reasoning. Student 4G was consistent with their assumption shown in the last question, in which they associated high potential to positive and low potential to negative, whilst 4I gave no reasoning in either this question or the last question. Surprisingly, students 4E, 4F, 4M and 4N correctly guessed the association in the last question, but the first three of these students reversed their association in this question, while student 4N did not attempt the question. As the remaining students provided no reasoning for their responses, it is unclear as to why they chose the charges as they did, but it is probable that it was due to guessing.

This section detailed the student's initial misunderstandings and limits of their understanding of work and potential. It was seen that when considering work in a field, when different paths can be taken from one point to another, the prevalent difficulties encountered by the students considering the distance travelled over the displacement travelled, or the considered the field to act with a tangible property providing a resistance for the work to overcome in moving a charge against the field. It was seen that the students could not rank the work done in moving a box when the force and displacement vectors were aligned in various directions, due to an inability to identify positive, negative or zero work, or consider the absolute value of the work done, regardless of whether it involved increasing or decreasing the energy in the system. The third question showed the students could predict the movement of positively and negatively charged objects under the influence of a potential difference, but several students associated positive and negative charge with high and low potential, which was not consistently observed in the last pre-test question.

6.2.2. Tutorial lesson: Work

The tutorial lesson on work began with a twenty-minute class discussion and presentation of the concept of work. The initial context used was the same used in the mechanics section of their physics course, in which a car moves from one point to another and the students were asked to explain what effect a force has on the car when pointing (i) in the direction of the displacement, (ii) against the displacement and (iii) directed towards the ground, as shown in Figure 6.6. Using a think-pair-share strategy, the student groups were able to explain that when the force and displacement were parallel, the velocity of the car would increase; when in opposite direction, the velocity would decrease, and when perpendicular the velocity would not change. The teacher reintroduced the terms positive, negative and zero work and applied explained them in terms of the student's responses, as these terms were used when they initially studied mechanics. The students were then presented with a formula for work, $W = Fs\cos\theta$, and required to substitute values in for the three force-displacement diagrams shown on the car. The students were not presented with vector notation due to their unfamiliarity with the notation in their mathematics course, and because the aim to develop their

understanding in the tutorial was primarily qualitative and conceptual in nature. The discussion repeated for contexts such as a ball on the end of a compression spring pushed from its equilibrium position to a compressed state, and a charged particle being pushed towards a positively charge dome. These contexts were taken from Conceptual Physics (Hewitt, 2009).

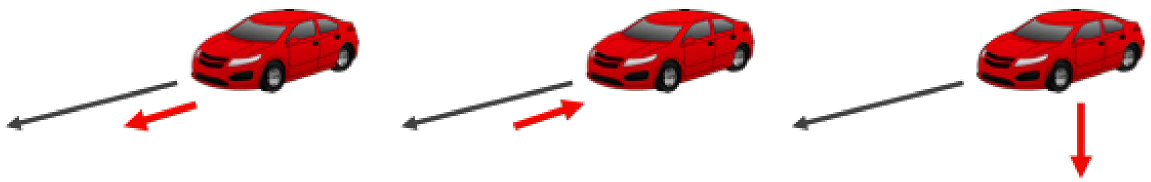


Figure 6.6. Initial context used to illustrate the concept of positive, negative and zero work.

The tutorial worksheet involved the students considering the path taken in getting from two points using different paths, as shown in Figure 6.7 (i). The students were required to explain the different between the distance travelled and the displacement. The students were then presented with a block of weight 100 N, that is pushed a total displacement of 6 m with 50 N of force. They were required to calculate the work done by the person pushing the block, and determine if any work was done by gravity, and if so, how it contributed to the movement of the block. The students were then required to consider the block being pulled with a force of 50 N through the 6 m, with the rope making an angle of 30° to the horizontal, as depicted in Figure 6.7 (iii). The students were required to resolve the force vectors, with acted along the 30° diagonal, its horizontal and vertical components, and determine which components contributed to the positive work and zero work. The students were required to calculate the net work done on the block, and were verbally asked to explain why the net work done, from the scenario presented in Figure 6.7 (iii) did not equal the work done when the force was applied horizontally, as presented in Figure 6.7 (ii), even though both blocks were moved with 50 N through a distance of 6 m.

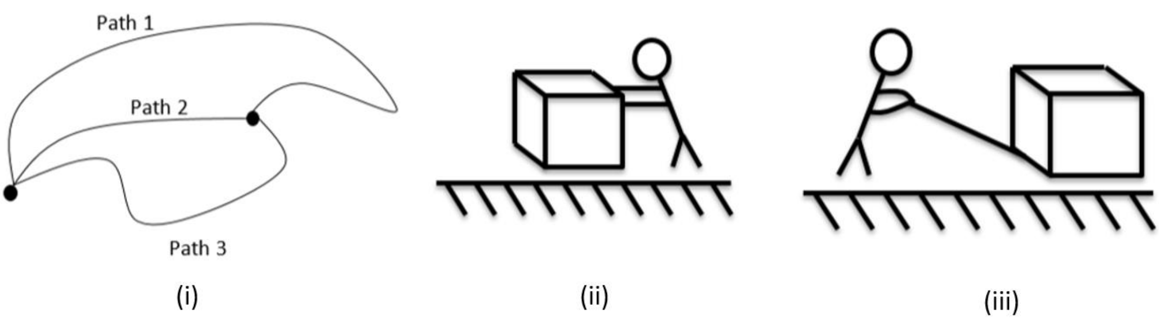


Figure 6.7. Diagram extracts from the work tutorial involving displacement and forces.

The students were then presented with the second pre-test question (see Figure 6.3), in which they had to rank the magnitude of the net work done in all four cases. They were encouraged to use

rulers to record relative magnitudes and, in some cases, resolve the force vectors into their horizontal and vertical components. Having discussed the concepts of positive, negative and zero work, as well as completing the previous questions in the tutorial, the students tended to produced rankings of $W_A = W_D > W_C > W_B$, or $W_A = W_B = W_D > W_C$. The students were encouraged to measure values from the diagrams by using a ruler and applying a scale of 1cm : 1N and 1cm : 1m, to determine the force and displacement respectively. In cases of the first ranking, students explicitly stated that the negative work in diagram B was less than zero, in cases of the last ranking, students stated that although the work was negative, it would have the same value as diagram A and D. One group displayed a unique error, consisting of students 4J, 4K, 4L and 4M, in which they added the force and displacement vector magnitudes, instead of multiplying them. This error is inconsistent with the work they completed in the previous section of the tutorial, in which all students multiplied the force and displacement magnitudes. One possible source of this error is the student's interpretation of the boxes Figure 6.3, in which they considered and applied vector addition when they observed the vector arrow pairs. If this is the case, this error came from the students misinterpreting the representation used, and were thus unable to correctly transfer the information to the mathematical symbolic representation, W = F.s.

Student 4H: 5.29 [J] is greater than 0 [J], which is greater than -5.29 [J]

Student 4K: The [horizontal] components in A, B and D result in 4 [J] because work is 2 + 2 which is 4, but because C has a vertical vector, the magnitude = 0.

In the last section of the work tutorial, the students were presented with the diagram shown in Figure 6.8, in which they were informed that a 1 kg mass was moved between different points.

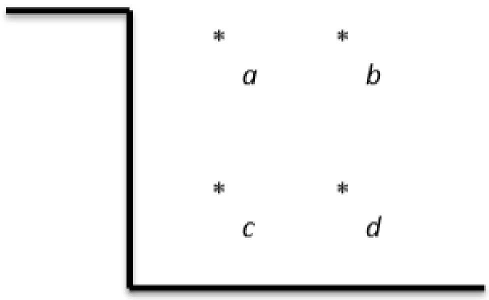
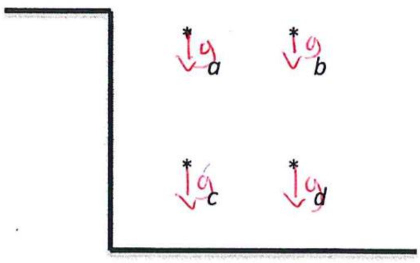


Figure 6.8. Diagram from work tutorial question focusing on positive, negative and zero work done by gravity.

They were required to identify the direction of the gravitational field, and determine if the direction of the displacement from *a* to *c* was parallel, anti-parallel or perpendicular to the gravitational field. This allowed the students to determine if the work done in moving from *a* to *c* was positive, negative or zero. From identifying the sign of the work done, the students had to

comment on the potential energy at the top and the kinetic energy at the bottom. The students then had to identify the work done in moving from *d* to *b* and *a* to *b*, and upon completion, were verbally asked to comment on the potential energy changes for these types of work done. Each group of students stated that negative work, with respect to the gravitational field, occurred when the body moved *d* to *b*, reasoning that the force of gravity was anti-parallel to the displacement travelled. They also reasoned that zero work occurred when a body moved from *a* to *b*, as the vectors were perpendicular. Having identified this, the groups verbally articulated that, with respect to the influence of the gravitational field on bodies between the points, positive work resulted a decrease in gravitational potential energy and an increase in kinetic energy, negative work resulted in an increase in gravitational potential energy and a decrease in kinetic energy and zero work resulted in no change in the gravitational potential energy. An example of a student's written response that displays most of these points is shown in Figure 6.9.

Student 4B:



The displacement is in the direction of the gravitational force. Positive [work moving from a to c] because the force is in the same direction of the displacement. The potential energy increases from c to a. The potential energy at a = to the kinetic energy at c.

Figure 6.9. Student 4B's response for section on work due to gravity in tutorial lesson.

This section discussed the student's progress throughout the work tutorial. It was observed that the students could identify instances of positive, negative and zero work. Having been guided through how to analyse and perform calculations to determine a value for work for instances in which force and displacement vectors diverge at an acute angle, all but one of the groups were able to apply this skill to a similar context. I suggest that the group's error was their interpretation of the information presented on the diagram, in which they decided to apply vector addition to the arrows, instead of multiplication of their magnitudes. The last section displayed that students could identify, in a field setting, positive and negative work, which was targeted so (a) they could apply it the concept of potential difference in an electric field context and (b) they could relate work the maximum potential and kinetic energies so they could apply conservation of energy calculations in electric fields, such as electrons moving in a cathode ray tube or x-ray tube.

6.2.3. Tutorial lesson: Potential difference

This section discusses the potential difference tutorial. The tutorial employs the use of field lines and displacement vectors to identify positive, negative and zero work. The students were then guided through developing the formal definition of potential difference, i.e., the negative of the work done by the field per unit charge when a charge is moved from point to another, through analysing a diagrammatic and mathematical scenario. The students were then given an electric field and were asked to complete calculations involving work, potential difference, and potential and kinetic energy.

Figure 6.10 presents the first electric field with various points marked. The students were required to identify the work done in moving through various combinations of the points, similar in nature to the exercise at the end of the work tutorial.

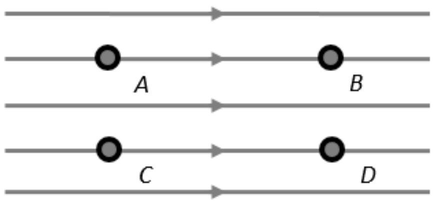


Figure 6.10. Diagram from potential difference tutorial focusing on positive, negative and zero work done on a charged body.

Having identified instances of positive, negative and zero work, the students were then required to consider paths taken in which involved a combination of positive and zero work, through analysing a dialogue between two hypothetical students.

Person 1: When we move the charge from A to D directly, there is less work done than moving it from A to C to D as we add up the work done moving from A to C directly to the work moving from C to D directly.

Person 2: When the charge if brought from A to C and C to D, the displacement has a vertical component which gives zero work. This makes the work done independent of the path taken.

By thinking about this dialogue the students would consider both the displacement between the initial and last point and how combinations of positive and zero work, or negative and zero work, can simultaneously contribute to a mobile test charge moving in a field. When the students were comfortable with applying the concepts of work to the electric field, the tutorial shifted to applying mathematics to moving charges in an electric field to develop their understanding of potential difference. They were presented with Figure 6.11, (i) and (ii), in which they initially considered the potential energy of a body lifted 3 m into the air, dropped to the ground.

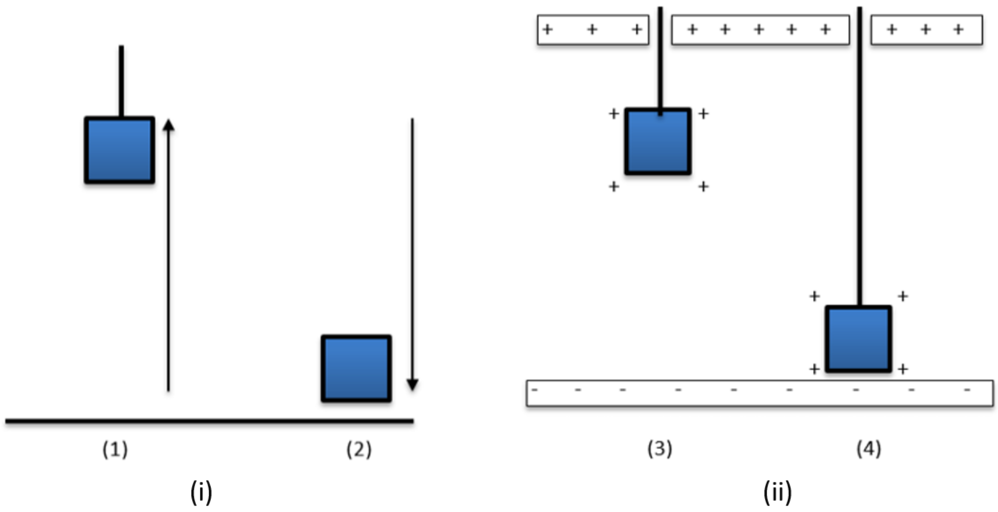


Figure 6.11. Diagram from potential difference tutorial comparing gravitational work with electrostatic work.

The students calculated the work done by gravity when 1 kg, 2 kg and 3 kg were dropped, and determined the ratio of work done per mass of the box. While there is no measured quantity for gravitational potential difference, this allowed the students to observe the fixed quantity nature of potential difference, in a context that is not abstract, in this case, the work done in moving 1 kg of a body. The students then compared the potential energy of the box at position (1) and the kinetic energy of the body at position (2) and expressed this in written form, and mathematically.

This set of activities was repeated using an electric field and a charged body, as shown in Figure 6.11 (ii). The students worked out the work done in moving charge bodies of +1 C, +2 C and +3 C a distance of 3 m. The students calculated the work done per unit charge in each case, observing that potential difference was a fixed quantity in the setup. The students had to summarize their finding by defining the ratio in their own words.

Student 4L: The work done is directly proportional to the charge → the ratios are constant.

Student 4B: Equal and proportional. The work divided by the charge is equal to 6/1, and is always constant.

Student 4I: 6 J of work is needed per +1 C of charge.

In the last section of the tutorial, the students were again presented with an electric field, and two points, A and B, as seen in Figure 6.12. The students were introduced to the formal definition of the potential difference, and the formula, $V = \frac{W}{q}$. The students were required to calculate the potential difference in moving a -1 C charge along path 1, and then path 2, and explain similarities between the two values. Most students said the potential difference was the same in each case, favouring explanations that reference the displacement between the points, instead of commenting on positive,

zero, and negative work reasoning, although one student, 4F, did identify the work moving from B to A would be positive but did not explicitly link it to either of the paths taken.

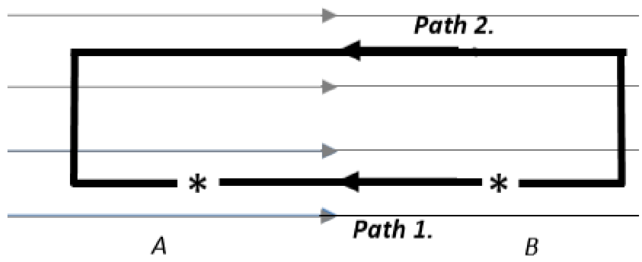


Figure 6.12. Diagram extracted from tutorial in which students apply work, potential difference and energy to different paths.

Using the potential difference value, the students were required to determine the work done in moving a -4 C charge and determine the magnitude of a charge if the field produces 36 J of work in moving the charge. They then applied their understanding of the conservation of energy to calculate the velocity of a body with mass 0.5 g and charge of magnitude -3 C as it moves from B to A. This allowed them to show that a body moving under the influence of a potential difference will convert potential energy, based on its position in a field, into kinetic energy. A sample of calculations is shown in Table 6.6. This demonstrates how the last section of the tutorial guided the students to relate the potential difference between two points in a field to apply the conservation of energy. By completing this, they demonstrate how to complete calculations involving potential and kinetic energy in electric fields. This is typical of the style of calculations the students complete when studying x-ray tubes, cathode ray tubes and particle accelerators, in which they need to utilise the potential difference of the device to calculate the energies and velocities of particles that move in the device.

Student 4H:	
$6 = \frac{w}{3}$ $w = 6\,x\,3 = 18\,J$	$w = \frac{1}{2}mv^2$ $18 = \frac{1}{2}(5\,x\,10^{-4})v^2$ $v^2 = 72{,}000$ $v = 268.33\,m/s$

Table 6.6. Example of calculations produced by student 4H.

6.2.4. Homework: Potential difference

The homework involved students using their understanding of attraction, repulsion and electric fields to build up an understanding of the behaviour of charged particles acting under the influence of a potential difference. As seen in the potential difference tutorial, in section 6.3.3, the students

initially explained the behaviour of a body acting under a gravitational field of high and low potential, and applied the reasoning to an electric field context, as seen in Figure 6.13 (i). They then extended their model, as seen in Figure 6.13 (ii) to incorporate the behaviour of negatively charged bodies in a potential difference, a phenomenon which has no equivalent behaviour in mechanics.

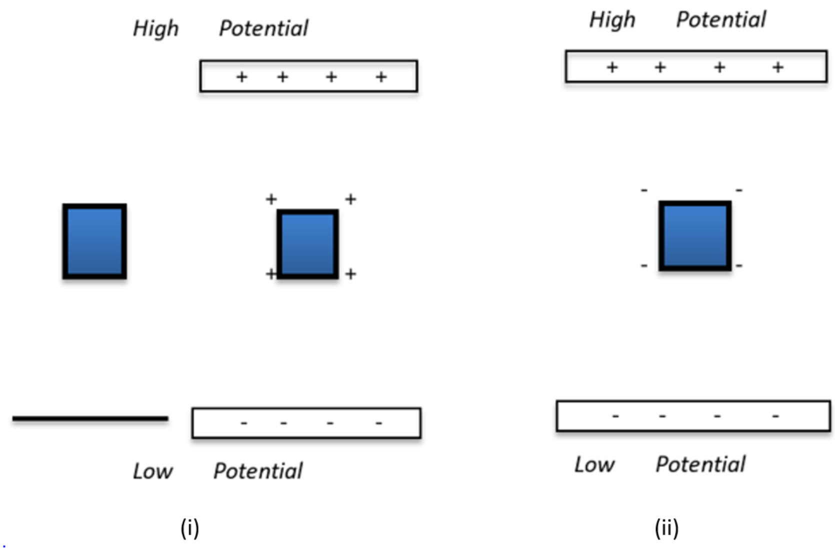


Figure 6.13. Diagram from homework for charged bodies moving under the infleunce of a potential difference.

It was seen that all students who completed the homework assignment explained that the positively charged box moves from high to low potential, generally noting how the box is attracted to the negative charged plate and reflects the behaviour of a body falling under gravity. They did not reference the interaction of charged box and the positive plate, and exclusively based their reasoning on the force of attraction they predicted. All the students acknowledged the negatively charged body would move up towards the positively charged plate and most could use this behaviour to determine that the negatively charged bodies would move from regions of low to high potential. However, student 4I and 4K contradictorily stated that it would move towards areas of low potential. When asked to explain this in feedback, they acknowledge that if the body is moving up towards the positive plate, it must move to high potential, and behave in the opposite fashion to the positively charged box. Student 4F explained that the body would build up potential energy, as it moved under the attraction to the positive plate. When asked to explain this, the student suggested that since the body was higher, it would have more potential energy. The student was asked to consider the phenomena as displaying the opposite behaviour to the positively charged box and asked to consider which of the two mimics gravitational fields and which acts in the opposite manner.

In the last section of the homework, the students were given a set of graphs with a charge layout, as seen in Figure 6.14. These graphs were to help students visualise and associated positively charged regions as areas of high potential and negatively charged regions as areas of low potential. The shapes of the graph were intended to show how relative potential decreases as the distance from a positive

charge increases, and the relative potential decreases as the distance as the distance from a negative charge decreases, as previously illustrated in Figure 6.1. Initially, the student had to explain the shapes of the graph, as the position from moves from left to right.

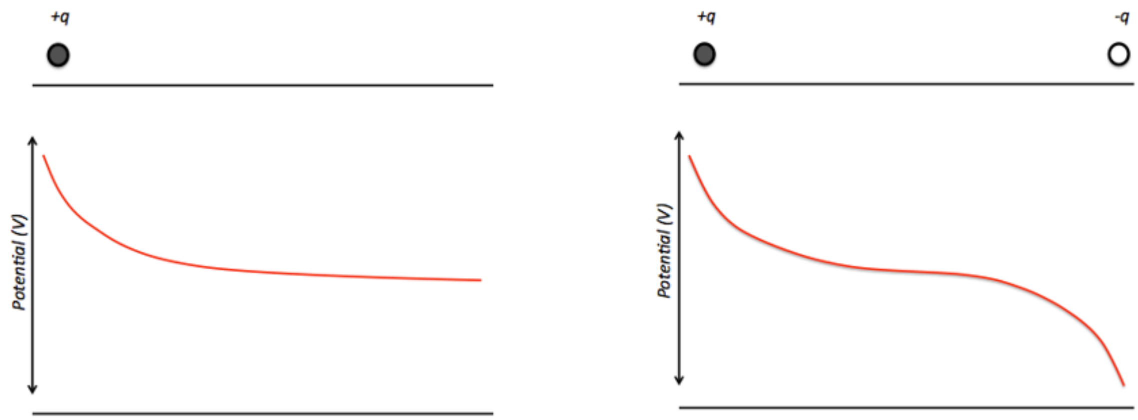


Figure 6.14. Graphs from potential difference homework.

The students had many difficulties with this section, and only two students (4B and 4G) explained that the first diagram showed variation from high to low potential while the second graphed showed variation from high to even lower potential, due to the negatively charged particle. Student 4K also indicated that moving from left to right showed moving from high to low potential but did not qualitatively compare how the presence of the negatively charged particle affected the potential.

A prevalent difficulty observed in the remaining seven student's responses was that they associated the negative particle as repelling the potential downwards or focusing on the shape of the graph and stating that the potential displays an inverse pattern that was stronger near the positive charge and weaker near the negative charge. The initial difficulty indicates a failure to understand potential as a property of an electric field produced by a charge. There is also an indication of attributing a tangible property to potential, which is commonly seen with field lines, but unexpectedly, this difficulty was translated onto a graphical representation. The latter difficulty shows an example of a student mathematically analysing the pattern shown, which accurate models the pattern seen in inverse relationships. The use of the terms weaker and stronger indicates a property of dominance between the two regions, typically seen in the superposition of vector quantities, such as force and motion. In both cases, students are attributing qualities and properties to the potential that the tutorial lesson and homework failed to address.

The students were then asked to graph the potential energy for the first set of charges in Figure 6.15, and then to sketch what types of charges could produce the graph seen the second half to the diagram.

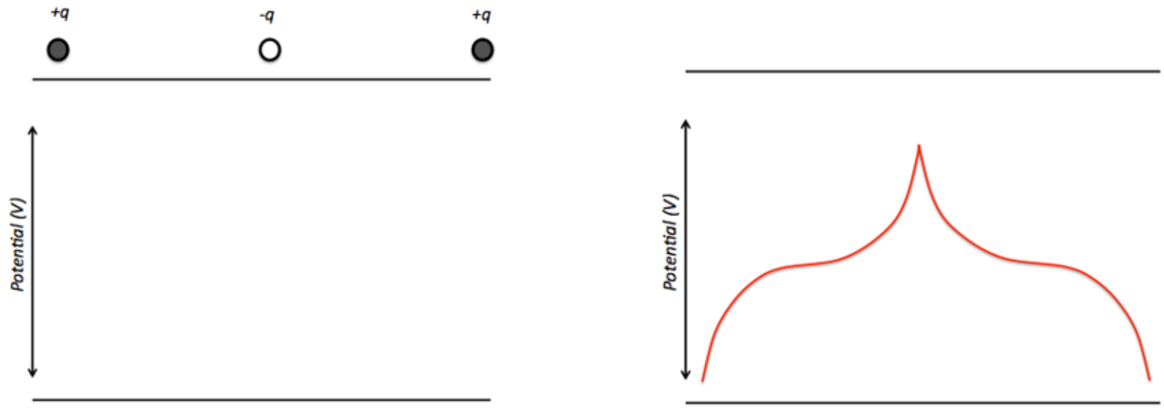


Figure 6.15. Graphs from potential difference homework, representing the variation of potential and charge layout.

All the students drew graphs of the correct shape for the first half of the diagram, with the two positive charges being regions of high potential, and the negative charge as a region of low potential. Additionally, all the students placed negative charges to the ends of the region and a positive charge in the middle, for the second diagram. However, the difficulties of attraction, repulsion and stronger and weaker potentials were also encountered in this section.

This section showed that using diagrammatic representations and drawing comparisons to between a body's behaviour in a gravitational field and a charged body's behaviour in an electric field can help student's construct their own understanding of how charged behave in a potential difference. It was also seen that the use of graphical representations can be employed, but difficulties about the nature of potential were seen and further instructional design would be required to address these difficulties.

6.2.5. Post-test: Work and potential difference

This section outlines the post-test results from the students, in which they apply their understanding of work in an electric field setup. The students also had to apply their understanding of the behaviour of charge under the influence of conducting spheres, connected by a conductor under the influence of an externally charged rod. The students were then required to use their understanding of vectors, forces, electric fields and potential difference to explain the behaviour of current in a closed circuit containing a battery and a wire. Two students were absent during the period this post-test was administered (N=12).

In the first post-test question the students were presented with an electric field, as shown in Figure 6.16, and asked to identify the direction of force acting on a negatively charged body placed at point *O*. A correct response for this question would be the student identifying that the force acts against

the field, due to the field pointing in the direction of a positive test charge. From this the students were asked to determine what type of work was done in moving a negatively charged particle to the points *A*, *B*, *C* and *D*. Correct reasoning would indicate the students were consider where the force and displacement vectors were parallel, anti-parallel or perpendicular to each other. The student results are presented in Table 6.7.

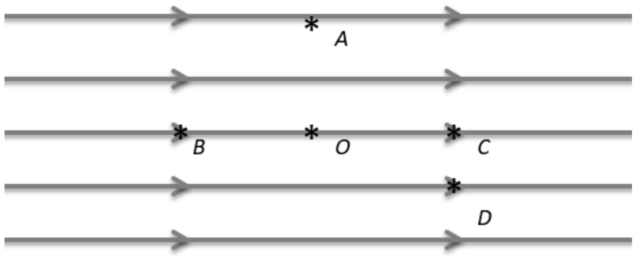


Figure 6.16. Diagram from post-test question involving work done in an electric field between various points.

Four of the students (4F, 4K, 4L, 4N) stated the stated that the negatively charged particle would follow the field line, not considering the convention that the field line points in the direction of force acting on a positively charged body. The remaining students correctly depicted the direction of the force acting on the negatively charged particle was anti-parallel to the field at the point *O*. Accounting for the error presented by the four students, it was observed that all the students consistently identified the work done was positive when the force and displacement vectors were parallel, negative when they were anti-parallel and zero when they were perpendicular. A summary of student's comparisons of the work done in the path (i) *O* to *C*, (ii) *O* to *D* and (iii) *O* to *C* to *D* is presented in Table 6.7.

The results show that only six of the students determined that the work done in moving the designated paths were equal. Four of these students (4D, 4G, 4M and 4N) produced explicit reasoning, in which they referred to the horizontal displacement producing non-zero work and vertical displacement producing zero work, or the horizontal displacement and force acting on the body being the same in all cases. Four students (4C, 4H, 4I and 4L) stated that only horizontal vectors need to be considered in this question, which alludes to their understanding of non-zero and zero work being done without being explicit about it.. Two of these students, 4C and 4H, produced the correct ranking with this reasoning. Students 4I and 4L stated that there is more work done in moving directly from *O* to *D* than there is in moving to *D* via *C*, using the same reasoning as students 4C and 4H. This suggests that these students applied the concept of non-zero and zero work when the vectors are explicitly parallel, anti-parallel and perpendicular, but struggle to apply this when the angle between the displacement and force vector is acute. This was seen in the case of the work done in moving from *O* to *D*, in which the displacement vector can be resolved into horizontal and vertical component, which contribute non-zero and zero work respectively, which was not considered by these students.

Responses.	**Students.**
$W_C = W_D = W_{CD}$	**4C, 4D, 4G, 4H, 4L, 4M.**
$W_D > W_C = W_{CD}$	4B, 4I, 4K, 4N
$W_D = W_{CD} > W_C$	4F
$W_D = W_{CD}$, W_C not defined.	4J
Perpendicular vectors produce zero work.	**4D, 4M, 4N**
Same force and same displacement in each path	**4G**
Only need to consider horizontal vectors.	4C, 4H, 4I, 4L
Displacement reasoning.	4B, 4F, 4I, 4K, 4N
Other	4J
N/a	4A, 4E

Table 6.7. Responses to post-test question involving ranking the work done in moving between different points in an electric field.

Two other students, 4K and 4N explicitly referenced that the displacement from *O* to *D* was the greatest but equated the other two paths. This suggests they also considered to apply the concept of non-zero and zero work to those paths but did not consider the displacement for *O* to *D* as a combination of horizontal and vertical vectors. Student 4F based their ranking on the absolute displacement for the start and end points of the paths. In this case, they ranked that *O* to *D* and *O* to *C* to *D* as having the same work done, but more than *O* to *C*, due to the smaller displacement in the last path. Student 4J also used the same reasoning for the first two paths but did not explain how the work done in the last path compared to the first two.

Another post-test question looked the student's ability to explain the behaviour of electrons moving under the influence of a potential difference, in a depiction of two metal spheres connected by a wire, with a galvanometer. The students were told that a rod of different charge is placed close to the spheres and that negative charge moved in the direction as shown on the galvanometer, as shown in Figure 6.17.

The students were asked to compare the potential on the two spheres in each of the three cases and explain their rationale. This question only references the initial potentials, just as the charges are being made to move, not the final potential when the charges have stopped moving and the potential is equal in all cases. A summary of the student's answers is presented in Table 6.8.

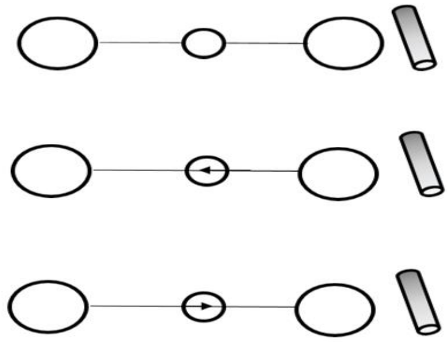


Figure 6.17. Diagram from post-test question eliciting understanding of the movement of charge in a potential difference.

Table 6.8 shows that the seven students stated the right dome had lower potential than the left when the charge moved to the left, and higher potential than the left when the charge moved to the right. This indicates that the students explained the direction of current moving is influenced by areas of low to high potential, in the initial moments of the scenario shown. One of the students (4G) used the movement of charge and considered how the charge built up on each sphere as time went on. For instance, in the middle diagram of Figure 6.17, their reasoning suggested that the right sphere was positively charged as it lost electrons. As it became positively charged, it developed a relative high potential. The opposite was true of the leftmost dome, in which it built up an excess negative charge and developed a relative low potential due to this build up.

Student 4C: The potential on the left is stronger than the potential on the right.

Student 4G: Low potential in the left [sphere] because it is negatively charged. High potential in the right [sphere] because it is positively charged.

Four students (4B, 4D, 4L and 4N) stated the opposite potentials for the spheres in the pictures, with reasoning that indicated they were considering the potential of the spheres are time moved on. For instance, in the last picture, in which the current moves to the right sphere, they indicated that as the charge builds up on the sphere to the right, it develops a negative charge and its potential lowers, whilst the sphere on the left builds up a positive charge and its potential rises. In the second case, they stated the opposite to be true. This suggests the students were considering the effect of potential at the spheres as the charges build up over time, although the question asked them to consider the potential at the start as the current just starts to move.

Student 4L: There is a higher potential at “A” [Left sphere] as the negatives have just travelled to “B.” [Right dome].

Responses	Students.
Both domes have equal potential	**4B, 4C, 4D, 4F, 4G, 4H, 4J, 4I, 4J, 4L, 4M, 4N.**
Right dome has low potential, left dome has high potential.	**4C, 4F, 4G 4H, 4J, 4I, 4J, 4M.**
Right dome has high potential, left dome has low potential.	4B, 4D, 4L, 4N.
Right dome has high potential, left dome has low potential.	**4C, 4F, 4G, 4H, 4I, 4J, 4M.**
Right dome has low potential, left dome has high potential.	4B, 4D, 4L 4N.
N/a	4A, 4E, 4K.

Table 6.8. Responses to post-test question involving the movement of negative charge under the influence of a potential difference.

All students who attempted this question could indicate that the potential of both sphere was equal when no current was observed on the ammeter, when a charged rod was placed beside the spheres for a long time. The reasoning was based on the observation that if no current was flowing, no potential difference exists between the two bodies. None of the students however mentioned how the potential difference observed in the first two setups reduced to zero as time moved on.

Student 4C: The potential is equal. Meaning that the electric charge wo not move.

Student 4L: They're equal, as there is no electricity moving.

The students were also presented with a graphing question, in which they were required to sketch the variance in potential for a series of positive and negative charges in various positions. The two graphs setups the students were presented with are shown in Figure 6.18, and the student results are summarised in Table 6.9.

The results clearly show that the students associated high and low potentials with positive and negatively charged objects, as guided in the homework exercises described in section 6.2.4. However, the general shape of the graphs drawn by the students were crude and did not fully represent the behaviour of potential for point charges, in which the inverse relationship was notable missing or badly represented. Only two students, 4G and 4H, produced at least one graph which were considered to reasonably show the relationships, with 4H producing two graphs which showed the inverse pattern that potential around a point charge displays. The remaining student made errors in

which the potential only changed between positive and negative charges but did not change before or after the charges had moved between the spheres. In these cases, when the potential dropped when close to a negative charge, the potential remained low and did not increase unless a positive charge was close. There were also instances in which this was shown for the positive charge. The third graph in Figure 6.19 illustrates this, where it is clear student 4I sketched a constant value for the potential until their graph moved into the region that represented the space between positive and negative charge, where the shape of their graph showed a decrease. However, as the potential decreases as the distance from a charged particle increases, the constant potential shown should not have been sketched.

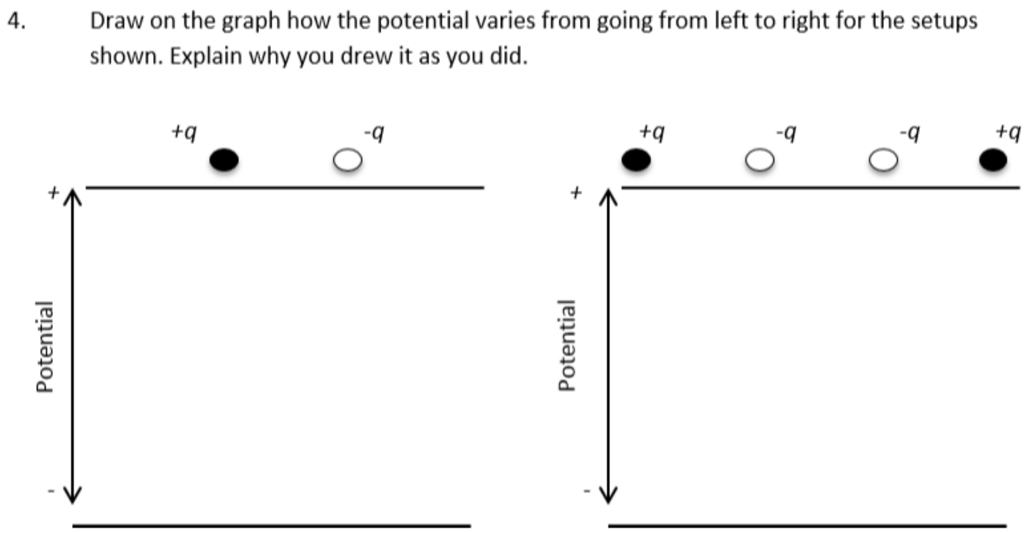


Figure 6.18. Post-test question, utilising graphical representations for potential.

Figure 6.19 shows the graphs produced by students 4G, 4H and 4I. Reasonable graphs can be seen by 4G and 4I, where they display not only the drop in the potential between the positive and negative charges, but also the increase and decrease in potential before and after the charges (i), and the increase in potential between the two negative charges (ii). In the final diagram, (iii), errors can be seen in which the potential was constantly high and drops between the positive and negative charge. In this case, an error is also seen in which the potential continues to drop as the graph moves to the left past the position of the negative charge. These errors were typical of those made by the remaining students. This indicates that students associate the positive and negative charges with high and low potential, but do not correctly consider the variation of potential around point charges.

Responses	Students
Positive is high potential	**4B, 4C, 4D, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N**
Negative is low potential	**4B, 4C, 4D, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N**
Positive is low potential	N/a
Negative is high potential	N/a
Reasonably correct shaped graph	**4G, 4H**
Notable errors in graph shape	4B, 4C, 4D, 4F, 4I, 4J, 4K, 4L, 4M, 4N
N/a	4A, 4E

Table 6.9. Responses to post-test question involving the association of high and low potential with charged bodies.

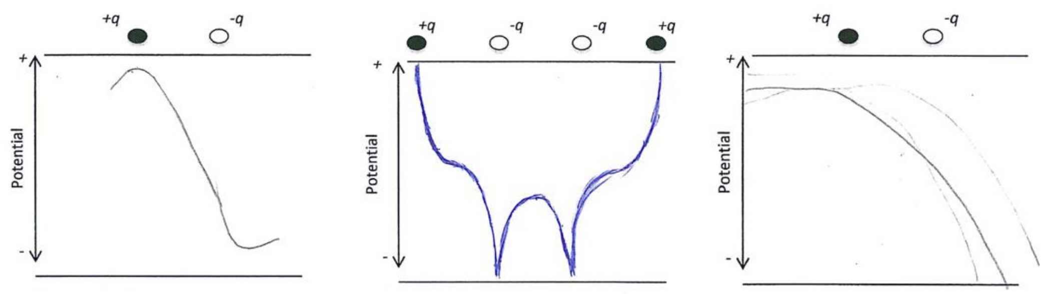


Figure 6.19. Examples of responses from students 4G, 4H and 4I.

In another post-test question the students were presented with the illustration of a battery connected by two wires from the positive to negative terminal shown in Figure 6.20.

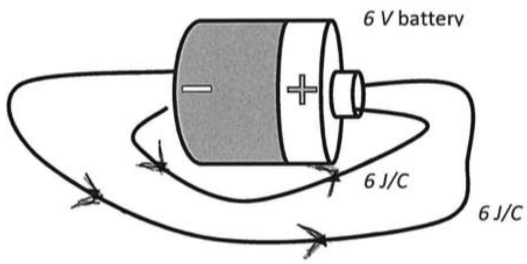


Figure 6.20. Diagram from post-test question requiring students to explain behaviour of current.

The students were required to explain the following two behaviours in the circuit, using the concepts they had learnt in the tutorial lessons:

1. Current (which is moving negative charge) flows from the negative to the positive terminal.

2. The *work done per unit charge* in moving charge from one terminal to the other is constant, regardless of the length / layout of the wire.

The students were given a list of key concepts, and asked to use their understanding of some, or all, of them to explain the two observations. The key concepts were work, potential, electric fields, electric field lines, vectors, the behaviour of negative charges in electric fields and the behaviour of negative charges between a potential difference. The forces of attraction and repulsion were explicitly omitted from this list, as to gauge how the students could apply the concepts mentioned in the list to explain the behaviour of current in a simple circuit. Although reasoning related to the attraction of the electrons in the current to the positive terminal, and repulsion between the electrons and the negative terminal are valid, the aim of the question was to elicit the other manners in which students could explain the behaviour of current. A summary of the student's concepts used in the first question is presented in Table 6.10. Students marked with an asterisk signify they referenced the concept but applied it incorrectly in this context.

Concepts referenced	**Students**
Attraction / Repulsion.	4B, 4C, 4D, 4G, 4H, 4J, 4M
Electric field.	4G, 4M, 4N*
Potential of plates.	4G, 4H, 4K*
N/a.	4A, 4E, 4F, 4I

Table 6.10. Responses to post-test question explaining the movement of current in a circuit.

The results show that seven of the students used reasoning based on the attraction of the mobile electrons to the positive terminal, and repulsion between the electrons and the negative terminal, of the battery. Six of these responses were exclusively based on these force interactions, without any further reference to the electric field or the relative potential of the plates suggested by the students. Student 4G expanded their explanation to include the behaviour of the negative charges in an electric field, in which they stated the direction of the field points from positive to negative and stating how the charge behaves in a field. They also stated how negative charges moves due to reasoning based on potential difference, stating that they flow from an area of low potential to high potential. Student 4H also submitted this reasoning based this potential difference, in addition to using attraction and repulsions reasoning. However, their explanation included a minor error, in which they referred to "potential" as "potential difference." The source of this error is unclear. Student 4M correctly used both attraction / repulsion reasoning, and reasoning based on the behaviour of the electrons in an electric field.

Student 4G: The negative current is repelled from the negative terminal of the battery and is attached to the positive terminal. Because it is negative, it is also attracted to the

high potential at the positive terminal of the battery. Negative charges always act in the opposite direction to the electric field.

Student 4H: Because the moving negative charge is attracted towards the positive terminal of the battery. Also, the negative charge goes towards the higher potential difference at the positive terminal of the battery.

Student 4M: Current flows from negative to positive because only negative charge moves and is attracted to the positive charge. The electric field lines go from positive to negative, but the charges go against the field lines. The is, the current flows from negative to positive.

Only two students, 4K and 4M, used reasoning that did not involve force. Both these students incorrectly used reasoning based on electric field lines and potential difference. Student 4M incorrectly stated that the field would point from the negative plate to the positive plate, which suggests they believed the electrons would move in a parallel direction to the field lines. Student 4K associated a high potential to the negative plate, and low potential to the positive plate, and indicated that charge would move from high to low potential. In both cases, it is observed the student's errors are rooted in incorrectly reversing the conventions of electric field and potentials, as they are applied to positive and negatively charged objects.

Student 4K: It travels from the high potential energy area to the low potential.

Student 4N: The field lines go from the negative to the positive.

The second part of this question required the students to explain why the work done in moving a unit of charge from one terminal to another in the circuit was constant. Again, the students were encouraged to use the key concepts listed in the question and apply them to the circuit to produce their explanation. A summary of their responses is presented in Table 6.11.

Concepts referenced	Students
References displacement between the plates	**4B, 4C, 4D, 4G, 4H, 4I, 4K, 4M**
References the force exerted on the current.	4G
Uses other reasoning.	4J, 4L, 4N
N/a.	4A, 4E, 4F

Table 6.11. Responses to post-test question explaining why the length of a wire does not affect the potential difference in the circuit.

As seen in the last part of this question, most of the students picked a concept with which they could explain the observation. The most prevalent response was that the terminals have a fixed displacement between them, so the work done in moving between them will be constant. Student 4G added to this by stating that the force acting on the charges would also be constant in both wires, presented in Figure 6.20. Although it is not clear what reasoning the student used to ascertain that

the force would be constant, as the net work done in both cases would be equal, this was considered a correct answer. Student 4J appeared to reference a model of current, referring to a constant flow in the wire, regardless of length. If referring to current by the flow, then the student infers that a constant current would associate constant work between the positive and negative terminal. Student 4L and 4N produced responses that reworded the observation of the direction of the current but provided no reasoning.

Student 4B:	The work done is constant, because no matter how long the wire is, the displacement between the positive and negative terminal is constant.
Student 4G:	The work done per charge is constant because the charge experiences the same force from the battery and moves a uniform displacement (straight line distance).
Student 4J:	The work done is constant no matter the length of wire or layout cause it going to be the same flowing from 1 terminal to another terminal.

This section presented and displayed the results of post-test questions undertaken by the students that help elicit their thinking of work and potential difference. The results indicate that the students can identify instances of positive, negative and zero work using field line and vector representations. However, in instances in which the force and displacement vectors produce acute / obtuse angles in which combinations of zero and non-work are presented, students can encounter difficulties. It was seen that the students can generally compare, and justify, the potential difference between two points based on the movement of current, and display that when electrons have travelled through a system, the potential will equalise. When using their understanding to explain the behaviour of current in a circuit, it was observed that the students generally focused on explanations that involved using only one concept, instead of trying to apply multiple concepts to model their observations.

6.2.6. Discussions

This section compares pre-test and post-test data for student's understanding of work and potential difference. It reveals student's gains in both reasoning and being able to either apply various representations to the different concepts or discern and interpret information from the various representations. The concepts addressed in this section are the ability to identify positive, negative and zero work, identify work with regard to the direction of displacement using an electric field line diagrams (Doughy, 2013), associate higher and lower potential to positively and negatively charged particles respectively (Hazelton, 2013), and explain the behaviour of charged particles under the influence of a potential difference (Guisasola, *et al.*, 2002; Maloney, *et al.*, 2003; Hazelton, 2013).

The first of these concepts is the student's identification of positive, negative or zero work. The first question in both the pre-test and post-test probed the student's understanding of work. A comparison of the student's responses is shown in Figure 6.21.

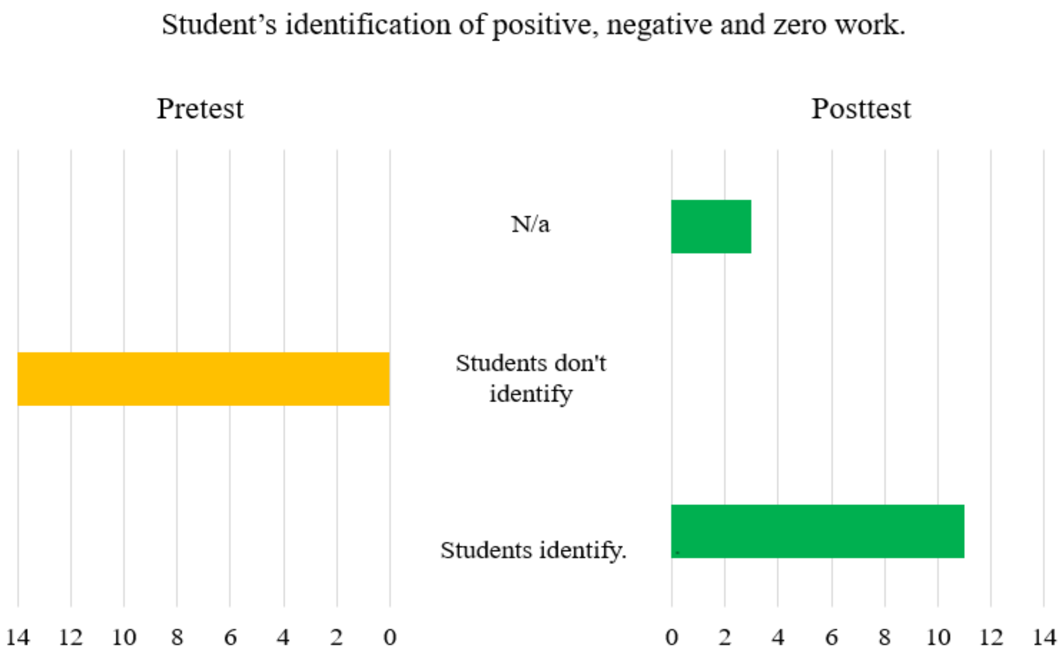


Figure 6.21. Comparison of student's ability to identify positive, negative and zero work.

In the pre-test, it was observed that none of the students identified instances of positive, negative and zero work. These students relied on using distance reasoning to determine the work done in moving from one point to another. In the post-test, all the students in attendance identified instances of positive, negative and zero work. Student reasoning focused on the displacement and force vectors being parallel, anti-parallel, zero or a combination of both. This progress reflects that observed by Doughty (2013). This shift in reasoning suggests that the tutorial was effective in enabling students to link their understanding of force, displacement and vectors and extend them to conceptually identify instances of positive, negative and zero work (Hewson, 1992; Konicek-Moran and Keeley 2015). The gain of 11 students developing their understanding indicates that moderate conceptual change occurred. Section 6.2.2. discussed the introduction of the concepts of positive, negative and zero work, and how the students applied this concept during the tutorial. The section illustrates an instance in which students were incorrectly applying the concept, but upon realising their error, they readdressed their understanding and figured out the application of work to produce the correct ranking (Posner, *et al.,* 1982). Section 6.2.3 presented the initial section of the potential difference tutorial, in which they applied this concept of work to an electric field context. Whilst there was a notable gain in the student's understanding from pre-test to post-test, it was seen that combinations of two types of work (work that has components that are both zero and positive/negative) caused difficulty for several students.

When students had to consider the absolute value of the work done, student's beliefs about the direction of the displacement of a charge in a field influenced their reasoning. Figure 6.22 shows students who considered work to be based on the total distance travelled by a charged particle in an electric field, and students who considered the work to be based on the net displacement of the charged particle.

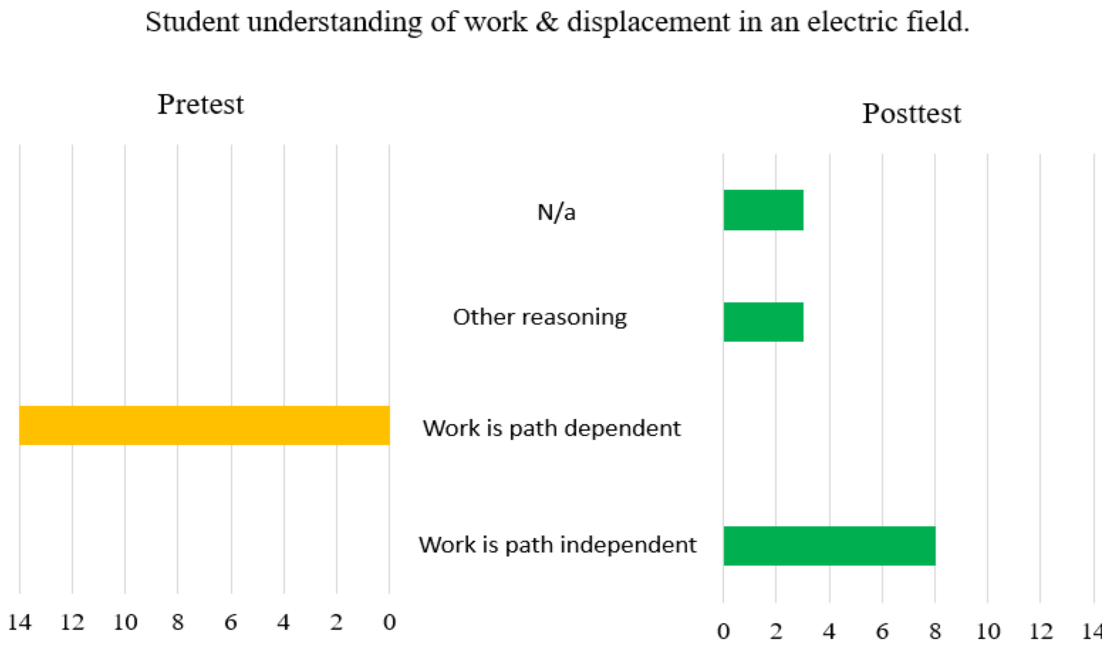


Figure 6.22. Comparison of student's understanding of the use of displacement in determining work done.

Even though the students had previously covered quantitative questions involving force, displacement and work during the mechanics section of their physics course, the pre-test results indicated they considered that the distance travelled affected the work, as opposed to the net displacement. To challenge this difficulty, the work tutorial addressed the quantities of distance and displacement using diagrammatic and verbal reasoning, as discussed in section 6.2.2. This allowed the students to apply both concepts to work, and become dissatisfied with the distance concept, and develop confidence to apply the displacement concept in an intelligible manner (Posner, *et al.*, 1982). Section 6.2.3 then illustrated how these concepts were applied to the potential difference context.

In the last post-test question, as discussed in section 6.2.5, it was seen that eight of the students shifted their thinking to consider the displacement between the start and end-point of a path in an electric field to determine the work done when current flows in an electric circuit. This indicates that conceptual exchange occurred and their conceptual understanding improves, as these students did not reference the errors observed in the pre-test and they applied the concept to an unseen context (Hewson, 1992; Konicek-Moran and Keeley 2015). The gain in eight students developing their understanding indicates that the extent to which conceptual change occurred was moderate. One difficulty that was persistent post-instruction was that some students reasoned that a constant current

in the circuit requires a constant voltage, regardless off the path taken. This reasoning would be erroneously transferred to electric circuits and does not account for variation of current caused by difference resistances in various branches of combinations of parallel and series components in circuits.

In the first question of the post-test, discussed in section 6.3.5, it was also observed that several the students did not consider the displacement vectors that were parallel and perpendicular to the field. Difficulty arose when students were required to consider displacements that were combination of parallel and perpendicular components. There was little difficulty in student identifying a path with two stages, the first being negative work and the second behind zero work. There was also no difficultly in students equating this work to a path in which only the first stage is taken. However, in a path which combined the positive and zero or negative and zero work, it was seen that students suggested the work would be greater than the two stages separately. This indicates that when considering parallel, antiparallel and perpendicular paths, the students analyse the problem in terms of positive, negative and zero work. However, when the paths taken make acute or obtuse angles to the field, the students shift their reasoning to think in terms of absolute displacement. The sources of difficulty with this thinking is the student do not consider parallel, anti-parallel and perpendicular displacement components separately, and relate them to the work concept. This difficulty is not directly reflected in the work done by Lindsey, *et al*., (2009), but in both cases, it was seen that a lackof understanding of whether displacement or distance travelled is the relevant concept when considering work can lead to student confusion.

The next section of the discussion associate relatively high and low potential to positively and negatively charged particles respectively. Figure 6.23 shows how the presents the students associations of potential to bodies of different charge, which was tested for using graphical representations.

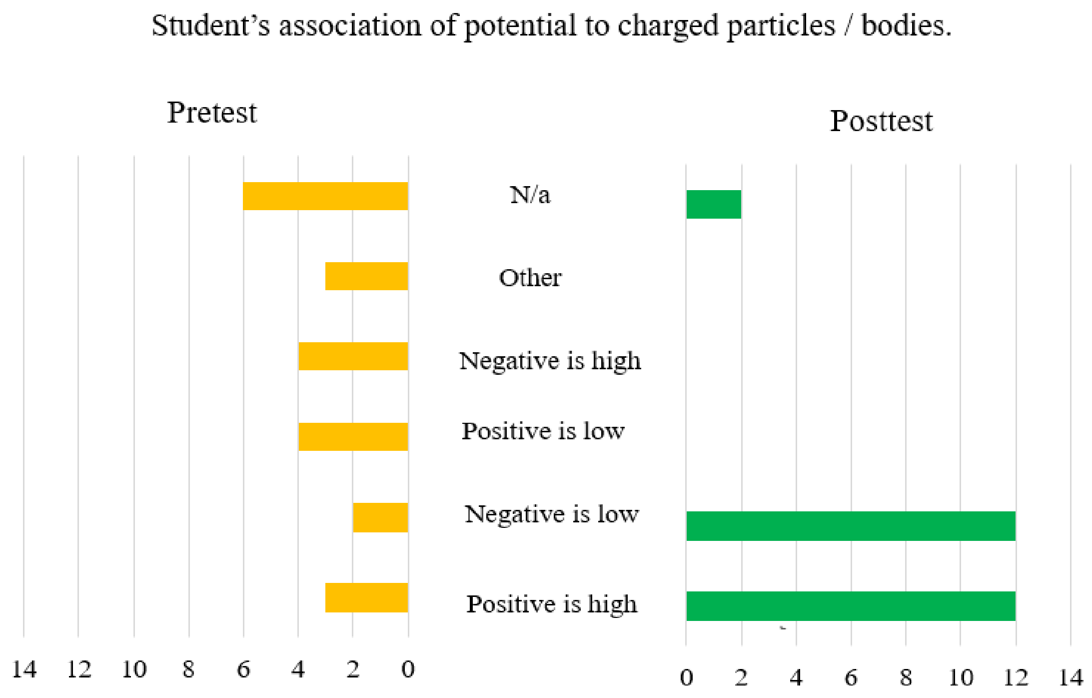


Figure 6.23. Comparison of student's association of potential with charged particles, using graphical representations.

The pre-test results clearly indicate that the students were unaware of the association of positive and negative charges to potential, with only two students successfully answering the question. In the post-test, there was a clear shift in student's associations to correctly apply the association to point charges. These results suggest that moderate conceptual exchange occurred, insofar there was a total gain of nine students correctly associating high potential with a positive charge relative to a low potential with a negative charge (Hewson, 1992). However, errors were observed in both the pre-test and post-test where students struggled to correctly represent the shape of the graphs correctly. However, the shape of the inverse patterns was not a target of this study and was not explored by the students during the tutorial.

One prevalent error in student's responses was that they appeared to represent the potential as constant until another charge increases or decreases it, or that the increase / decrease in potential continues past the point where the charges are placed, as shown in last student response of Figure 6.19. These students may have been considering a uniform electric field, which does not vary with distance, and applying this property to potential. As sections of the tutorials used contexts involving uniform electric fields, using field lines, more so than varying ones, this is not unlikely, but further work would need to be completed to help students separate these two types of thinking.

The last section of this discussion discusses the student understanding of the movement of a charge body under the influence of a potential difference. Figure 6.24 compares the student's pre-

test and homework results for this concept. In this case the homework assignment was used in lieu of a post-test question.

In the pre-test, it was observed that the students could predict the behaviour of charged bodies in a potential difference, and a drop in the student's correct responses was observed in the homework assignment. However, in the pre-test, the predominant strategy employed by the students was to assume the charges of the high and low potential plates. This is in line with difficulties presented by Guisasola, *et al.*, (2002). The homework exercise aimed to address this, by initially asking students to answer in terms in charges, and then answer in terms of potential. This appeared to enable some of the students to develop their understanding, but gravity-like thinking, in which all bodies move from high to low potential, was persistent in some of the student's responses, even when they directly contradicted their previous responses.

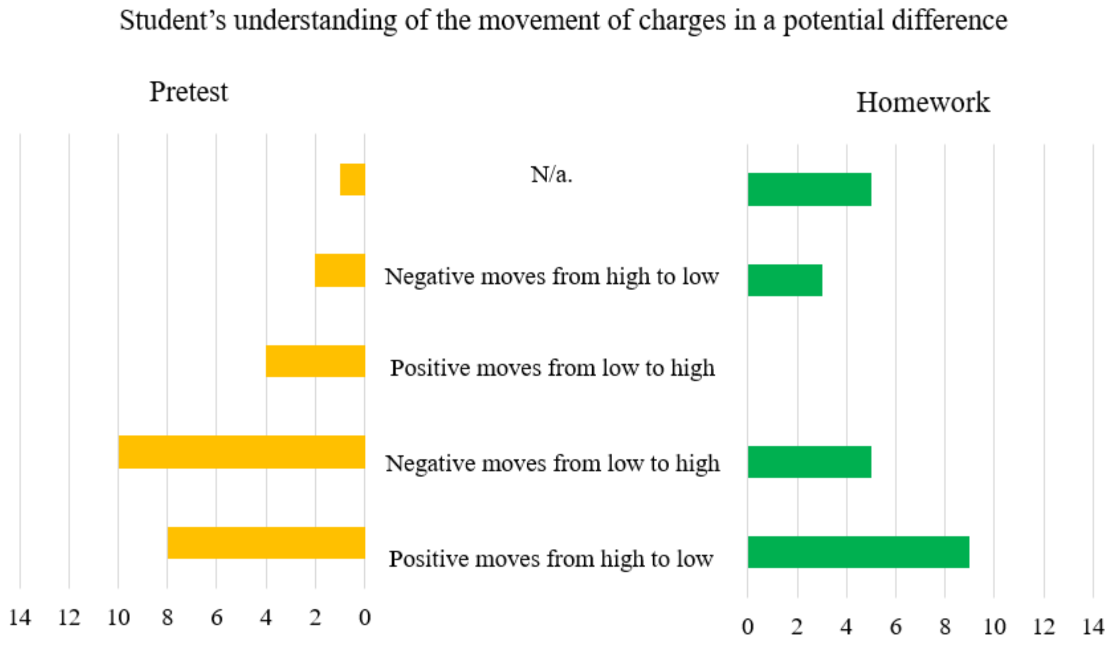


Figure 6.24. Comparison of student's understanding of the movement of charge under the influence a potential difference.

The reasoning used in the homework indicated that more students were thinking in terms of the potential difference of the scenario they were presented with, than the relative location of positive and negative charges in the presented scenario. This suggests that the tutorial helped promote conceptual exchange (Hewson, 1992), as there was a shift in the focus from considering charges to considering the potential difference of the setups presented in the pre-test homework assignments. Further time to develop this concept may be required, using a combination of static and current electricity contexts, to encourage the remaining students to focus on thinking and applying potential difference.

In the final pre-test question shown, the students were asked to explain the behaviour of charge in a field. The most common responses by the students involved the interaction of the charge with the charged plates, referring to the attraction and repulsion force experienced. Guisasola, *et al.*, (2002) stated that student's resort to using charge models to explain interactions when possible, even then other manners such as potential are appropriate or required. Attraction and repulsion was not listed on the key concepts to answer the question with, but the students favoured it. However, other responses in the student's homework and post-test questions suggest they could explain the behaviour under the influence of potential difference, when a question prompts them to directly. The student's familiarity with attraction and repulsion, and their interpretation for the apparent ease in using this reasoning may explain their reliance on using it. As the students demonstrated they could associate high and low potentials in line with the learning expectations of the tutorials, but their application of the reasoning was typically rooted in attraction and repulsion, the extent to which conceptual change occurred was determined to be partial.

This section presented the discussions on student's development of work and potential difference. Whilst the approach adopted in the tutorial helped the students develop understanding, there is still space for development. The use of field lines and vectors showed gains in the student's ability to reason situation of positive, negative and zero work, but difficulties were seen in correctly applying the displacement vector in situations where the force and displacement were neither parallel or perpendicular. The use of graphs and diagrams help students develop understanding of potential and the behaviour of charges in potential difference, but the use of graphs without the context of a mathematical formula produced errors for the student's understanding of potential and led to difficulties indicative of thinking about uniform electric field. The use of diagrams to enable students to develop understanding of the behaviour of charged bodies in potential difference opened the opportunity to use charge-based reasoning to explain the student predictions, and not focus on the potential difference concept.

6.3. Conclusions

This chapter seeks to address the following research question:

- To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

This question is addressed by addressed the following considerations:

- To what extent does the use of vectors and field lines, representing force and displacement, enable students to identify positive, negative and zero work?
- What affect does the use of graphs and diagrams have on students understanding of potential difference
- What difficulties are encountered by the students during this transfer to a potential difference context?

The approach developed used tutorial lessons that employed the use of vector and field lines to help students develop their understanding of work. Mathematically, work is a scalar concept that employs a dot product to produce a scalar from two vector quantities. This mathematics is not covered by the students and instead, the approach adopted looked to develop conceptual understanding. The work tutorial employed the use of vectors to enable the students to consider the relationship between force and displacement and produced an opportunity for them to deconstruct parallel and perpendicular component vectors, so they could be considered to produce non-zero and zero work respectively (Doughty, 2013). This skill was seen in the student's use of work in the potential difference tutorial, in which they could identify the sign of the work done in an electric field. However, component deconstruction was observed to be a difficulty in answering conceptual questions in the post-test and requires focus in future lessons for the students to correctly apply the skill. Additionally, how the work done in electric fields, as completed in the research, did not address the concept of how work increases or decreases the energy of a system.

The use of graphs and diagrams was seen to promote conceptual understanding of potential difference. The graphical method provided ease for students to develop an association of high and low potential to positive and negative charges. By students constructing their own sketches in line with the initial examples in the homework, they developed an intelligible method to apply the association of relative potential to positively and negative charged bodies in similar contexts, to allow them to apply and engage with the concept (Posner, *et al.,* 1982; Konicek-Moran and Keeley 2015). The use of diagrams provided a simple model for students to consider when thinking about the movement of charged bodies in a potential difference. Guiding student's reasoning to consider attractive and repulsive forces employed their prior knowledge and extended it by getting them to consider the potentials involved in the positively and negatively charged regions. This approach also reinforced their association of high and low potential to charged plates.

The representational approach highlighted difficulties in the student's models of potential difference. The use of graphs in the post-test questions elicited that students thought that potential drops remains constant along a path unless another charge increases or decreases it. This model is in error for point charges, although it is useful in explaining and representing potential along conducting wires in circuits with components (Reeves, 2003). The use of diagrammatic representation presents

the opportunity for students to be over-reliant on the use of charge interactions to explain processes that can be explained in terms of potential difference (Guisasola, *et al.*, 2002). In combining the association of potentials to charges, and the movement of charge under a potential difference, this model could be employed to help students understand the behaviour of a capacitor discharging, combining diagrammatic models involving potential and current, and graphing data of potential difference vs. time and current vs. time, showing many processes occurring using the different representations.

The extent to which the student's developed their understanding of work and potential difference is displayed in Figure 6.25. A legend of the codes used can be found in Appendix F.

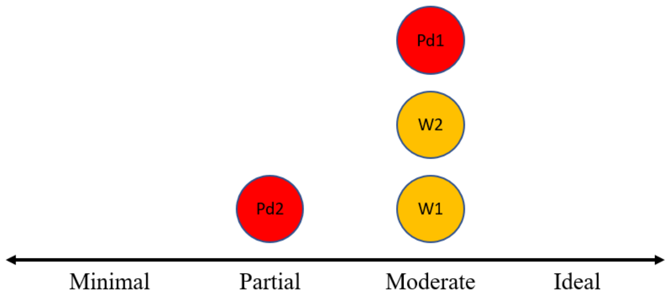


Figure 6.25. Line plot of extent of conceptual change for student's understanding of work and potential difference.

It was seen that the student gains in understanding from completing the tutorials were mostly moderate. The student's understanding of work and potential difference was rooted in enabling them to explain the behaviour of charged bodies under the influence of a potential difference. As the student's mathematical abilities are limited, calculus and vector mathematics was not employed in the tutorials and limited the depth of understanding developing of the students. Therefore, the developed student reasoning was phenomenological in nature, which impeded the mental models constructed by the students.

Overall, the use of multiple representations through structured inquiry tutorials encouraged conceptual change in the student's understanding of work and potential difference. There were persistent student difficulties observed that could be addressed in future research, and extensions to other topics to use multiple representations to explain the behaviour of different processes. Examples of such an extension study student's application of vectors and field lines to various processes in electromagnetism, applying tabular data, graphs, strobe diagrams and mathematical symbolic representations to developing students understanding in mechanics topics such as motion and the conservation of energy.

Chapter 7. Conclusions and implications.

This section summarises the main findings from chapters 4, 5 and 6, and discusses these findings in the light of the research questions presented in the last section of chapter 1. Section 7.1 presents the conclusions for the student's understanding of vector concepts, the inverse square law and field lines. Section 7.2 presents the conclusions related to the use of multiple representations to promote student's understanding of Coulomb's law and the electric field. Section 7.3 presents conclusion related to the use of field lines, vectors, graphs and diagrams in developing student understanding of work and potential difference, section 7.4 discusses implications for learning and section 7.5 presents further conclusions.

This thesis addressed 5 research questions in relation to student development of conceptual understanding in electrostatics. The 5 research questions were as follows:

RQ 1. To what extent does the use of a structured inquiry approach develop student understanding of vector concepts?

RQ 2. To what extent does the use of a structured inquiry approach develop student understanding of the inverse square law relationship, by employing multiple representations?

RQ 3. To what extent does the use of a structured inquiry approach develop student understanding of the field, when utilising the field line representation?

RQ 4. To what extent does the use of a multi-representational structured inquiry approach develop student understanding of Coulomb's law and electric fields?

RQ 5. To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

The approach adopted in these research studies was structured inquiry tutorials. A small number of individual questions were directly taken from Tutorials in Introductory Physics (McDermott and Shaffer, 2003), but otherwise, the suite of pre-tests, tutorials, home-works and post-tests developed in this research are of original design. As discussed in Chapter 2, the literature that references student difficulties primarily revolves around research in third level, typically with undergraduates studying introductory physics. Published research that utilised the approach of Tutorials in Introductory Physics (McDermott and Shaffer, 2003) also revolves around undergraduates, while there is little

research published that looks at adopting the approach to the second level context. One piece of research that uses the approach at second level adopted two tutorials in a current electricity context and reports the gains by students by comparing the pre-test and post-test results (Benegas and Flores, 2014). Their study presents quantitative data in terms of student gains between pre- and post-tests but gives little indication of how the student's developed their understanding or what difficulties they encountered during the tutorials. In contrast, the studies in this research used specifically designed tutorials, along with many methods of data collection. This approach allowed me to probe, develop and assess the student understanding, by analysing and interpreting the qualitative data collected. The tutorials in this research were of original design to ensure the targeted concepts were accessible to the students at an appropriate level for their ability. The use of this approach enabled the collection of evidence to identify instances where conceptual change did/did not occur in the student's understanding of vector concepts, the inverse square law, field lines, Coulomb's law, the electric field, work and potential difference. The findings of these studies allowed for the determination of the type of conceptual change occurred, identifying instances of conceptual extinction, exchange and extension (Hewson, 1982). The approach adopted also allowed the extent to which conceptual change was achieved, ranging from minimal, partial, moderate to ideal.

7.1. Vector concepts, the inverse square law and field lines

When attempting to discern student difficulties when developing their understanding of Coulomb's law, the electric field, work and potential difference, it can be difficult to determine if the difficulties are rooted in the topics themselves or in prerequisite concepts. To address this, the approach that was taken in this research allowed the students to first develop their understanding of vectors, the inverse square law and field lines in a mechanics context, in the first set of tutorial lessons as discussed in chapter 4. Research questions 1, 2 and 3 address the development of the student's understanding for these concepts.

The inquiry tutorial focused on developing student's understanding of vector magnitude, their application of vector constructions and their conceptual understanding of vector addition in terms of superposition of the vector components and how it can affect the resultant magnitude of two, or more, vectors. The results indicated that moderate conceptual exchange was observed over the course of the tutorial lessons. The tutorials focused on students developing a conceptual understanding of vector concepts, without utilising specific vector notation or operations. The student's mathematical ability, or lack thereof, impeded the depth of understanding to which the tutorial lessons could target. Had the students been more familiar with vector mathematics, the opportunity to develop a richer understanding of vectors in a physics context may be been possible. Considering this limitation to construct the boundaries of the vector concepts targeted in this research, the results indicate that the

tutorials approach was effective in promoting conceptual change in the student's understanding. Such an approach would be fruitful for other teachers to adopt into their practise, to develop students understanding of vectors at upper secondary level.

The tutorials which focused on the inverse square law employed a multi-representation approach, which used a diagrammatic model involving scaling, tabular data and graphical analysis. The diagrammatic model which utilised area scaling enabled students to explain the behaviour of phenomena that follow an inverse square law. The use of multiple representations during the tutorials enabled the students to explore the inverse square law relationship both qualitatively and quantitatively. The extent to which conceptual change was observed in the tutorial lessons was in the minimal and partial range. This indicates that the students encountered difficulties in transferring between the representations and relating their qualitative reasoning to their quantitative findings. Two possible explanations and implications from this study are presented here. The first implication is that the student's unfamiliarity with inverse quadratic functions in their mathematical education could have hampered their learning. In lower secondary level mathematics, the student's encounter linear, quadratic and exponential functions and briefly explore inverse proportionality. Joint planning between mathematics and physics teachers in a second level setting could provide a better opportunity for students to develop a more consistent understanding of inverse relationship and mathematical operations related to them. This cooperative planning could also be extended to include understanding of inverse square relationships. The second implication relates to the use of the number of representations and the dual reasoning between quantitative and qualitative required during the tutorials. As there is a lot of information transferred between representations, it is plausible that the student's working memory was saturated with information and therefore they were unable to develop a coherent understanding of the inverse square relationship (Reid, 2009). Marzec (2003) states that multiple exposures to contexts that involve the inverse square law may be required before learners can develop an intelligible understanding of the relationship. The implementation of tutorials that allow students to encounter this relationship in various contexts would support upper secondary students develop their understanding. Over time, this could allow the students to cluster the information from the multiple representations and would allow them to free up their working memory capacity. This would enable students to incrementally form a coherent understanding of the inverse square law.

The findings from the studies involving field lines showed moderate and ideal gains in the student's conceptual understanding and their ability to use field line representation. The tutorials facilitated students to associate the field line density with relative field strength. Students were able to determine the direction of the field at various points, and to show that the path taken by a body in a field is influenced by, but not identical to, the pattern of the field lines (Törnkvist, *et al.*, 1993; Galili, 1993 and Cao and Brizuela, 2016). The tutorials focused on students transferring from vector to field line representations and explore the behaviour of bodies acting under the influence of the

field. As the students have limited knowledge of vector mathematics, transferring between vectors to field lines involved students primarily applying rules, such as the force is tangential to the field lines, without developing an appreciation for why this is based on mathematical equations using vector notation. In line with this constraint, moderate and ideal change was observed in the student's understanding of the field line representation. The students developed clear reasoning to explain the behaviour of bodies under the influence of a field, linking the representation to relevant vector concepts such as velocity, momentum, acceleration and force (Konicek-Moran and Keeley, 2015). Overall, the approach adopted in these tutorials was effective in promoting the students understanding of the basic behaviour that bodies display under the influence of fields, in a manner that is appropriate for teaching and learning at upper secondary level.

7.2. Coulomb's law and electric fields

Research question 4 addressed the development of the student's understanding of Coulomb's law and the electric field, by employing different representational tools for the topics. This section presents the conclusions of how the students revisited vectors, the inverse square law and field lines as they applied them to Coulomb's law and the electric field. The use of structured inquiry tutorials not only enabled me to gather evidence of the students as they developed their understanding of these electrostatic topics, but also to gather evidence of successful transfer of vectors, the inverse square law and field lines to the electrostatics context.

The prior learning of vector concepts developed in completing the tutorials presented in chapter 4 was not initially demonstrated in the electrostatic context, based on the pre-test results presented in chapter 5. This suggests that the students struggled to apply their understanding in a new context and/or the status they associated with the conflicting concepts was not in line with their initial understanding of these concepts. This is not surprising, as the students working memory must process concepts relating to vectors and simultaneously interpret information about electrostatics. If their initial understanding of vector concepts, discussed in chapter 4, were not clustered together, it is plausible the students working memory was full and therefore they were unable to apply the vector reasoning to new contexts (Reid, 2009). Revisiting these concepts in the tutorial lessons gave the students an opportunity to develop clusters in their working memory. Over the course of the Coulomb's law and electric field tutorials, the students revisited the concepts from the initial vector tutorial and transferred them to this context. Moderate instances of conceptual change were observed in the student's application of vector concepts to Coulomb's law and the electric field.

When students were applying the inverse square law to Coulomb's law and the electric field, partial conceptual change was observed in the students understanding. Using both qualitative and

quantitative reasoning in the application of the inverse square law in the electrostatics context, it was observed that there was a lack of coherence between the student's understanding and their ability to use the inverse square law mathematically. This suggests the students were operating without complete mental models but were effective with using the inverses square law to approach and solve quantitative mathematical problems. As discussed in section 7.1, multiple exposures to the inverse square law in context in tutorial classes could allow the students to cluster information regarding the inverse square law, freeing up working memory capacity to develop a deeper understanding of the relationship. The reasoning displayed by the students during teaching and learning interviews, discussed in section 5.5.3, indicated that when the students were prompted to focus on the change in dimensions in the area model, they displayed deeper conceptually coherent reasoning than was previously recorded in the initial tutorial, as discussed in chapter 4. This indicates that exposure to multiple explorations of the inverse square law across various contexts can incrementally develop student's understanding.

Difficulties were recorded with the student's transfer of their understanding of field line concepts to the electrostatics context. Gains recorded in the field lines tutorials and post-tests in the initial section of the research, as discussed in chapter 4, were not observed in the electrostatic field lines pre-test with similar frequency. Having completed two tutorials revolving around these representational tools, moderate conceptual change was observed to have occurred, which allowed them to effectively apply the concepts to electrostatic contexts. The Coulomb's law and electric field post-test results indicated that the students were proficient in interpreting field lines and predicting the behaviour of charged bodies in electric fields. As mentioned in section 7.1, the students lack understanding of using vector mathematics and this hindered the depth to which the students could explain the conventions they applied to using field lines. The direction of force being tangential to the field was referenced considering the definition of electric field strength, but the student's application of this was primarily phenomenological. Taking this into account, the tutorial lessons were effective in enabling the students to use field line representations to describe simple systems of charge particles, and could be adopted into teacher practise, not only for electrostatics, but also electromagnetism.

The difficulties targeted for conceptual change were chosen from reviewing literature, as discussed in section 2.3.1. The findings of the research indicate that the tutorial lessons were effective in addressing these difficulties, although persistent difficulties did remain in some cases. While not generalizable, the findings of this section of the research could be used by teachers who wish to develop the structured tutorial approach in their own practise. The findings presented could be used to inform the development in teaching and learning materials for other teachers who would wish to address these difficulties with their own students.

7.3. Work and potential difference

Structured inquiry tutorials were used in the teaching and learning of work and potential difference. In these lessons, the structured tutorial approach employed field lines, vectors, symbolic and graphical representations. This allowed for multiple collections of evidence to gauge the student's conceptual understanding across the different representations. The results can also be used to illustrate instances of conceptual extension, evidenced by the successful transfer of vectors concepts and field lines to work and potential difference.

The pre-test and tutorials on Work and potential difference highlighted that initially the students had conceptual difficulties with the implications of the dot product and did not consider the relative parallel / perpendicular components of the displacement and force vectors. The post-test results discussed in section 6.2 indicate that conceptual change occurred in the student's understanding of the relationship between force, displacement and work, but some difficulties persisted post-instruction. Conceptual extension occurred in which the students applied vectors and field lines to a mechanical work context, and exchange occurred as the tutorial was effective at developing student's understanding of positive, negative and zero work. However, scenarios which involve combinations of zero and non-zero work by parallel and perpendicular components of forces proved difficult for the students. Further development of student's understanding of displacement in terms of components, and its application to work could help alleviate this difficulty in students understanding.

The use of multiple representations in the context of potential difference allowed students to develop the concepts targeted in the research. The use of diagrammatic models allowed for a relatively easy manner for the students to determine the behaviour of charged bodies acting under the influence of a potential difference. However, some students continued to rely on reasoning based on the force of attraction and repulsion between a charged body and oppositely charged stationary plates. This suggests that conceptual extension did occur, but not in a manner that was envisioned during the design of the potential difference tutorial, as the majority of the student's relied on reasoning based on force interactions, as opposed to reasoning potential difference which the students only referenced if explicitly asked.

The employment of graphical representations was successful in helping students associate the high and low potential to point charges. In this manner, the graphical representation enables the students to engage in conceptual exchange, through their interpretation of the graphs. The sketches of the student's graphs provided evidence to indicate the student's association of the relative high and low potential of areas surrounding positively and negatively charged particles. However, some difficulties persisted, as a number of the students could not accurately represent of the variation of potential with distance for simple systems of two, or more, point charges and suggests further revision in this area would be warranted to complete the student's conceptual exchange. Their

understanding could be further developed by employing tabular data for various arrangement of charges and guiding the students to explain the variation of potential, before constructing graphs of their own design. This approach has proved fruitful in the inverse square law and Coulomb's law tutorials.

Prior to, and during, the carrying out of these studies, the participating students had not completed calculus mathematics - which is a necessary tool required to develop a complete understanding of potential and potential difference in an electrostatic context. Electrostatic potential is an abstract topic, with very little accessible examples that upper secondary physics students can relate to. Mathematics is an important tool in helping students develop an understanding of such abstract physics topics. Therefore, this study was limited to students developing a phenomenological understanding of work and potential difference in the electrostatic context. Partial and moderate conceptual change was observed in the students understanding of work and potential difference, but it is acknowledged that their mental models are likely incomplete due to the lack of possession of the required mathematical tools to fully explore these concepts.

As the difficulties targeted for work and potential difference were recorded in research literature, the findings of this research could be used to aid in the design of teaching and learning sequences in other classrooms. This could allow other students to address the common difficulties when learning about work and potential difference.

7.4. Implications for classroom teaching and education policy.

The use of the inquiry-based learning approach in this work enabled the students the opportunity to overcome difficulties related to electrostatics. As discussed in section 2.1.2, Mestre (1991) and Roth (1990) suggest this opportunity would likely not have been provided in traditional lesson sequences, and it is likely the difficulties in student understanding would have been unknown to both the students and teacher as the students progressed through their physics course. The approach adopted allows for the specific identification of student difficulties in upper secondary level electrostatics topics, and difficulties in student's ability to transfer conceptual understanding between representations. When these difficulties were addressed, there were gains in student understanding evidenced, that varied from conceptual extinction to conceptual exchange and conceptual extension.

Having covered the various representational tools in lesson before electrostatics, it allowed for discussions involving these concepts and tools, to help students develop deeper understanding than it typically afforded in traditional approaches. The use of multiple representations in inquiry learning, as presented in this research, allowed multiples dimensions to collect evidence of student understanding of electrostatics. Comparisons of the student's responses in the different

representations can gauge if they were consistent in their reasoning when addressing the same concept in different ways, or if the representation influenced the reasoning used by the students. This also has implications for assessing student's understanding in other topics in physics, and some aspects of mathematics.

The use of inquiry-based learning that employ multiple representations is not limited to developing student understanding of electrostatics concepts. The student's understanding of vectors, field lines and the inverse square law and utilisation of graphs, tabular data, diagrams and mathematics can be applied to other domains of secondary school physics, such as mechanics, optics, radioactivity, sound and electromagnetism. By employing inquiry-based learning in these contexts, a teacher may gather evidence of student learning, such as pre-test and post-test data, as well as insights from the tutorials themselves, to make a judgement as to the efficacy of the teaching and learning that occurred. By determining the overall impact that the approach had on student learning, they could adjust their lesson planning for future iterations to tackle difficulties they notice with their own students. The tutorials not only allow for the teacher to address difficulties in student learning, but also to explore how students to engage in tasks, in which they develop reasoning to address difficulties and engage in conceptual change.

In addition to the use of this approach in teaching upper secondary physics, this research highlighted other issues with student's development of their understanding of Coulomb's law, electric fields, work and potential difference. There were several instances in which students lacked the appropriate mathematical knowledge and understanding to develop a complete understanding of the electrostatic concepts targeted and this impeded their ability to form complete mental models, as discussed in the conclusions of chapters 5 and 6. The student's ability to perform calculations based on the inverse square law, vector and calculus mathematics and understand the mathematical concepts underpinning these is necessary to develop a coherent model for understanding electric fields, work and potential difference. Numerous actions could be considered to address this. Educational Policy on curriculum and assessment could align the learning outcomes of mathematics with the outcomes of science, technology and engineering subjects to ensure that the students also explore these concepts in their mathematics courses, enabling them to develop and apply their understanding. This coherence in policy would enable consistency in teaching and learning in classrooms, and promote students developing links across different subjects. Another action could be to require physics teachers to dedicate lessons to focus on the development of mathematical understanding and continuously build on student's understanding over the implementation of the physics course. A final proposed action would be for education policy makers to review the level of coherence between the mathematics and the physics syllabi. For example, in Ireland there is a shift towards learning outcomes-based curricula and this provides an opportunity to align the two emergent specifications. In the cases where this cannot occur, there is an argument to make that certain topics in physics may not be appropriate for secondary level and it would be more appropriate

to be taught in third level. At this level, the students would have acquired the necessary mathematics to develop conceptually accurate mental models of these concepts. While these actions have their advantages and drawbacks, they are worth considering by policy makers and educational stakeholders.

Another implication arising from this research addresses the role of assessment in physics education. Assessment can be a strong influencer on what pedagogical approach teachers use in their classrooms. As shown in Figure 1.1, the assessment of upper secondary physics in Ireland relies heavily on content that can be memorised and uses qualitative problems that can be solved using algorithmic procedures (SEC, 2015). The inclusion of a broader range of questions, such as qualitative conceptual questions, two-tier ranking questions or diagnostic questions in physics could enhance the assessment of student understanding and influence teacher's classroom practice. This approach could provide evidence of the student's development of mental models and problem-solving strategies.

7.5. Implications for research.

Possible extensions to this work would be to conduct the research with a bigger sample size. As these research studies were completed with a small group of students, the findings of this research would not be generalizable to the wider population of students learning physics at upper secondary level. However, a series of research studies in which gathers data from a larger group would be informed by both the student gains and student difficulties presented in this thesis and could be used to reliably determine how frequent each of the gains and difficulties occur with different groups of students. If multiple teachers were to adopt the approach, the research could also gauge teacher attitudes to the use of structured inquiry tutorials and multiple representations in their own practise, and gauge how teacher implementation of the approach affect's student's understanding. Another extension would be to gauge the efficacy of adopting structured inquiry tutorials and multiple representations in other domains of Physics at second level. The topics covered in this research would allow for extensions into electromagnetism and mechanics topics.

Research regarding the use of tutorial lessons primarily focuses on the development of conceptual understanding in third level, but it is less studied at secondary level. One example of such research is Benegas and Flores (2014), who implemented tutorial lessons with upper second level students in Argentina, in which they presented quantitative analyses of pre-test and post-test results, with little insight into the student's conceptual development. The research presented in this thesis presents qualitative findings with upper second level students, developed using the tutorials approach, which is relatively unreported in the literature. This research represents a novel approach

in employing the use of the structured tutorials at secondary level and highlights the opportunity for further research in this area. A coordinated research study that uses both qualitative and quantitative findings would validate the efficacy of this approach and illustrate how this approach can enable students to develop their understanding and form coherent mental models of various topics of physics. Tutorials in Introductory Physics (McDermott and Shaffer, 2003) address multiple topics in Physics and they could also be used as a guide to draft and develop tutorials that adopt the tutorial approach in the second level context, as they were for this research. The findings of this research could also be used to guide future research at lower secondary level science and upper secondary chemistry and biology. If the approach were to be used at lower secondary level, it may be advisable to limit the representations to diagrammatic, graphs and tables, to help students understand how a process occurs, but remove any complex symbolic representations, as their mathematical ability to interpret them effectively would likely be underdeveloped. At upper secondary chemistry and biology, a focus for the structure tutorials on visualising complex processes could help students link observations between the atomic-scale, micro-scale and macro-scale.

A conceptual change model was employed as the underpinning theoretical framework employed in this research. This is not the only framework that could have been utilised. A framework revolving around developing and assessing the student's mental models could have been employed. The various types of evidence collection used in this research and use of multiple external representations could be used as indicators as to the mental models the students possess. However, unlike the closely related topics of magnetism (Borges and Gilbert, 1998) and current electricity (Borges and Gilbert, 1999), there is not an abundance of literature of naïve models of electric fields in which to compare to compare the student's mental models to. A future extension to this research could be to probe students understanding and establish descriptors of naïve mental models of electric fields, in a similar manner to that used by Borges and Gilbert (1998, 1999). Upon completion of the tutorial lessons, the analysis of the collected evidence could be used to determine what initial mental models the students were operating with, and how these mental models changed over the course of the lessons. The student's overall conceptual understanding would still be developed and assessed, without any critical changes to the tutorial lessons required to take place. Employing a modelling framework to the approach taken in this work could extend the research presented in this thesis and align it with trends in modelling research currently taking place in the physics education research sphere.

Chapter 8. References

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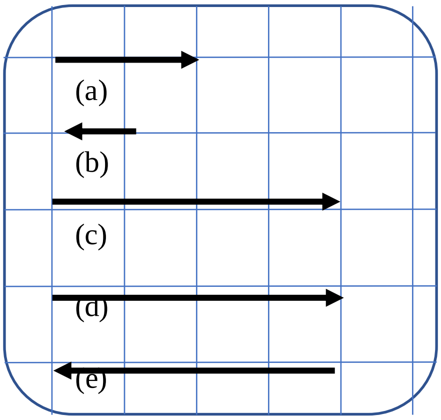
Appendix A

Vectors tutorial materials.

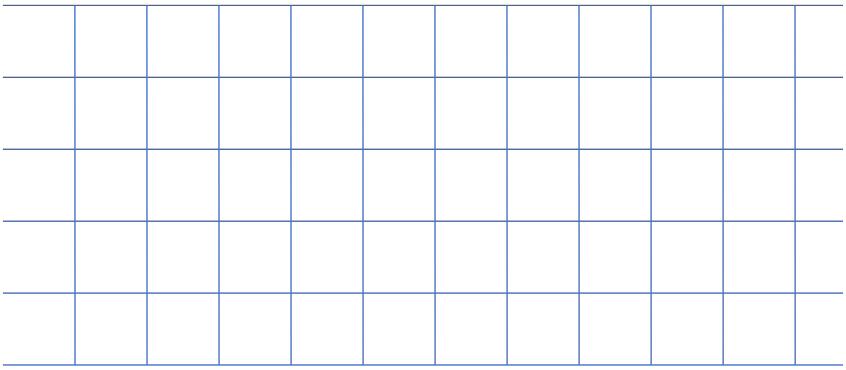
1. Rank the following vector arrows (a – e) from weakest magnitude to strongest magnitude.

Ranking:

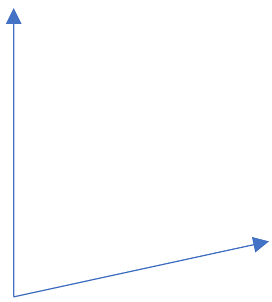
Explanation:



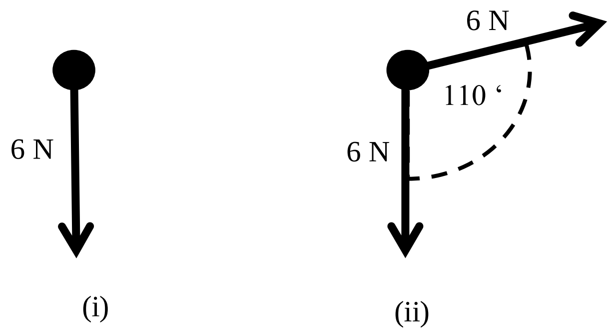
2. In the space below, Construct the resultant vector of (a) and (c) from question 1.



3. Construct the resultant vector of the following two vectors.

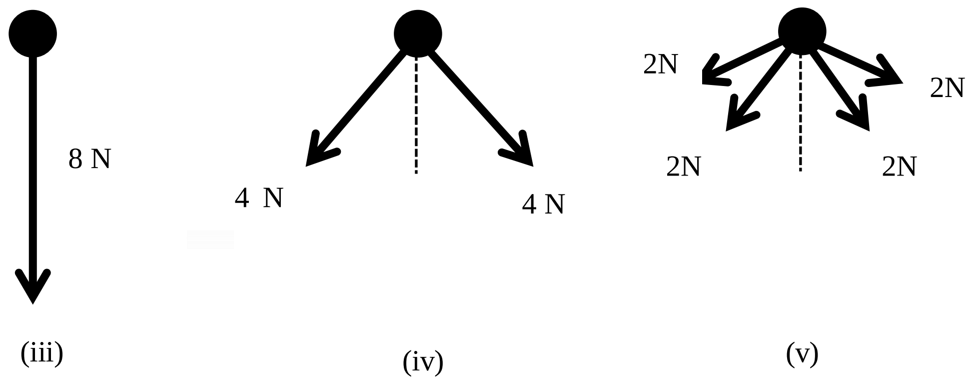


4. A charge experiences a force as shown in diagram (i). An equal charge experiences 2 forces as shown in diagram (ii).



Which charge experiences the most force? Explain your choice. You may draw on the diagrams if necessary.

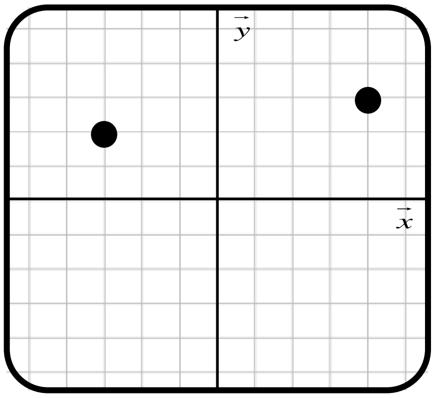
5. The following set of diagrams shows charges experiencing multiple forces. Which, charge, if any, experiences the most force? Explain. If they are the same, state so explicitly and explain why. (Angles shown are in (iv) are 45’ and in (v) are 45’ and 75’)



Writing vectors in terms of components using vector notation.

(i) Starting from the origin, explain to get to point "a" as shown.

(ii) Write how to get to "a" in terms of steps in the x direction and y directions. (e.g. 2 steps in the x and 1 step in the y is written as "$2\vec{x} + \vec{y}$")



(iii) Circle which of the following correctly maps how to get from the origin to the point labelled "b."

(a) $3\vec{x} + 2\vec{y}$ (b) $2\vec{x} - 3\vec{y}$ (c) -$3\vec{x} + 2\vec{y}$ (d) $2\vec{x} + 3\vec{y}$

Explain why you picked the answer you did. (It may help to explain why the other answers are incorrect)

(iv) On the diagram above, draw arrows from the origin to the points "a" and "b." We will call these vector arrows $\vec{a}$ and $\vec{b}$ respectively.

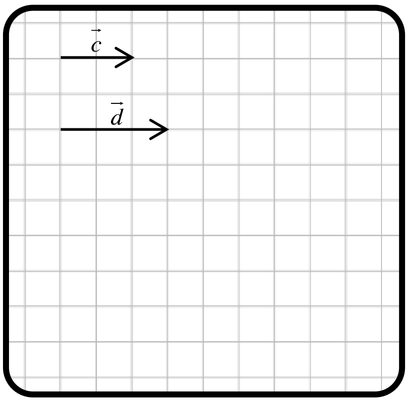
(v) On the diagram above, draw $\vec{a}$ as a combination of arrows along the $\vec{x}$ and $\vec{y}$ axis. Do the same for $\vec{b}$.

(vi) Using either trigonometry or co-ordinate geometry, explain how you could find the magnitude (length) of the vectors $\vec{a}$ and $\vec{b}$.

(vii) Find the length of $\vec{a}$ and $\vec{b}$.

II. Adding vectors.

Two vectors, $\vec{c}$ and $\vec{d}$, are shown to the right.

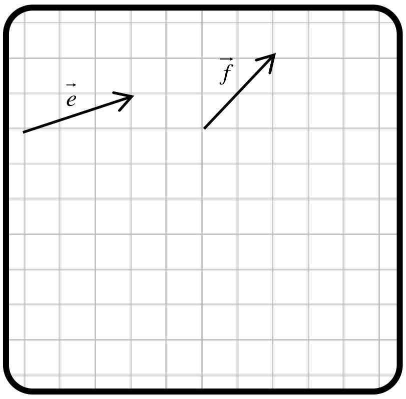


(i) Write both vectors in terms of their horizontal and vertical components. (eg, $\vec{a} = 4\vec{x} + 3\vec{y}$)

(ii) By connecting the tail of one vector to the tail of the other vector, show the resultant vector of $\vec{c} + \vec{d}$, on the diagram.

(iii) Add the horizontal components of $\vec{c}$ and $\vec{d}$ together. Add the vertical components of $\vec{c}$ and $\vec{d}$ together. Do the horizontal and vertical components define the resultant vector you drew in part (ii). Explain.

Two vectors, $\vec{e}$ and $\vec{f}$, are shown to the right.



(iv) Write both vectors in terms of their horizontal and vertical components. (eg, $\vec{a} = 4\vec{x} + 3\vec{y}$)

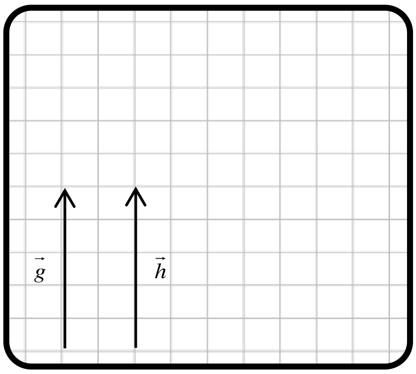
(v) By connecting the tail of one vector to the tail of the other vector, show the resultant vector of $\vec{e} +$
→

(vi) Add the horizontal components of $\vec{c}$ and $\vec{d}$ together. Add the vertical components of $\vec{c}$ and $\vec{d}$ together. Does the horizontal and vertical components define the resultant vector you drew in part (ii). Explain.

(vii) Explain how adding vectors head to tail is the same as adding vectors by adding their components.

III. Determining how horizontal and vertical vectors affect the resultant vectors.

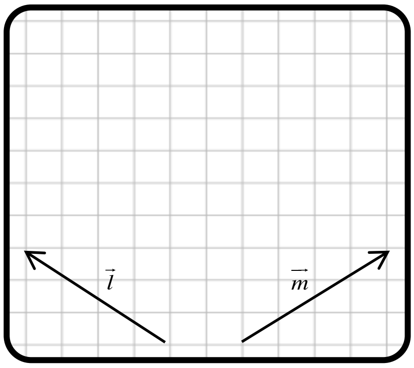
Two vectors, $\vec{g}$ and $\vec{h}$, are shown to the right.



(i) What is the magnitude of $\vec{g}$ and $\vec{h}$?

(ii) Draw in the resultant vector of the $\vec{g}+\vec{h}$. What is the magnitude of the resultant vector you drew?

Two vectors, $\vec{l}$ and $\vec{m}$, are shown to the right.



(iv) Show the magnitude of $\vec{l}$ and $\vec{m}$ is 5 units for each vector.

(v) Draw in the resultant vector of the $\vec{l}+\vec{m}$. What is the magnitude of the resultant vector you

(vi) Write both vectors in terms of their horizontal and vertical components.

(vii) What is the result when the horizontal components are added together?

(viii) What is the result when the vertical components are added together?

(ix) From your results in (vii) and (viii), explain why the magnitude for the resultant you got in (v) is less than 10 units. (i.e. directly adding the magnitude of both vectors; 5 + 5 = 10)

I. **Drawing vectors.**

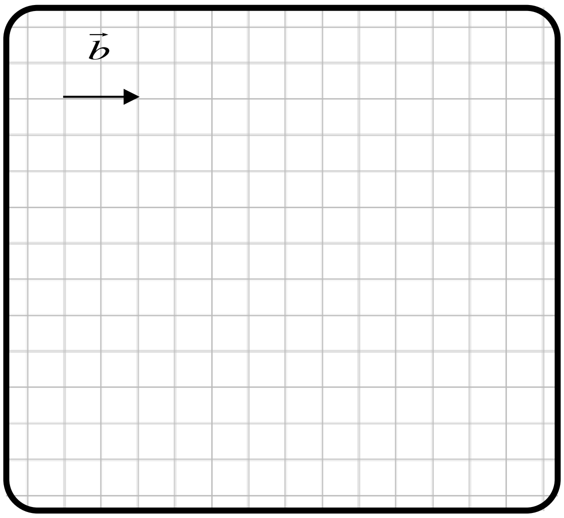
The arrow in the diagram to the right shows a vector, $\vec{b}$. In the extra space, draw the following vectors.

(i) $2\vec{b}$

(ii) $4\vec{b}$

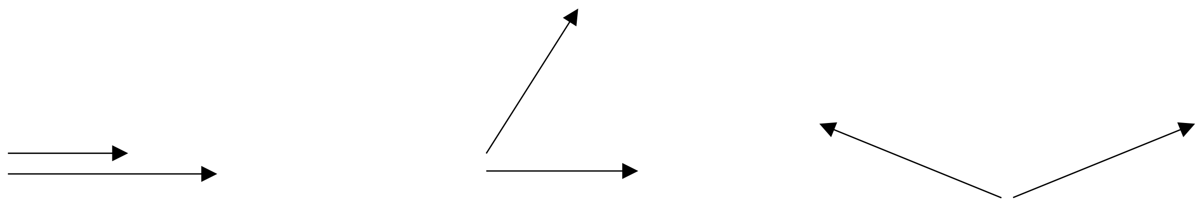
(iii) $-\vec{b}$

(iv) $-3\vec{b}$



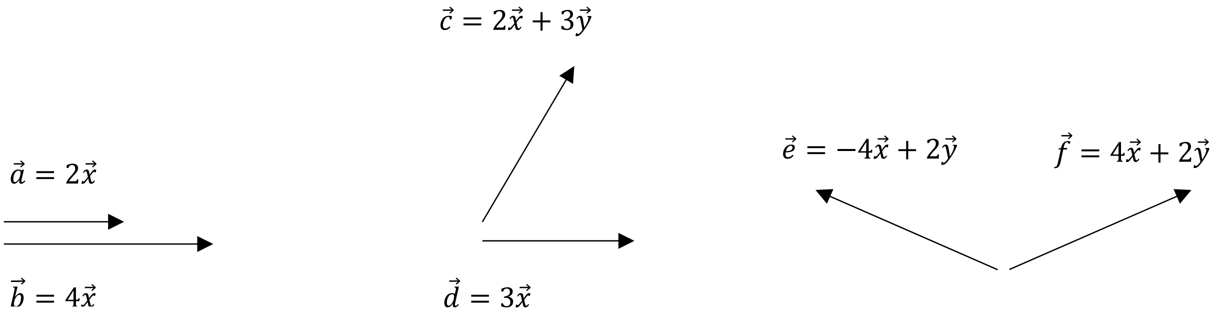
I. Adding vectors.

Use the tip to tail, or the parallelogram rule to add the following pairs vectors. Clearly show any construction lines you draw.



(i) (ii) (iii)

II. Adding vectors using components.



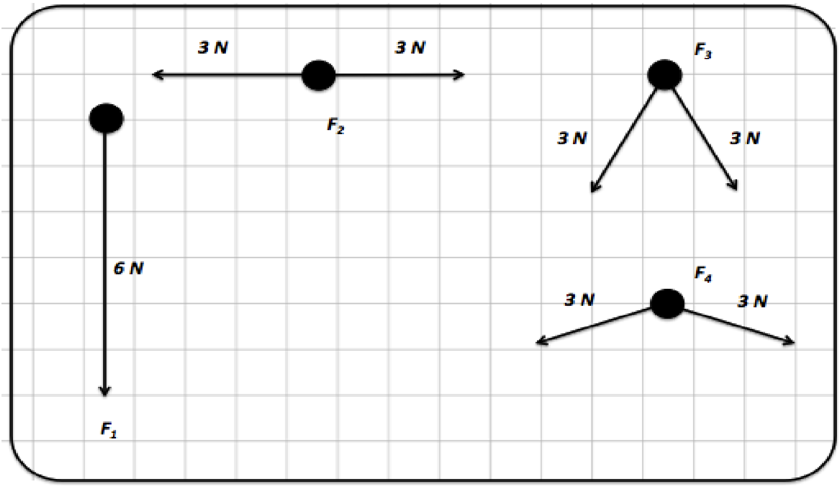
(i) Using the vector notation given, add the following vector pairs:

$\vec{a} + \vec{b}$, $\quad \vec{c} + \vec{d}$, $\quad \vec{e} + \vec{f}$

(ii) Use Pythagoras' theorem to find the magnitude of $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$.

(iii) Explain, referring to the addition of horizontal and vertical components, explain why the magnitude of $\vec{c} + \vec{d}$ is greater than $\vec{c}$ and $\vec{d}$, individually, but the magnitude of $\vec{e} + \vec{f}$ is less than $\vec{e}$ and $\vec{f}$ individually.

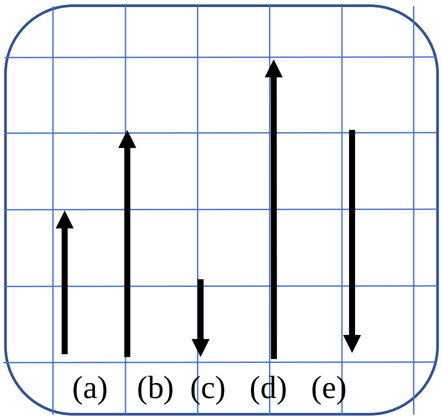
III. Looking at how horizontal and vertical components affect the resultant.



A small ball has a force or numerous forces acting on it as shown in the diagram to the right. The net force (resultant force) is labelled as F_1, F_2, F_3 and F_4.

(i) Rank the net forces acting on the ball, from highest to lowest. Explain your ranking. (Refer to either the tip to tail / parallelogram rule or refer to horizontal or vertical components, or both to give a full answer)

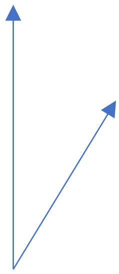
1. Rank the following vector arrows (a – e) from weakest magnitude to strongest magnitude.

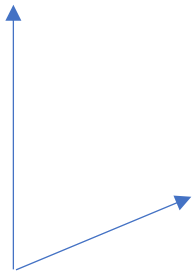


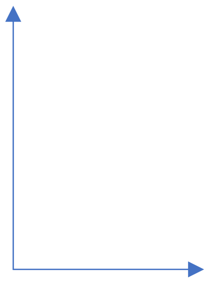
Ranking:

Explanation:

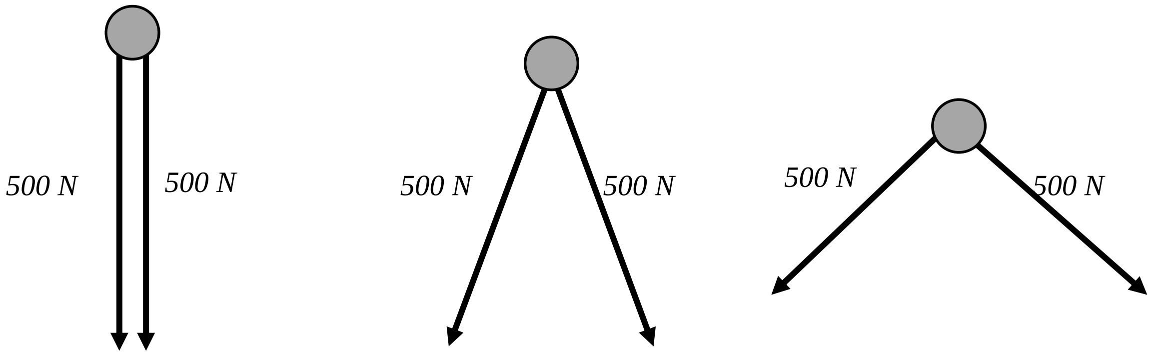
2. Show how to construct resultant of the following vectors.







3. A truck is broken down and needs to be pulled. A number of different cars can be used to pull the truck to a safe spot. All cars have the same pulling strength as shown buy the force vectors in the 3 diagrams below. The circle represents the centre of mass of the truck.



(i) Which diagram represents the vectors that will result in the strongest magnitude of force pulling the truck?

(ii) Explanation for part (i).

(iii) Which diagram represents the vectors that will result in the weakest magnitude of force pulling the truck?

(iv) Explanation for part (iii).

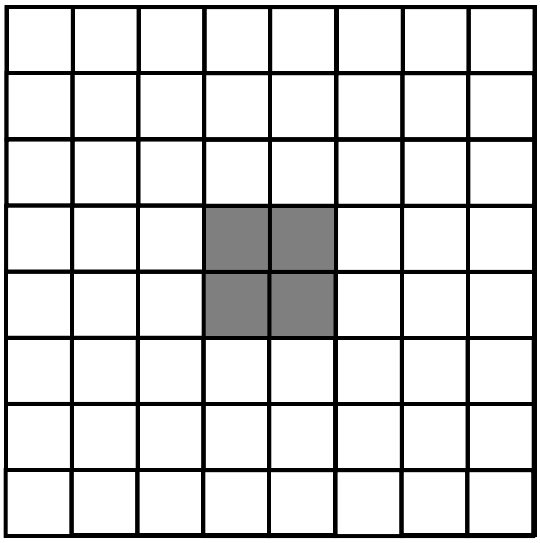
Appendix B

Inverse square law tutorial materials.

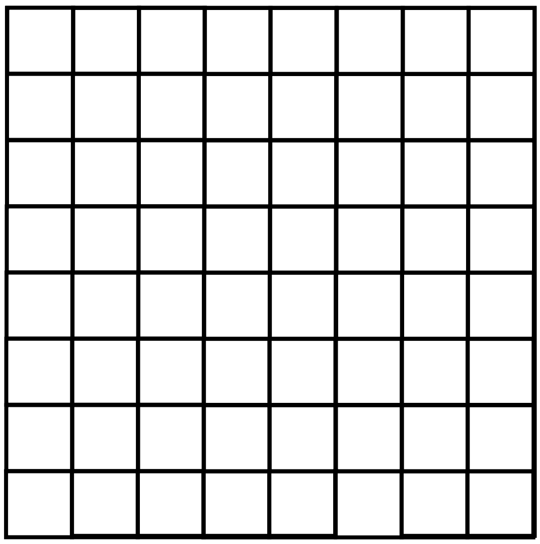
I. On the graph, sketch a graph of the pattern seen when you graph the function $y = k\frac{1}{x^2}$

Explain why you drew the pattern as you did.

II. A bulb shines on a wall from 1 m. The wall has an 8 x 8 grid on it



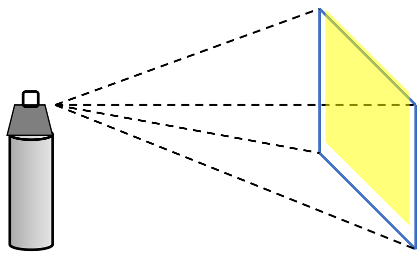
(i) If the bulb were moved to 3 m, shade in the shape would look like on the *8 x 8* grid below. (next page)



(ii) Explain why you drew it at you did.

III. A bulb is held 2 m from a light sensor. The sensor record the light intensity to be 100 Wm^{-2}. If the bulb is moved so that it is 4 m from the light sensor, what reading will the light sensor read?

I. Spray paint "Intensity."



A can of spray emits 100 drops of paint in 1 second.

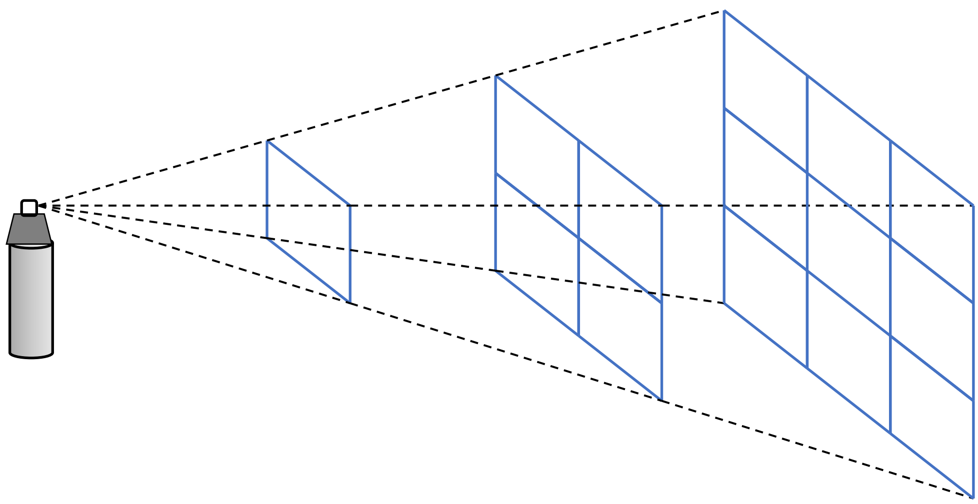
A metal frame, in the shape of a square, is placed in path of the paint spray so that all of the droplets of paint passes through it is area.

The square frame has a width of 10 cm and each of the

(i) Calculate the area, in m^2, of the metal frame

(iii) Determine how many paint droplets pass through the $1m^2$ frame in 1 second. Choose an appropriate unit for your answer.

II. How distance affects spray paint "intensity."



A second metal frame is placed away from the can, so that all it is outer corners are 10 cm from the can of paint. This metal frame is made up of smaller frames that are identical to the frame discussed at the top of the page.

(i) Explain why this second frame has an area that is four times bigger than the original frame used on the last page.

(ii) What is the value of the area of the overall frame that is 10 cm from the can.

(iii) Determine the how many droplets of paint pass through 1m^2 frame in 1 second, for this overall frame.

A third metal frame is placed away from the source, so that all it is outer corners are 15 cm from the can of paint. This metal frame is made up of smaller frames that are identical to the frame discussed at the top of the previous page.

(iv) Explain why this third frame has an area that is nine times bigger than the original frame used on the last page.

(v) What is the value of the area of the overall frame that is 15 cm from the can.

(vi) Determine the how many droplets of paint pass through 1m^2 frame in 1 second, for this overall frame.

(vii) As the distance from the paint can increases, the number of droplets of paint passing through a 1 m^2 area in 1 second decreases. Using your answers from the previous questions, explain why this occurs.

The relationship between distance from source and spray paint "intensity" is an example of an inverse square relationship. Inverse square relationships have the general equation:

$$y = k\frac{1}{x^2}$$

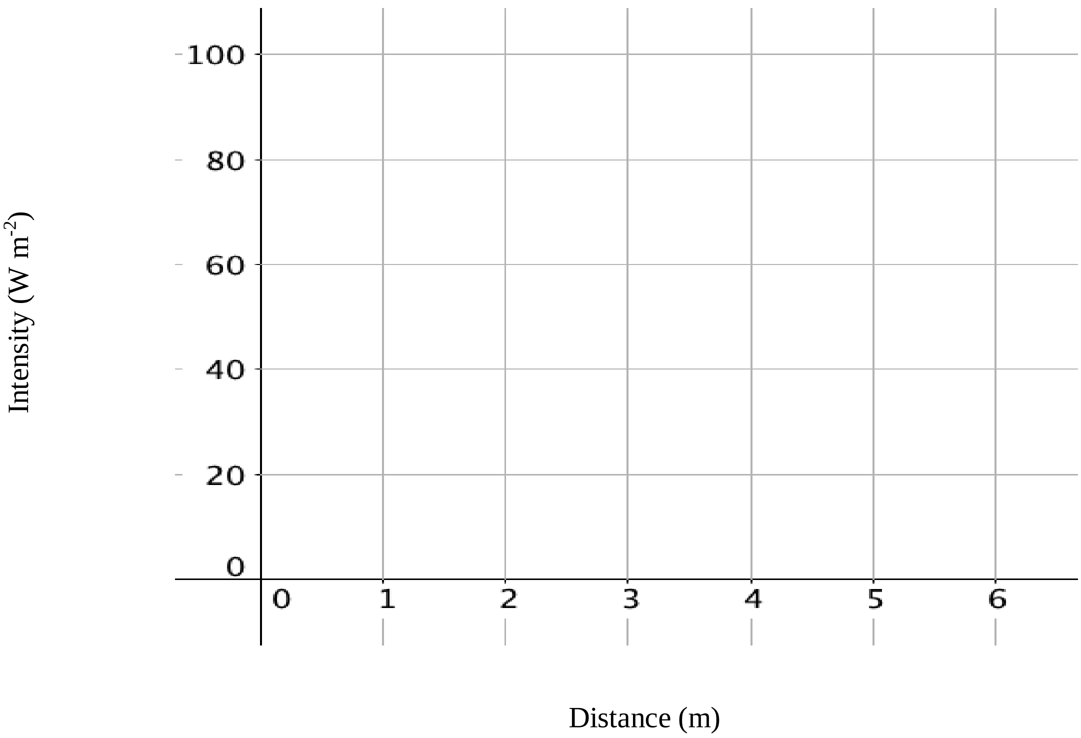
Note that as the *x* values increase, the *y* values decrease. This is due to the position of the x as a denominator on the right hand side of the equation. We also can note that the power on the *x* is x^2, which separates the inverse square relationship from an inverse relationship. We will now explore this mathematically.

III. Exploring the inverse square relationship.

The following table shows spray paint intensity given by a the can that emits 100 droplets of paint per second.

Plot this data on the graph below.

Distance (d)	**Intensity (I)**
1	100.0
2	25.0
3	11.1
4	6.3
5	4.0
6	2.8



(i) Describe the pattern shown using the following criteria: shape, increasing / decreasing, change in the slope. Include others if you think of them.

(ii) As the distance from the bulb increases, does the intensity increase, decrease or stay the same? Explain with reference to the shape of the graph.

(iii) Use your graph to determine the intensity of the light at a distance of 1 m from the bulb, and 2 m from the bulb

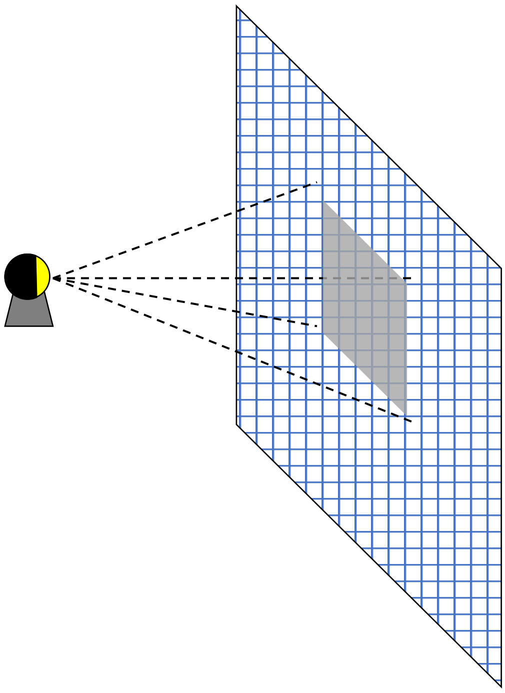
(iv) By what factor is the intensity at 2 m smaller than the smaller/bigger than the intensity at 1 m?

(v) Use your graph to determine the intensity of the light at a distance of 1 m from the bulb, and 5 m from the bulb

(vi) By what factor is the intensity at 5 m smaller than the smaller/bigger than the intensity at 1 m?

(vii) Using your answers, and the reasoning you developed on the first two pages, explain why this is an example of an inverse square law.

I. Light Intensity.

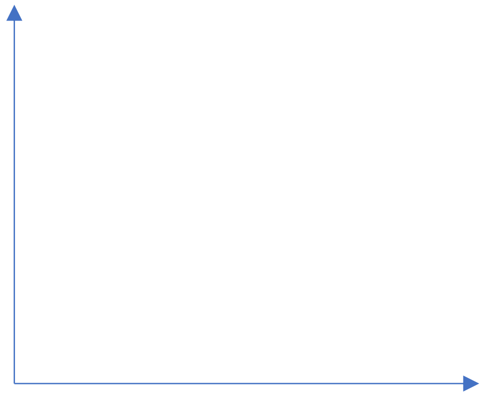


(i) Calculate the light intensity at a point 2 m from the bulb. (Formulae: $I = \frac{P}{A}$, $A = \pi r^2$)

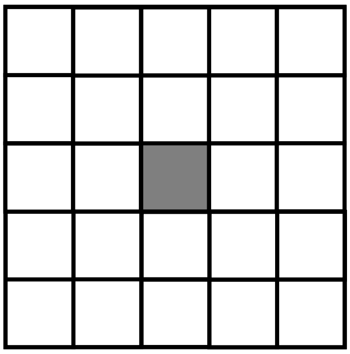
(ii) If you increased the distance from the wall to to the bulb, would the light intensity increase, decrease or stay the same? Explain your reasoning. (review what you covered in the worksheet for spray paint “intensity” if you need to)

(iii) Sketch a graph to show the relationship you explained in part (ii) and explain how it accurately shows the relationship from the formula you used in part (i)

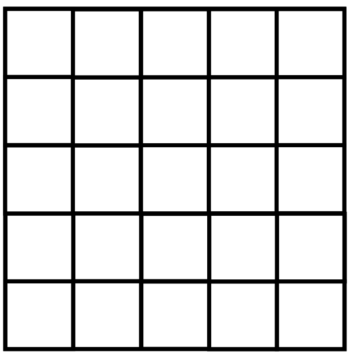
How it shows the relationship:



The photograph of the wall shows the following shape of the light on the wall. This is the shape of the light when the *6 x 6* grid is looked at head on.



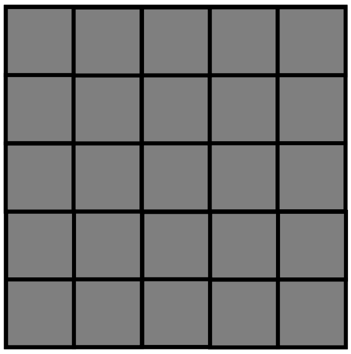
(iv) If the bulb were moved to 4 m, shade in the shape would look like on the *6 x 6* grid below.



(v) Explain why you drew it at you did.

(vi) Has the light intensity on the grid increased, decreased or remained the same? Explain your reasoning (consider the effect of moving the bulb back, in conjunction with your sketch in (iv)).

(vii) The bulb is moved to a distance where the shape of the light is shown on the grid on the next page. Determine how far the bulb is from the wall. Ensure you show how you figured it out.



III. Calculations. (Formulae: $I = \frac{P}{A}$, $A = \pi r^2$)

(i) A 200 W bulb is placed 5 m from a light sensor. Calculate the light intensity that the light sensor.

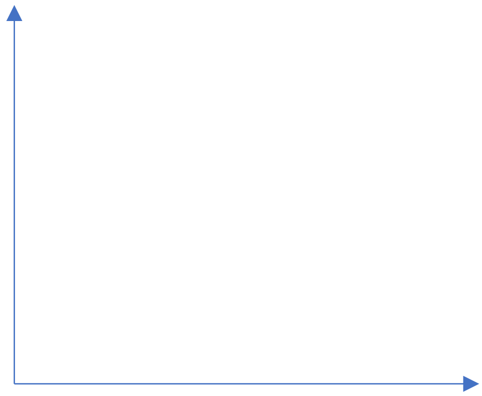
(ii) The 200W bulb is moved to 10m from the light sensor. Calculate the light intensity that the light sensor.

(iii) Use your answers from (i) and (ii) to show that light intensity follows an inverse square law.

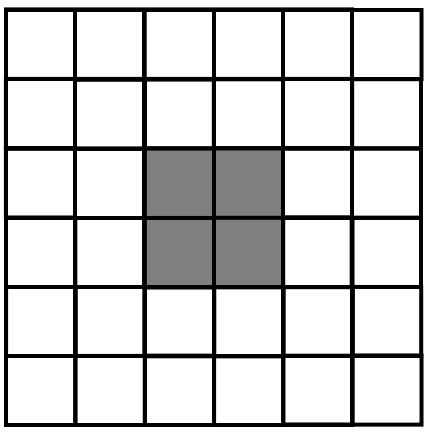
I. A can of spray paint emits 200 droplets of paint per second from the nozzle. The amount of droplets from a can of spray paint that fall on a given area (intensity – I) is given by the formula: $I = \frac{200}{0.125\,\pi r^2}$.

Draw a sketch of the graph that represents the relationship between spray paint intensity (I) and the distance from the nozzle (r), and explain how it shows the relationship.

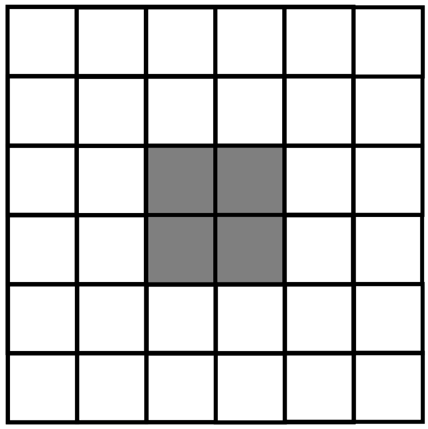
How it shows the relationship:



II. The can is held 2 metres from a wall that has squares marked on it like a grid. The can is then sprayed for 1 second. This is the shape of the paint landing on grid is looked at head on, is shown in the diagram below.



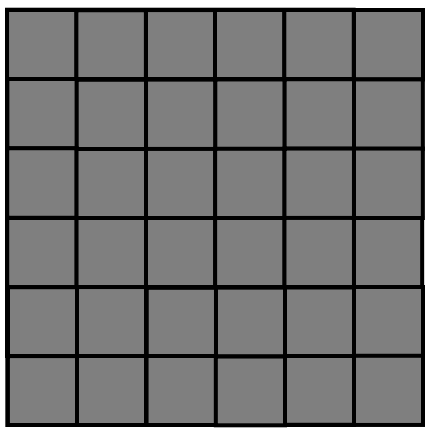
(iii) If the can was moved to 4 m, shade in the shape would look like on the *6 x 6* grid below.



(iv) Explain why you drew it at you did.

(v) Has the intensity of the droplets per square on the grid increased, decreased or remained the same? Explain your reasoning

(vi) The can is moved to a distance where the shape of the paint is shown on the grid below. Determine how far the can is from the wall. Ensure you show how you figured it out.



III. Calculations.

(iv) The can is placed 5 m from a wall. Calculate the paint intensity, using the formula given on the first page.

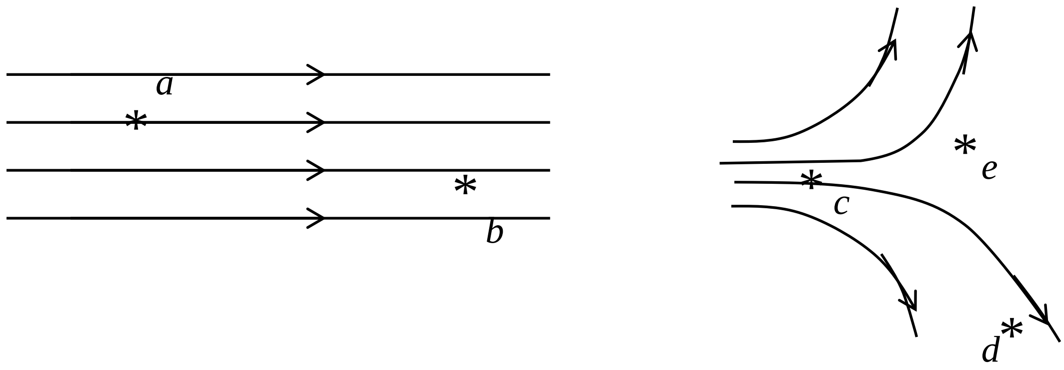
(v) If the distance from the can to the wall is doubled, what is the new "paint intensity?" Explain how you got your answer?

(vi) Explain how your answer from (ii) shows this is an example of an inverse square law.

Appendix C

Field lines tutorial materials.

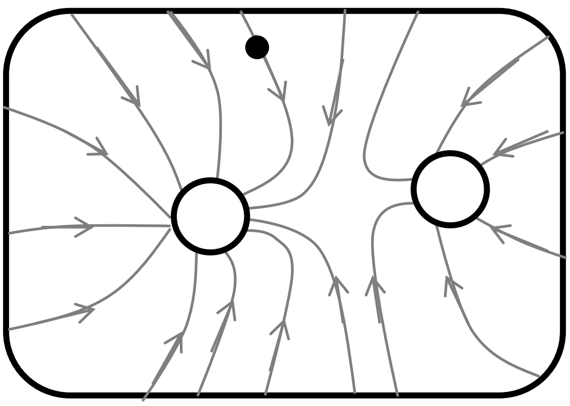
1. Here are two examples of field lines.



(i) Rank the field strength, from highest to lowest, for the points (a) to (e). Explain your reasoning.

(ii) Use vector arrows to show the direction of force acting on an object if it were to be placed at (a) to (e).

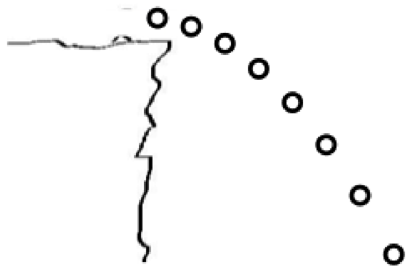
2. A meteor is placed at rest between two planets, that are close to each other. The gravitational field of both planets is shown in the diagram using field lines.



On the diagram, show the path taken by the meteor as it accelerates under the influence of the gravitational field. Explain why you drew the path as you did.

I. Gravitational field (Acceleration due to gravity).

A ball is thrown off a cliff, with an initially velocity *10 ms⁻¹*. A camera takes a photo every quarter of a second and all the pictures of the ball are combined into one picture as shown.



(i) Can you tell from the diagram whether the magnitude of the velocity of the ball is changing? Explain.

(ii) Can you tell from the diagram whether the direction of the velocity of the ball is changing? Explain.

(iii) Can you tell from the diagram alone, whether the object accelerates in the horizontal direction or vertical direction. Explain.

The ball has a mass 0.5 kg.

(i) Calculate the force of gravity acting on the ball. (Take mass of earth as 6×10^{24} *kg*, radius of earth as 6.4×10^{6} *m* and the gravitational constant as 6.7×10^{-11} $N\,m^2\,kg^{-2}$.

(ii) Is the force of gravity acting on the ball constant during the duration of the fall? Explain.

(iii) Using $F = ma$, determine the acceleration, due to gravity, acting on the ball.

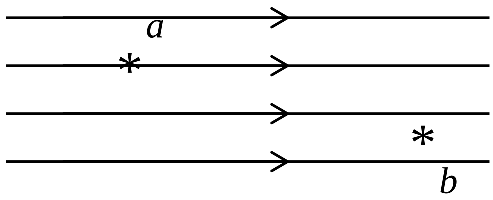
(iv) Is the acceleration, due to gravity, acting on the ball constant during the duration of the fall? Explain.

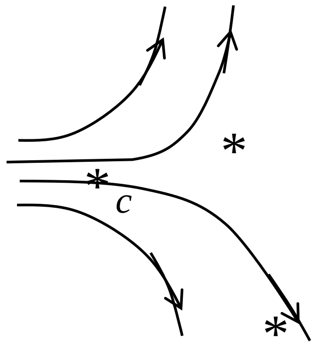
(v) Draw vector arrows on the all the balls to represent your answer from (iv). (8 arrows in total, one from each ball).

II. Gravitational field (lines).

In the last question of section II, you were asked to draw 8 vector arrows to represent the acceleration acting on the ball. Drawing vectors arrows in some cases can be clumber-some, and it is sometimes easier to use field line representations instead. Field lines represent the direction and strength of the force felt by objects that interact with those fields.

These are continuous lines that, when a point in picked, a tangent to the line at that point denotes the direction of force at that point. The closer field lines are together, the stronger the force is and a body does not have to be on a field line to feel a force. Here are two examples of field lines.





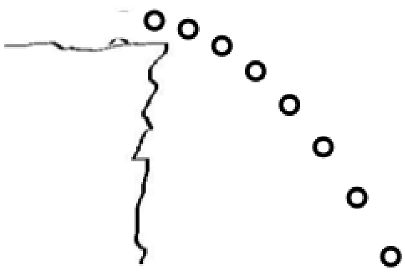
(i) Rank the field strength, from highest to lowest, for the points (a) to (e). Explain how you used the field lines to justify your ranking.

(ii) Use vector arrows to show the direction of force acting on an object if it were to be placed at (a), (d) and (e) (use a ruler).

A uniform field is described as a field that always points in the same direction, and has a constant field strength.

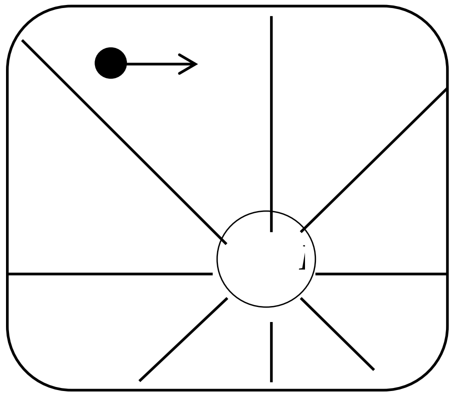
(iii) Which field (left or right) shows a uniform field? Explain how the field shows it is uniform.

(iv) Use field lines to sketch the field that causes the ball to fall. Explain if the field is uniform or not. (refer back to section I)



III. Gravitational field of the earth.

The diagram shown contains a small meteor that is passing close to the earth, shown as a small black circle. It has a velocity of 10 *km/s*, shown by the vector arrow. The earth is shown as a big circle, and it is gravitational field is sketched. At no point, does the meteor collide with the planet.



(i) Is the strength of the gravitation field caused by the ***earth*** the same everywhere? Explain.

(ii) Using $F = mg$, and $F = G\frac{Mm}{r^2}$, show that the acceleration due to gravity is given by $a = G\frac{M}{r^2}$

(iii) Explain how the field lines represent that gravity follow an inverse square law.

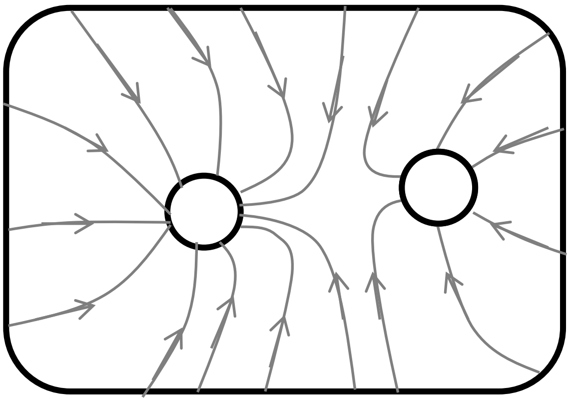
(iv) Use arrowheads to show the direction the field-lines point. Explain why you drew them as you did.

(iii) Draw the path followed by the meteor, under the influence of the gravitational field. Explain your reasoning for drawing it as you did. **(Remember, it has an initial velocity as shown with the vector arrow in the diagram) (Explain why your path follows / does not follow the field lines)**

IV. Gravitational field between two planets, very close together.

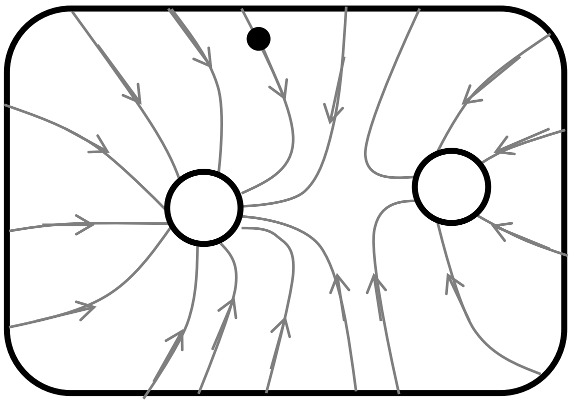
The diagram to the right shows the field between two planets.

(i) Using the field lines, determine whether the mass of one of the planets is bigger than the other, or if they are the same. Explain your reasoning.



Consider the following student dialogue, between two students (S_1 and S_2) concerning a small meteor initially at rest placed at a location shown by the small black circle.

S_1: *The field lines indicated the direction of the force, so the meteor will be forced along the line until it hits the left planet*

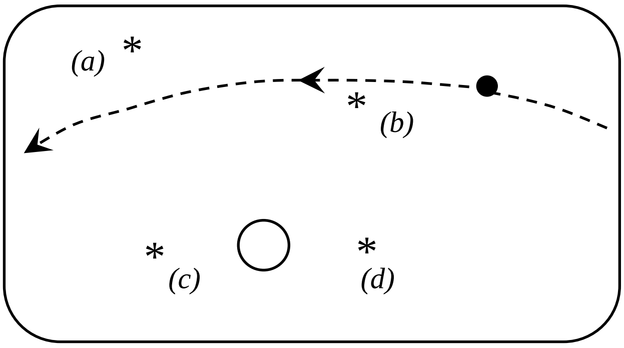


S_2 *As the small meteor begins to accelerate, its gained velocity will make it move away from the field line that it was on originally, so we can be sure it'll*

(iii) Which student, if any, do you agree with. Explain why your reasoning.

(v) Using your answer from (iii), draw in the likely path followed by the meteor on the diagram above.

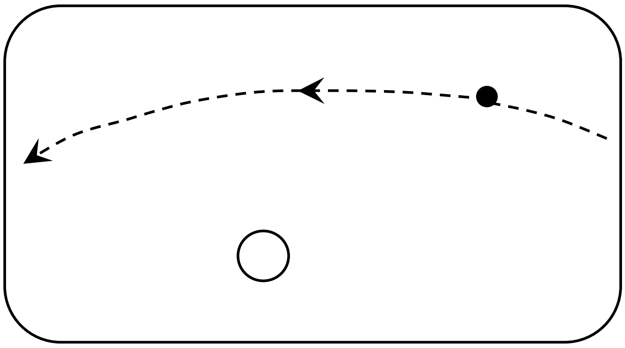
1. The following diagram shows a small planet, denoted by the white circle. A meteor passes by the planet in the path shown. A number of points (a) – (d) are also highlighted. The planet has mass of 3×10^{24} *kg*.



(i) Use field lines to represent the gravitational field caused by the planet.

(ii) Rank the field strength, from lowest to highest, at the points (a) – (d). Justify your ranking, ensuring you reference the field you drew in (i).

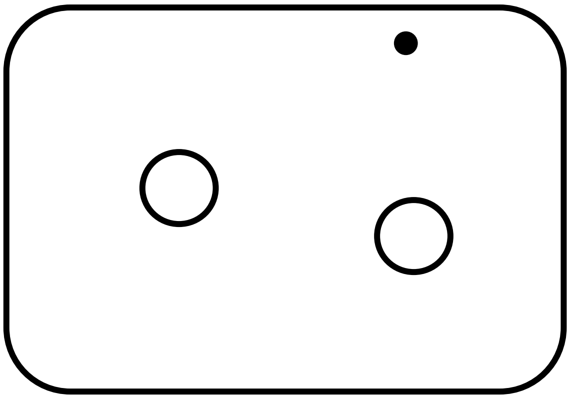
(iii) If the planet had a mass of 6×10^{24} *kg*, draw in the field lines that represent that this planet has an increase in mass on the diagram below.



(iv) Draw in the new path taken by the meteor in this scenario, in which the mass of the planet has increased. Explain why you drew the new path as you did. (If you think it follows the original path, which is shown in the diagram, explain exactly why you think this).

(v) What effect, if any, does increasing the mass of the meteor have on the path taken? Explain. (If you think it does not affect the path taken, explain exactly why you think this).

3. A meteor is placed at rest between two planets of equal mass, that are close to each other.

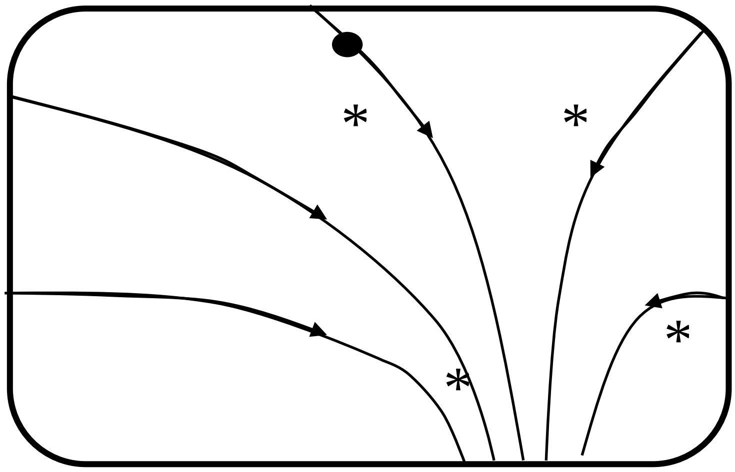


(i) Draw the field lines to represent the gravitation field caused by both planets in the diagram above.

(ii) On the diagram on the top of the page, show the path taken by the meteor as it accelerates under the influence of the gravitational field. Explain why you drew the path as you did.

(iii) Highlight the point between the two planets where the gravitational field is zero using a circle. Explain why this point exists.

1. Here is a small snapshot of a section of field lines.



(i) Trace you finger to the end of any field line, starting where the lines are closest together, so that it travels against the direction of field line. How the field strength varies as you move your finger. Justify your answer.

(ii) Use vector arrows to show the direction of the force the points highlighted.

(iii) A small body is placed at the point marked with a black circle. show the path taken by the body as it accelerates under the influence of the gravitational field. Explain why you drew the path as you did.

Appendix D

Coulomb's law and electric field tutorial materials.

Coulomb's Law explains the attraction or repulsion between two charges. It is given by the formula

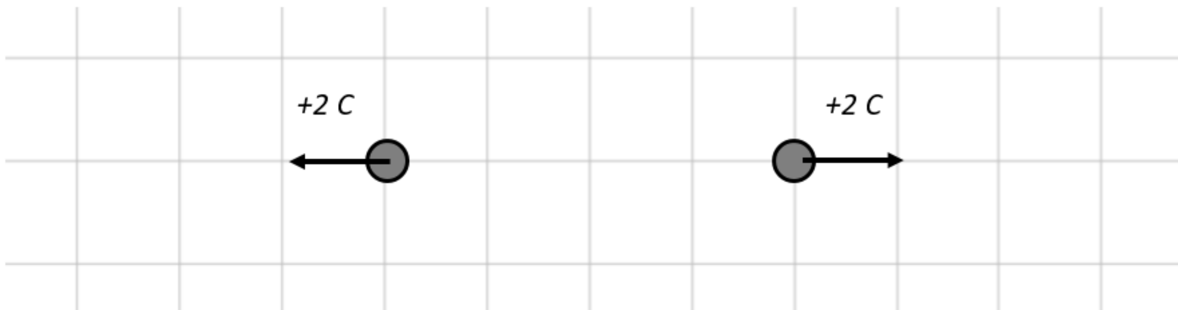
$$F = k\,\frac{q_1 q_2}{d^2}$$

1. Using the formula above, explain the relationship between the force between the two charges and their magnitudes.

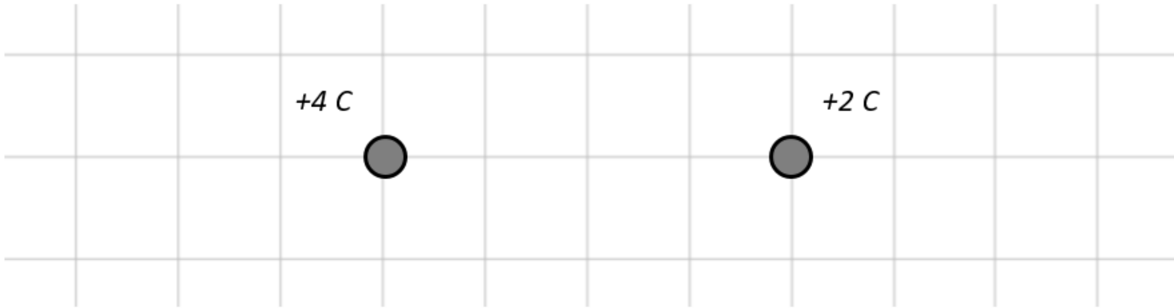
2. Using the formula above, explain the relationship between the force between the two charges and the distance between them.

3. Two *+3 C* charges are held a distance of *10 cm* from each other, and the force acting on both charges is *10 N*. If one of the +3C charges is replaced with a +9C charge, what is the new force acting on the charges? Explain how you know what the change in force is.

4. A *+2 C* and a *+2 C* charge are placed 20 cm from each other. The vectors to show the force are shown in the following diagram.



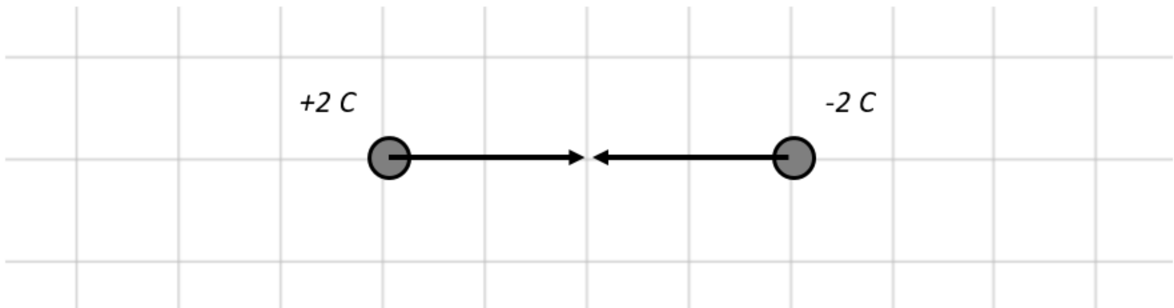
If the +2 C charge on the left is replaced with a +4C charge. Draw the vectors to represent the forces now acting on the charges.



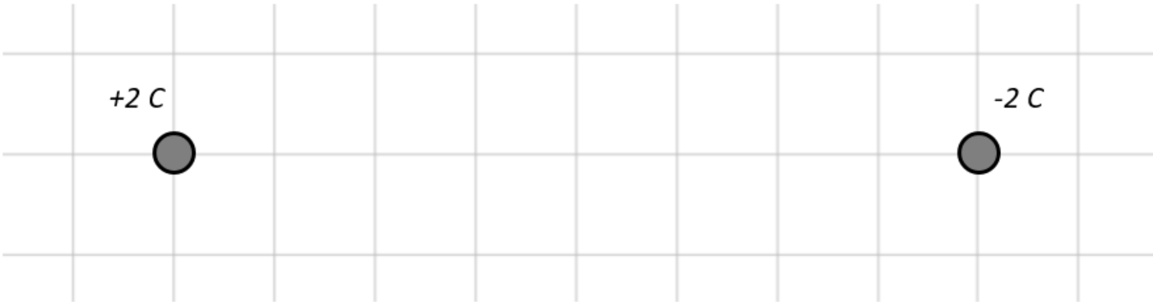
Explanation:

5. Two *+8 C* charges are held a distance of *10 cm* from each other, and the force acting on both charges is *90 N*. The charges are moved so the distance between then is now *30 cm*. What is the new force acting between the charges? Explain how you know what the change in force is.

6. A *+2 C* and a *-2 C* charge are placed 20 cm from each other. The vectors to show the force are shown in the following diagram.

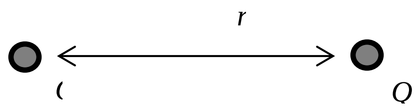


If the distance between the charges is increased to *40 cm*, draw the vector arrows to show the force acting between the charges now.



Explanation:

I. **Looking at the relationship between the charges and the force in Coulombs' Law.**



Two charges are held a distance apart from each – other, a constant distance "r." There is a force exerted between the charges. Each charge, q_1 and q_2, are replaced and with various stronger charges and the forces are recorded as shown. (The product of the charges (q_1*q_2) is shown in the third column)

q_1 (C)	q_2 (C)	$q_1 q_2$ (C^2)	F (N)
1	1	1	2
1	2	2	4
2	1.5	3	6
4	1	4	8
1	5	5	10
2	3	6	12

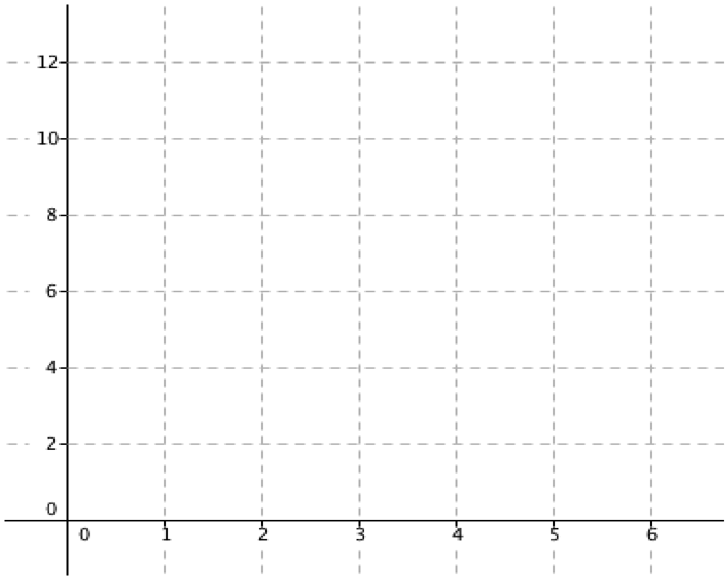
(i) Can you see any pattern between the first column (q_1), the second column (q_2) or the third column (q_1q_2) with the fourth column (F)?

If so, how would you describe this pattern.

(ii) Linear patterns through the origin follow the form y = mx, where m has a constant value. By letting the values for column 4 (F) represent your y values, and which-ever of the other three column you chose to in (i) for the x-values, show that $\frac{y}{x}$ is a constant.

y	F (N)	2	4	6	8	10	12
X							
$\frac{y}{x}$	$\frac{F\ (N)}{\ }$						

If you identifies any patterns in sections (i) – (iii), graph it / them on the graph shown. (Label the axis)



(iii) Explain how this graph shows a directly proportional relationship.

(iv) What force is exerted between the charges when the product of the charges is 2 C^2?

(v) What is the force exerted between the charges when the product of the charges is 6 C^2?

(vi) From part (iv) and (v), explain what affect does tripling the product of the charges have on the force exerted between the charges?

(vii) Is this in agreement to your answer from part (iii) that this is a directly proportional relationship? Explain your answer.

II. Calculations involving a directly proportional relationship.

The force, F_A, exerted by the two charges shown, can be calculated in the manner shown below, if q_1 has a magnitude of $6\ \mu C$ and q_2 has a magnitude of $3\ \mu C$, and the distance between the charges is 1 *cm*. (Use $F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$, where $\varepsilon = 8.85 \text{ x } 10^{-12}\ F\ m^{-1}$)

A

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

$$F = \frac{1}{4(3.14)(8.9\ x\ 10^{-12})}\frac{(6\ x\ 10^{-6})(3\ x\ 10^{-6})}{(1\ x\ 10^{-2})^2}$$

B

$$F = \frac{1}{1.12\ x\ 10^{-10}}\frac{(1.8\ x\ 10^{-11})}{(1\ x\ 10^{-4})}$$

C

$$F = (8.9\ x\ 10^{9})(1.8\ x\ 10^{-7})$$

D

$$F = 1{,}602N$$

(i) Explain the mathematical step that occurs in A. (include explanation as to the use of scientific notation for the values)

(ii) Explain the mathematical steps that occur in B (there are 3).

(iii) Explain the mathematical steps that occur in C (there are 2).

(iv) Explain the final steps that occur in D.

(v) From your steps outlines in (i) to (iv), calculate the force, F_B, exerted between the charges if the 3 μC charge is replaced with a 9 μC charge.

(vi) By what factor is the force between the charges increased?

(vii) How does your answers show that the force experienced by the charges is directly proportional to the product of their magnitudes?

(viii) Explain how there would there have been a quick way for you to determine the new force after the replacement?

III. **Looking at the relationship between the distance and the force in Coulombs' Law.**

Coulomb's law states that the force between two charges is directly proportional to the product of the magnitude of the charges, and inversely proportional to the square of the distance between them. This is given by the following formula.

$$F = k\,\frac{q_1 q_2}{d^2} \qquad k = \frac{1}{4\pi\varepsilon}$$

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

In the last 4 pages, you learnt how to show the first relationship by, between the force exerted and the product of the charges by using tables, graphs and calculations. Using what you learnt, your task is to prove the relationship between the force exerted between the charges, and the distance between them, using whichever methods you choose. Attempt all of them. If you get stuck with a method, ask for help in using it.

You can use the following to help you.

Directly proportional general equation: $y = mx,\quad \frac{y}{x} = m,\quad \frac{y}{x} = constant$

Directly proportional to square equation: $y = ax^2,\quad \frac{y}{x^2} = a,\quad \frac{y}{x^2} = constant$

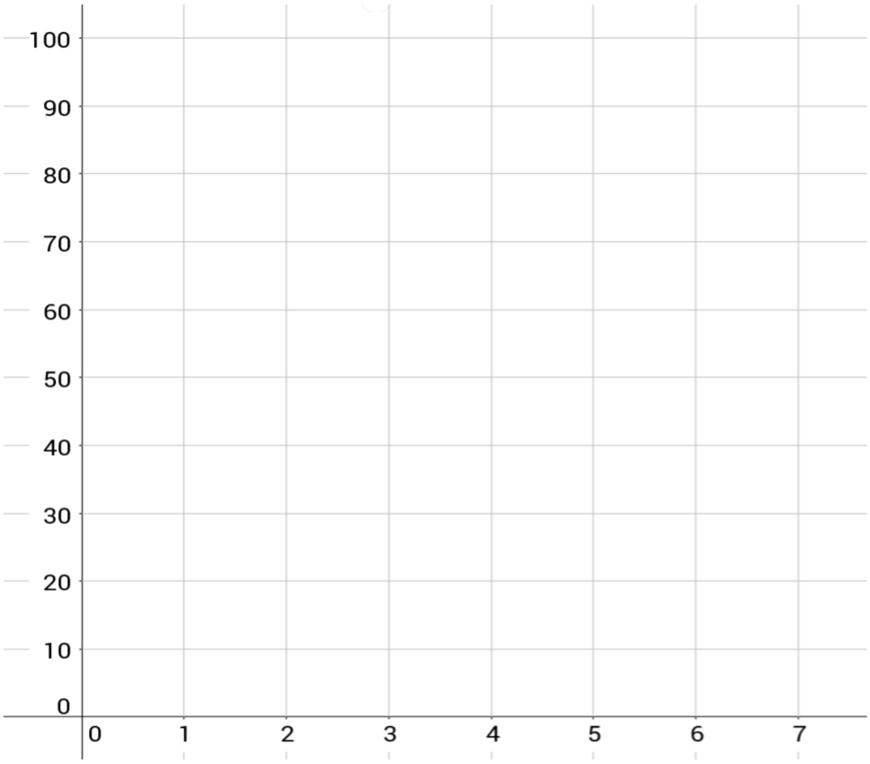
Inverse proportional general equation: $y = \frac{k}{x},\quad xy = k,\quad xy = constant$

Inverse square proportional equation: $y = \frac{k}{x^2},\quad x^2y = k,\quad x^2y = constant$

Table:

y	F (N)	100	25	11.11	6.25	4	2.78
x	d (m)	1	2	3	4	5	6

Graph:



Equation and values for calculation:

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

$\varepsilon = 8.85 \times 10^{-12}\ F\ m^{-1}$, $q_1 = 6 \times 10^{-6}\ C$, $q_2 = 4 \times 10^{-6}\ C$.

$d_1 = 4\ cm$, $d_2 = 8\ cm$

Which method do you think is the most effective? Why?

Which method is the easiest do use? Why?

Which method would you use, if you have to choose one? Why?

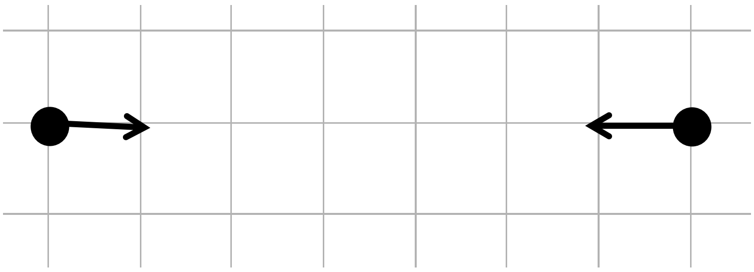
I. **Numerical calculations involving Coulomb's Law. (Formula is in equation tables, as is value for Ɛ)**

A *+3 C* and a *-3 C* charge are placed 10 cm from eachother.

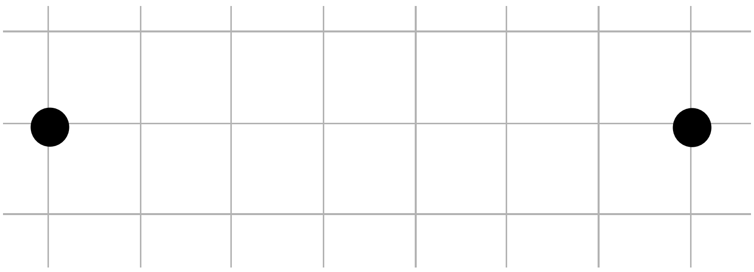
(i) Calculate the force between these two charges.

(ii) If one of these charges is replaced with a *+9 C* charge, what is the new force acting between these two charges.

The force between the two charges from (i) is shown in the following diagram.



(iii) Using your answers from (i) and (ii), draw the vector arrows to how the force acting between the charges from (ii).



(iv) Explanation how your answers from (i) – (iii) show the force between two charges is directly proportional to their magnitude.

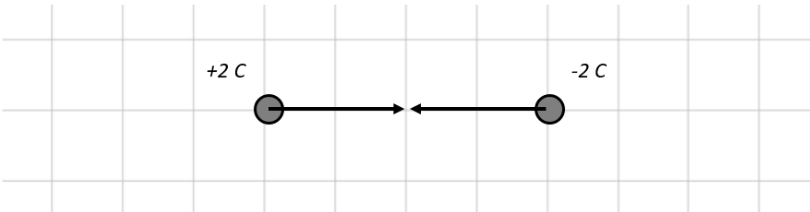
I. **Numerical calculations involving Coulomb's Law. (Formula is in equation tables, as is value for Ɛ)**

A *+2 C* charge and *-2 C* charge are placed 20 cm from eachother.

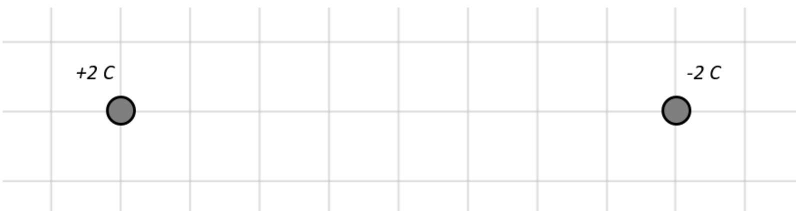
(ii) Calculate the force between these two charges.

(ii) These charges are moved to 40 cm from eachother. Calculate the new force acting between these two charges.

The force between the two charges from (i) is shown in the following diagram.

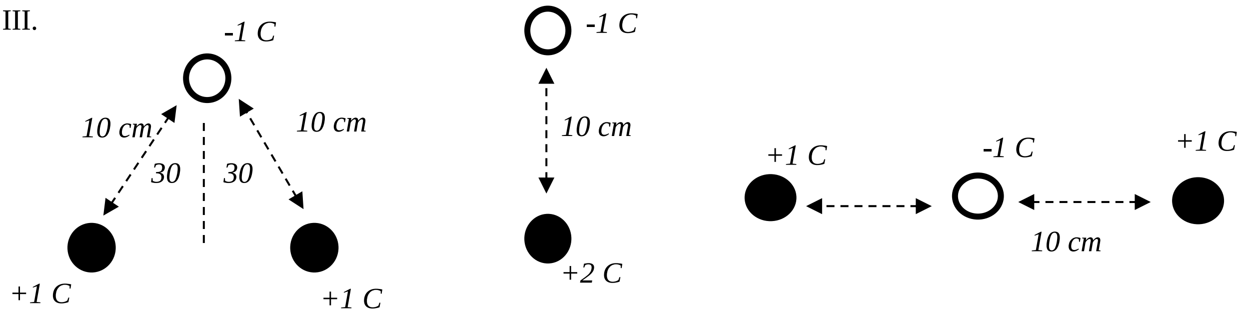


(iii) Using your answers from (i) and (ii), draw the vector arrows to how the force acting between the charges from (ii).



(iii) Explanation how your answers from (i) – (iii) show the force between two charges is inversely proportional to the square of the distance between the charges.

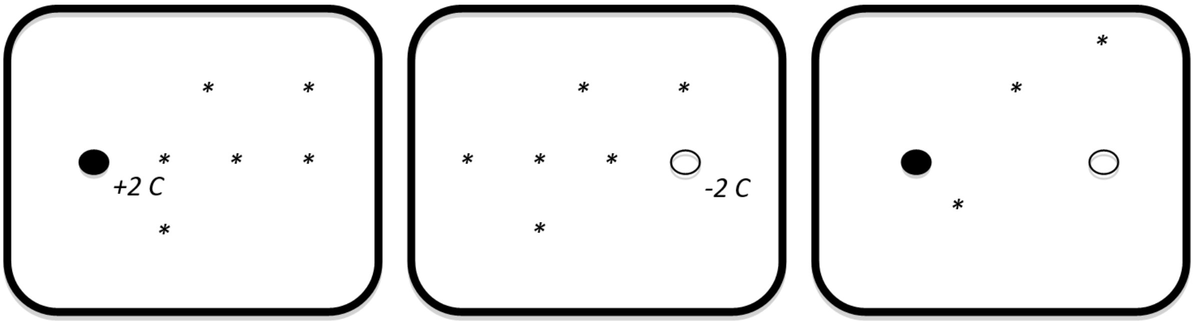
III.



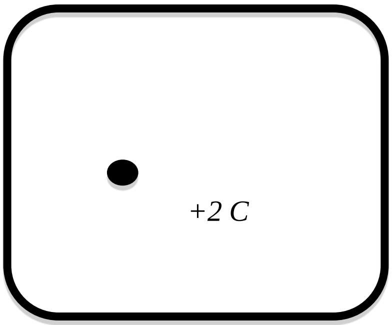
(a) (b) (c)

(i) 3 setups of charges are set up as shown in the above diagram. How does the magnitude of the net force acting on the negative charge (white) in setup (a) compare to the magnitude of the net force acting on the negative charge in (b) and (c). Explain your answer, using vectors, calculations or any other manner you see fit.

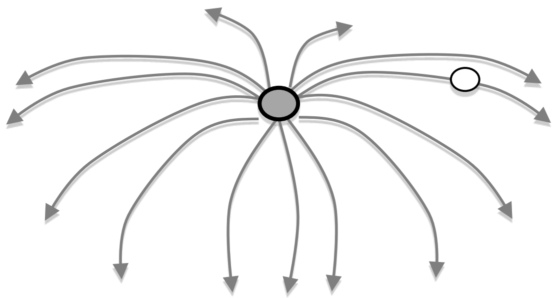
1. Use vector arrows to draw the electric field surrounds charges, at the points highlighted with stars, in the following three diagrams.



2. Represent the first diagram you drew, for the +2 C positive charge, using field lines.



3. A collection of unknown charges are located in the grey circle, which produce an electric field as shown to the right. An electron is placed at the position marked with the while circle.



(i) Draw in the path followed by the electron as a result of it is position in the force field.

(ii) Explain why you drew the direction as you did.

(iii) Rank the magnitude of the electric field strength, from highest to lowest, between a, b and c. Explain your ranking.

(iv) Draw vector arrows to represent the electric field at the points a, b and c on the diagram above.

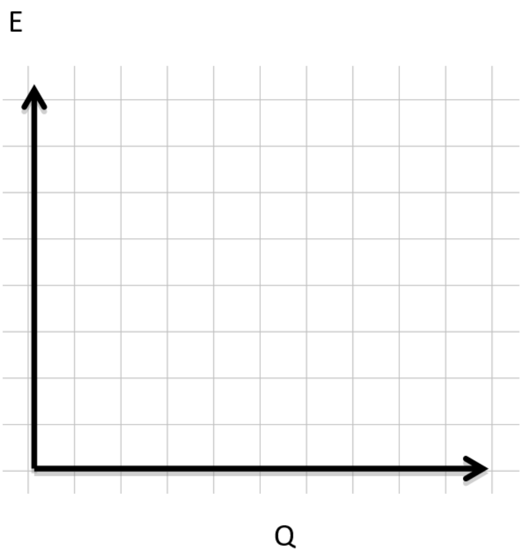
4.. The electric field strength around a charge is given by the formula $E = \frac{1}{4\pi\varepsilon}\frac{Q}{d^2}$.

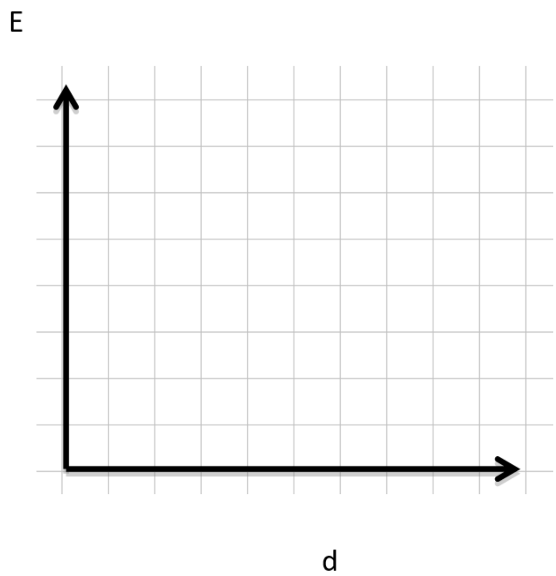
What is the relationship between the electric field strength and (i) the magnitude of the charge causing it, and (ii) the distance from the charge.

(i)

(ii)

Draw these relationships on the graphs below.

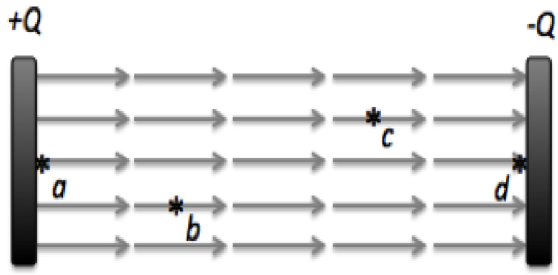




I. The forces experienced by a particle by an electric field – Vector treatment.

An electric field between two charged plates is depicted using vector arrows as shown. 4 points (*a, b, c* and *d*) are highlighted as shown.

(i) How would you describe variation of direction and strength of the electric field?



The picture above represents a ***uniform electric field***. This means that the strength of the electric field is the same at all points, and is in always in the same direction.

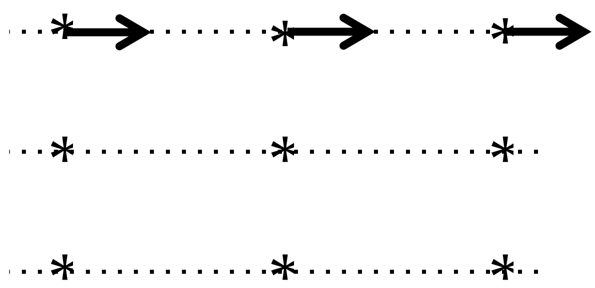
(ii) How does the representation show that the electric field strength is the same at all points?

(iii) Would the force experienced by a *+4 C* charge placed at *b* be *stronger*, *weaker* or *the same* compared to it being placed at *c*. Explain.

(iv) If the electric field has strength, $E = 20{,}000\ N\ C^{-1}$, find the force experienced by a *+4 C* charge when placed in the field.

(v) On the diagram on the right, the first line shows vector arrows, going left to right, for an electric field that is uniform.

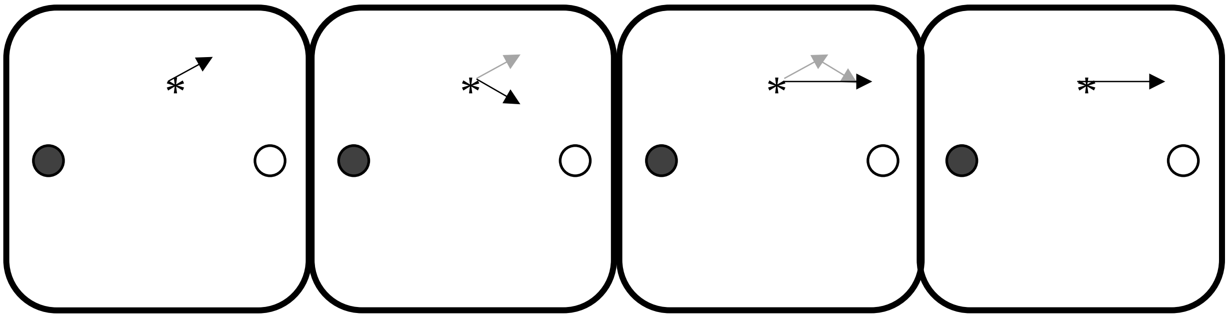
On the second and third line, use vector arrows to show (a) an increasing electric field and (b) a decreasing electric field.



II. Electric field vectors – Principle of Superposition.

One positive and one negative charged particles are placed a small distance from each – other. Both charged particles are have an equal magnitude. The electric field at a point marked with a star (*), are shown using vectors in the following order.

- The first field vector represents the direction of the electric field based on the position of the positive charge only.
- The second diagram represents the direction of the electric field based on the position of the negative charge, but the vector from the first diagram is represented with a grey arrow.
- The third diagram shows the net electric field vector, at this point, based on the two individual electric field vectors shown in the first and second diagram.
- The final diagram shows the net resultant electric field vector without referencing the two vector arrows that were used to construct it.

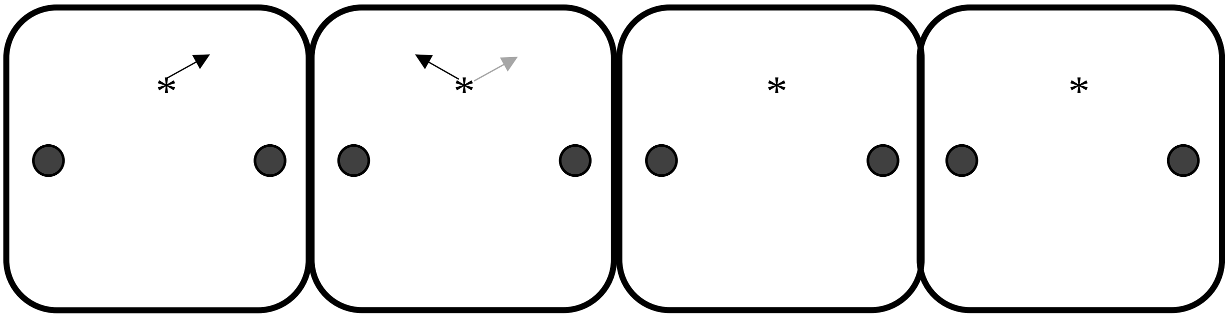


(i) Explain the process in the third diagram that allows us to find the net electric field vector, at this point.

(ii) In the first and second diagram, we can see the electric field vectors point in diagonal directions. However, the net vector in the third and fourth diagram is only horizontal. Explain why this is the case reference the horizontal and vertical vectors of the first two vectors.

(iii) If we measure the magnitude of the vectors in the first, second and third diagram, we will find the net electric field in the third diagram has a weaker magnitude than the sum of the vectors in the first and second diagram. Explain why this is the case, referencing the horizontal and vertical vector components of the first two vectors.

The negatively charged particle is replaced with a positively charged particle of equal magnitude. The first and second diagram represent the electric field of the positive charge on the left and right respectively.

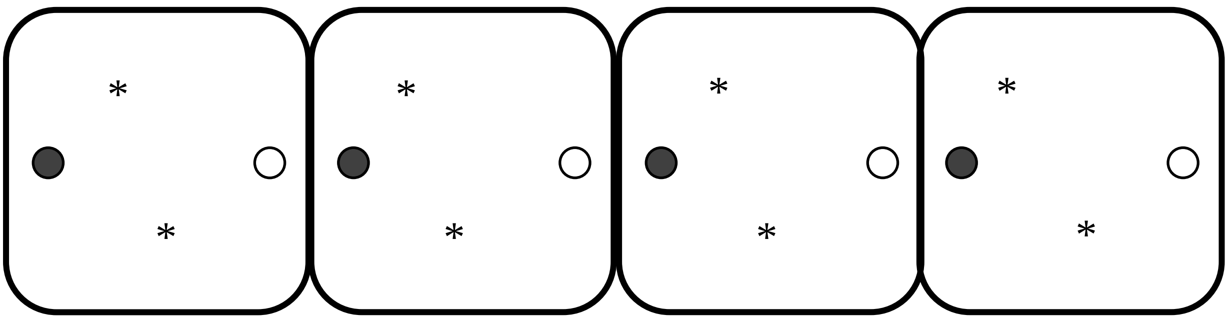


(iv) Draw the net electric field at this point in the third and fourth diagrams using the vector arrows presented in the first two diagrams, in the same manner as shown on the previous page.

(v) Is the net electric field you drew pointing in a horizontal, vertical or diagonal direction? Explain why it points in this direction, referencing the horizontal and vertical components of the first two vectors.

(vi) If we measure the magnitude of the vectors in the first, second and third diagram, we will find the net electric field in the third diagram has a weaker magnitude than the sum of the vectors in the first and second diagram. Explain why this is the case, referencing the horizontal and vertical vector components.

(vii) In the diagram below, use vectors to show the net electric field at the two points points shown, between a positive and negative charge. Show the initial component vectors and how you use them to construct the net electric field in the fourth diagram.



III. The forces experienced by a particle by an electric field – Field treatment.

When representing an electric field, it can be easier to use field lines. We have seen in mechanics that we can use field lines in replacement, or combined with vectors to show the direction of the force at a point, and the relative strength in a field.

The electric field always points in the direction that a small positive charge would feel a force. From this, we determine that an electric field line will always point away from a positively charged object, and towards a negatively charged object.

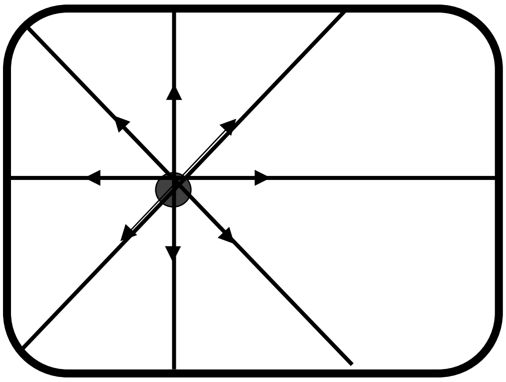
The rules you've already covered about field lines is as follows.

- The closer field lines are, the stronger the field.
- When a field line curves, the direction of the force is tangential to the field lines.
- The field line represents the direction of force acting on a body, no the path taken by a body in the field.

Other rules for using field lines are as follows:

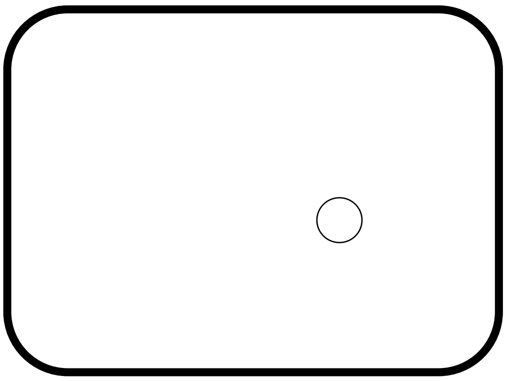
- Field line patterns do not overlap or intersect. Two bodies that cause a field both contribute to one overall field surrounding both objects.
- Field lines do not finish, or terminate. They should extend to infinity / off the page / to the end of the diagram boundary.

(i) Taking this into account, identify the charge on the following particle. Explain how you can tell.



(ii) Does the electric field strength increase, decrease or stay the same as you move away from the charge? How can you tell?

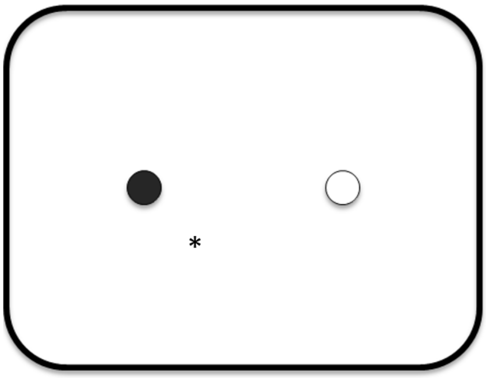
(iii) Draw the electric field for an oppositely charged particle, as shown to the right.



(iv) Explain the differences and similarities for the field for the two charges.

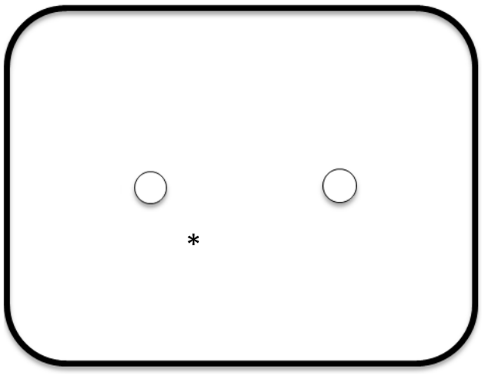
IV Electric field vectors – Principle of Superposition.

(i) Using the principle of superposition (how you determined the net vectors on pages 2 and 3) sketch the electric field between the positive and negative charge of equal magnitude. Ensure that you draw one field line going through the star.



(ii) If a positively charged particle were to be placed at rest at the star, sketch the path it would take. Assume the initial two charges do not move. You can represent the path taken in any manner you see fit (a bold line, a strobe diagram, vectors, etc). Explain why you drew the path as you did.

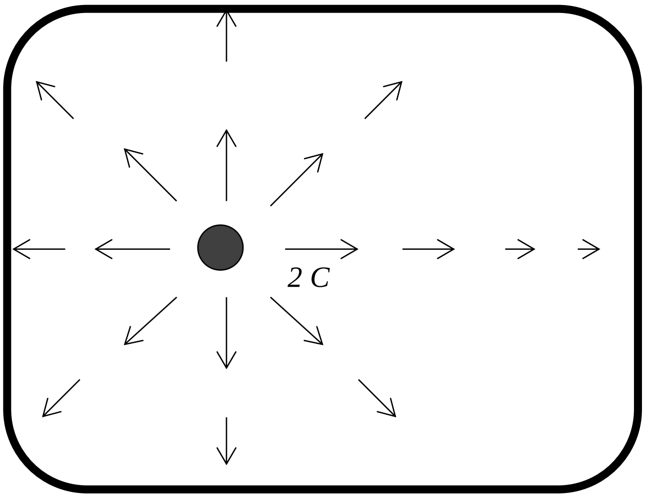
(iii) Using the principle of superposition (how you determined the net vectors on pages 2 and 3) sketch the electric field between two negative charges of equal magnitude. Ensure that you draw one field line going through the star.



(iv) If a negatively charged particle were to be placed at rest at the star, it would take. Assume the initial two charges do not move. You can represent the path taken in any manner you see fit (a bold line, a strobe diagram, vectors, etc). Explain why you drew the path as you did.

I. Electric field of a particle.

The electric field surrounding a *2 C* charge is shown in the diagram to the right, using vectors.

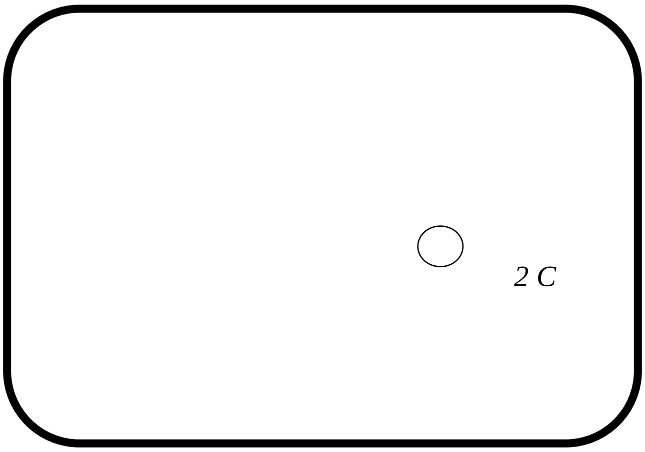


(i) What can you tell about the electric field strength as the distance from the charge increases. How can you tell?

(ii) By using the arrow directions, detemine whether the 2 C charge is positive or negative.

The charged particle is removed and an particle that **is oppositely charged** is placed down, as shown in the following diagram.

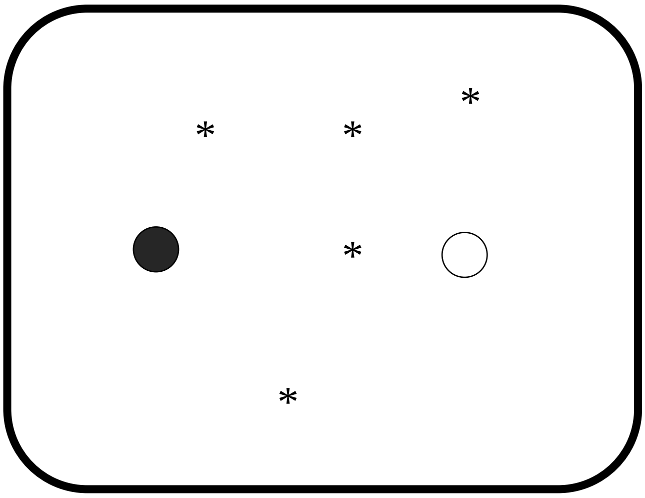
(iii) Sketch the electric field, ***using vector arrows***, the electric field around the new charge.



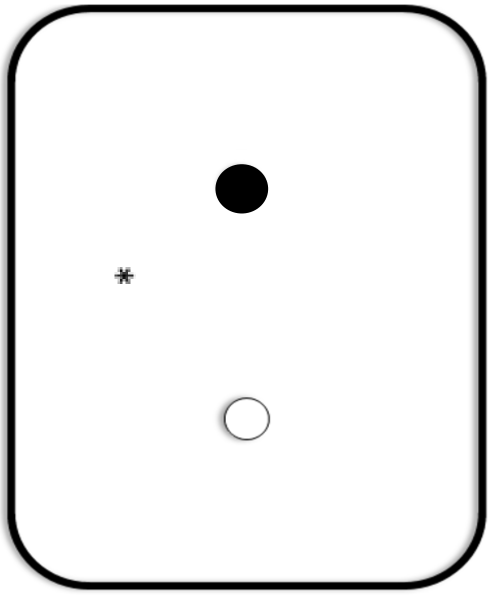
(iv) Explain how your arrows show the change in electric field strength, if any, as you move away from the electric charge.

(v) Explain why you drew your arrows pointing either ***towards or away from*** the negative charge.

(vi) ***Construct vector arrows*** at the points marked with stars to show the electric field around a positive and negative charge. (use the diagrams you did already in ***I(i)*** and ***II(i)*** to help you)



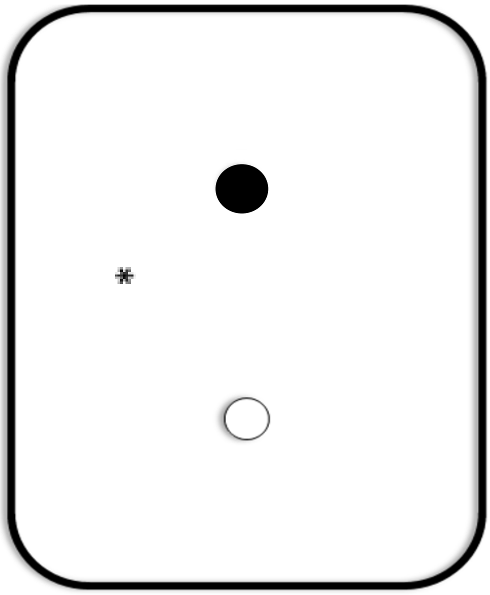
II. Path taken by a charged object in an electric field.



(i) In the above diagram, represent the electric field between the two charged particles, using any manner of your choosing. In this setup, the positively charged particle (black) has twice the magnitude of the negatively charged particle.

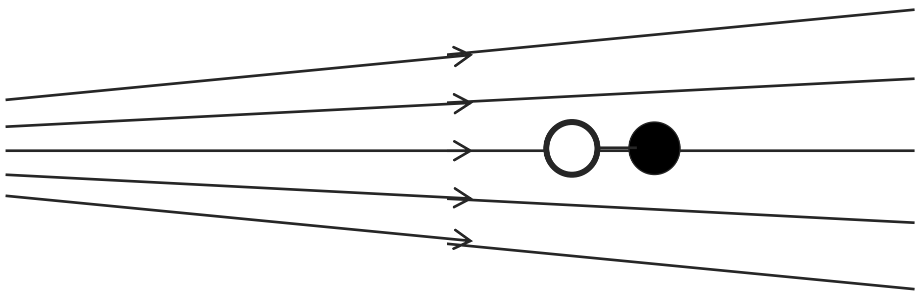
(ii) Draw in the path a positively charged particle would take, if it were placed at the star. Explain why you drew it as you did.

(iii) On the second diagram, draw in the path the particle would take if it has an initial velocity to the right, as shown in the diagram with the arrow. Explain why you drew as you did.



III. Behaviour of charges in an electric field.

An electric field is shown in the following diagram. Within this electric field, a negatively charged particle (white circle) is attached to a positively charged particle (black circle) so they cannot be separated. They are initially at rest.



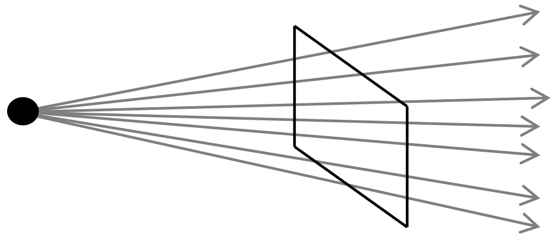
(i) As you go from left to right, does the electric field strength increase, decrease or stay constant? Explain how you determine this?

(ii) What direction will the force on the negatively charged particle act, caused by the electric field? Explain.

(iii) What direction will the force on the negatively charged particle act, caused by the electric field? Explain.

(iv) Using your answers from (i) to (iii), determine whether two charges will move to the left, right or will remain at rest. Justify the outcome you pick.

I. Investigating the variation of electric field strength with charge.



The above diagram uses lines to represent an electric field, of a *+1 C* charge, passing through a *1 m^2* frame. While not accurately sketched, assume there are 100 field lines coming from the charged particle passing through the frame. ***There should be more lines coming symmetrically from the charge in all directions, but for simplicity, we have not drawn on the diagram above.***

(i) If we doubled the magnitude of the charge (*+2 C*), we would double the amount of field lines that are coming from the charge. How many lines pass through the *1 m^2* frame?

(ii) If we had a charge of +4 C, how many lines would pass through the *1 m^2* frame?

(iii) What effect does increasing the charge generating a field have on the electric field strength at a point? Use (i) and (ii) to justify your answer (include the type of relationship observed).

(iv) If we used a *2 m^2*, frame, we would see 200 lines passing through it, as the lines are coming out of the charge symmetrically. Does using a bigger frame change the intensity of how many lines pass through a 1 m^2 frame? Explain your answer (may help to consider how many lines are in a *3 m^2*, *4 m^2*, etc frame)

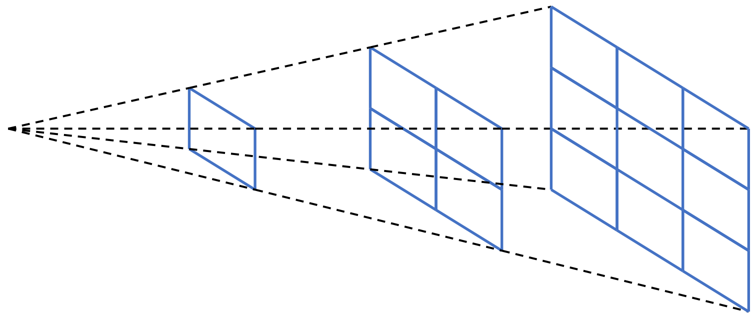
(v) Using your answer from section I (iv) and section I (iv), explain why changing the magnitude of the test charge used to measure electric field strength has no effect on the electric field strength at that point.

(v) If the +1 C charge was replaced with a – 1 C charge, what affect, if any, would you have to make to the field lines?

(vi) How would replacing the positive charge with a negative charge affect the number of field lines passing through *1 m²* frame?

(vii) Using your answer from (v) and (vi), explain why changing the sign of the test charge used to measure the electric field strength at a point has no effect on the electric field strength.

II. Investigating the variation of electric field strength with distance.



A second frame is placed away from the charge, so that all its outer corners are twice the distance from the charge This frame is made up of smaller frames that are identical to the frame in section II.

(viii) Explain why this second frame has an area that is four times bigger than the original frame used on the last page.

(ix) Determine the how many field lines through each of $1m^2$ frames, for this second overall frame.

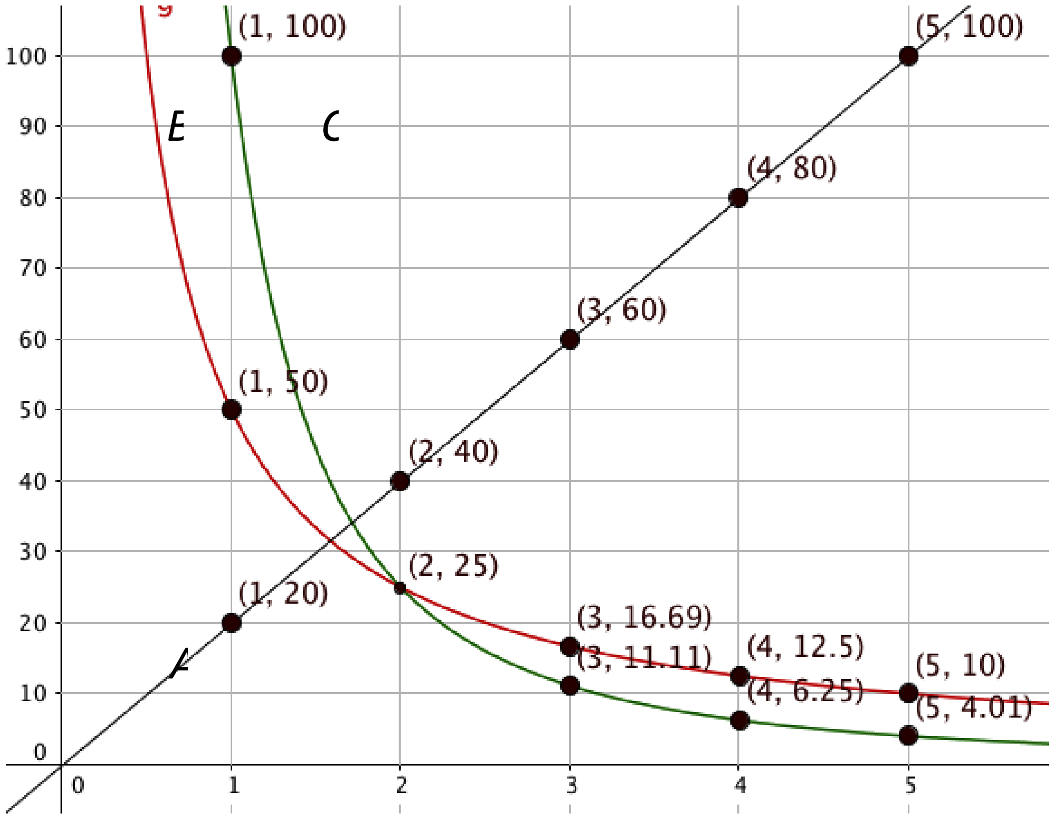
A third frame is placed away from the charge, so that all its outer corners are triple the distance from the charge. This frame is made up of smaller frames that are identical to the frame in section II.

(x) Explain why this third frame has an area that is nine times bigger than the original frame used on the last page.

(xi) Determine the how many field lines through each of $1m^2$ frames, for this third overall frame.

(xii) As the distance from the charge increases, the number of lines passing through a 1 m^2 area decreases. Using your answers from the previous questions, explain why this occurs. (Explain the relationship involved)

1. The following graph shows 3 patterns for different types of functions, *A*, *B* and *C*.



(i) Determine which of the three functions is of the form: $y = mx$. Justify your answer using whatever reasoning you wish.

(ii) Determine which of the three functions is of the form: $y = k\frac{1}{x^2}$. Justify your answer using whatever reasoning you wish.

(iii) Which graph represents the relationship between the force (y-axis) and distance (x-axis) between two charged objects following Coulomb’s law?

(iv) Explanation:

(v) Which graph represents the relationship between the force (y-axis) and product of two charged objects (x-axis) following Coulomb’s laws?

(vi) Explanation:

2. A $+2\ \mu C$ charge and a $+3\ \mu C$ charge are held 20 cm away from each other. The force of repulsions between the two charges is 1.35 N (rounded to two decimal places). (remember, $1\ \mu C = 1 \times 10^{-6}\ C$)

(i) If you replaced the $+3\ \mu C$ charge with a $+\ 12\ \mu C$ charge, what would the magnitude of the force of repulsion be?

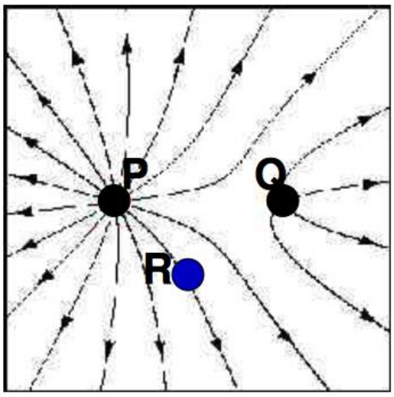
(ii) Use Coulomb's law (equation on last page) to calculate (to 2 decimal places)the force of repulsion between the $+2\ \mu C$ and $+\ 12\ \mu C$ charge at a distance of *20 cm* from eachother. Use your answer to verify your answer from (i) .

The original +2 μC charge and a +3 μC charge are held 20cm away from each-other again.

(iii) If the distance between the charges was increased to 40 cm away from each-other, what would the magnitude of the force of repulsion be?

(ii) Use Coulomb's law to calculate (to 2 decimal places)the force of repulsion between the $+2\ \mu C$ and $+\ 3\ \mu C$ charge at a distance of *40 cm* from each-other. Use your answer to verify your answer from (iii).

3.



Consider the electric field above, where P and Q are charged particles and R is a point in the electric field.

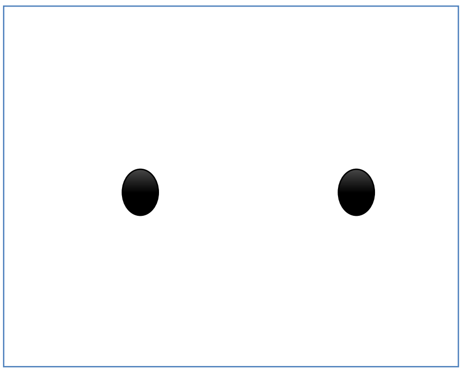
(i) Determine the charge on P and Q. Explain how you can tell.

(ii) Is the magnitude on the charge P greater than, equal to or less than the magnitude on the charge Q. Explain how you can tell.

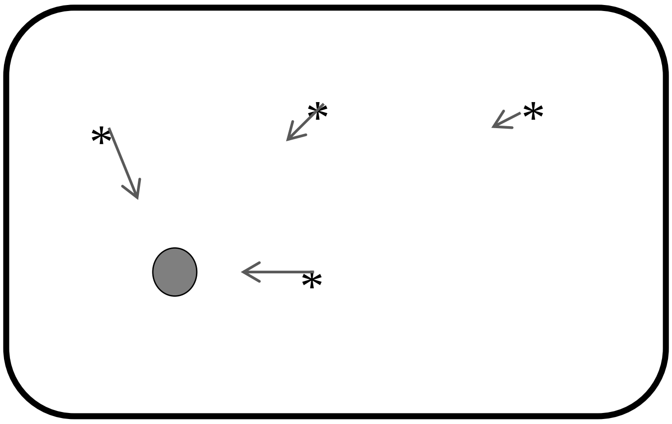
(iii) Place a finger on P and follow one of the field lines coming out of P. As you trace out the path, does the electric field strength increase, decrease of remain unchanged? Explain how you can tell.

(iv) If a negatively charged particle was placed at R, draw on the diagram the path you think the particle would take. Explain why you think it would take this path.

(v) Use vector arrows to above to represent the field shown above, at the following points.



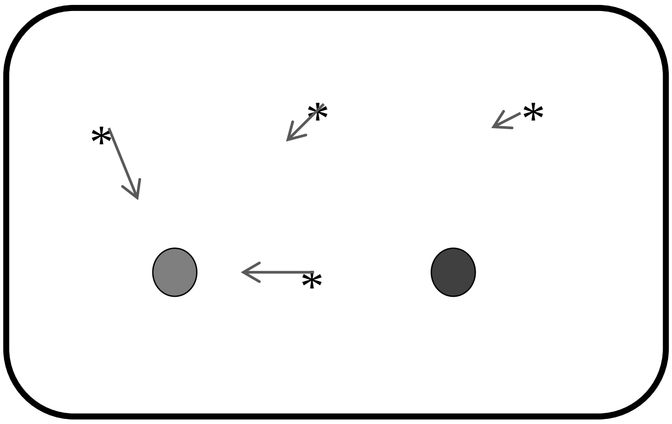
4. An electric charge is nailed down and the electric field at points around it is shown in the diagram, using vector arrows.



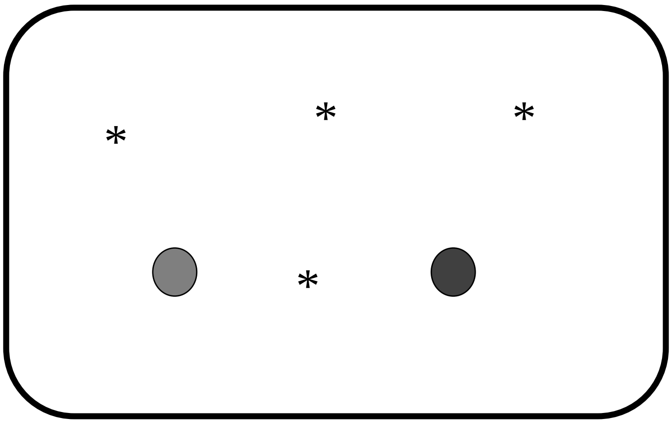
(i) What sign is the charge? Explain how you can tell.

(ii) Explain why the lengths of the vector arrows are not all the same.

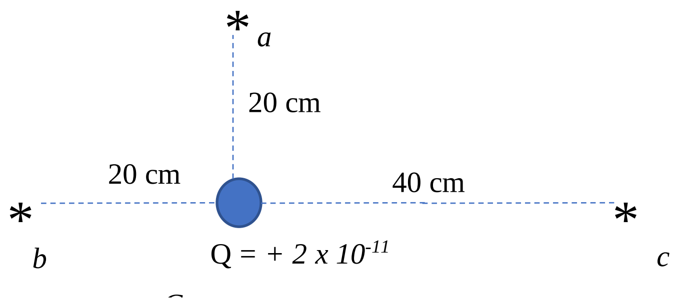
(iii) Another charge, of opposite sign and equal magnitude, is placed at the position shown in the following diagram. ***Construct*** the vectors that show the magnitudes and directions of the electric field of the two charges.



(iv) Use field lines to show the field produced by these two charges.



5. A positively charged object is placed down as shown above. 3 points are marked, a – c, are marked around the charge. The magnitude of the charge, and the distances to the points from the charge are shown on the diagram.

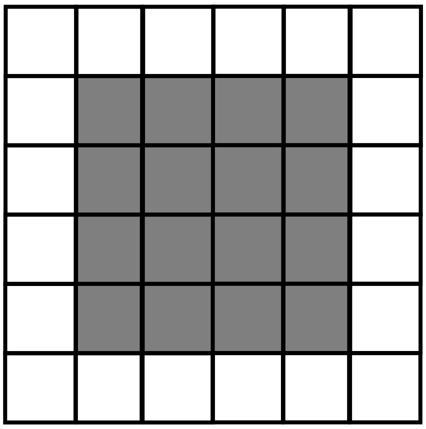


(i) Rank the strength of the electric field, *from lowest to highest*, at the positions *a*, *b* and *c*. Justify your ranking.

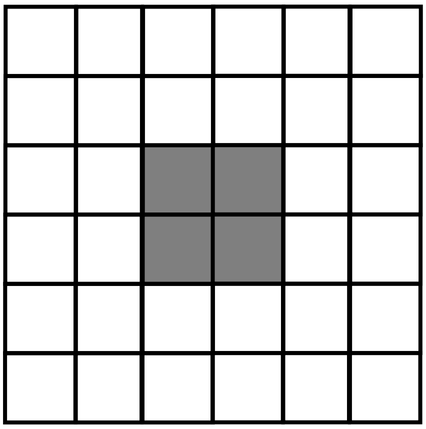
(ii) Show that the electric field at a point around an electric field is given by the formula $E = k\frac{Q}{d^2}$, where $k = \frac{1}{4\pi\varepsilon}$.

(iii) What is the ratio of the magnitude of the electric field at *c* to *a*? Show your workings.

6. A charge is placed down and a square shaped grid is held a distance of 10m from it. 100 electric field lines go through the shaded area of the grid.



(i) At what distance do I need to move the grid from the charge to get the 100 electric field lines to pass through the following shaded area. Explain your answer.



Distance to get 100 lines in this area:

Justification:

Formulae:

You may still want to have your mathematical tables handy, unless I missed something but these are the formulae you should need to complete this test.

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

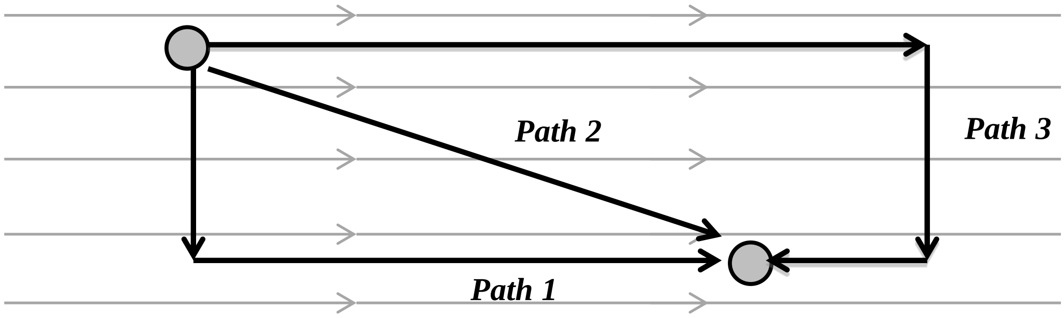
$$E = \frac{F}{q}$$

$$E = \frac{1}{4\pi\varepsilon}\frac{Q}{d^2}$$

Appendix E

Work and potential difference tutorial materials.

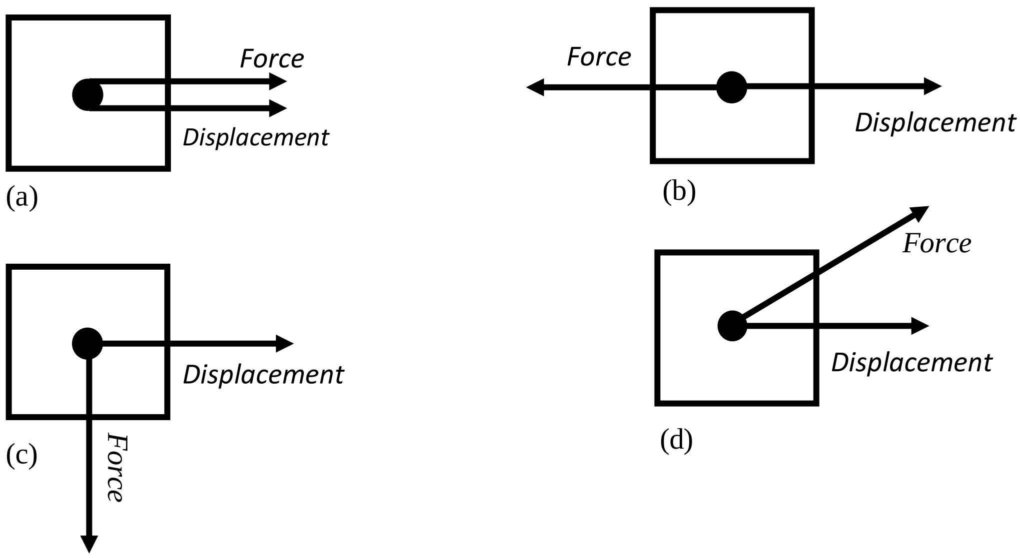
1. The electric field shown is uniform. A positively charged particle can be moved from point *a* to point *b* in one of the three paths as shown. Rank the net work done, by the field, in moving the charge from point *a* to point *b* in the different paths it can take. Justify your ranking.



Ranking:

Justification of ranking:

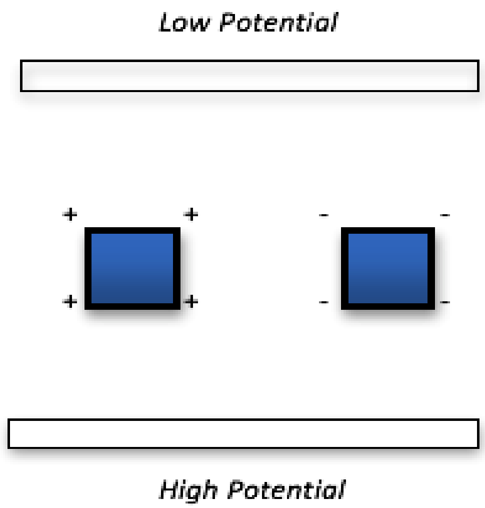
2. Rank the magnitude of the work done for the following pairs, (a) to (d), of Force – Displacement vectors.



Ranking:

Justification for ranking:

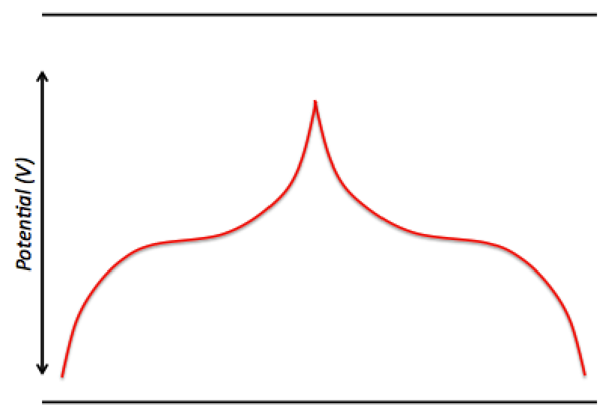
3. A positively charged box and a negatively charged box are suspended between two charged plates, one which has high potential and the other has low potential.



(i) When the positively charged box is released, will it move (a) towards the high potential or (b) towards the low potential. Explain

(ii) When the negatively charged box is released, will it move (a) towards the high potential or (b) towards the low potential. Explain

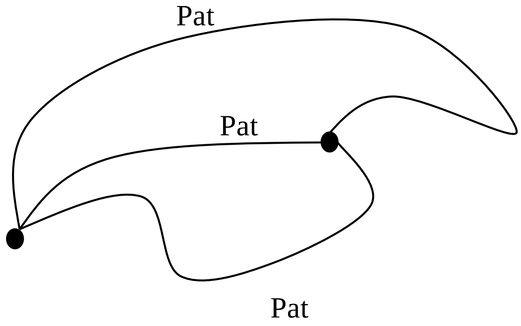
4. On the top line, draw the charges that need to be placed down to show the change in potential as you move from left to right.



Explain why you drew as you did:

I. Distance vs Displacement.

A person can move from point *A* to point *B* using one of the three paths shown.



(i) In which path, if any, is the distance travel greatest?

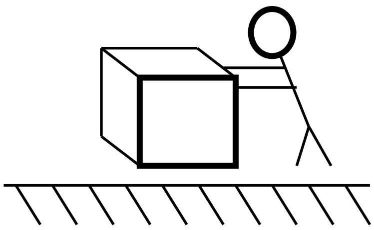
(ii) In which path, if any is the displacement greatest?

(iii) Explain why the distance travelled is not the same as the displacement.

II. Calculating work.

A person pushes a cube shaped block and ground as shown. The block weighs *100 N* and the person is pushes it with a force, F_{Push}, of *50 N*. The block moves a total displacement of *6 m*.

(i) On the diagram, draw vector arrows, to show the direction of the force and the direction of the displacement.



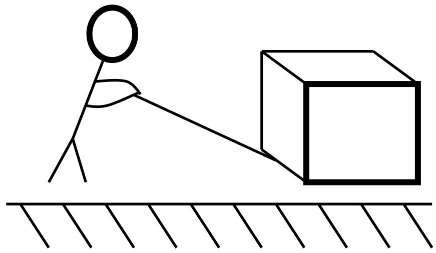
(ii) Calculate the work done by ther person in pushing the block.

(iii) What work is done, if any, on the block by gravity? Explain

III. Resolving a force vector into component vectors.

The person attaches a rope to the block to pull the block, to the left, with a force, F_{Pull}, of 50 N. The rope makes an angle, θ, of 30° with the ground. The block is pulled a displacement of *6 m,* to the left.

(i) On the diagram, draw vector arrows, to show the direction of the force and the direction of the displacement.



(ii) From your answers in (vi), is the work done in pulling the block *positive*, *negative* or *zero*? Explain.

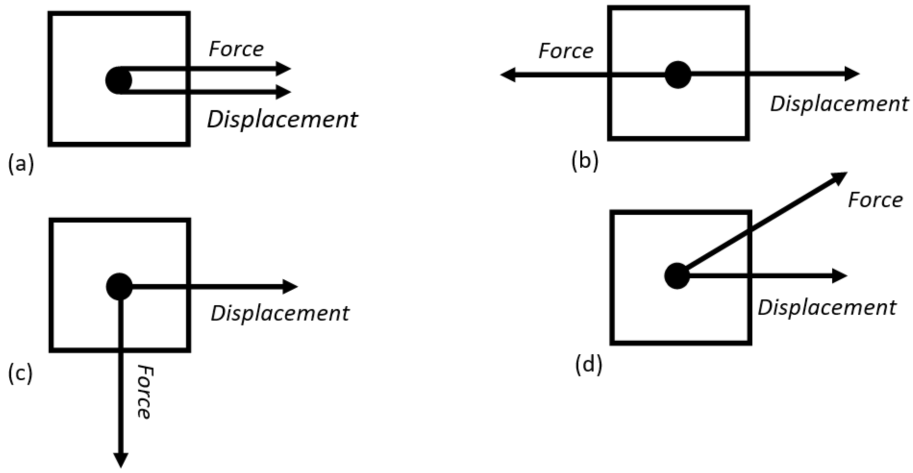
(iii) Copy the force vector into the box to the right. Resolve the vector into it is horizontal, F_x and vectical, F_y, components.

(iv) Which component, if any, contributes *zero* work to moving the block to the left.

(v) Show that the horizontal force moving the block can be given by $F_x = F_{Pull} Cos\theta$

(vii) From part (v), show by multiplying the horizontal force, F_x, component by the displacement, *s*, that the work done on the block is 260 J.

(viii) Rank the magnitude of the work done for the following pairs of Force – Displacement vectors. You may use a ruler to record the relative magnitudes, if necessary. If required, resolve the force vectors in the following into horizontal components.

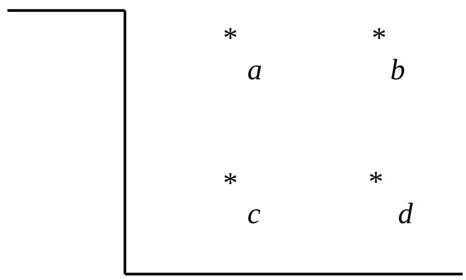


Ranking:

Justification:

III. Work done in a gravitational field.

An object, of mass 1kg, is moved between the points shown in the diagram.



(i) What is the direction of the gravitational force acting on the mass when it is held at *a, b, c* and *d?*

(ii) When the mass is moved from *a* to *c*, is the displacement *in the direction of, against the direction of* or *perpendicular* to the gravitational force?

(ii) Is the work done by the gravitational field, in moving the mass from *a* to *c*, *positive, negative* or *zero*. Explain.

(iii) When the *1 kg* mass is moved from *c* to *a*, does the potential energy of the mass increase, decrease or remain the same.

(iii) When the *1 kg* mass is moved released from *a* to fall to *c*, how does the kinetic energy at *c* compared to it is potential energy at *a*.

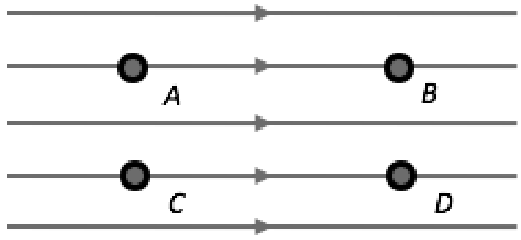
(iv) When the mass is moved from *d* to *b*, is the displacement *in the direction of, against the direction of* or *perpendicular* to the gravitational force?

(v) Is the work done by the gravitational field, in moving the mass from *d* to *b*, *positive, negative* or *zero*. Explain.

(vi) Is the work done by the gravitational field, in moving the mass from *a* to *b*, *positive, negative* or *zero*. Explain.

I. Work done by a field on a charge.

A positive charge ($+Q$) is placed in a uniform electric field, and is moved in the following paths.



(i) In which paths is the net work done on the charge, by the field, positive, negative or zero. Explain you reasoning.

A to *B*

C to *B*:

D to *C*:

A to *B* to *D* to *C*:

A to *C*:

A to *B* to *C* to *D:*

(ii) How does the <u>*net work done on the charge, by the field*</u>, compare when it moves from *A* to *B* as when it moves from *C* to *D*? Explain.

(Iii) How does the <u>*net work done on the charge, by the field*</u>, compare when it moves from *A* to *D* as when it moves from *A to C to D*? Explain.

Two people are considering what occurs when the charge in moved through the two paths outlined. Their understanding is shown below.

Person 1: *When we move the charge from A to D directly, there is less work done than moving it from A to C to D as we add up the work done moving from A to C directly to the work moving from C to D directly.*

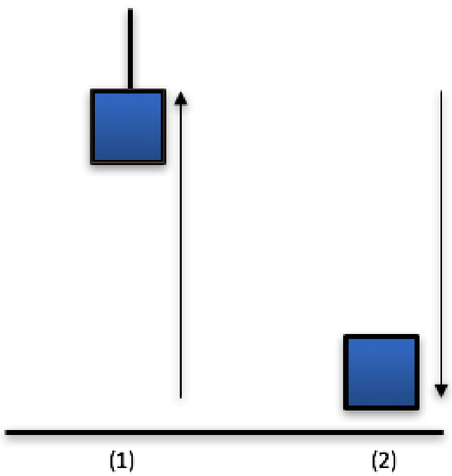
Person 2: *When the charge if brought from A to C and C to D, the displacement has a vertical component which gives zero work. This makes the work done independent of the path taken.*

(vi) Which person do you agree with?

What error in understanding did the other person make?

II. Work done on mass in gravitational field.

A 1 kg box is lifted a distance of 3 m into the air, as seen in (1). The box is then released so it falls to the ground (2). (W = Fs)



(i) Calculate the work done, by gravity, when the 1 kg box falls to to the ground.

(ii) Calculate the work done, by gravity, if the box had a mass of (a) 2 *kg* and (b) 3 *kg*.

(iii) Find the ratio (fraction) of *work done* per *mass of the box* for your answers in (i) and (ii), and write them in their simplest form.

(iv) What do you notice about all of the ratios?

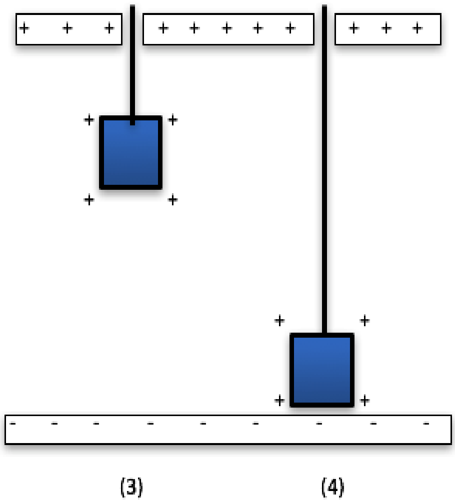
(v) How does the potential energy of the box in (1) compare to the kinetic energy of the ball just before it hits the ground in (2).

(vi) Express your answer from (v) mathematically.

*****Checkpoint****

III. Work done in a charge moving between plates.

A $+1\ C$ box is lifted a distance of $3\ m$ towards a positive plate (3) and then released towards a negative plate (4). The electric field between the two plates is uniform and has a magnitude of $2\ N\ C^{-1}$. ($W = Fs$, $F = qE$)



(i) Calculate the (a) force and then (b) the work done, by the electric field, when the $+1\ C$ box is brought to the negative plate.

(ii) Calculate the work done, by gravity, if the box had a charge of (a) $+2\ C$ and (b) $+3\ C$.

(iii) Find the ratio (fraction) of work done per mass for your answers in (i) and (ii).

(iv) What do you notice about all of the ratios?

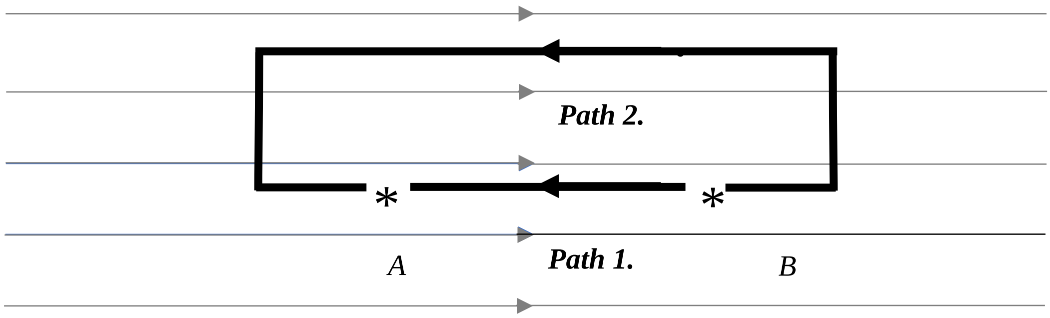
(v) How would you define this ratio, in your own words?

*****Checkpoint*****

In electricity, there is a special name given to the ratio you just calculated. This is called **potential difference (V),** and is defined as *the work done in moving a unit of charge (+1 C) between two points in an electric field or electric circuit.*

$$V = \frac{W}{q}$$

A uniform electric field is shown below. Two points, A and B, are highlighted. There are two paths between the points, also shown on the diagram.



(i) Explain how you can tell it is a uniform electric field.

(ii) It takes *6 J* of energy to move a -1 C charge from *B* to *A*, along *Path 1*. What is the potential difference between *A* and *B*?

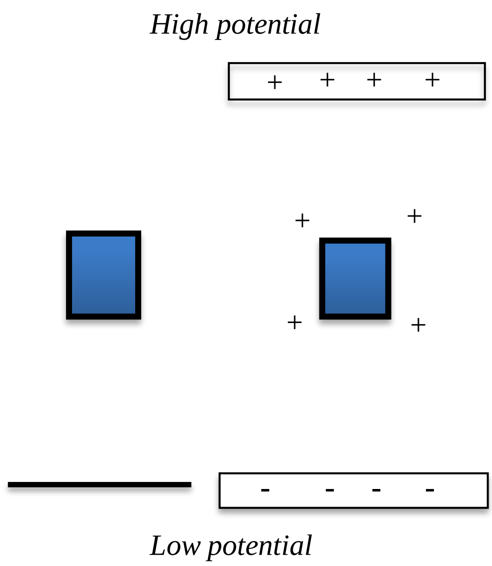
(iii) What is the work done in moving the *-1 C* charge from *B* to *A*, along *Path 2*? Explain your answer.

(iv) How much energy would it take to move a *-4 C* charge?

(v) If I use 36 J moving a charge from *A* to *B*, what is the magnitude of the charge moved?

(vi) If a charged particle of magnitude -*3 C* has a mass of *0.5 g*, determine the magnitude of the velocity it would have at *A* when it travels along *Path 1* from *B to A*. ($W = {}^{1}/_{2}\,mv^2, V = \frac{W}{Q}$)

I. Positive and negative charges in a potential difference.



(1) (2)

A box held is held a height above the ground, as seen in (1).

(i) As it is height above ground increases, what change is seen in it is gravitational potential energy?

(ii) When it is released, will the box move up, fall down or remain at it is height?

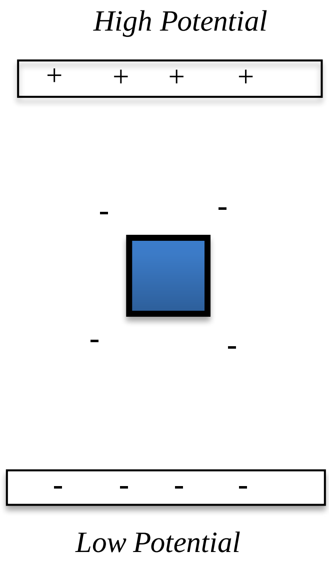
(iiI) From your answer in (i) and (ii), does the box move from (a) low to high potential (b) high to low potential or (c) is not affected. Explain.

A positively charged box is held between a postively charged plate and a negatively charged plate. We associate a high potential with positively charged plate and a low potential with the negatively charge plate.

(iv) When the box is released, will it move towards the positively charged plate, or the negatively charged plate. Explain your answer, ***referring to either the forces involved or the electric field between the plates.***

(v) In terms of potential, is the positively charged box moving from an area of high to low potential or low to high potential? Explain.

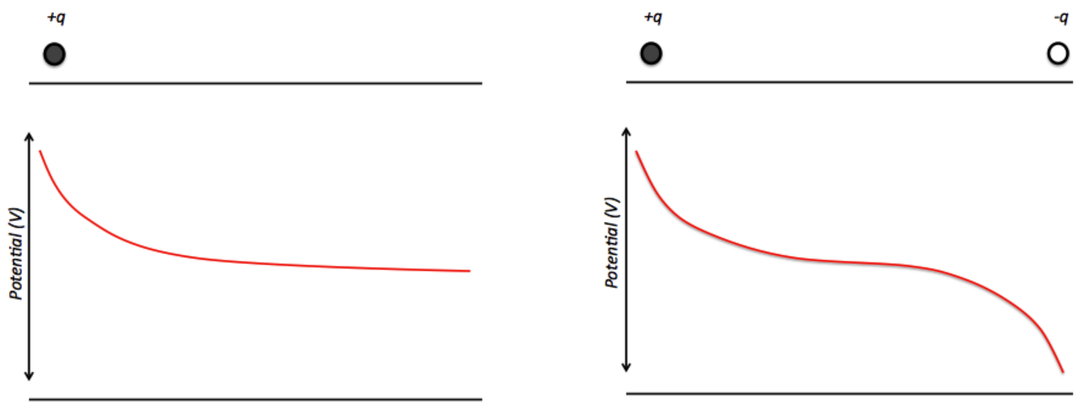
(vi) If the box was replaced with a negatively charged box, which way would it move?



Explanation (refer to potential):

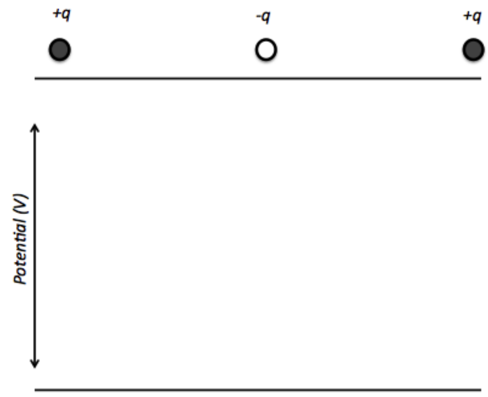
II. Graphing potential.

We have seen that regions of positive charge are considered to have high potential and areas of negative charge are areas of low potential. Consider the following graphs of potential for the setups shown, as you moves from left to right in each.

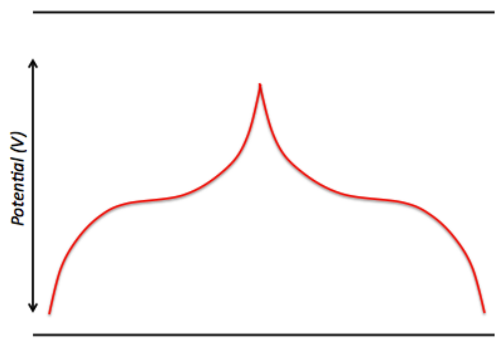


(a) *(b)*

(i) Explain the shape of both graphs as you move from left to right.



(c)



(d)

(ii) Draw on the graph how the potential varies from going from left to right for the setup shown in (c). Explain why you drew it as you did.

(iii) Draw the setup that produces the graph for potential as seen in (d). Explain why you drew the setup as you did.

III. Understanding a battery.

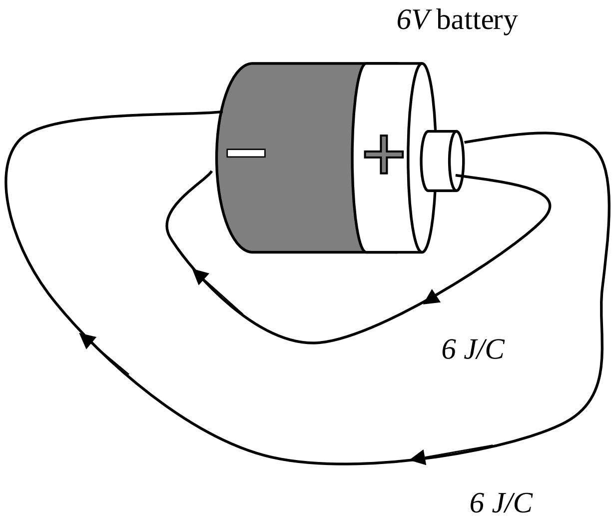
In a battery, there is a positive and a negative terminal. The potential difference that exists between these is usually printed on the battery (typically *1.5 V*, *6 V*, *9 V* and *12 V*). Use your understanding of some or all of the following:

- work,
- potential,
- electric fields,
- electric field lines,
- vectors,
- the behaviour of negative charges in electric fields,

- the behaviour of negative charges between a potential difference

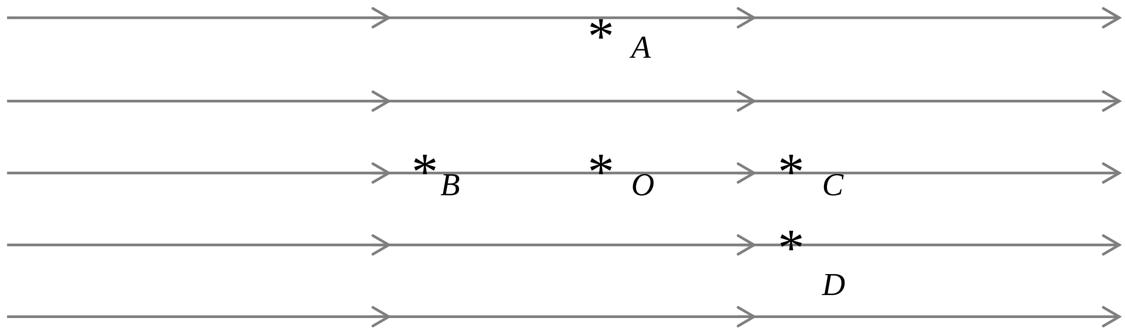
to explain, In the space below, explain why:

3. Current (which is moving negative charge) flows from the negative to the positive terminal.
4. The *work done per charge* in moving current from one terminal to the other is constant, regardless of the length / layout of the wire.



Relevant formulae: $E = \frac{F}{Q}$ $\quad W = F.s \quad$ $V = \frac{W}{Q}$

1. A ***negative charge*** is placed in an electric field as shown, at position *O*. It can be moved to any of the positions marked on the field as shown.



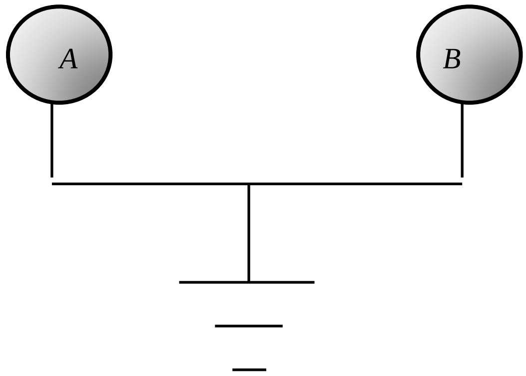
(i) Draw in the direction of the force acting on the negative charge, when placed at *O*.

(ii) State whether the work done in each case is *positive, negative or zero,* when ***the negative charge*** is moved from *O* to *A*, from *O* to *B,* from *O* to *C* and from *O* to *D*. Explain.

(iii) How does the work done, by the field, in moving the charge from "*O* to *C"* compare moving from "*O* to *D"* and "*O* to *C* to *D*?" Explain.

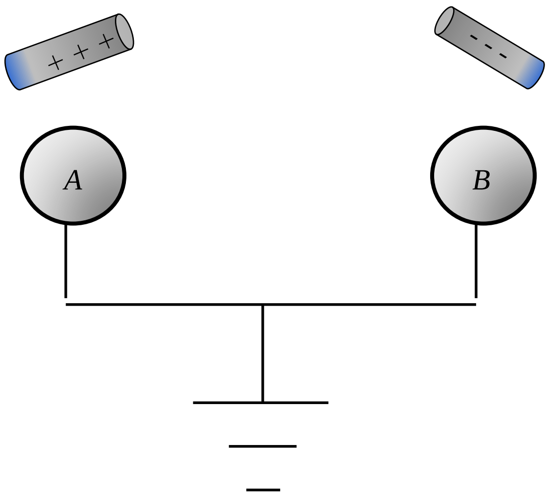
(iv) A student says that the potential difference between the B and O, is the same as the potential difference between O and C. Do you agree with this student? Explain why you think they are correct / incorrect.

2. Two conducting balls are connected to the ground as shown in the diagram.



(i) How does the potential at *A* and *B* compare to each-other and the ground. Explain.

A positively charged rod is held about *A* and a negatively charged rod is held above *B*.



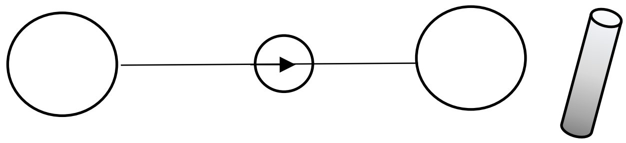
(ii) What effect does this have on the potential of *A*, initially.

(iii) After some time passes, what type of charge will build up on *A*. Explain, **referencing the potential** on *A* you gave in (ii).

(v) What effect does this have on the potential of *B*, initially.

(vi) After some time passes, what type of charge builds up on *B*. Explain, **referencing the potential** on *B* you gave in (iv).

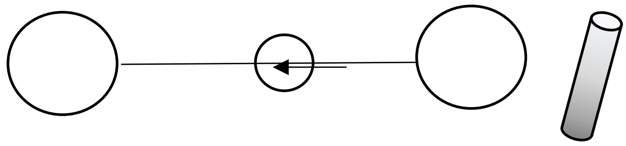
3. Two metal domes are connect with a conducting wire. A device with shows the direction of the movement of charge is placed on this wire.



When a charged rod is placed beside the rightmost dome, the negative charge begins to move, in the initial moments, in the direction as shown on the device.

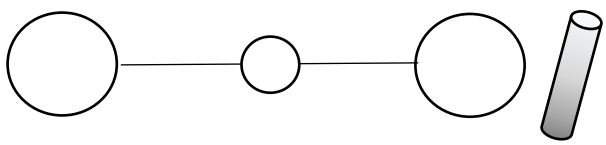
(i) Using this information, compare the potential on the left dome to the potential on the right dome. Explain your comparison.

The rod is taken away replaced with a rod that causes the negative charge to move, in the initial moments, in the direction as shown on the device.



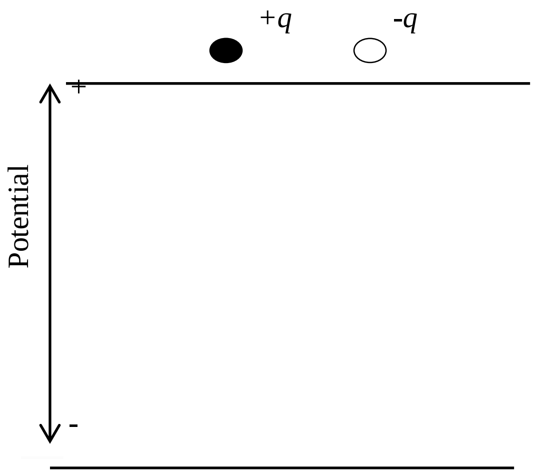
(ii) Using this information, compare the potential on the left dome to the potential on the right dome. Explain your comparison.

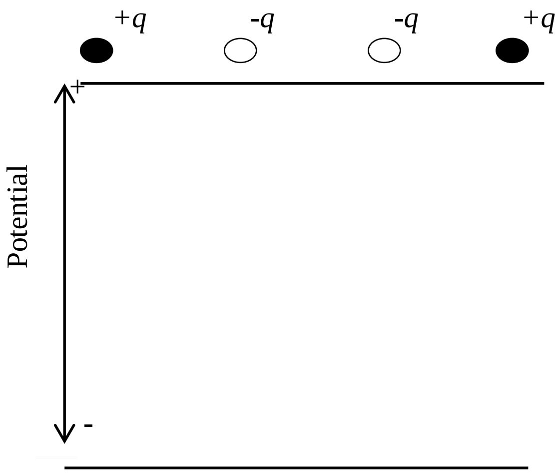
After a long period of time has past, the device registers that there is no electric charges moving, as shown in the diagram.



(iii) What does this tell you about the potential of both domes? Explain.

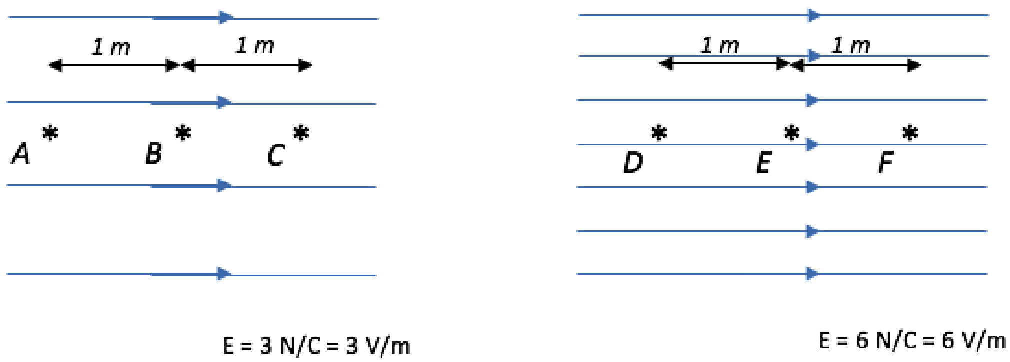
4. Draw on the graph how the potential varies from going from left to right for the setups shown. Explain why you drew it as you did.





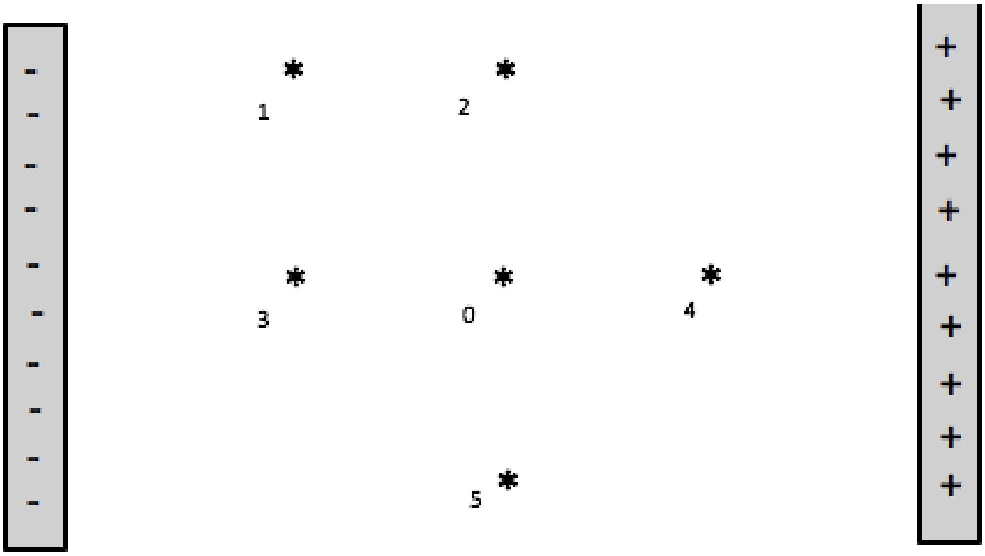
Explanation:

5. Two electric fields are shown in the diagram below. Both fields uniform. The field strength and distance between the points is shown on the diagram.



(i) Rank the Potential difference, from highest to lowest, between AB, AC, DE, and DF. Justify your answer using whatever reasoning you think is necessary (there are multiple ways to justify your answers correctly – you need only use one. If you choose to try use calculations, use a test charge of magnitude +1 C)

6. A positively and negative charged plate at held across from each-other. A uniform electric field exists between these plates. A number of positions are shown between them.



(i) If a positive charge is laid down at position 0, rank the magnitude, from lowest to highest, as it is from along the following paths:

[01], [02], [03], [04] and [05].

[01] means the particle starts at 0 and moves directly to 1. You may draw on the diagram in any manner you see fit.

Ranking:

Justification:

(ii) Rank the potential difference, from lowest to highest, between the points [01], [02], [03], [04] and [05], where [01] means the potential different between the point 0 and the point 1.

Ranking:

Justification:

Appendix F

Legend for codes used in line plots to display extent of conceptual change.

Code.	Learning outcome	Figure.
V 1	Ranking vectors based on magnitude.	4.47
V 2	Use of vector constructions.	4.47
V 3	Consideration of vector components in vector addition	4.47
In 1	Graphically representing inverse square relationship.	4.47
In 2	Reasoning for change in area using scale model.	4.47
In 3	Use of inverse square relationship mathematically.	4.47
Fl 1	Using field line density to determine relative field strength.	4.47
Fl 2	Drawing vectors are points based on field line diagram.	4.47
Fl 3	Reasonable sketches of trajectories of bodies under influence of a field.	4.47
V 4	Variation of electric field strength represented with vectors	5.43
V 5	Use of superposition principal using vectors and electric fields.	5.43
In 4	Use of inverse square relationship mathematically in electric fields.	5.43
Fl 4	Direction of force on negative charge in electric field.	5.43
Fl 5	Reasonable sketches of trajectories of charged bodies under influence of an electric field.	5.43
Fl 6	Using field line density to determine relative field strength.	5.43
W 1	Identification of positive, negative and zero work.	6.25
W 2	Application of work and displacement to electric fields.	6.25
PD 1	Association of relative high and low potential to areas surrounding positive and negative charges.	6.25
PD 2	The movement of charge under the influence of a potential difference.	6.25

Developing and assessing student’s conceptual understanding of electrostatics in upper secondary Physics

Richard Moynihan, B.Sc. (Hons)

This thesis is submitted to Dublin City University for the degree of Doctor of Philosophy

School of Physical Sciences
Dublin City University

July 2018

Supervised by Dr. Eilish McLoughlin, Dr. Paul van Kampen and Dr. Odilla E. Finlayson.

Declaration

I hereby certify that this material which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work, that I have exercised reasonable care to ensure the work is original and does not to the best of my knowledge breach any law of copyright, and has not been taken from the work of others, save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed:

ID No.: 54312678

Date:

Contents

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I would like to sincerely express my gratitude to CASTeL for providing the funding for this research. The opportunity to partake in post-graduate study at relatively little cost is a privilege not afforded to many and for this, you have my thanks.

To my supervisors, Eilish and Paul. You have both given insight and guidance that has helped me develop as a researcher, as you did when I was an undergraduate student a decade ago. While at times it was frustrating when feedback came in the form of questions, it ultimately made me a better researcher and encouraged me to learn more. The advances I made in the later stages of this research were the fruits of you both always encouraging me to push myself as a researcher.

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Publications.

1. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. (2016) *Developing student understanding of attraction between charged and uncharged objects in a lower secondary setting.* School Science Review, No. 363, pg 101-108.

2. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. (2016) *Helping students explore concepts related to the electric field at upper secondary level science education.* Proceedings of GIREP-EPEC 2015, pg 195 – 201.

3. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. (2017) *Developing second level student's understanding of the inverse square law, and electric fields.* Proceedings of GIREP-ICPE-EPEC 2017. (Currently under review).

Conference presentations.

1. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *The application of tutorial based worksheets to enhances student understanding of static electricity and magnetism at lower second level education.* SMEC, 2014.

2. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *Helping students explore concepts related to the electric field at upper secondary level science education.* GIREP-EPEC, 2015.

3. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *Progress and difficulties in student's understanding of vector and field concepts in electrostatics: A qualitative study of a small group of upper secondary students*. SMEC, 2016.

4. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *Developing second level student's understanding of the inverse square law, and electric fields.* GIREP-ICPE-EPEC, 2017.

Abstract

Richard Moynihan - 54312678

Developing and assessing student's conceptual understanding of electrostatics in upper secondary Physics

This thesis presents research studies carried out with upper secondary level physics students (n=14) over a timeframe of four months, with the aim of promoting their conceptual understanding of the electrostatic concepts, Coulomb's law, electric fields, and work and potential difference. Teaching and learning materials were developed that adopted Inquiry Based Learning (IBL) with structured inquiry tutorials and Multiple Representations (MR) approaches to examine the extent of conceptual change in the student's understanding of these topics. The research utilizes a case study methodology, with various sources of evidence recorded. These sources include pre-test and post-test comparison, retrieval of student artefacts used during lessons, audio recordings of student's discussions during lessons, teaching and learning interviews and teacher field notes.

In the research design, vector concepts, the inverse square law and field line representations were identified as concepts that students need to be familiar with, to develop their understanding of electrostatics. The students completed structured inquiry tutorials for these topics in the context of mechanics. The findings of these studies show that conceptual change generally occurred in the student's understanding, for these three concepts, with evidence of conceptual extinction and extension occurring. However, the findings of these studies show students encountered difficulties in transferring their understanding of vectors, the inverse square law and field lines to the electrostatic context. By revisiting these concepts during the electrostatic tutorial lessons, evidence of conceptual exchange and extension was observed in the student's understanding of Coulomb's law, the electric field and work and potential difference.

The research presents evidence that the use of structured inquiry tutorials and multiple representations can be an effective approach in promoting conceptual change in student's understanding of electrostatics in upper secondary physics. The overall findings of this research suggest that this approach may have significant benefit for the teaching and learning of other physics / science topics at both upper secondary and lower secondary levels.

Chapter 1. Introduction

As a branch of science, physics is a human endeavour to study the nature of energy, matter and their interactions. It allows for the construction of models, theories and laws that explain observable phenomena on scales as small as the sub-atomic to as large as galaxies and beyond. Physics helps generate fundamental knowledge needed for technological progression that can directly be used to develop new products or influence the progression of other disciplines. In Ireland, physics and physic-trained people underpin a range of sectors from medical technologies to ICT and web-services (Institute of physics, 2012). There are many branches of physics; the ones encountered by upper secondary school students in Ireland are optics, thermodynamics, mechanics, electricity and electromagnetism with introductions to quantum and particle physics. These branches of physics can give students a fundamental foundation that they can use as a platform for further study in more advanced fields of science, technology, engineering and mathematical fields.

Electrostatics concepts are important domains of science teaching that deserve attention. In Ireland, the Chief Examiner’s Reports (2013; 2010; 2009; 2008; 2005a; 2005b) for physics at senior level and science at junior level show that students have above-average difficulties with electrostatics. If possible, they tend to avoid questions relating to electricity, as seen in the reports. As the domains of static and current electricity are intrinsically linked, the teaching and learning materials developed in this thesis aim to develop student’s understanding of electrostatics concepts. The expected outcome of this approach is that students can transfer their understanding of these concepts to concepts in current electricity, such as the behaviour of current, potential-difference and resistance in circuits. This can aid the development of the student’s understanding of these domains, instead of treating electrostatics and current electricity as two separate and exclusive phenomena.

The use of multiple representations of these physics concepts has been recognised as beneficial to learners. Hestenes (1996) explains that a complete understanding of a model requires a student to be able to coordinate information between multiple representations to complete their understanding. Although this research does not employ modelling methodology explicitly, the use of a structured inquiry approach is used to support student’s development of models. The utilisation of multiple representations in this work aims to develop similar gains in the student’s understanding of electrostatics to those that a modelling methodology would aim to produce. Jackson, *et al.*, (2008) note that the use of a modelling method involves students developing conceptual understanding through graphical and diagrammatic representations, before moving onto the algebraic treatment of problem-solving. In the design of the teaching and learning materials in this research, this instruction method is employed in numerous lessons, to aid student’s conceptual development.

Using multiple representations in developing education materials can help achieve other goals. Student difficulties that may not be apparent in one representation may easily be assessed in others. McDermott, *et al*., (1986) showed students can have a relatively good grasp of concepts, but numerous issues of interpretation can cause confusion when interpreting graphs. This inability to successfully translate between representations can hinder student's ability to completely understand a concept. Kozma (2003) notes that that experts are fluid in their transitions between different representations when problem solving, while novices typically use one or two. Students focus on surface features of the concept and don't develop understanding at a deep level. This suggests that helping students translate between representations could enable them to develop expert–like understanding, and tackle problems using multiple methods.

1.1. Context and background

This thesis concerns the teaching of electrostatics in the Irish secondary system. Irish secondary education comprises a Junior Certificate and a Leaving Certificate programme. The Junior Certificate programme is a three-year programme, in which students typically aged 12-15 take ten subjects, including Science. In some schools, Science is mandatory, in others it is optional. At the time this research was completed, Junior Certificate students completed the 2003 science syllabus (NCCA, 2003). The Leaving Certificate programme is a two-year programme, in which students typically aged between 16-18 take seven subjects. Leaving Certificate Physics is optional to all students who undertake the programme, subject to availability of facilities and suitably qualified teachers. Leaving Certificate Physics follows the 1999 syllabus (NCCA, 1999), which is briefly discussed in this section. The physics course is set at two levels; ordinary level and higher level. The difference between these two levels is the following:

- Ordinary level consists of a defined set of concepts. Higher level consists of the ordinary level concepts, additional concepts and a particle physics or applied electricity optional section.
- Ordinary level provides an overview of physics and its applications, while at higher level, there is a deeper, more quantitative treatment of physics.
- At ordinary level, equations must be used, while at higher level, some equations must be used and derived. Calculations for higher level are more challenging that those found at ordinary level.

(NCCA, 1999)

As electrostatics concepts are the focus of this research, Table 1.1 provides an extract from the physics syllabus and a discussion of the extract and a commentary of an examination question, shown in Figure 1.1, from the state exams commission (SEC, 2015) is presented.

In recent years, the teaching of Science and Physics in Ireland has undergone changes in both classroom syllabus and examination style. First examined in 2002, the Leaving Certificate Physics Syllabus, reduced the content to be covered in mechanics and electrostatics. This was to accommodate for the introduction of a new section on modern physics. The Junior Certificate Science syllabus (NCCA, 2003), first examined in 2006, also reduced the amount of content to be covered, to allow time for an investigative approach in the delivery of the syllabus. This was justified under the rationale that it allowed for the allocation of class time to allow for an investigative method of learning, but it has been seen that typically, traditional methods of teaching still prevailed in classrooms (Wemyss, 2009).

In developing teaching and learning materials for this research, the learning outcomes for the students, as listed in the Leaving Certificate Physics syllabus (NCCA, 1999) were used to consider what concepts should be addressed. The syllabus lists the topics to be completed in the Leaving Certificate Physics course, the depth of treatment the topics are to be taught to, and suggested activities and examples of science and technology that are appropriate for the topics. There is no prescribed methodology suggested for the teacher to use in completing the syllabus. For instance, there is no indication of suggested explanatory models to understand the concepts underpinning Coulomb's law and the electric field. The Leaving Certificate Physics syllabus is designed to be completed over two academic years, in approximately 180 hours of class contact time. This contact time generally breaks down to just under three hours a week to include laboratory work, teaching and learning of content and concepts and practice solving numerical problems in physics. The syllabus is assessed in a three-hour written examination, which is drafted by the State Examinations Commission and is corrected anonymously. The Physics examination papers give little weight to student's conceptual understanding of the content covered. Typically, students are given mathematical questions which are typically solved using algorithmic problem-solving methods. Any questions that are conceptual tend to revolve around the direction of an electric field in a given setup, with one or two charges, or explaining how Coulomb's law is an example of an inverse square relationship. Table 1.1 presents an extract of the electric field section of the Leaving Certificate Physics syllabus. Items denoted in bold indicate that they are part of the higher-level physics course only.

Topic	Depth of treatment	Suggested activities	Science, Technology and Society
Static Electricity			
1. Force between charges.	Coulomb's law - $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$ - As an example of an inverse square law. **Forces between collinear charges.**	**Appropriate calculations.**	
2. Electric fields.	Idea of lines of force. Vector nature of electric field to be stressed. **Definition of electric field strength.**	Demonstration of field patterns, using oil and semolina, or other method. **Appropriate calculations – collinear charges only.**	Precipitators Xerography Hazards: effect of electric fields on integrated circuits.
3. Potential difference	Definition of potential difference: work done per unit charge to move a charge from one point to another. Definition of the volt. Concept of zero potential.	Appropriate calculations.	

Table 1.1 Extract from Leaving Certificate Physics syllabus (NCCA, 1999) detailing static electric learning outcomes.

When considering student understanding of potential and potential difference, it is defined in the syllabus as work done when moving a unit of charge from one place to another in an electric field. However, it has been shown that students at third level have difficulties in the understanding of concept related to work (Doughty, 2013). This has also been seen to be a topic of difficulty to students exploring potential difference (Hazelton, 2012), so it is not unreasonable to speculate that some of the difficulties could be allayed by helping students to develop a conceptually sound understanding of work and potential at second level. Figure 1.1 presents an extract from the 2015 Leaving Certificate Physics examination paper, which illustrates a number of these points.

Define electric field strength. (6)

Both Van de Graaff generators and gold leaf electroscopes are used to investigate static electricity in the laboratory.

Draw a labelled diagram of a gold leaf electroscope.
Describe how it can be given a negative charge by induction. (20)

A Van de Graaff generator can be used to demonstrate point discharge.

Explain, with the aid of a labelled diagram, how point discharge occurs.
Describe an experiment to demonstrate point discharge. (18)

The polished spherical dome of a Van de Graaff generator has a diameter of 40 cm and a charge of +3.8 μC.

What is the electric field strength at a point 4 cm from the surface of the dome? (12)

Figure 1.1. Electrostatics question from Leaving Certificate Physics examination, 2015. (SEC, 2015)

The initial question requires students to define electric field strength, in which they need to recall that it is the force experienced per unit charge in an electric field. Acceptable answers to this question were in written word, or mathematical notation in which the variables were explained. The following pair of questions required students to sketch a diagram of a gold leaf electroscope and describe how it can be given a negative charge by induction. While the latter question can be answered by understanding the principle involved in this process, both questions can be answered by recalling rote-learned material. This also applies to the following pair of questions which required the students to describe the process of point discharge and describe an experiment to demonstrate the process. The final question required students calculate the electric field strength at a point from the surface of the surface of the Van der Graaff generator. Relevant formulae for this question are provided to the students in the examination. The student must correctly determine the distance from the point to the centre of the Van der Graaff generator, and otherwise, substitute variables into the formulae and evaluate the expression on their calculators. A student who has spent time rote-learning the theory for the initial questions and practised the substitution and calculation process could score highly on this question, without demonstrating the depth of their conceptual understanding to the examiner marking the paper.

1.2. Research approach

In this research, the researcher is also the teacher that develops the teaching and learning materials used to promote the student's understanding of electrostatics. At the time the lessons in this thesis were implemented, I had been employed as a full time second level teacher for 8 years. In this

time, I gained experience in teaching multiple cohorts of students the Junior Certificate Science, Junior Certificate Mathematics and Leaving Certificate Physics syllabi. As a teacher, I developed classroom activities that in which the students would predict the outcome of observable phenomena, share reasons for their predictions, discuss alternative outcomes with their peers. When completing quantitative exercises, I would ask students to consider whether numerical answers appeared reasonable, based on their understanding of underpinning theory they were applying. As the second level syllabi were prescriptive in the material that is to be learned, opportunities to develop lessons in which the students were given a high degree of autonomy in their learning were rare. This research provided an opportunity to develop a sequence of such lessons and gather data on the student's understanding of electrostatics.

The aim of this research is to develop a suite of research-informed and research-validated educational materials to improve student learning of electrostatics concepts, by improving their conceptual understanding of Coulomb's law, the electric field, work and potential difference. The teaching and learning materials employed during the tutorial lessons in this research are, unless referenced otherwise, of my own original design with the primary inspiration for their design coming from *Tutorials in Introductory Physics* (McDermott, *et al.,* 2003) and *Conceptual Physics* (Hewitt, 2009) In this research, the students completed tutorial lessons in groups for three/four, in which they explore different concepts. This allows opportunities for peer tuition between the students, as they discuss and debate different ideas relating to the content in the tutorials and reach conclusions that the group agrees to. The teacher gets real-time insight to student ideas about the concepts covered in the tutorial, as they circulate the classroom, reviewing the student's responses and using probing question. This is discussed in-depth within the specific electrostatics topics covered in chapters four, five and six. Analysis of student's responses and worksheets also allows the opportunity to critically evaluate the efficacy of the tutorials and for redesign of the materials. The theory underpinning this style of research is discussed in chapter 3. There has been no published research completed on the teaching and learning of these electrostatics topics in the Irish second level context, but the use of activity and inquiry learning, has been shown to be useful in the effective teaching of science in Ireland (Wemyss, 2009; Broggy, 2010; Flynn 2011).

The design of the educational materials utilises a multi-representational approach (discussed in section 2.2.3), to promote the student's development of their conceptual understanding. Representations are anything that symbolize an object, a collection of objects, interactions, etc (Rosengrant, *et al.*, 2007). Different representations can be used to display different information and can have various levels of difficulty for learners to interpret information from. Ainsworth (1999) states when learners encounter multiple representations related to a topic, the different representations in which the topic is presented allows for different functions in learning. They can show different processes, different information, constrain the learner's interpretation of the material presented and enable them to construct deep understanding. Using multiple – representations,

abstract concepts can become more accessible to learners (Dienes, 1973). Kozma (2003) notes that experts can transition from one representation to another without difficulty, while novice learners struggle with this skill. The use of multiple representations in lessons can help learners develop this skill. For instance, learners may initially struggle to understand a relationship displayed in an equation, but scaffolding using tabular data and graphs can allow students to consider common characteristics between the representations and equations and to establish the relationship in the symbolic form (Project Maths, 2011). To develop student's understanding of Coulomb's law and the electric field, the representations utilised were vectors, field lines and multiple representations of the inverse square law, which were used to develop their understanding of the properties of both topics and relate the concepts to mathematical formulae. The inverse square law, as a mathematical function, allowed for opportunities to employ the use of diagrammatic model, tabular data and graphical representations. The use of field lines, vectors, diagrammatic models and graphs were employed in the lessons for work and potential difference.

1.3. Research Questions

As presented in Sections 1.1. and 1.2, there are difficulties encountered by students developing their understanding of vector concepts, the inverse square law and field lines. To help promote student's understanding of these topics, structured inquiry with multiple representations was used. Therefore, this thesis was designed to focus on the following research questions (RQs).

RQ 1. To what extent does the use of a structured inquiry approach develop student understanding of vector concepts?

RQ 2. To what extent does the use of a structured inquiry approach develop student understanding of the inverse square law relationship, by employing multiple representations?

RQ 3. To what extent does the use of a structured inquiry approach develop student understanding of the field, when utilising field line representations?

These first three research questions address these topics outside of an electrostatic context. The lessons on these topics were completed during the mechanics section of the Leaving Certificate Physics course. The student's understanding of the application and transfer of these topics to an electrostatic context is addressed in the following two research questions.

RQ 4. To what extent does the use of a multi-representational structured inquiry approach develop student understanding of Coulomb's law and electric fields?

RQ 5. To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

Chapters 4, 5 and 6 address these questions, and discuss the considerations that were employed in drafting each of these research questions. The considerations were used to both design the instructional materials and construct a boundary to which these questions could be answered. An overview of the evidence collection methods employed to address these questions is presented in the following section.

1.4. Research Design

In designing this research, it was decided that the students would encounter vector concepts, the inverse square law and field lines outside of the context of the electrostatic topics, and then apply the concepts during the later tutorial lessons. This in line with lesson sequences for how the Leaving Certificate Physics course was delivered to student groups taught in years prior to this research. Student groups first encountered vectors and the inverse square law in mechanics, as an introductory topic and as a concept related to Newton's Universal Law of Gravitation respectively. As field lines can be applied to gravitational contexts, this representation was also explored by the participants of this research in this topic. This allowed the students to develop their understanding of the three representations in relatively familiar contexts. They then later applied the representations to electrostatics to develop their understanding of the Coulomb's law, the electric field, work and potential difference. Figure 1.2 illustrates the design of the lesson sequence employed over the course of this research.

Students commence the Leaving Certificate Physics programme, at age 15-16 years after they have completed a three-year Junior Certificate programme (depicted in blue in Figure 1.2). As part of the Junior Certificate Science syllabus, students explore forces, motion and static electricity (NCCA 2003). In the Junior Certificate Mathematics syllabus, they explored algebra operations, linear and quadratic patterns, trigonometric ratios and graphing functions (NCCA, 2012). These Junior Certificate topics lay a foundation for Leaving Certificate Physics. The prior learning of the students is explained in-depth in Section 3.5.

Figure 1.2 shows how I sequenced core syllabus topics relevant to this research. The five concepts, depicted in green, represent the topics addressed by this research. The sequence of teaching the first three of these concepts were first introduced to the students while teaching of some different

topics. The final two are specific to teaching the electrostatics section of the Leaving Certificate Physics syllabus.

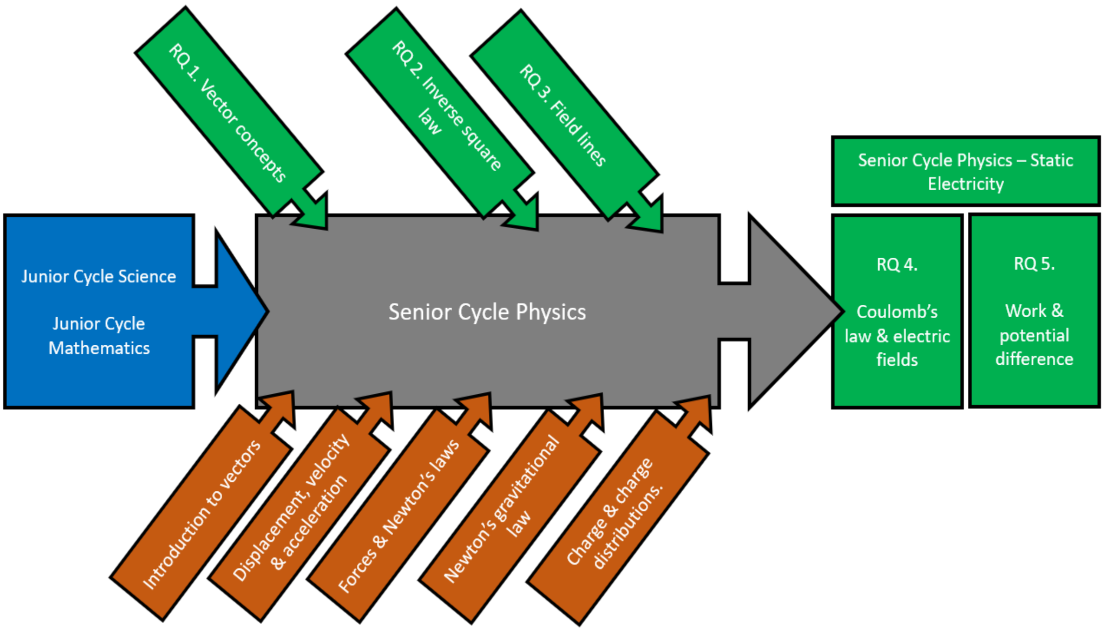


Figure 1.2. Leaving Certificate syllabus topics relevant to this research.

In my classroom, I started by defining vectors and basic mathematical operations with them. The first tutorial lesson was implemented after this initial introduction, in which students were given the opportunity to develop their understanding of vector magnitude, vector addition and vector components. This allowed for evidence collection to address RQ 1. Upon completion of the vector concepts tutorial, the students applied these concepts to the context of motion. The students explored the concepts of displacement, speed, velocity, and acceleration. They practised quantitative problems involving the three equations of linear motion. They used $s = ut + {}^{1}/_{2}\, at^2$ to experimentally calculate the acceleration due to gravity, by using apparatus designed to record the fall-time as a ball bearing is in free-fall. Upon completion of motion, I gave them a lecture-style introduction to momentum, law of conservation of momentum, forces and Newton's three laws of motion. The students were presented with definitions, formulae, videos and scenario's involving collisions between bodies in which they were to predict the outcome based on their understanding of the topics. The students also experimentally demonstrated Newton's second law and the conservation of momentum. Motion, forces and momentum are treated as 2-dimensional quantities but typically only 1-dimensional calculations are completed. The students completed various problem-solving exercises involving these quantities before being introduced to their next topic, gravitational forces, where the second tutorial lesson was implemented, the inverse square law.

This point was chosen as Newton's gravitational law is the first opportunity to apply an equation of the form $y = k\frac{1}{x^2}$ to a physics context. The students would not have used an equation of

this form in their prior formal education, so the tutorial was designed to employ a multi-representational format to allow the students to develop conceptual understanding of the inverse square law. This allowed for evidence collection to address RQ 2. After the tutorial, they applied their understanding to Newton's gravitational law and completed mathematical exercises.

Proceeding from Newton's gravitational law, the students completed a tutorial on field lines, in a gravitational context. While field line representations are not required for the Leaving Certificate Physics course for gravitational contexts, this presented an opportunity for students to use the representation before learning the electrostatics topics in this research. It also allowed for students to differentiate the behaviour of forces, acceleration, velocity and displacement in a gravitational context, and evidence was collected to address RQ 3.

In the interim between the field line tutorial and the initial lessons on charge and charge distributions, lessons on remaining mechanics topics were completed. As these topics were outside the scope of this research, they do not appear on the lesson sequence shown in Figure 1.2. When the students completed the introduction to charge and charge distributions, they completed the tutorials on the electrostatic topics. This allowed for evidence collection to address RQs 4 and 5. These topics were presented in a series of tutorial lessons. The students were required to employ their understanding of vector concepts, the inverse square law and field lines to Coulomb's law and electric fields. They also were required to apply vector concepts and field lines to work and potential difference.

This section overviewed the overall research design as the tutorial lessons were implemented into the student's Leaving Certificate Physics course. The next section details the overall structure of the thesis.

1.5. Structure of the thesis

This section briefly outlines the structure of this thesis. Following this introductory chapter, chapter two presents the theoretical basis for the research and presents the literature on developing student's conceptual understanding in physics, the use of inquiry learning in physics, difficulties encountered by students in vectors, the inverse square law, field lines and their transfer to electrostatics and potential difference, how students learn and a review of various inquiry teaching and learning materials that were used to influence the design of the tutorial lessons employed in this research. Chapter three discusses the research methodology utilised in this research. It discusses case study methodology, justifications and limitations for the use of case studies and the various sources of evidence collected for analysis are identified. This is followed by the implementation

methodology, which illustrates what a tutorial lesson looks like and why it was chosen to be used in the research lessons. The chapter finishes with an overview of the analysis methodology used and a description of the participants in the study.

Chapters 4, 5 and 6 present student's developments and understanding of vectors, the inverse square law, field lines, their application to Coulomb's law, the electric field, work and potential difference. Chapter 4 presents the results of the tutorial lessons in the topics of vectors, the inverse square law and field lines. For each of these topics, a narrative and discussion of the pre-test results, the lesson tutorials, homework assignments and the post-test results are given. These were composed using the sources of evidence presented in Chapter 3. These are followed by a discussion comparing the results, highlighting evidence of student development and persistent student difficulties. Chapter 5 uses a similar structure to present the findings from the tutorial lessons used during Coulomb's law and the electric field. In addition to the student's understanding of these topics, the chapter discusses the student's ability to transfer and apply vectors, the inverse square law and field lines to these two topics. There is also some discussion on the student's ability to transfer between vector and field line representations. Chapter 6 follows the same structure as Chapters 4 and 5 but looks at students understanding of work and potential difference in an electrostatics context.

Chapter 7 presents the final conclusions of this research, in which answers to the five research questions, presented in Section 1.4 are discussed. Implications for teaching and avenues for further research are identified.

Chapter 2. Research Basis

2.1. Introduction

In this chapter, a review of literature detailing how students learn concepts in Physics is presented. The initial section of this chapters discusses how students learn, discussing the information processing model, constructivism and the use of scaffolding in teaching and learning. This is followed by how student develop conceptual understanding and the use of Inquiry as a method to promote conceptual understanding.

The middle section of this chapter presents the theoretical framework that underpins this research. It defines conceptual change, based on the work of Hewson (1992), and describes the conditions necessary for conceptual change to occur in teaching and learning. How inquiry is employed in this research, as a method to promote conceptual change is discussed. As multiple representations are employed in this research, the use of multiple representations in teaching and learning is also discussed.

The final section of this chapter details student difficulties recorded from literature that relate this research. Student difficulties in vectors, the inverse square law, field lines, Coulomb's law, the electric, work and potential difference are detailed. Finally, a pedagogical framework is presented to inform the implementation of the teaching and learning materials over the course of the lessons for this research.

2.1.1. How students learn

Learning can be defined as the process through which relatively permanent change in behaviour or knowledge occurs because of experience (O'Donnell, *et al.*, 2009). Joyce, *et al.*, (2002) state that students learn in "human settings" which are assemblies of teachers and students in environment created for learning purposes. They state that effective schools gather students together to learn, but also engage in specific kinds of inquiry (Rutter, *et al.*, 1979; Mortimore, *et al.*, 1988 and Levine and Lezotte, 1990). As classroom engagement tend to be at the centre of education discussion, the question of how to teach is central to discussions of effective methodologies. Yet traditional "chalk and talk, drill and recite" (CTDR) and didactic teaching still dominate the methods used by teachers, and the minds of critics of education (Race and Powell, 2000; Joyce, *et al.*, 2002). This contrasts with student's perceptions of their own learning: they feel learning occurs when they engage in activities such as class debates, and little learning occurs during intervals when the teacher is talking

(Jensen and Kostarova-Unkovska, 1998). Joyce, *et al.*, (2002) suggest that, in the US, the combination of recitation and lectures contributes to approximately one third of learners being unable to complete secondary education, and approximately one fifth being unable to read and write to a standard that allows them to acquire professions that require literacy. They also suggest that evidence supports that this situation is reflected in Great Britain. In these type of classroom environments, Cooper and McIntyre (1996) suggest that students tend not to have strategies for learning, and instead adopt a passive role, in which it is the teacher's responsibility to ensure they learn, and they are "made to work" in the classroom.

Regardless of the way a teacher attempts to engage the student in leaning, the information-processing model (IPM) can explain how students learns (Huffman, 2004). The IPM describes how learners develop internal representations of the external world. It illustrates how stimuli from the environment is transferred from the sensory memory, to the short-term memory (STM) and to the long-term memory (LTM). The model is presented diagrammatically in Figure 2.1.

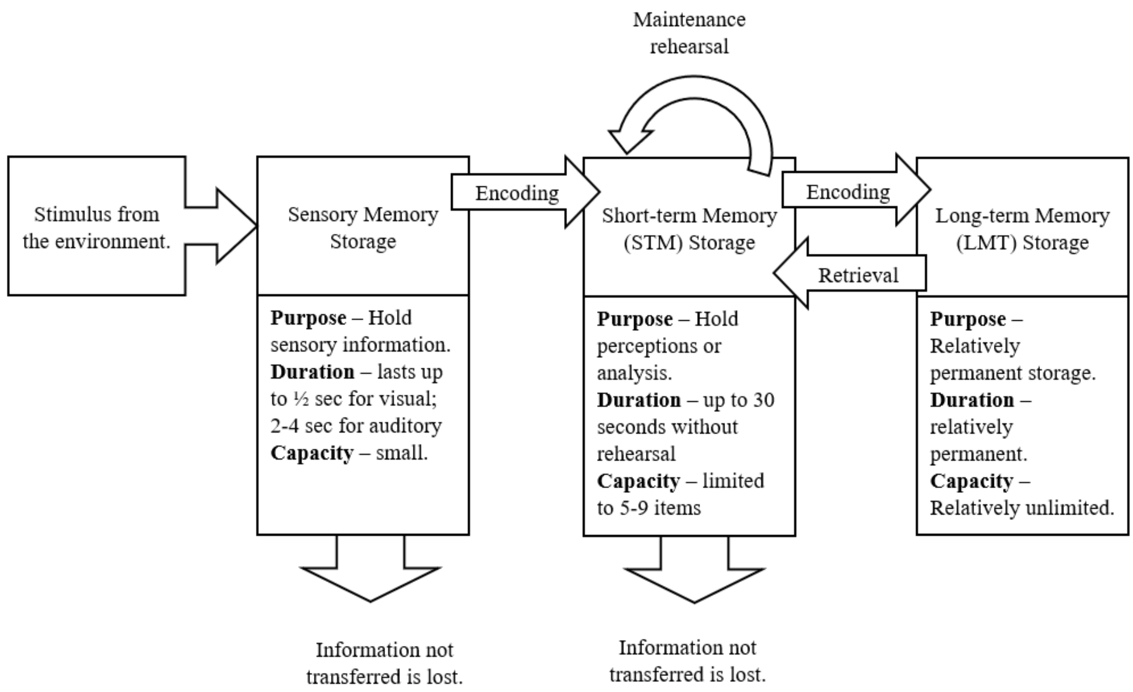


Figure 2.1. Information-processing model (O'Donnell, et al., 2009), reproduced from Huffman (2004).

Sensory memory (SM) is very brief and tends to be applied in two forms in the classroom, visual and auditory. This information is passed to the STM, where if it is not processed, it is lost. However, the information in the memory can be rehearsed to keep it active, in the STM. This information is subject to decay, when information not used is lost, and interference, when something else gets in the way of the recall. Information that is transferred to the LTM potentially has permanent and unlimited storage for the life-time of the learner. O'Donnell, *et al.,* (2009) explain how LTM can take the forms of episodic (events), semantic (verbal information), declarative (details about a structure) and procedural (details about how to do something). This model suggests varied and

complex processes occurring cognitively in the learner, but didactic teaching and CTDR can be limited in how they allow students to engage in the encoding process to transfer from SM to STM to LTM.

Reid (2009) references Yuan, *et al.,* (2006), stating that there is a consensus that STM is a function of working memory (WM). Reid (2009) expands upon this point that the STM not only retains information, but also allows the information to be worked on. Information can be retrieved from the LTM into WM and worked on, pending the capacity of the WM is not overloaded. The capacity of WM can be considered in terms of the number of items that can be processed at any one time. Reid (2009) states that people aged 16 years and older can have 7±2 items of information available to work on in their WM capacity, and processes that involve using more of these can lead to capacity overload, in which the learner can no longer effectively process all the information and will be unable to complete the task they are participating in. Over the course of their education, a learner can "chuck" information together into one item, and there is virtually no limit to how much information a person can "chuck" together to form an item. This frees up WM items for the learner to process new information and complete their task. If the learner is unsuccessful in this, their ability to develop understanding ceases as there is too much information for them to process.

Joyce, *et al.*, (2002) list numerous models for learning based on the work of Jean Piaget's theory constructivism. Constructivism is a theory about how students learn, and has many sub-theories related to information-processing, the construction of knowledge from prior knowledge and new experiences and knowledge construction because of social interactions (O'Donnell, *et al.*, 2009). In the theory of constructivism, Piaget (1967) discusses learners use of mental processes, called schemas, to organize their knowledge and experiences.

Through observations and experiences, children learn about the world they live in, in which prior knowledge and understanding are used as a lens through which they view new knowledge and experiences. In constructivist learning, no information learned is independent of the experience of the learner and the context in which it is learned. Hein (1991) outlines some principles of constructivist learning as the following:

- Learning is an active process, in which learners use sensory input and construct meaning out of it.
- People learn to learn, as they learn. Learning consists of both constructing meaning and constructing systems of meaning.
- Constructing meaning is a mental action. Hands on activities are not sufficient unless the mind is also engaged.
- Learning is a social activity. We are more likely to be successful in instructional efforts if we recognise the use of conversations, interactions with others and the application of knowledge as an integral part of learning.

- Learning takes time. It is not an instantaneous process and for significant learning, we need to revisit ideas, ponder them, try the out and use them.
- Motivation is a key component in learning. Unless we know "the reasons why," we may not be very involved in using the knowledge learned.
- Learning is contextual. We do not learn facts and theories in an isolated fashion but instead link them and relate them to other facts and theories.
- One needs knowledge to learn. It is not possible to assimilate new knowledge without first having knowledge to build upon.

Transmission models of learning (Mestre, 1991) tend to ignore learners' prior knowledge and experience, while constructivist models provide opportunities for students to perceive an event, and the student's perception informs what learning occurs for them. Generally, students make sense of these perceptions and assimilate them into, and extend, their mental schemata. Green and Gredler (2002) note that from Piagetian and Vygotskyian perspectives, constructivism helps learners develop logical thinking, self-regulated attention, conceptual thinking and logical memory. They note constructivist classrooms may employ student-directed experimentation, or interaction between students and subject matter concepts to develop advanced cognitive capabilities. The learners manipulate objects and ideas, which may lead to cognitive conflict between one's ideas and experimental results, and they can interact with the teacher to develop conscious awareness of, and mastery of, one's thinking and learn to think in subject matter concepts.

The degree of prior knowledge required to process an event and complexity of the event being perceived can lead to difficulty in extending a mental schema in an accurate fashion. Vygotsky (1978) presents a model for learning based on student's development that addresses this. Students have 3 zones in this model; the zone of actual development, the zone of proximal development and the zone of no development. If a student is presented with tasks that they can complete unaided, the skills required to do so are in their zone of actual development. When a student requires assistance or instruction to complete a task or develop a skill, it is said to be in the zone of proximal development. If a task or topic is in the zone of no development, this is currently beyond the capabilities of the student, and no form of instruction will aid the student. This model is useful for educators as it allows them to account of what students can do, what students are capable to doing with aid, and consider what teaching and learning needs to occur for the students to be able to complete more difficult aspects of their courses.

The zone of proximal development is of importance as this is where cognitive development occurs. To support students in this zone, scaffolding can be employed by the teacher. Scaffolding is the guidance, support and tutelage provided by a teacher during social interaction, designed to advance student's current level of skill and understanding (O'Donnell, *et al.,* 2009). Scaffolding provides support for the learner, extends the range of what the learner can do to enable

accomplishment of tasks that would otherwise not be possible. It can be reduced or removed from lessons as learners develop the skill and understanding to learn, or complete a set of tasks, on their own. Lynn, *et al.,* (2013) discusses 4 central tenets of scaffolding in science education, in which it can be used to make science accessible to the learner, make thinking visible, help students learn from others and promote autonomy and life-long learning.

Lynn, *et al.,* (2013) explain that making science, and scientific reasoning, accessible to learners using scaffolding can involve using tangible examples familiar to students, as opposed to abstract models that are employed in science. This allows learners to develop scientifically normative views and explain them using familiar contexts. Learning can be made visible as learners can be encouraged to explain their ideas to others. Multiple representations utilise various models for the students to engage with. Scaffolding can involve helping students listen to their peers, to take advantage of the collective knowledge of the group. Class discussions where students are required to respond to each-other and critique to one another can allow for individual learning, in learning to consider alternative views, expand their own knowledge and engage in effective communication with one another. Autonomy and life-long learning is promoted through scaffolding, as who students who reflect and explain their ideas learn more (Chi, Bassok, *et al.*, 1989), and gain a more robust understanding when revisiting concepts in new contexts.

2.1.2. Developing student's conceptual understanding

As discussed in Section 2.1.1, traditional methods of instruction tend to focus on transfer of facts from teacher to student, while little emphasis on students constructing their own knowledge and understanding. When discussing traditional instruction Mestre (1991) discusses the transmission model of education and notes that it is not a model of learning, but an instructional practice. He highlights that a central assumption of the instructional practice is that the message that the student receives is the same in which the teacher intended for them to learn. In this practice, difficulties in student's understanding are due to the manner the material is presented and the teacher needs to augment their presentation of the material. Roth (1990) notes that primary and secondary level education has struggled to develop student's conceptual understanding when traditional methods of teaching and learning are employed. She notes that while students in these environments are proficient in memorizing facts and procedures, they struggle to build arguments, make predictions and explain observable phenomena, both inside and outside the classroom. When considering conceptual development in a traditional classroom, she states that student's conceptions are largely invisible to both teachers and students, the instruction focused on student's learning explanations and terminology already developed by scientists and test questions revolve around repeating these ideas. Students are rarely asked to apply the concepts to explain everyday situations and the emphasis of

the learning was students developing the “right” answer rather than exploring the nature of the student’s conceptions. As detailed in section 2.2.1, Roth (1990) suggests a conceptual change model of instruction that promotes the student’s abilities in these areas, allowing them to overcome the difficulties that traditional instruction do not address.

To develop learner’s conceptual understanding, Mestre (1991) suggests a constructivist approach, which considers the principles discussed in Section 2.1.1, which sees learning as a process of constructing knowledge and understanding. Meaningful learning occurs when students interpret and apply knowledge in novel contexts, in which the students are mentally engaged. Roth (1990) suggests that the teaching and learning materials should focus on allowing the students to develop deep understanding of a limited number of concepts, that they can apply in novel contexts, which does typically not occur in traditional classrooms. A constructivist approach allows the students to engage with new concepts and can provide opportunity for students to resolve their prior understandings/misunderstandings with new concepts. Johnston (2010) suggests strategies to successfully engage students in developing their conceptual understanding in the classroom, such as actively engaging students and providing regular feedback, focusing on the observable phenomena, explicitly exploring misconceptions, using various problem-solving skills and strategies and providing homework tasks that involve qualitative and conceptual analysis of phenomena. This sort of constructivist environment promotes students to use active learning to support their construction of knowledge and understanding. The students are not being viewed as passive recipients of knowledge but of active participants in its creation (McDermott, 1991).

Regardless of the teaching style of the teacher, students will have constructed their own models of understanding from both their formal education and their interactions and observations of the world. This prior learning can result in their models of the world not being scientifically accurate, and the students cannot be considered “blank slates” when they partake in science lessons (Knight, 2004). Students have relied on these models to explain the world for some time before entering classrooms. These models, which may contain misconceptions, can be resistant to change, so constructing a conflict between the student’s model and the scientific model may be required multiple times before successful change takes effect.

The approach chosen to facilitate the student’s conceptual development in this manner is structured inquiry. This approach is discussed in the Section 2.1.3.

2.1.3. Inquiry learning in Physics

To promote development of conceptual understanding as discussed in Section 2.1.2, inquiry-based learning (IBL) can be utilised (Tabak, *et al.*, 1995; Blanchard, *et al.*, 2010). There are many

definitions for IBL in science education. The National Research Council present a general definition for inquiry that does not necessarily preclude the use of any one specific teaching / learning method.

> *Scientific inquiry refers to the diverse ways in which scientists study the natural world and propose explanations based on the evidence derived from their work. Inquiry also refers to the activities of students in which they develop knowledge and understanding of scientific ideas, as well as an understanding of how scientists study the natural world.*

(National Science Education Standards, 2006, p23)

An aspect of this definition is that student's experiences in a classroom setting should reflect scientists' experiences when seeking to expand the human scientific knowledge. In using inquiry in the classroom, learning involves the students thinking scientifically. While this is not an exhaustive list, inquiry can take the form of designing and critiquing experiments, performing research, engaging in scientific debate, discourse with peers, construct models, search for information, and/or some combination of these (Lynn, *et al.*, 2013).

As there as many classroom activities that can be used to engage students in inquiry, there is a wide interpretation of what inquiry is. Banchi and Bell (2008) present inquiry on a four-point scale, "limited, structured, guided and open." Limited inquiry involves following a set of predetermined instructions to arrive at a pre-determined conclusion and is typically referred to as "cook book". The aim is for students to confirm something they have already learned. Structured inquiry occurs when there is no pre-determined conclusion to the task. It is purely based on the student's construction of knowledge through whatever activity was completed, such as an investigation. Guided inquiry has no predetermined method presented to the students, and they must determine how to complete the task, as well as draw conclusions from it. In a manner, guided inquiry involves presenting students with a learning objective, such as a phenomenon to be investigated, but allowing them to complete the objective in a manner in which they see fit. Open inquiry involves giving the students no predetermined questions to answer, and instead they generate their own questions which they answer themselves.

The choice of inquiry to use in a set of lessons can depend on the outcomes the teachers wishes for their students to achieve. Tabak, *et al.,* (1995) and Blanchard, *et al.,* (2010) showed structured and guided inquiry practices are beneficial for students to develop conceptual knowledge and engaging with nature of science skills. Research into open inquiry showed that learners gained independence and autonomy over their learning, relying less on their instructors (Krystyniak and Heikkinen, 2007). Students may develop meta-cognitive reflection skills, engage in higher order thinking and improved motivation to complete investigations (Berg, *et al*., 2003). The several types of inquiry allow students to engage with several types of autonomy (Stefanou, *et al*., 2004). In guided and open inquiry, students are given an opportunity to display an elevated level of cognitive

autonomy. They can, amongst other things, discuss multiple ways to think about an issue, debate thoughts freely, re-evaluate errors and ask questions. Open inquiry, additionally, allow students to display an elevated level of procedural autonomy, such as design investigations freely, choose how to demonstrate competency and design their learning outcomes.

Inquiry learning is not accepted as best practice by all academics in the educational sphere. Clark *et al.,* (2012) argue when learners are presented with information, they should be fully instructed on what to do and how to do it, basing their argument on a review by Mayer (2004) and overloading short term memory (STM). Mayer’s review identifies numerous studies since the mid twentieth century which compared unguided to guided instruction. In all cases, the results indicated that guided instruction resulted in better students gains, over discovery learning, in the tasks specified in the studies. However, the studies referenced by Mayer do not consider the various inquiry types (Banchi and Bell, 2008), and when considering these, the studies can be interpreted as utilising structured and guided inquiry. The argument that Clark, *et al.,* (2012) make regarding STM relates to the limited number of elements it can process at any one time. They state that instruction should aim to impart as much knowledge and skills into a learner’s long-term memory (LTM) as they have relatively limitless access to this, as opposed to their STM. They suggest a “worked-example” approach, in which the learner uses their STM resources to develop comprehension instead of both comprehension and discovery which directs attention to storing essential information and understanding. However, they state that this approach is only fruitful for introducing new knowledge, and it can have a detrimental effect for material the learner is familiar with. Their article does not appear to address limited or structured, in which case the design of the teaching and learning materials can present learners with the introductory concepts and knowledge, to which the learners initially only need to comprehend. After this introduction, the learners can then apply the initial concepts and develop a deeper understanding in a limited or structured environment.

Rocard, *et al.,* (2007) identifies IBL as a method of best practice for implementation in school classrooms. Bevins and Price (2016) state that there is a wide range of empirical evidence that reports positive outcomes for learners in terms of achievement, enthusiasm, ownership and scientific skills development. They reject that inquiry must be entirely student driven and unsupported by the teacher. They propose that inquiry can develop a new sophisticated model, to reap further benefits. They propose a three-dimensional model to inquiry practices, as shown in Figure 2.2.

The first dimension focuses on the body of knowledge in science. This informs how scientists think about the natural world and generate questions for inquiry. The second dimension focuses on procedural knowledge. These allow for the reliable generation of data and evidence, ensure they are reliably interpreted, and the data and evidence is communicated appropriately. The third dimension focuses on learner’s psychological energy, in which they engage with the inquiry process, which generates energy to create and manage authentic inquiry process. Bevins and Price propose that these

three dimensions are not intrinsically interlinked but instead the model of inquiry in made up of these dimensions as the sum of their individual parts.

An early implementation of structured inquiry as defined by Banchi and Bell, which the authors however term guided inquiry, is the Physics by Inquiry worksheets developed by McDermott, *et al.,* (1995). It comprises a series of modules across various physics topics. Each module is structured so students develop their understanding based on their own observations and encourages them to develop explanatory models. There are gaps between each narrative that each student must bridge as they complete the experiments and exercises in the module. The primary emphasis is on teaching by questioning rather than the transmission of knowledge. This helps students to see physics as a process of discovery. The student is central to the learning process, developing sound qualitative understanding, which compliments and improves student's ability to tackle quantitative problems, and overall, sets a higher standard of learning (McDermott, 2001). However, the time required to complete a module in any one topic can be quite demanding, making it inappropriate for direct implementation in a secondary level setting where multiple topics are to be taught in a relatively short amount of time.

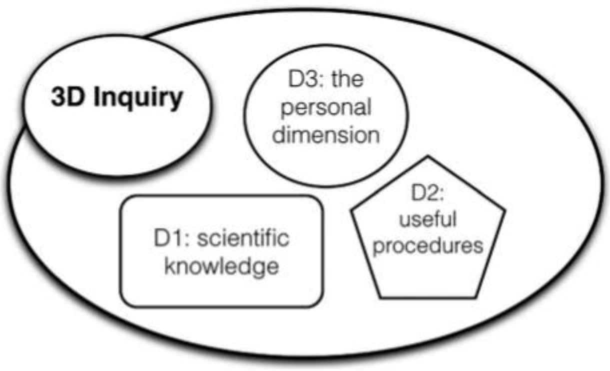


Figure 2.2. Three-dimensional model for inquiry (Bevins and Price, 2016).

Within the field of Physics Education research, there have been numerous studies in recent years (Shaffer and McDermott, 2005; Close and Heron, 2010; Heron, *et al.*, 2004; Hazelton *et al.*, 2012) show that student who learn through inquiry methods develop better conceptual understanding of physics than students who learn using traditional approaches. These approaches focused on development of mental models, student reasoning, conceptual reasoning and conceptual change.

To illustrate the efficacy, the first of these four cited studies is reviewed which employed a structured inquiry methodology for teaching and learning. When looking at student's understanding of vectors, Shaffer and McDermott (2005) showed students can have trouble with one dimensional vector subtraction when shown in a real-life context, such as collisions, and these student difficulties in vectors appeared to increase when transferred to a two-dimensional problem. Their lesson was designed to engage students with using vectors to identify changes in velocity and determine the

direction of acceleration as a result. In a separate lesson, students apply the same skills in a two-dimensional context to provide commentary about velocity and acceleration for a body in motion at points on a closed loop path. They found upon completion of the lessons, they helped not only undergraduate students transfer vector analysis from one-dimension to two-dimensions, but also the postgraduate students who acted as teaching assistants during the tutorials. This example illustrates how structured inquiry lessons can help inform instructors of student difficulties and provide an environment for students to overcome them.

2.2. Theoretical framework

This section discusses the theoretical framework that underpins this research. As a mechanism to promote conceptual change, this research employs the use of structured inquiry and multiple representations in the design and implementation of the teaching and learning material used in this work.

2.2.1. Conceptual change

Konicek-Moran and Keeley (2015) describe traditional forms of instruction as promoting "literal understanding". Literal understanding allows students to memorise and reproduce knowledge without understanding the meaning behind it, or the power to use it to argue, predict or delve deeper into the ideas involved. They argue that to do these things, a learner must develop a conceptual understanding of what they learn. Although they do not give a formal definition, Konicek-Moran and Keeley (2015, pg. 6) state that when a learner develops, or is developing their conceptual understanding, they can partake in or demonstrate any of the following attributes with their conceptual knowledge:

1. Think with a concept.
2. Use the concept in areas other to that in which they learnt it.
3. State it in their own words.
4. Find an analogy or metaphor for it.
5. Build a mental or physical model of it / to explain it.

To develop conceptual understanding, the learner must be exposed to a concept in some manner. Concepts are packages of meaning; they capture regularities, patterns, or relationships among objects, events, and other concepts (Novak and Canas, 2006). They are formed when a student engages in an intellectual function in which memory, attention, inference and language all

participate. From experience, students naturally construct their own conceptions, which may or may not be scientifically accurate, to make sense of the interactions they see in the world around them. This can also result from methods of instruction that do not aim to reduce the occurrences of students developing misconceptions (McDermott and Shaffer, 1992). Assessments and evaluation based on recall of content and solving quantitative problems can hinder development of conceptual understanding, as educators tend to teach to the test instead of focusing on their students developing coherent understanding of the material they learn in the classroom setting. For this reason, misconceptions do not tend to be addressed by traditional instruction (Dykstra, *et al.*, 1992). As students spend considerable time and energy constructing these concepts, they can develop an emotional and intellectual attachment to them, which is not easily overcome. To overcome such an attachment, a conceptual change model of instruction is proposed.

Hewson (1992) discussed conceptual change as three mechanisms, conceptual extinction, conceptual exchange and conceptual extension.

- **Conceptual extinction** is when an alternate idea is challenged to the point where it is no more, and all that remains is the new learned concept.
- **Conceptual exchange** occurs when a student is presented with a conception that challenges their current understanding. Through the learning process, the status they associated with their current understanding is lowered and the status of the new conception is raised. The original idea is still present, but the student becomes aware of its inaccuracies or limitations and disregards it in favour of the more useful robust concepts that were learned.
- **Conceptual extension** is when a person goes from not knowing an idea to knowing the idea. A person makes connections between the things they already know and extends their understanding to account for new material achieves this.

To promote conceptual change in the context of conceptual exchange and extension, the following proposed four conditions generally tend to be present:

- *There must be dissatisfaction with existing conceptions.*
- *The new conception must be intelligible*
- *The new conception must be initially plausible*
- *A new conception should suggest the possibility of a fruitful research programme.*

(Posner, *et al.*, 1982, pg. 214)

Under these conditions, student's models are effectively challenged and allowed to fail. This allows the students to adjust their models of understanding of the topic being studied. Vosniadou and Brewer (1994) consider conceptual change as it applies to a student's overall understanding of a topic and see it *"as the product of the gradual lifting of constraints, as presuppositions, beliefs, and mental*

models are added, eliminated, suspended or revised during the knowledge acquisition process". By encouraging students to evaluate their initial ideas over time and adjust it by adding the elements of scientific explanation, students are facilitated to allow their models to undergo conceptual change to create models that extend to more fields of study and possess greater explanatory power.

In some cases, cognitive conflict arises between the new knowledge and the existing schema (Piaget, 1975, Posner, *et al.*, 1982, Chan, *et al.,* 1997). This results in the student altering their schema to accommodate the new knowledge. This reorganization of mental schema is required to make sense of the world, as they learn more about the world around them. Schemata can be altered or replaced with other schemata that better explain the environment the learner is in. This results in the construction of new knowledge that furthers learner's understanding of a particular experience / concept / etc. In this view, constructivism can be understood "in terms of a shift in the location of the meaning of what is found in our environment" (Taber, 2011, pg. 40). However, the understanding constructed by the students is not always what the teacher wishes, which can lead to the construction of misconceptions, which can be difficult to deconstruct.

The following sections discuss structured inquiry as a method to promote conceptual change in the classroom, the use of various inquiry resources employed in the development of the research instruments and the use of representations as an aid to promote conceptual change.

2.2.2. Approach adopted in this research

Structured inquiry is adopted as the approach used in the research to promote conceptual change for the student's understanding of Coulomb's law, the electric field, work and potential difference. Structured inquiry generally allows students to develop conceptual understanding (Tabak, *et al.,* 1995; Blanchard, *et al.,* 2010). As seen in the pre-test discussions in chapters 4 and 5, many students produced answers consistent with difficulties recorded in literature. Therefore, structured inquiry was chosen to be utilised in this research. Section 3.3 discusses how structured inquiry was implemented in this research, through the development of tutorial lessons. To avoid confusion, the term inquiry is used as an umbrella of the inquiry types used in the materials developed, instead of identifying each type of inquiry as they arise.

In developing the materials, numerous approaches to inquiry tasks that could have been employed in the lessons were reviewed. After consideration of the research questions, learning objectives of the Leaving Certificate Physics curriculum, the Leaving Certificate Physics examination format and ability of the students, the following approaches were considered for use in the development of the teaching and learning materials designed in this research to be employed in the second level classroom: Tutorials in Introductory Physics and Conceptual Physics.

Tutorials in Introductory Physics (McDermott, *et al.*, 2003) is a set of supplementary activities to accompany lectures or a standard textbook in a standard university physics course. The emphasis is on student understanding of concepts and scientific reasoning skills, as opposed to rote learning theory or solving quantitative problems. A tutorial consists of a pre-test, worksheets, homework assignments and a post-test. The pre-test is typically given after the lecture to determine what concepts the students do understand, and what they are expected to understand at the end of the materials. This indicates student's initial conceptions that can be targeted for conceptual change. The worksheet questions are designed to guide students to construct concepts and apply them to real world situation, when contextually appropriate. These are completed in groups to allow for peer tuition when constructing answers. When students run into difficulty, the teacher uses prompt questioning to guide student thinking instead of volunteering answers themselves. While the teacher can explicitly state when a student's reasoning is diverging from what is intended, they ultimately encourage the students to find their own correct answers. Through completing the worksheet and engaging in discussions, students encounter the conditions for conceptual change to occur (Posner, *et al.*, 1982). The homework exercises are designed to reinforce what is covered in the worksheet, applying the concepts in both familiar and unfamiliar contexts, and in some cases, extend student knowledge. Questions used in the pre-test may or may not appear in the tutorial lesson, or the homework assignment, depending on the design of the materials. This can be used for comparative purposes, and to allow the students to apply their developed understanding to a previous question they may have struggled with or completed in error. The post-test can then be used to gauge any development in student understanding. To be effective, the post-test is written to emphasize the concepts and reasoning skills used in the tutorial lesson and can be used as a comparative tool to determine how a student's understanding has developed since completing the pre-test. Student responses can be individually, or as a group, compared with the pre-test responses. This allows for the identification for the extent of conceptual change that occurred, and determine if the conceptual change was extinction, exchange or extension (Hewson, 1992). Any difficulties persistent in both the pre-test and the post-test can then be redesigned in a future edition of the tutorial, with the research informed of specific difficulties encountered by students. This cycle of results-based redesign has been used to develop more robust materials that increase the number of students able to access difficult concepts (McDermott and Shaffer, 1992; Wosilait, *et al.*, 1998).

The use of materials based on Tutorials in Introductory Physics as the intervention allows for a strong targeting of specific concepts and topics (Ambrose, 2004). For this reason, the tutorial lesson format of Tutorials in Introductory Physics Tutorials in Introductory Physics (McDermott and Shaffer, 2003) address multiple topics in Physics and they could also be used as a guide to draft and develop lessons that adopt the tutorial approach in the second level context, as they were for this research is adopted in this research. However, Tutorials in Introductory Physics is aimed at the introductory undergraduate level, instead of upper secondary level. The material that is covered is

too advanced for a second level classroom; and the worksheets developed in this research were of original design, utilising the structured inquiry approach utilised in the Tutorials in Introductory Physics materials.

Another resource that was considered was Conceptual Physics (Hewitt, 2009), as the material is more accessible to a second level student, and utilises familiar everyday contexts, representations and analogies to illustrate physics concepts.

In 1964, Paul Hewitt began teaching at City College, San Francisco, in which he taught Physics to non-scientists. His approach focused on teaching concepts and relationships in physics using English words, and using little or no mathematics (Hewitt, 2011a). The approach uses analogies and imagery from real-world situations to promote student conceptual understanding of physics principles. When students explore equations, they learn to reveal information about the relationships involved and then develop the ability to manipulate formulae to substitute values in during the last step. This allows students to observe relationships that are otherwise not typically seen when the equations are represented in their standard form.

> "When problems are couched in symbols, and the numbers held for later, a student's task calls for thinking that calculators cannot supply. They think concepts."
>
> (Hewitt, 2011b, p. 264)

By giving students a solid foundation in the concepts involved in Physics, they are equipped to understand various formulae, and to make connections between the concepts of physics and their everyday world.

Exercises from the Conceptual Physics practice book, 10th edition (Hewitt, 2009), were used to develop the contexts used in exploring concepts related to the electric field and potential difference. The material presented in the practice book was further developed, to align it with the current Leaving Certificate Physics curriculum, so the students would not be at a disadvantage when they complete their terminal examinations at the end of their second level education.

2.2.3. Using representations to develop conceptual understanding

A representation describes something that symbolizes or stands for an object, a collection of objects, interactions and/or a process (Rosengrant and Etkina, 2007). Representations can come in many forms, such as words, diagrams, mathematical equations, tables, graphs, animations, simulations, etc. In this research, the use of multiple external representations (MERs) is employed as an aid in the tutorials, to promote student's conceptual development of the different electrostatic

topics. Presenting the same concepts in separate ways provides the learner with the opportunity to build abstractions about mathematical concepts (Dienes, 1973). This is a fundamental step in successful learning. Numerous studies show the benefits of using MERs (Cox and Brna, 1995; Mayer and Sims, 1994; Tabachneck, *et al.,* 1994). The use of MERs is not a "silver bullet," as others have failed to find benefits (Chandler and Sweller, 1992; Van Someren, *et al.*, 1998). This section briefly discusses design parameters to consider when using MERs, cognitive tasks related to the use of MERs and the functions of using MERs in a learning experience for learners.

Ainsworth (2006) states in designing educational materials that employ the use of MERs, the designer should consider number, information, form, sequence and translation. The number of representations must be at least two, but more can be employed. They can be presented simultaneously to the learner, or in sequence. The designer must consider the information they wish to convey to the learner. MERs allow for flexibility in how information is distributed across representations. Information can be redundant, where each representation presents specific information that is isolated from others, partially redundant, where some information is common to all representations and some in unique to each representation, or all the representations express the same information and the only difference is how the information is represented. This must influence the design of teaching and learning materials, as redundant information in representations can make it difficult to transfer between representations, relate representations together and overload their cognitive load capacity. The form of MERs considers what type of representations are chosen and how the learner will interpret how the interactions interact with each other upon completing their lesson. When considering the learner's translation when using MERs, the designer needs to consider how the learner will interpret the information and consider if the student will increase their understanding of the representational format chosen, or the domain knowledge it represents.

When leaners are presented with, or use their own MERs to complete a task, they must face and master several cognitive tasks (Ainsworth, 2006). These cognitive tasks are presented in order, but they are not necessarily the order a learner should approach them.

1. Learners must understand the form of a representation. They must know how a representation presents information, and how to use the operators of the operation. Learners can have difficulties in both learning the operators of a representation and how the operators are applied in the contexts the representation models. An inability for students to use representation operators can hinder the learners' ability to complete a task, understand the concepts and/or context they are studying or achieve the targets learning from an activity that employed MERs.
2. Learners should understand the relation between the representation and domain. Interpretation of representations is a contextualized activity. For instance, students need to be aware that the slope of a line on a distance – time graph represents the velocity, as opposed to the height of the graph (Leinhardt, *et al.,* 1990). This can be a challenging task during a learning process, as opposed to problem solving, as the learner is applying MERs upon incomplete domain

knowledge. Deciding how the learner will use the representation to model the context of the domain knowledge needs consideration. Otherwise, difficulties could be encountered by the students, and the envisioned learning using MERs may be hindered.

3. Learners need to understand how to select an appropriate representation. Learner may have the opportunity to select which representation is most appropriate to complete a task, and they consider factors such as the representation, task characteristics and outcomes and individual preferences. While students are often competent in this (diSessa, 2004), selecting appropriate representations tends to be more difficult for novices as opposed to experts (Chi, Feltovich and Glaser, 1981). Understanding, and accounting for, learners limited ability to select appropriate representations can be used to inform the design of the tasks that employ MERs.
4. Learners need to understand how to construct appropriate representations. In certain tasks, learners may need to construct their own representations. DiSessa (2004) argues that learners are good at doing this, and Grossen and Carnine (1990) showed that children solved problems more effectively when they constructed their own diagrams, rather than select from a series of pre-drawn diagrams. This can be incorporated in material design by providing learner the opportunity to model a process using a representation and have them explain how it represents the process.

When MERs are employed in a teaching and learning sequence, it is important the designer considers why they wish to employ MERs. When carefully planned and applied, such as being cognisant of the cognitive tasks involved in using MERs and their limitations, Ainsworth (1999) suggest the use of MERs allows for the employment of multiple parallel functions in learning. However, without considering the 4 cognitive tasks that Ainsworth (2006) outlines, the result of using MERs could range from fruitful to detrimental to the desired learning outcomes. To consider the functions of MERs, she developed a functional taxonomy of multiple representations that illustrates these functions, shown in Figure 2.3.

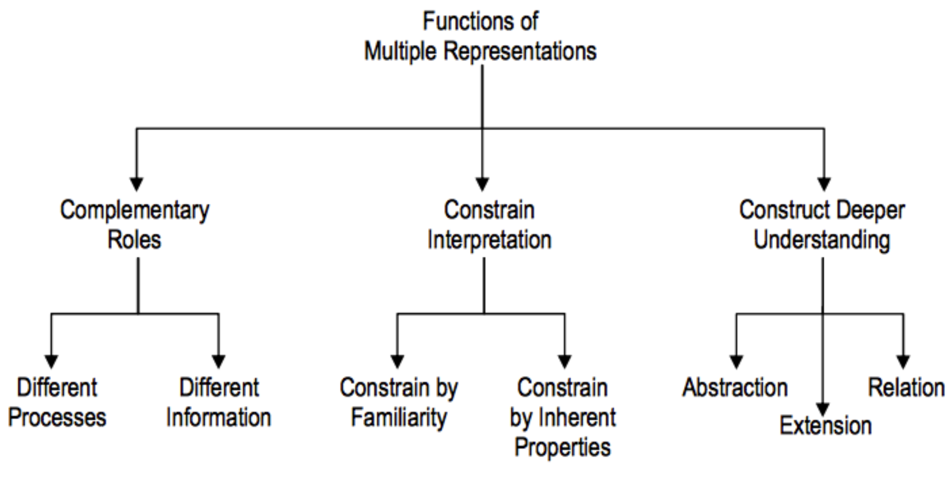


Figure 2.3. Functions of Multiple representations (Ainsworth, 1999)

To understand the complementary roles of multiple representations, consider how different information can be represented in diverse ways, as is appropriate to that information. Representing the velocity and acceleration against time is appropriate in a table or a graph, while inappropriate for representing the mass of the body. Different processes used in multiple representations allow the student to take advantage of their computational properties. Diagrams allow students to group together relevant information, graphs and tables allow students to determine values from reading off, patterns can be determined by analysis of the table and graph, while algebraic equations summarize the relationships between various variables in a given context.

Constraining information involves using the properties of one representation to focus on information taken from another. This allows the familiar representation to be used as a scaffold to show the same or similar concept in a more difficult representation. For instance, using a matching game with several types function in graph form, tables of graphs typical of those functions and general equations for those functions can give students the opportunity to identify key coefficients, variables and parameters in the equations that are characteristic of each type of function. When designed at the appropriate level, there are only a limited number of combinations and reasoning the student can arrive at, and can further be facilitated by instructional guidance.

Multiple representations can also support deeper understanding of the material covered by reinforcing concepts and information that is common to multiple representations of the same material, but also by highlighting features that is most notably prominent in a representation. Velocity – time graphs for non-uniform displaying increasing and decreasing motion can be relatively easy for students of all levels to understand and interpret, but when given various algebraic equations, with corresponding domains for time can be quite difficult to interpret for students. The use of multiple representations in this research primarily aims to employ this construction of deep understanding. Many sections of the tutorials require the students to represent information as tables, graphs, apply algebraic reasoning, and draw and interpret vector and field line diagrams.

Kozma (2003) discussed one difference between understanding concepts by novices and experts, being that experts are fluid in their transitions between representations while novices typically use one or two. Students focus on surface features of the concept and do not develop understanding at a deep level. To advance their understanding, Hestenes (1996) suggests that a complete understanding of a model of a physical system requires a student to be able to transfer between multiple representations. This ability to engage in transfer between different representations leads to increased conceptual understanding.

2.3. Overview of the research study

2.3.1. Teaching and learning electrostatics

As discussed in chapter 1, this research focuses on upper secondary level student's understanding of vectors, the inverse square law, field line representations, Coulomb's law, electric fields and work and potential difference. This section presents a review of literature related to these domains, detailing difficulties and misconceptions typically encountered by students in electrostatic forces, fields and potential difference. Section 2.1.3.1 discusses issues of vector concepts vector addition and vector components, and then difficulties of using vectors in Coulomb's law and electric fields. Section 2.1.3.2 details issues around the inverse square law, focusing on learner difficulties surrounding scaling. Section 2.1.3.3 discusses difficulties related to field lines, and how they are applied to represent electric fields. Finally, section 2.1.3.4 discusses issues related to work and potential difference.

2.3.1.1. Difficulties in understanding of vector concepts and their application to the electrostatic context

Student difficulties are encountered in the understanding of vector concepts. Flores, *et al*., (2008) showed that highlighting the vector nature of forces, and acceleration, in kinematics can increase student's ability to use vectors to solve problems that otherwise prove difficult, but that the overall improvement of vector understanding is quite a challenge. Nugyen and Meltzer (2003) showed that students have difficulties with vector addition, in cases of collinear vectors and vectors in two dimensions. Illustrations of these difficulties are shown in Figure 2.4. Difficulties seen with collinear vectors included adding vectors to form two-headed arrows, connecting vectors "tail to tail" or "tip to tip," incorrectly attempting to find the resultant between two vectors. They also showed another difficulty in student's understanding, such that when finding the superposition of two vectors, students re-orientate vectors arrows if they were not perpendicular to each-other to apply Pythagoras' theorem to determine the resultant vector. This shows an inability for students to correctly combine vectors, both diagrammatically and mathematically. Conceptual difficulties with vector addition in two dimensions include adding magnitudes as in scalar addition instead of vector addition, not taking into account direction of a vectors, or the angles between which the vectors act, not conserving vertical / horizontal components or not acknowledging their contribution to the resultant vector and using a "split the difference" algorithm, in which the resultant is always along the bisector of two vectors, regardless of their magnitude.

Students must be aware of how vectors sum to form resultant vectors, by the principle of superposition. Flores, *et al.*, (2008) showed students have difficulties applying vector concepts to forces in which they treat them as scalar quantities, in the domain of mechanics. It is reasonable to postulate these difficulties would transition to electrostatics.

As an overall aim of the research was to improve student's overall development of the electrostatics, an isolated tutorial covering vector concepts was developed. It was expected that the students could transfer their understanding to electrostatics and reduce the cognitive load on their working memory. By initially developing understanding of vector concepts, this would free up items in the student's working memory capacity, to be devoted to Coulomb's law and the electric field. Nguyen and Meltzer (2003) highlighted the common errors that were likely to occur, and these were considered in both the design of the materials, and as conceptual difficulties to identify in student responses as they progressed through the vectors teaching and learning materials.

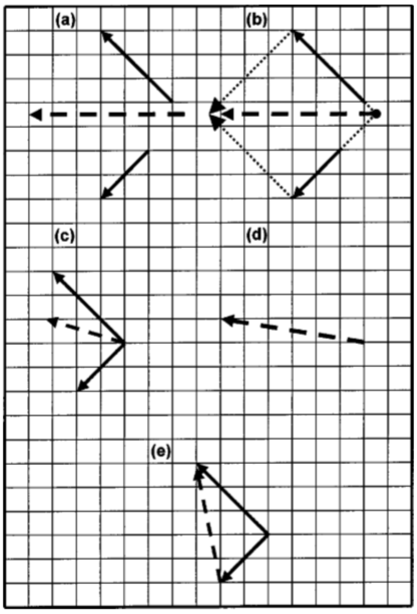


Figure 2.4. depictions of student errors during vector addition (a) zero vertical vector components (b) "split the difference" (c) incorrect parallelogram addition (d) incorrect horizontal component and (e) top – to – toe error (Nguyen and Meltzer, 2003).

Maloney, *et al.*, (2001) showed that undergraduate students can struggle with vector concepts such as superposition in an electro-static context. In question 6 of the Concept Survey of Electricity and Magnetism (CSEM), as shown in Figure 2.5 (i), students were asked to find the direction of the net force acting on the charged particle labelled B. In question 8, shown in Figure 2.5 (ii), the students were required to determine the outcome that adding a charge +Q at (b, 0) would have on the charged particle, q_1.

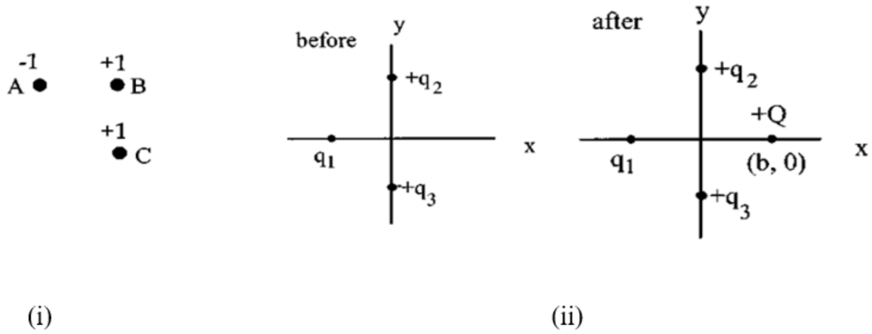


Figure 2.5. Question 6 (i) and question 8 (ii) from the CSEM (Maloney, et al., 2001).

These questions rely on the understanding of vector magnitude, vector direction and the superposition of vector quantities, and their application to electric forces and fields. They noted that in question 6, during the post-test, 33% of students in an algebra-based course and 27% in a calculus-based course could not determine the superposition of the force acting on the charged particle, B. They also reported that noted, in question 8, 47% and 34% of students respectively could not determine how the addition of a new charged particle would affect the direction of the force felt by q_1. The majority of incorrect answers suggested there would be a change in the magnitude and direction of the force, suggesting student struggled to differentiate between the quantities, and not consider the interaction of vertical and horizontal components.

2.3.1.2. Difficulties in understanding of the inverse square law and its application to the electrostatic context

Coulomb's law is considered the fundamental law of electrostatics. It is the first mathematical treatment that students in Leaving Certificate Physics engage with, after completing charge transfer and demonstrate some charge phenomena qualitatively. The Leaving Certificate Physics syllabus (NCCA, 1999) outlines that students should appreciate it is an example of an inverse square law, without explicitly defining what constitutes as an appreciation. Marzec (2012) and Arons (1997) notes that students find it difficult to understand unless exposed to it repeatedly. Maloney, *et al.*, (2000) found that students have difficulty understanding the mathematical implications of Coulomb's law pre-instruction. Post-instruction, they found that student gains in understanding were not as one would expect, for those which have completed exercises on Coulomb's law.

The application of the inverse square law to electrostatic forces was disputed for 40 years (Heering, 1992), it has since been proven and can now be demonstrated in a laboratory using charged domes and an electronic balance of precision to 4 decimal places (Cortel, 1999) or measuring the distance between charged and uncharged pith balls (Wiley and Stutzman, 1978). A simple approach for students to investigate the inverse square law is to use an electric field sensor held at various

distances from a charged body and plot the electric field against distance and analyse the data. This approach was taken for light intensity (Bohacek and Gobel, 2011). Other methods include presenting students with data and getting students to determine the relationship based on graphing activities (Hestenes and Wells, 2006). However, data alone does not enable students to develop a conceptual understanding of the inverse square law, nor provide a tangible context for them to relate the relationship to. In Conceptual Physics – 10^{th} Edition (Hewitt – 2009), the context of a spray paint can spraying drops of paint over different areas is used to illustrate the inverse square law. This model is easily understandable by students and adopted in the approach taken in this research.

Research literature also looks specifically at students understanding of the inverse square law, as applied to electric fields. Cao and Brizuela (2016) demonstrated that learners can qualitatively develop an appreciation that the closer charged particles are together, the stronger the force exerted between the charged particles. Conversely, Maloney, *et al.,* (2001) delivered a question involving Coulomb's law where the distance between a pair of charges was increased by a factor of three. It was seen after instruction, 54% and 32% of students struggled to apply the inverse square law to the increase in distance, with the most prevalent error submitted by the students was that the force would reduce by a factor of three. Marzec (2012) suggests that students misunderstanding of spherical scaling could lead to student to difficulties in this area. While there are many methods to experimentally demonstrate the inverse square law, research looking at students conceptual understanding of the law are scarce. Previous findings in this research, as part of an initial pilot study (Moynihan, *et al*., 2015) noted the that students have an over-reliance on the use of formulae and tend to unintentionally ignore the index value on the distance variable in the equation for Coulomb's law. These difficulties appear to be in line with Arons (1999), in which he notes that learners struggle understanding proportional scaling for area when individual dimensions of a geometric shape increase. For example, in the absence of formulae, learners struggle to explain how increasing the sides of a square by a factor of 3 results in an increase in the area by a factor of 9. When this reasoning is applied to a gaussian sphere to model an electric field, these struggles can indicate that scaling can cause difficulty in understanding the inverse square law.

2.3.1.3. Difficulties in understanding of field line concepts and their application to the electrostatic context

Törnkvist, *et al.,* (1993) paraphrased the work of Newton, Faraday and Maxwell in explaining that field lines presented an explanation of "action at a distance" for non-contact forces. For some, field lines presented a physical medium in which the force could act, where others saw them as representations. For a course that introduces the use of field lines, understanding the relationship between force, field and trajectory provides solid foundation for field theory. Greca and Moreira

(1999) showed that in a small group of undergraduate students taking an introductory course in electromagnetism, the students that could form working models of electromagnetic fields were comfortable exploring the concepts of a field mathematically or used models for field lines and solve problems using constructed images. Conversely, students who struggled to form working models of electric and magnetic fields appeared to overly rely on definitions without understanding their implications. They were unable visualise a field, using field lines or vectors, and in some cases, represented the fields incorrectly. Student's inability to apply the concept of field hinders their understanding and ability to progress to more advanced applications of electromagnetism in physics, electrical engineering and other relevant avenues of study.

As a diagrammatic representation of a vector field, a lot of information can be gleaned from a field pattern, that otherwise, would require complicated calculations to determine. Such examples are determining relative field strength at various positions, identifying relative magnitudes of charges / masses of bodies, identification of charge types, construction of reasonable paths taken by various charged and uncharged bodies in the fields and identification of charge on a particle moving in a field. While such a simple representation can be used to determine a vast amount of information, this also presents opportunities for students to develop confusion about the representation and errors in understanding (Maloney, *et al*., 2001).

Furio and Guisasola (1998) presented their student's difficulties in differentiating between field intensity, and the force acting on a particle in a field. One possible source of this difficulty is suggested by them that interpreting the definition of the magnitude of the electric field strength, $E = \frac{F}{q}$, the students interpret the proportionality between the electric field strength and force for an equivalence. Similar findings were presented by Cao and Brizuela (2016) who demonstrated that students can accurately produce canonical electric field lines, but struggle to attribute meaning such as force directions, velocity directions and trajectories to them. In some cases, students can attribute combinations of these meanings simultaneously to the field lines. These difficulties contrast the

convention that field intensity is based on the charge / mass generating the field and is independent of the test-charge / small-mass placed in the field. This is typically represented by the density of the field lines. Törnkvist, *et al.,* (1993) note that students struggle to transfer between field line representations and vector representations, in which some students produce curved vector force arrows to represent a curved field line, instead of producing a tangential arrow. They also showed that student errors include field lines overlapping, instead of use of the superposition principle to show a resultant field. Galili (1993) found that some students perceive field lines to be real tangible structures, which mediate the force acting on a particle in a field. Both Galili (1993) and Törnkvist, *et al.,* (1993) showed student represent the path taken by a body in a field to follow the curvature of a field line.

While the difficulties presented be used to inform development of instructional materials, Cao and Brizuela (2016) discuss how students do not always express in-depth reasoning initially but can develop sophisticated understandings whilst explaining and reinterpreting their work. They illustrate an example of a student making additions to their work during an interview to expand on their initial explanations and form an accurate model of force, motion trajectories using electric field line representations, but point out that they do not imply that students should be expected to develop sound reasoning without the aid of some manner of instruction.

2.3.1.4. Difficulties in understanding of work and potential difference

This section discusses literature related to student's understanding of work, potential and potential difference. Literature on these topics tends to focus on the work-energy theorem and how it applies to system interactions. Potential and potential difference are looked at from a calculus-based context, or their applications to electrical circuits. As this research focused on second level students, the literature reviewed focuses on students conceptual understanding of the mathematical implications of work, applying it to potential difference, and applying potential difference to systems, using an electrostatics context and reasoning, as opposed to current electricity.

Doughty (2013) conducted research with undergraduate students, in which she presented student difficulties in determining whether work done was positive, negative or zero when a test charge moves between various points in the electric field. Difficulties were observed for student determining the direction of the electric field, and force, relative to the displacement and assuming the direction of the force is collinear and in the same direction to the displacement. During student interviews, she noted students use energy conversion as a mechanism to explain the concept of positive work (increase in translational KE), negative work (decrease in translational KE) and zero work (no change to gravitational potential energy). This reasoning can be applied to systems, in which work can increase or decrease the energy in a system. Lindsey, *et al.*, (2009) conducted a study with over 4000 undergraduate students over numbers of years, where they developed tutorials focusing on systems involving mechanical work and springs. They identified numerous difficulties in student's understanding of work, such as belief that energy in any system is constant, thinking in terms of kinetic and potential energy and ignoring the cause and effect of the work-energy theorem, and associating work with change in kinetic or potential energy, as opposed to a system. They also identified specific difficulties related to work, such as treating the sign of work as dependant on a coordinate system and failing to consider the displacement of the point at which the force is applied. The approach taken in this research does not discuss systems, or internal energy, so it is expected to see students associate work with the change in kinetic and potential energy of charges in electric field, in the models presented to the students. Other approaches that were taken include a multi-

representation approach by van Heuvelen and Zou (2001), which focused on the use of verbal, pictorial, graphical and mathematical representations, and showed graphs and charts can be useful in helping student develop understanding of these processes.

Hazelton (2013) showed that students struggle to associate higher and lower potential, relative to the ground, to metal spheres with charged rods adjacent. While students could associate high and low potential to positive and negative charge, they incorrectly applied them to the metal spheres, both as the charged rod was initially placed over the sphere, and as time went on so charge was no longer moving between the ground and the rod. Student also demonstrated that they associate the potential with the charge on the body, but do not consider the overall system when multiple bodies of various charge are present. Maloney, *et al.*, (2003) showed that students struggle to understanding how a negative charge will move based being at a position of high potential, but not being aware of the potential in other regions, indicating they do not consider the movement of charged to be based on potential difference, as opposed to potential. Some of these difficulties can be explained by work completed by Guisasola, *et al.*, (2002), in which they found that, when phenomena are related to the process of charge, students feel more comfortable when they talk in terms of charge, rather in terms of potential. This can lead to difficulties when students must consider the interactions with objects outside the system they are dealing with, and resort to imagining some form of contact or interaction with external bodies.

2.4. Implementation of lessons

The last section of this chapter discusses the overall teaching and learning approach adopted in this research. Section 2.3.1 highlighted that there are many issues and difficulties in the teaching and learning of Coulomb's law, electric field and work and potential difference. Difficulties and misconceptions were identified from literature and assessed using pre-tests to identify student's initial conceptions that can be targeted for conceptual change. Inquiry methods of learning were employed to direct the learning to promote conceptual change in the lessons (Tabak, *et al.*, 1995; Blanchard, *et al.*, 2010). The use of inquiry is employed in the research using tutorial lessons. The tutorial lesson structure is based on the lesson format as described in Tutorials in Introductory Physics (McDermott and Shaffer, 2003) and the design of some of the questions was influenced by Conceptual Physics (Hewitt, 2009).

Multiple external representations are utilised in these tutorial lessons, using various representational tools to enable the students to encounter information and display their understanding of the concepts underpinning Coulomb's law, the electric field and work and potential difference. It was aimed that the students would have been enabled to iso-morphically transfer the understanding

concepts in these topics between all the representations employed in the tutorial lessons, i.e., students can transfer proficiently between all representations without difficulty in any case. The tutorial lessons utilise representations to allow the students to (a) explore concepts and relationships in different manners, (b) glean different information from similar scenarios and/or (c) predict the behaviour of charged particles in different scenarios using the different representations. Homework and post-tests gauge the extent to which conceptual change occurred, and gather evidence for indications of conceptual extinction, exchange and/or extension. The details of how the tutorials lessons are implemented are discussed in detail in chapter 3.

Vygotsky's (1978) zones of development were considered to determine the order of the lessons. Teaching Coulomb's law, electric field, work and potential difference, without having covered any other underpinning topics would not be feasible, as without some foundational domain knowledge, these topics are in the zone of no development. In this research, vectors, the inverse square law and field lines are identified as element topics, as in they are elementary to the understanding the electrostatic topics. Once their understanding of these element topics is developed, the student's zones of proximal development extends to encompass the electrostatic topics. The student's development of the element topics is discussed in chapter 4, and the development of their understanding of the electrostatic topics is discussed in chapters 5 and 6.

Table 2.1 summarises this section in a pedagogical framework for the implementation of these lessons. It presents the several topics covered by the research and links them to the research questions that underpin this research. The target concepts within each topic are identified and the representations are presented. The later topics of Coulomb's law, the electric field and work and potential difference display which element topics are employed, and the necessity for the students to have completed them prior to the electrostatic topics. The various teaching and learning (TandL) materials that were developed and utilised for this research are listed for each topic, with a reference to a copy of each set of the TandL materials listed in the Appendix column.

	Contextual / conceptual domain of physics	Research questions	Target concepts	Representations & elements used	Teaching & Learning (T&L) Materials	Appendix
Elements	Vectors	RQ 1	• Vector magnitude • Vector addition constructions • Horizontal / vertical component superposition.	Mathematical / algebraic. Vector diagrams.	Pre-test. Tutorial lesson. Homework. Post-test.	Appendix A
	Inverse square law	RQ 2	• Increase in x-variable reduces the y-variable • Change in y variable is invers square proportional to x-variable • Area and paint model to explain concepts.	Diagrammatic model. Tabular data. Graphs. Mathematical / algebraic.	Pre-test. Tutorial lesson. Post-test.	Appendix B
	Field lines	RQ 3	• Difference between force and field. • Field line density for strength. • Path taken not represented by field lines.	Field lines.	Pre-test. Tutorial lesson. Homework. Post-test.	Appendix C
Application of elements	Coulomb's law & Electric field	RQ 4	• $F \propto q_1 q_2$ • $F \propto \frac{1}{d^2}$ • Vector addition of electrostatic forces. • Difference between force and field. • Field line density for strength. • Path taken not represented by field lines. • Negative charge moves against field lines. • Field patterns for attraction and repulsion. • Drawing vectors from field lines. • Drawing field lines from vectors.	Tabular data. Graphs. Mathematical / algebraic. Diagrams. Vectors. Field lines.	Pre-test. Tutorial lesson. Homework. Post-test. Pre-test. Tutorial lesson I. Homework I. Tutorial lesson II. Homework II. Homework III. Post-test.	Appendix D
	Work & potential difference	RQ 5	• Use of vectors to differentiative between displacement and distance. • Use of vectors and field lines to identify positive, negative and zero work. • Potential difference as a mathematical ratio.	Field lines. Vectors. Mathematical / algebraic.	Pre-test. Tutorial lesson I. Homework I. Tutorial lesson II. Homework II. Post-test.	Appendix E

Table 2.1, Pedagogical framework for the research studies.

Chapter 3. Research Design

3.1. Introduction

This chapter presents four sections: the research methodology, the implementation methodology, analysis methodology and description of participants. The research methodology overview focuses on the case study approach and outlines how and why it was adopted in this research. The propositions used to focus the research are outlined and the types of evidence collected for analysis to explore the propositions are detailed. The implementation methodology introduces the background to the tutorial lessons used in this project, outlines what occurs in a tutorial lesson, and justifies the use of structured-inquiry tutorials as an educational method. The ethical considerations for the students participating in the case for research is also discussed. The analysis methodology discusses the use of qualitative analysis in this project, whilst the description of participants illustrates the background of the fourteen students who took part in this project.

3.2. Research methodology

This section of this chapter discusses the research methodology employed in this research. It discusses published research on the use of case studies, primarily looking at the work of Yin (2009). The chapter then discusses how the case study methodology is adopted in this research, identifying the case central to the research and several propositions of the research. The final part of this section overviews the several types of evidence collected during this research and provides a descriptive commentary of each.

3.2.1. The use of case study in qualitative research

A case study is a study of something that occurs over time, with the subject case being a person, a group of people, an organization, other possible groups and even specific events. As empirical research into a chosen phenomenon within a context, a case study can gather both qualitative and quantitative data that can be analysed to construct conclusions (Given, 2008). Factors that affect this research include the sample size of the case being studied, and whether a single-case study or multiple case studies take place. As case studies look at individual contexts, it is not always possible to identify and control all variables that could affect the outcomes in the study, especially when research

involves multiple cases. While this can be considered a weakness of using case studies, they afford a unique opportunity to gather in-depth data that other research methods may not afford; moreover, it is often simply not possible to control all variables.

Yin (2009, p13) suggests a two-fold technical definition for a case study:

> "*A case study is an empirical inquiry that investigates a contemporary phenomenon in-depth and within its real-life context, especially when the boundaries between the phenomenon and context are not clearly evident*"

and

> "*The case study inquiry copes with the technically distinctive situation in which there will be many more variables of interest than data points and as one result, it relies on multiple sources of evidence, with the data needing to converge in a triangulating fashion, and as another result, benefits from the prior development of theoretical propositions to guide data collection and analysis*."

A case study is an appropriate methodology when the following considerations are applicable to the research:

1. A "how" or "why" question is being asked.
2. The events being researched are contemporary, i.e., they are events occurring in the present.
3. They are events in which the investigator is unable to control all the relevant variables that may affect the outcome. (Yin, 2009)

Other methods of qualitative research were considered for this research, such as social experiments, surveys, archival analysis and narrative historical accounts. However, due to the small sample size of the student cohort, the evidence collection employed in the research and the propositions use to guide the research questions discussed in section 1.3, a case study methodology was deemed appropriate.

When considering the rigor and reliability of using case study, the following advantages were identified: they

1. cope with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result,
2. rely on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result;
3. benefit from the prior development of theoretical propositions to guide data collection and analysis. (Yin, 2009)

3.2.2. Case study design

A case study is a research study of a person, group or situation over a period of time. Nisbet and Watt (1984) note that there are many strengths and benefits to the case study methodology. Case studies catch unique features that may otherwise be lost in large data-sets, and these features may hold the key to understanding the situation. As case studies tend to use relatively small sample sizes, numerous sources of data are recorded and analysed which can give a wide set of results that show the interaction of many factors being studied. They are strong on reality, in that they provide insight into an event being studied and seek to analyse the event. They can also provide insights into other, similar situations, thereby assisting interpretation of similar cases. Unlike alternative forms of research, case studies can be undertaken by a single researcher, instead of requiring a team of research for completion. Finally, they can embrace and build in unanticipated events and uncontrolled variables. When designing a case study, Yin (2009) suggests the following aspects of the research need to be considered, to implement the method efficiently:

- The research questions.
- The propositions of the research.
- The units of analysis.
- Linking of data and propositions.
- The criteria for interpreting the findings.

The research should be guided by an overall research question or set of research questions. This can guide the researcher to perform an effective literature review to inform and aid their research design. As case studies can produce a wide amount of varied data, when the researcher develops a research question, they should also identify propositions that guide the data analysis and provide a structured purpose to their research. As the sample size in this research is small, any hypothesis developed to guide the research would not be verifiable / rejectable when considering the general population. For this reason, the research is framed around propositions. Baxter and Jack (2008) explain that in a case study, propositions can be equated with hypotheses. This is justified as both have a predictable power to determine possible outcomes of the experiment/research study. Propositions are useful in that they allow the research to place limits on the scope of the study and increase the feasibility of completing the project, with specific propositions allowing for the construction of boundaries in a case study. Propositions may come from the literature, personal/professional experience, theories, and/or generalizations based on empirical data (Baxter and Jack, 2008). A case study may contain several propositions to guide the study. They are distinct from each-other and allow for a specific purpose when determine what data to collect and how to use that data to inform discussion. Each proposition serves to focus the data collection, determine

direction and scope of the study and together the propositions form the foundation for a conceptual structure/framework (Miles and Huberman, 1994).

The unit of analysis involves identifying the case that is to be researched and how it will be analysed. Over the course of research, the unit of analysis can change, due to improved designed research questions, or new opportunities for inquiry presenting themselves. When data is collected, the results can be linked back to the initial proposition stated in the research design. This allows for purposeful data analysis that links to the overall research question(s). The last consideration is criteria for interpretation, which allows the researcher to develop a manner for interpreting the analysed data to ensure it reflects what is being asked by the research. In the implementation of a case study, many sources of evidence can be collected, such as documentation, archival records, interviews, direct observation, participant-observation and physical artefacts (Yin, 2014).

Documentation involves the analysis of any documented material related to the case study. It can take the form of letters, memoranda, agendas, administrative documents such as reports, formal studies and/or evaluations. Archival records involve the analysis of archival data such as "public use files" (example, census data), service records and organisational records. Interviews allow for direct inquiry with the research participants and allow for a fluid generation of qualitative data from the interview participants. Direct observations involve directly observing the case participants, using data collection activities that range from casual to formal. In participant-observer data collection, the researcher assumes a role within the fieldwork situation and may participate in the actions being studied and record their own field notes as the research progresses. Finally, physical artefacts are the retrieval of physical evidence from the case study, with can be analysed upon completion of the evidence collection. The different strengths and weaknesses of each of these evidence collection types are presented in Table 3.1.

Using multiple sources of evidence allows for triangulation of data. The requirement for multiple evidence sources in case studies far exceeds that of other research methods (Yin, 2014), due to the method's limitations. Multiple evidence corroborating the same finding gives more weight to their validity. By triangulating data, the researcher constructs validity in their case study, and increases confidence that the research accurately renders the event being studied. Figure 3.1 illustrates the convergence of triangulated data to the same findings, and the non-convergence of un-triangulated data from separate sub-studies.

As every classroom is different and student's experiences and prior knowledge may be different, it is difficult to control all the variables that could affect student performance. To determine the progression of student's conceptual development, multiple sources of evidence are employed, discussed in-depth in section 3.4, such as pre-test – post-test comparisons, analysis of student's artefacts, teacher observations, teacher feedback, student feedback and teacher-student interviews.

This allows for the identification of patterns in the student's progression in their conceptual development and attribute this to the students completing the materials.

SOURCE OF EVIDENCE	Strengths	Weaknesses
Documentation	• Stable – can be reviewed repeatedly. • Unobtrusive – not created as a result of the case study. • Specific – can contain exact names, references, and details of event. • Broad – can cover a long span of time, many events and many settings.	• Retrievability – can be difficult to find. • Biased selectivity, if collect is incomplete. • Reporting bias – reflects (unknown) bias of any given document's author. • Access – may be deliberately withheld.
Archival records	• *[same as documentation]* • Precise and usually qualitative.	• *[same as documentation]* • Accessibility due to privacy reasons.
Interviews	• Targeted – focuses directly on case study topics. • Insightful – provides explanations as well Insightful into interactions and personal views (e.g., perceptions, attitudes and meanings)	• Bias due to poorly articulated questions. • Response bias. • Inaccuracies due to poor recall • Reflexivity – interviewee gives what interviewer wants to hear.
Direct observations	• Immediacy – covers actions in real time. • Contextual – can cover the case's context.	• Time – consuming. • Selectivity – broad coverage difficult without a team of observers. • Reflexivity – actions may proceed differently because they are being observed. • Cost – hours needed by human observers.
Participant – observation	• *[same as direct observations]* • Insightful into interpersonal behaviour and motives.	• *[same as direct observations]* • Bias due to participant-observer's manipulation of events.
Physical artefacts	• Insightful into cultural features. • Insightful into technical operations.	• Selectivity. • Availability.

Table 3.1. Six sources of evidence: Strengths and Weaknesses (Yin, 2014)

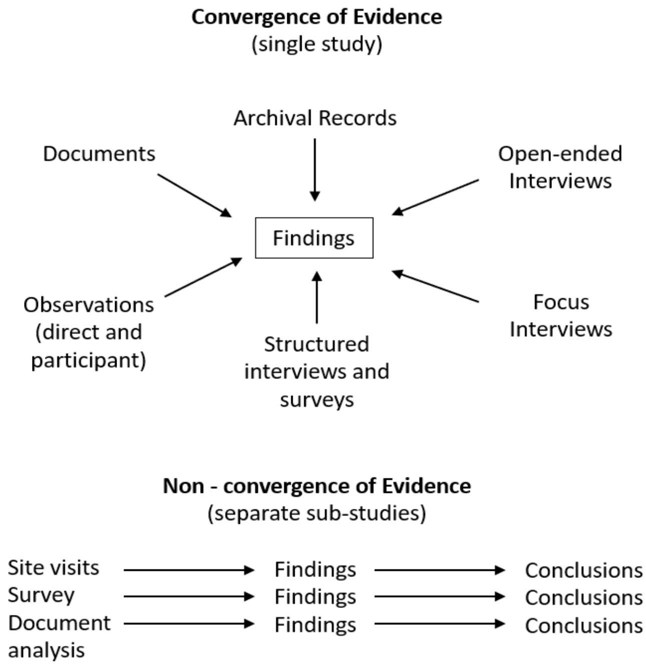


Figure 3.1. Convergence and non-convergence of evidence (Yin, 2014)

3.2.3. Case study limitations

Case studies produce unique research that can allow for understanding complex phenomena in particular contexts (Simons, 1996). The contexts tend to be unique and dynamic, allowing for investigation into complex and unfolding interactions of events, human relationships and influences and other unique factors (Cohen, *et al.*, 2000). However, the case study methodology is not without its limitations. Some researchers consider it to be a less than desirable form of research methodology, to which Yin (2009) suggests the following reasons:

- The lack of rigor of case study research, possibly due to a lack of literature on the subject when compared to other research methodologies.
- Single-case studies do not provide enough evidence for generalization.
- The time required to complete case studies, which can produce massive unreadable documents.
- Case studies do not provide enough evidence to determine “causal” relationships.

Nisbet and Watt (1984) note that results from this research (i) may not be generalizable to the wider population but may be useful to studies in which the contexts are similar, (ii) are not easily open to cross checking, and may be selective, biased, personal or subjective and (iii) prone to problems of observer bias. Simons (1996) notes that the first weakness is often cited as a case study

weakness, but this view assumes a polarity in research. However, an alternative perspective of this is that case studies allow for the development of descriptions and explanations to the case being studied, whilst acknowledging the numerous variables in the context that can affect the study. While it is outside the scope of this research to attempt to resolve these limitations of case study research, limitations of the case study methodology are acknowledged.

3.2.4. Applying a case study to the research

In this research, the case to be studied is the conceptual development and understanding of one class of upper secondary level physics students in the academic year 2016/17, in the topics of vectors, inverse square law and field lines, and how they transfer their understanding of these topics when learning about Coulomb's law, electric fields and potential difference. As described in-depth in section 3.5., the group consists of mixed gendered, mixed ability students which varies from gifted to weak students.

A case study was chosen for this group for the following three reasons. The sample size of the case being studied consists of fourteen students. Any findings generated from data from a sample of this size are not reliable to make any generalisations that apply to the wider population of upper secondary physics students. Outside of instructional design, student abilities, student attitudes and previous learning experience from different teacher influences are factors that could influence the outcomes of this case study. A case study is an appropriate method to use when extra variables that influence the findings cannot be controlled. As the influence of these variables changes from school to school, the research can also define the case as a contemporary grouping of students, which is difficult to study in other forms of qualitative analysis.

As suggested by Yin (2014), numerous sources of evidence are collected for triangulation to determine conceptual development and link how the tutorials promoted the development. Using all the evidence collected, would produce a report of unreasonable length. Collating the data and providing a varied sample of student responses results in a concise overview of student's progressions and difficulties encountered when completing the tutorial lessons. However, it is acknowledged that in some cases, the omission of some students work in favour of other's, may not provide a complete picture of what occurred during the lessons, but it is endeavoured to keep this to a minimum. A final consideration is that while the tutorials are likely the only intervention the students use to explore concepts related to the research, it is acknowledged the evidence can only provide indications that attribute the developed materials to the student's conceptual gains but cannot provide a causal link between the two.

In drafting the propositions that underpin this research, relevant sections of the thesis that cover the background used to inform them are referred to. The propositions used in the case study to frame the research and determine how to collect data are:

1. Tutorial lessons are a good teaching methodology to enable students to develop conceptual understanding in physics topics. (Section 3.1)
2. Students that learn a concept through, and develop the ability to transfer between, multiple representations to develop their understanding of that concept. (Section 2.2., section 2.3., section 3.3
3. Vectors, the inverse square law relationship and field line representations are tools, that when mastered by students, will enable them to describe electric fields, and the interactions of charges with electric fields. (Chapter 4)
4. Vectors and field line representations are tools, that when mastered by students, will enable them to describe the work done in moving a charge in an electric field, and conceptually explain the potential difference between two points in an electric field. (Chapter 5, chapter 6)

3.2.5. Evidence collection in this study

In this study, five types of evidence were collected. These evidence types were categorized as primary and secondary evidence. The primary evidence is used in all aspects of the data analysis through all concepts covered by the students and is used to glean the conceptual understanding developed by the students. The secondary evidence is used to support of the findings from analysing the primary data, but in and of itself, it not rigorous enough to construct meaningful findings on their own. The primary evidence types are used to pre-tests / post-tests, student artefacts and teacher – student interviews, while the secondary evidence are recordings of student dialogues and teacher reflections.

3.2.5.1. Pre-tests and post-tests

Pre-test / post-test comparisons are used in this research to gauge conceptual development of student's models over the course of the inquiry lessons. Each question on the pre-test is designed to elicit student's understanding of one aspect of a concept. An ideal pre-test question is structured in a manner that only correct understanding of the concept being asked about can bring about a correct answer. However, when asked to predict outcomes or produce rankings, it is possible for students to guess a correct answer, and so students are also required to display the reasoning they used to inform

their answer. The contexts of the questions are typically simple ones, so students are not influenced by the context itself. Additionally, by designing the pre-test questions to be as simple as possible, the students would be less likely to be put off answering the question by the perceived difficulty of the pre-test.

The post-test questions were designed in the same format as the pre-test questions. In some questions concepts were also asked in a manner that only one aspect of a topic was tested in any one question and correct understanding of the concept was required answer to the question correctly. The questions are not designed to guide students through the question, as they are in the tutorial. Additionally, there were questions asked that required the use of multiple concepts to answer correctly. By comparing student's answers between their pre-tests and post-tests, indications to the level of conceptual change that occurred could be observed. This provides evidence to judge the effectiveness of the instructional materials and the determination of what revisions need to be considered for redesigning future materials.

Students completed pre-tests in class, before completing the tutorial and after the teacher presentation of the topic. These tests were scanned and stored on an external hard-drive. Each pre-test was analysed, and student responses were categorized based on concepts / misconceptions used in answering the questions. These categories were used in the post-test questions, allowing for direct comparison between student answers in both tests. Further analysis of student's understanding was then gleaned from specifically looking at the student's articulation of their answers and their direct application of the concepts when answering questions. In this case, student's understanding must be interpreted from their written responses, especially in cases where the student used common colloquial language, instead of physics specific terminology and it is acknowledged that while this is a strong indication of student thinking, it cannot provide a complete representation of it.

3.2.5.2. Student artefacts

Samples of the student tutorial materials and the homework assignments were collected and scanned. The homework assignments were categorized in the same manner as the pre-tests and post-tests, while the tutorials were reviewed to identify where particular students encountered difficulties. When this occurred, students were requested to not destructively erase their earlier writing, and instead use extra space, to allow for the comparison of incorrect and correct reasoning produced by the student. This gives insight into student thinking, that is not afforded in the pre-test/post-test comparison. The use of the pre-test/post-test model can struggle to identify the source of difficulties in student reasoning. In analysing student artefacts, it is possible in some instances to pinpoint where student thinking diverged from what was the intended reasoning. In class, this was used by the teacher

to consider what line of questioning and prompts were required to enable the student to revise their thinking and develop news lines of thought to arrive at the correct conceptual understanding.

The homework assignments allowed students to review and extend their understanding of concepts covered in the tutorial. In some cases, questions from the pre-test appeared in the homework assignments. This gave the opportunity to observe how students answer the same question to determine if there is any change in their answers. When the homework assignment was designed to allow students to review the lesson material, it was possible to gauge whether there was divergence in student thinking when in a group setting in the tutorial or reaffirm correct reasoning between both the tutorial and the homework assignment.

3.2.5.3. Teacher-student interviews

The teacher-student interviews were conducted after the students completed the tutorial lessons. Students who completed the interviews were generally identified as having difficulties with concepts covered in the tutorials and post-tests. This allows for the use interviews as an extra mechanism for the student to help students alleviate their difficulties and provide insight to the source of their difficulties.

A small 20-minute tutorial worksheet is developed for these interviews, which explore the same concepts as those seen in the tutorials, but in an unseen context. As the interviewer, I provided prompts and scaffolds to the students, as they complete the worksheet. As students answered questions, I could ask the students to explain the reasoning used by the students to develop their answers. This gave insight into what the students were thinking and allowed them to articulate their reasoning more in depth than the reasoning some students submitted in the tutorials. I could also observe the students discussing the questions with each-other and take note of how the students developed and supported each-other’s reasoning over the course of the teaching and learning interview. The interviews also allowed me to produce a record of what interventions and prompts I used that were effective in facilitating student conceptual development.

Engelhardt, *et al.,* (2004) highlight this as an advantage of using teaching-learning interviews. The teaching-learning interview also provides an opportunity to continually probe student’s understanding in a manner not afforded when using post-tests. When students provide assertions, the interviewer can question the nature of these assertions. If the student’s explanation is not complete, the interview can take time to explore their explanations, so the student clearly articulates their thinking, in a manner that they do not consider in the tutorial lesson. However, as this focus on student’s specific explanations of their reasoning is atypical of the depth feedback generally provided in the tutorial lessons, the interview does not accurately reflect the learning experience the student

encounters in a classroom. Chini, *et al.*, (2009) note that this is a limitation of the interview, in that, what interventions work in this setting may not work in a classroom setting.

3.2.5.4. Recordings of student dialogues

Student conversations that took place during the lessons were recorded. As the students worked together to complete the materials, developments in student's conceptual understanding that were verbalised may not be recorded in the artefacts. Having recorded student's conversations, it is possible to analyse the reasoning used by students in their discussions to observe how student difficulties were overcome. Due to the massive amount of data generated, and the time required to analyse the data, the students were asked to record the time on the dicta-phones when they felt that other students helped them overcome difficulties they encountered during the lessons. The teacher also recorded the time in which they used questions and prompts to help students develop and consolidate their thinking.

3.2.5.5. Teacher reflections

Upon completing of the tutorial lessons, the teacher drafted reflections, based on his observations and feelings on what felt worked and did not work in the lesson. In a teacher reflection, the teacher engages in a cognitive process, which involves providing a commentary on difficulties encountered during the sequencing of the lesson. Reflective thinking generally addresses practical problems, allowing for doubt to be used as a mechanism to identify problems and shortcomings before solutions are reached (Hatton and Smith, 1995). By reviewing the teacher reflections, a comparison of the teacher's feedback and recordings of student dialogues can allow for the identification of student difficulties to take place. Specific difficulties that the teachers encountered during implementation can also be identified and open a conversation space to discuss ways to alleviate encountered difficulties.

3.3. Implementation

This section of the research discusses the implementation methodology for this research. It describes different influences used to develop the tutorial lessons implemented in this research. The structure and format of a tutorial lesson and details their implementation are then presented. The justifications of using tutorial lessons with the second level students is discussed. Additionally, as

the participants in this research were under 18 years of age, the ethical considerations that were made, and the approval by DCU's ethics committee is outlined in this section.

The approach adopted in this research is the use of structured inquiry. Instead of students being presented with the content and required to learn it off, learning occurs through students answering questions, solving problems and working through challenges (Lynn, *et al*., 2013). As seen in section 1.3, improving student's conceptual understanding is central to the research questions of this research, and inquiry has been shown to be effective in this pursuit (Tabak, *et al.*,1995; Blanchard, *et al.,* 2010). The tutorial lessons developed in this research as the teaching and learning materials, are based on the approach from Tutorials in Introductory Physics (McDermott and Shaffer, 2003). The use of tutorial lessons has been shown to promote gains in student's conceptual understanding of physics concepts (McDermott and Shaffer, 1992; Wosilait, *et al.*, 1998). As stated in section 2.2.2, exercises from the Conceptual Physics practice book, 10th edition (Hewitt, 2009), were used to influence the design of the tutorials in this work, as the way Hewitt approaches contexts would be more accessible to the second level students. Sections 3.3.1 and 3.3.2 discuss how inquiry was employed in this research, by detailing what a tutorial lesson looks like, and justifying the use of inquiry in this research.

3.3.1. What does a tutorial lesson look like?

The following description of a tutorial has been patterned after Tutorials in Introductory Physics – Teachers Guide (McDermott and Shaffer, 2003). It summarizes what occurs when students complete a tutorial lesson, from start to finish. The logistics of implementing tutorial lessons into a second level classroom are also described.

A tutorial lesson is designed to supplement lectures and textbook materials in Physics courses. Students do not focus on reciting definitions, listing how to complete demonstration experiments, or completing quantitative problems. Instead, the focus is shifted on students developing their understanding of physical concepts and the use of their scientific reasoning skills. Tutorials provide a structured format for students to determine what they do and do not understand when learning physics. In small groups, students complete a series of questions in which they are guided through the reasoning necessary to construct scientifically valid models of a chosen topic in physics. They provide opportunity for students to interpret and represent concepts using a variety of representations, such as formulae, graphs, diagrams and verbal descriptions. Students take part in the tutorial lesson after they have been initially introduced to a concept through a lecture, presentation and/or laboratory, but can be used to introduce a concept or extend it.

As discussed in-depth in section 2.2.2., this research developed tutorial lessons to promote the student's conceptual understanding. The tutorial lessons comprise of a pre-test, worksheet assignment, homework assignment and post-test. The pre-test examines student's prior knowledge and understanding of a concept. This allows for the identification of difficulties to be targeted for conceptual change. The worksheet assignment is completed by the students in small groups. They work through the exercises together, engaging in dialogues between themselves or with the teacher when difficulties are encountered. The conditions of conceptual change (Posner, *et al.*, 1982) are generally encountered and met at this stage. The homework assignments are completed by the students, which gives them the opportunity to revisit the concepts from the worksheet or in some cases extend them. The context of the homework may or may not be altered from contexts in the worksheet exercises. The post-test is given at the end of the tutorial lesson. It is written to focus on the concepts and skills used in the tutorial lesson. Comparisons between the pre-test and post-test allow for the generation of evidence that can be used to identify and indicate the extent to which conceptual change occurred.

With the participants of this research, they completed a lesson in which concepts were introduced through power-point presentations, demonstrations and practicing qualitative problems and classroom discussion. These lessons occurred in either a single lesson period, (40 minutes in length) or a double lesson period (80 minutes in length). The students then completed tutorial lessons in a double lesson period. The materials themselves were written with the materials designed to take up to 50 minutes of class time, which affords time at the start of the lesson to be used to take the pre-test, and time at the end for a quick round up with the class in their entirety.

When covering inverse square law, vectors and field lines, the students completed one tutorial a week, and the pre-tests, homework assignments and post-tests are completed around the tutorial. During the section on electric field, a traditional lecture style class introduced concepts related to charge and Coulomb's law, and the pre-test and tutorial on Coulomb's law followed. Students were then given another lecture style lesson introducing the electric field and completed quantitative problems. Students then completed all the electric field tutorials in succession and a post-test based on Coulomb's law and the electric field was completed. When the students completed the tutorial lessons on work and potential difference, they completed the pre-test before completing the work tutorial, engaged in two tutorial lessons, one on work and one on potential difference, before completing the final post-test for these two topics together. An in-depth timeline for the implementation of the tutorials is presented in Chapters 4, 5 and 6.

3.3.2. Justifications for using inquiry tutorials

In section 2.1, the role of inquiry in science and physics education was discussed, and in the previous section, the application of inquiry in this research, using tutorial lessons, was outlined. This section provides justification for the use of structured inquiry in this research, in terms of the efficacy in promoting targeting and promoting conceptual understanding, how it aligns to some of the aims of the Leaving Certificate Physics syllabus (1999) and how structured inquiry allows for balance between content requirements and conceptual depth of treatment.

Structured inquiry was utilised in this research to promote conceptual change in the students understanding. Structured inquiry was chosen as it has been shown to be a favourable method to develop conceptual knowledge in students (Tabak, *et al.*, 1995; Blanchard, *et al.*, 2010). Ambrose (2004) notes that it is effective for targeting specific concepts for conceptual change. Section 2.1.3 illustrated the effectiveness of structured inquiry tutorials by detailing one example from research (Shaffer and McDermott, 2005) and referencing others (Close and Heron, 2010; Heron, *et al*., 2004; Hazelton, *et al.*, 2012). This research initially targets students conceptual understanding of vector concepts, the inverse square law and field line and then targets how they employ these topics to develop their understanding of Coulomb's law, the electric field, work and potential difference. Structured inquiry is employed in this research as evidence from the research literature suggests it is an effective method to achieve the teaching and learning outcomes that unpin this research.

Tutorial lessons involve learners working in small groups to discuss their thoughts, ideas and understanding of concepts that underpin the topics they are studying. This provides them the opportunity to articulate their thinking and communicate to each other to collate their understanding. This is in line with the aims of the Leaving Certificate Physics syllabus (NCCA, 1999, pg2) which states students should develop "an understanding of the fundamental principles of physics" and "develop the ability to observe, to think logically, and to communicate effectively." The tutorial lesson format provides students the opportunity to attain these aims.

Finally, as tutorial lessons allow for the targeting of specific concepts, the lessons were developed in a manner that the core learning outcomes for the Leaving Certificate Physics syllabus (1999) were met. The depth of understanding targeted by the research was balanced with the content the students were required to learn for completion of the course. This way, the students have completed the required content to complete questions on their terminal examination. Other forms of inquiry, such as guided and open, can result in students learning content not targeted by the teacher, and may miss specific content designated by the learning outcomes of the course they follow, but develop their competencies in their cognitive and procedural autonomy (Stefanou, *et al*., 2004). Guided and open inquiry was not employed in this research, as developing these competencies were not targets in this research.

However, there are notable difficulties in applying general inquiry methods to a classroom. These difficulties include, but are not exclusively, the workload involved, lack of resources and the effect the increased workload has on the teaching time required to finish a syllabus (Higgins, 2009). To address these difficulties, the materials are designed and prepared during school breaks, to not interfere with the time allocation to the teacher in preparing lessons. The topics in which the lessons are used, electric field and potential difference, take up a relatively small part of the overall physics course, and thus, the time allocated to completing the classes is minimal. Ambrose (2004) notes that structured tutorials allow for an inquiry implementation in a relative short duration of time. As the tutorials are mainly paper and pen focused, no special laboratory equipment needed to be procured to complete the material and any practical equipment the can be of use is generally readily accessible in the standard fit-out of a second level physics laboratory.

3.3.3. Ethical considerations for research involving second level students

To conduct this research, ethical approval was sought from the Ethics Committee of Dublin City University. As this project involved the analysis of data taken from students, it was important for the university to ensure that every effort was taken to ensure the educational experience provided to the students was well planned and would not hinder their academic progress in their physics course. The allotted time that could be spent on the research was also considered, to ensure they experienced minimal disruption to the lessons required to complete their physics course.

Other issues considered when seeking ethical approval for this project was honesty in reporting results, both positive research and negative results, identification of any potential bias, as the funding body are involved in the promotion of inquiry based learning, and human subjects protection, provided as an opt out of the research and communicating with the school care team regarding student welfare during the project, in the event that students were to find the teaching approach adding undue stress to their educational environment. Approval for this project was granted December 2013 (DCUREC/2013/197) for the pilot trail of the materials and amended September 2014, to include the additional schools for garnering data for external validation purposes.

All students were given two copies of an informed consent form, two plain language statements and a cover letter briefly outlining the aim and rationale of the project. As the students were under 18 years of age, their parents / legal guardians were required to sign the consent forms and return one copy to the teachers involved. Parents were informed of the aims of the project and given the opportunity over a two-month time frame to contact me with regards to any concerns regarding the project, prior to its commencement. Parents were also informed that their children could opt-out of the research at any time, in which any data recorded would be destroyed electronically, and paper copies of materials would be shredded.

Each student's name was known only to the teachers involved in the classrooms. This allowed for the comparison of pre-test data, post-test data, class worksheets and homework worksheets of individual students. Each student was given a unique reference code, which is used to identify him or her in this document. The reference code is made up of a number and a letter. The number referred to the year of the research and the letter referenced the student. In the case of the 2014/15 academic year, the second group to complete the material were given two letters. For example, student 2C took part in the 2014/15 editions of the materials, is from the second group of students to complete the materials and is the third person on the class roll as denoted by C being the third letter of the alphabet.

3.4. Analysis Methodology - Qualitative explanatory and qualitative descriptive

There are numerous types of case studies, as outlined by Baxter and Jack (2008), such as explanatory, exploratory, descriptive, intrinsic, instrumental and collective. The type of case study devised in this research contain elements of both an explanatory and a descriptive case study. Explanatory is when a case study would be used if you were seeking to answer a question that sought to explain the presumed causal links in real-life interventions that are too complex for the survey or experimental strategies. In evaluation language, the explanations link program implementation with program effects (Yin 2003). Descriptive is a type of case study is used to describe an intervention or phenomenon and the real-life context in which it occurred (Yin, 2003).

As the sample size of the students is small, quantitative data and quantitative data analysis tools cannot be used reliably. Instead descriptive qualitative data analysis is used, as it gives insight into understanding the context, participants and interventions encountered when the research was conducted. In this manner, a collection of data that allows for understanding the learning process that occurs during the tutorials lessons applied during the tutorial lessons could be generated. The data analysis allows for the drawing of patterns based on the progression of student conceptual development and provides illustrative explanations of their conceptions, based on their individual responses.

In the research, the educational instructional intervention used in the research is the developed inquiry materials used in a tutorial lesson. A case description (Yin 2009) is developed on the theoretical propositions discussed in section 3.2.2. Over the course of chapter 4, 5 and 6, the implementation of the tutorial lessons is described as an observational narrative. An analysis of the student's conceptual developments in vectors, inverse square law and field line representation is detailed in chapter 4 and provide a commentary on how these developments enable students to explore Coulomb's law, the electric field and potential difference.

Through this narrative, content analysis (Carley, 1993), is used to describe how the tutorial lessons promoted conceptual gains in the student models, using multiple forms of evidence gathered, as described in-depth in section 3.2.3. In the content analysis, the evidence was primarily used to populate matrices of student responses and concepts/misconceptions apparent in the data, to determine whether students are using key concepts in their correct contexts or overlapping their understanding (Yin, 2003). These matrices were analysed to produce the results tables shown in Chapter 4, 5 and 6. As a consideration to remove potential bias, initial blind marking was used when populating these matrices. This was done by removing the codes from the scanned student artefacts and initially recorded the frequency of the concepts used and difficulties observed in the student's responses. The artefacts were then analysed again, with the student codes intact, and were used to populate the student responses, as shown in the tables of results throughout chapters four, five and six.

An assumption of the research that students developing a functional conceptual understanding of vectors, inverse square law and field line representation will result in positive conceptual gains in electric field and potential difference, as explained in various sections of chapter 2. This is tested for using a type of pattern matching, determining whether the concepts used in the later topics are transferred from the former topics, and provide a narrative on how the students use these initial concepts to build their understanding of the electric field and potential difference. In the discussions of each section, instances of conceptual change will be discussed, collating evidence from as many of the various the evidence types as possible to identify conceptual extinction, exchange and extension where possible. If conceptual change did not occur, difficulties are identified and suggestions for redesign are given.

Four descriptors are selected in this research to indicate the extent of conceptual change that occurs over the course of this research. As shown in Figure 3.2, these descriptors are based on the total number of 14 students that were studied to measure their conceptual change for a given concept. Minimal conceptual change refers to instances in which between one and three students demonstrated that conceptual change occurred. Partial conceptual exchange refers to instance where between four and seven students were observed to have engaged in conceptual change. Moderate conceptual change is referred to when between eight and eleven students engage in conceptual change and ideal conceptual change is referred to when between twelve and fourteen students demonstrate conceptual change.

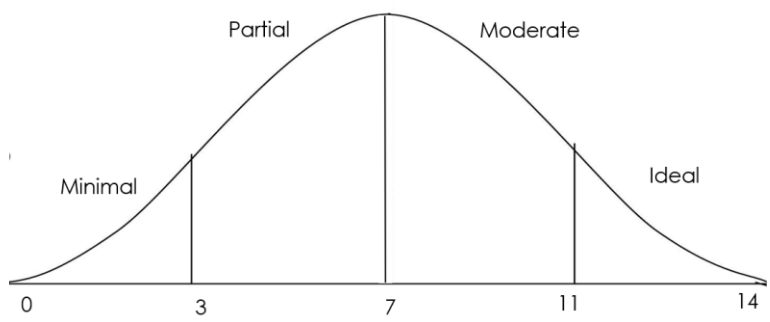


Figure 3.2. Illustration of descriptors used to describe extent of conceptual change within a group of 14 students.

3.5. Description of participants

The group of students that undertook this research was a group of 14 mixed ability students, aged 16 – 17 years old. The group is mixed gendered, but predominantly male (female = 4 students, male = 10). Three of the students speak English as a second language but are fluent in their use of the language. One of the students took part in the educational system of New South Wales, Australia for all the years they spent in formal education, before enrolling in the upper secondary Irish educational system in the academic year 2016/17. To illustrate the general ability of the students in science and mathematics, Table 3.2 shows the student's results from their lower secondary level final examinations in mathematics and science, which were produced by the State Examinations Commission. A copy of the results was obtained from the school where the research took place. These exams are unseen by students and teachers in Ireland, and all students partake in these so-called Junior Certificate examinations at the exact same time. The grades are presented as categorical ordinal data, with A being the highest grade and NG being the lowest grade. The table also includes their results from an in-house Christmas physics examination. This is an indication of the general ability of the students, having studied Leaving Certificate Physics for 4 months. This examination took place after the vector, inverse square law and field line tutorial lessons, but before the electrostatics tutorials. The grades for the in-house test are presented as categorical ordinal data, with H1 being the highest grade and H8 being the lowest grade, mirroring the grading system of the Leaving Certificate exam they undertake at the end of 6th Year.

Figure 3.3 presents the Junior Certificate exams results in bar charts, and the in-house Physics examinations results are shown in Figure 3.4. This allows for identification of the clusters of the results, to create a profile of the student's academic achievements in these exams. As both charts show the cluster of data centralized towards the higher grade, the student's results suggest that the students are a mix of high achieving and average achieving students.

Student Code	Junior Results **JC Science**	 **JC Maths**	Senior Results **5th Year Christmas**
4A	C	C	N/a
4B	B	B	H1
4C	B	B	H2
4D	B	D	H5
4E	A	B	H3
4F	N/a	N/a	N/a
4G	A	A	H1
4H	B	C	H4
4I	B	B	H1
4J	C	D	H2
4K	A	B	H3
4L	C	D	H6
4M	B	D	H6
4N	A	A	H2

Table 3.2. Summary of student's results pre-research.

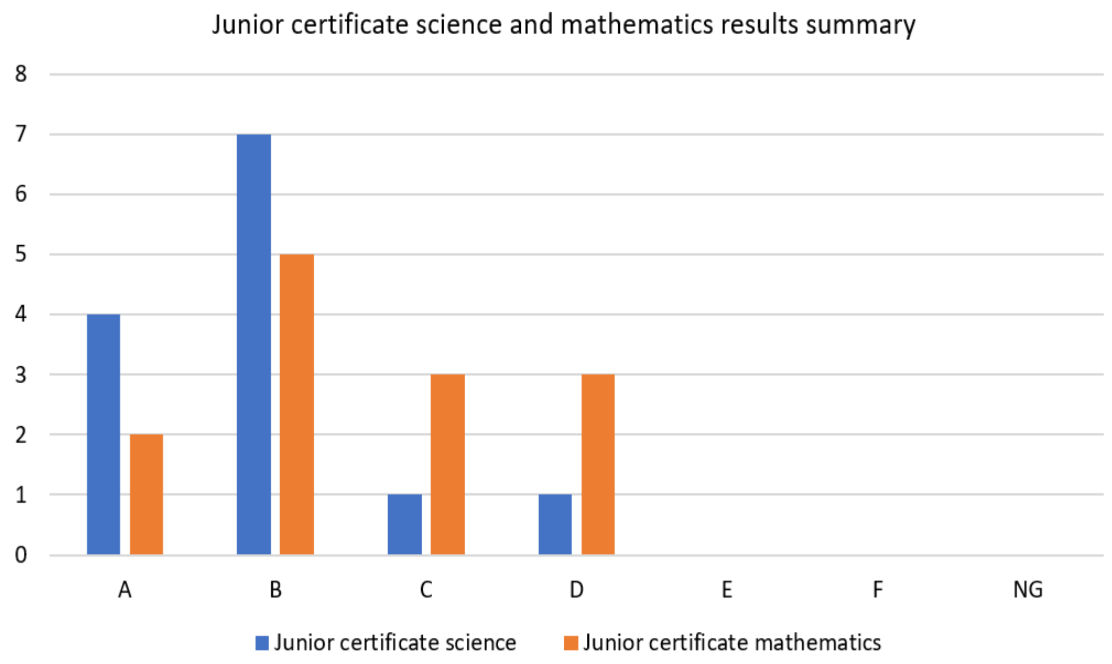


Figure 3.3. Student's results for Junior Certificate Science and mathematics examinations.

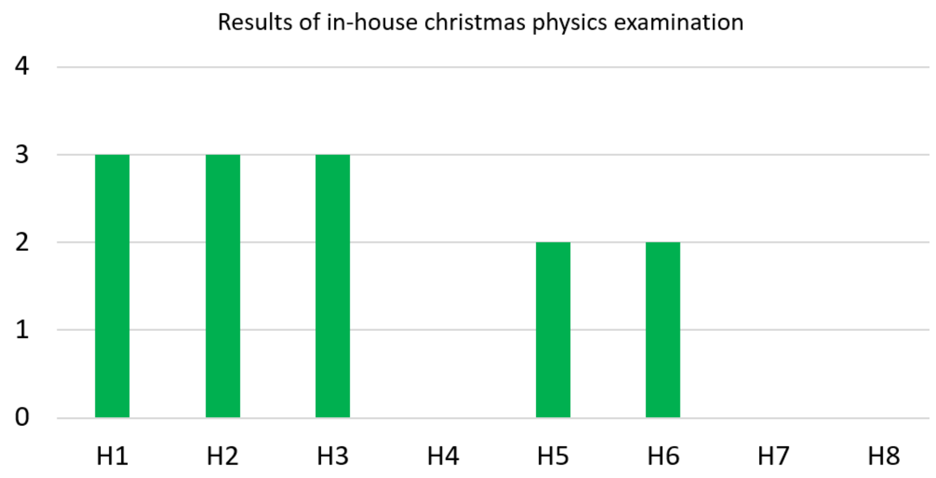


Figure 3.4. Student's results of in-house physics Christmas examination.

3.5.1. Participants' prior learning

This section discusses aspects of the student's prior learning in formal education, for lower secondary level science and mathematics. As the lower secondary level science and mathematics course are delivered for a duration of three years, this section will only reference the content covered in these subjects that are relevant for this research. As the students had different teachers for lower secondary science and mathematics, the aims and learning objectives from the syllabi for Junior

Certificate Science (NCCA, 2003) and Junior Certificate mathematics (NCCA 2012) will be referenced to illustrate the student's prior learning in these subjects.

According to the Junior Certificate Science syllabus (NCCA, 2003), the study of science contributes to a broad and balanced educational experience for students, extending their experiences at primary level. The course aims to promote student's development of scientific literacy, science process skills, and an appreciation of the contribution that science had on the humanity and the planet. The junior science course was designed to be investigative and activity based, and the courses' aims (NCCA, 2003) seek to encourage student development of manipulative, procedural, cognitive, affective and communication skills. Student are to be provided with opportunities for observing and evaluating phenomena and processes and for drawing valid deductions and conclusions, which aids to enable students to acquire a body of scientific knowledge appropriate to their age, and an understanding of the relevance and applications of science in their personal and social lives. The nature of the activities and investigations also aim to help students develop a sense of enjoyment in the learning of science. However, the extent to which teachers use lessons that allow student to engage in activity or investigative based exercises is not always in line with the extent intended by the syllabus (Weymms, 2008).

No matter the manner of instruction chosen by a lower secondary science teacher, the students must have completed the same learning objectives, as part of their lower secondary science course. The design of the tutorial materials identified the following objectives are relevant to the research, and considered the following to be required prior learning completed by the students:

- appreciate the concept of force; recall that the newton is the unit of force; describe forces and their effects.
- investigate examples of friction and the effect of lubrication.
- investigate the relationship between the extension of a spring and the applied force.
- recall that weight is the force due to gravity and that weight can vary with location; recall that mass in kilograms multiplied by 10 is approximately equal in magnitude to weight in newtons on the surface of the earth.
- carry out simple experiments to show attraction and repulsion between magnets and test a variety of materials for magnetism.
- plot the magnetic field of a bar magnet.
- demonstrate that the Earth has a magnetic field and locate north and south.
- use simple materials to generate static electricity; demonstrate the force between charged objects and the effect of earthing.

In the Junior Certificate mathematics syllabus, (NCCA 2012), it states that mathematics is the study of quantity, structure, space and change. In Mathematics, students develop skills in numeracy,

statistics, basic algebra, shape and space, and technology that have many uses in society. These skills can be utilized to allow students to make calculations and informed decisions based on information they come across in their everyday lives. Students also develop the skills to become good mathematicians, which allows them to compute and evaluate a calculation, follow logical arguments, generalize and justify conclusions, and apply mathematical concepts learned to real-life situations.

In the design of the tutorial materials, I took note of the learning objectives (NCCA, 2012) in the mathematics syllabus as relevant to the research. The mathematical aspects of the tutorial lessons were designed so they would align with the student's prior knowledge. This include students understanding co-ordinate geometry, the application of the rules of indices, the use of scientific notation. The algebraic understanding included the use of tables to represent a repeating-pattern situation, generalizing and explaining patterns and relationships in words and numbers. Writing arithmetic expressions for terms in a sequence and using tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations. When using tables, graphs and equations, the students would have learned to distinguish those features that are appear in the different representations, use the representations to reason about situation from which relationships can be derived and communicate their thinking to others and recognize problems involving direct proportion and identify the necessary information to solve them without formulae. Finally, the students have completed the using of graphs to represent various phenomena in different contexts including motion graphs, interpreted quantitative graphs, make connections between the shape of a graph and the story of a phenomenon and describe both quantity and change of quantity on a graph. In addition to considering the learning objectives, all the mathematical understanding and operations related to algebra (NCCA, 2012, pg. 28) is considered prior learning for this research.

This section of the description of participants illustrated the student cohort who agreed to undertake in this research. A description of the students was presented, results from both state examinations and an in-house exam were presented to help gauge the progress of the students in formal education in science, mathematics and physics thus far. An in-depth look at the specifics of the Junior Certificate Science and mathematics courses was presented, highlight the aims of both courses, and the objective completed by the students which are relevant to this research.

Chapter 4. Vectors, inverse square law and field lines

4.1. Introduction

This chapter addresses the development of student's understanding of vector concepts, the inverse square law, and field line representations as they completed the tutorial lessons. This chapter aims to address the first three of the research questions:

1. To what extent does the use of a structured inquiry approach develop student understanding of vector concepts?
2. To what extent does the use of a structured inquiry approach develop student understanding of the inverse square law relationship, by employing multiple representations
3. To what extent does the use of a structured inquiry approach develop student understanding of the field, when utilising field line representations?

These questions are important to the overall research as vectors, field lines and the inverse square law are the foundational topics for promoting conceptual understanding of Coulomb's law, the electric field and potential difference. We use structured inquiry tutorials to develop the student's understanding of three representations, as this allows us to (i) target specific concepts for the students to develop, (ii) design lessons that can easily be designed to be completed in a 60-minute timeframe and (iii) put the emphasis on the students to explore and develop their own understanding of vectors, field lines and the inverse square law. Having developed an understanding of these topics, the cognitive demand on the students should be lessened when they apply and utilise them in a electrostatics context, and instead they can focus on their understanding of the electrostatics concepts. In Vygotskian terms, when the students come to develop understanding of electrostatics, the required concepts for vector representation will have been moved from the student's zones of proximal development to actual development (Vygotsky, 1979). Multiple representations are employed, as Ainsworth (1999) shows that they can have many positive roles in the learning process, such as displaying difference processes and information, constraining the content and concepts learned by the nature what the representation can communicate, and constructing deeper understanding.

The tutorials, in the style of Tutorials in Introductory Physics (McDermott and Shaffer, 2003), utilise scaffolding questions to help students construct and develop understanding of a particular topic or concept. Breaking a topic down into small questions prevents the students from being overloaded by trying to assimilate too many new concepts at any one time. This approach also allows

for the observations of student difficulties as they progress through the tutorial, allowing for identification of the cause of the conceptual difficulty.

Figure 4.1 depicts where the topics fit into the student's broader understanding of Coulomb's law, the electric field and potential difference. Items presented in the left box are completed at Junior Certificate, items at the bottom are completed during Leaving Certificate Physics course, items on top are addressed specifically by this body of research and items on the right are the electrostatic topics covered by this research, discussed later in chapter 5.

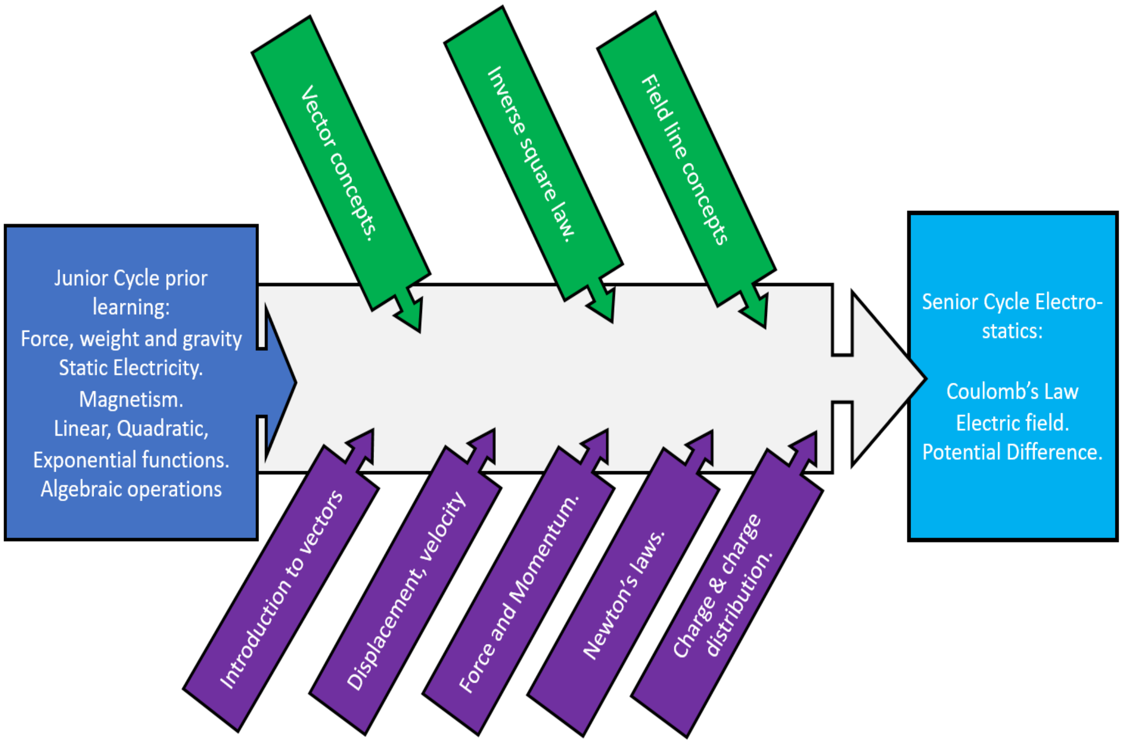


Figure 4.1. Flowchart depicting how the topics in this chapter contribute to electrostatics.

The timeline of this section of the project is shown in Table 4.1. Sections in bold refer to materials covered as they related to the research. Sections that are not presented in bold are required to be covered for completion of the required syllabus for Leaving Certificate Physics, but did not contribute to the collection of data used in this research. Week 1 denotes the beginning of the students studying mechanics, in their physics course. This coincided with the commencement of the first half of the research. These classes ran from the second last week in October to the second week in December 2016.

The order of the tutorials, (vectors, inverse square law and field lines) followed the order in which I normally teach the Leaving Certificate Physics course. This allowed for minimal disruption to the annual planning of topics. The vectors tutorial was implemented first, as it is one of the initial mechanics topics presenting in the Leaving Certificate Physics syllabus (NCCA, 1999). The inverse

square law and field lines tutorials followed later as Newton's gravitational law was contextually the first topic these representations could be applied to.

Week 1.	**Vectors Pre-test, vectors worksheet, vectors homework and vector constructions.**
Week 2.	Topics unrelated to project: Inclined planes, Momentum and Forces
Week 3.	**Vector Post-test.** Topic unrelated to project: Forces and Motion.
Week 4.	**Inverse square law pre-test**. Newton's law of gravitation. **Inverse square law tutorial.**
Week 5.	**Field lines pre-test. Field lines tutorial. Field lines homework. Further Newton's law of gravitation.**
Week 6.	**Inverse square law and field lines pre-test.**

Table 4.1. Timeline of the implementation of the first section of the research.

Section 4.2 covers the development of the student's understanding of vectors. Section 4.3 reports on the intervention used to promote the student's understanding of the inverse square law. Section 4.4 presents the development of the student's understanding of the use of field line presentation. In each of these three sections, a brief introduction presents common difficulties seen in literature by students used to form the learning objectives of the tutorial lessons. This is followed by a discussion of the student's initial understanding is elicited from the pre-test results, their development of the concepts is presented with a narrative of the tutorial lessons and homework assignments and their final understanding is delivered through an analysis of the post-test results. Each of the sections close with a discussion that compares the pre-test and post-test results. Difficulties targeted for conceptual change are identified and instances of Posner's conditions for conceptual change (1982) are referenced. Discussions of the post-test illustrate instances of conceptual change, when apparent. Finally, Section 4.5 presents the conclusions of this chapter, which discusses the progression of the student's understanding of the three different representations and implications for the use of the concepts in Coulomb's law, electric fields and potential difference.

4.2. Vector concepts

This section presents a narrative and analysis of the development of student's understanding of vectors. Section 2.1.3.1 detailed difficulties encountered by learners in understanding vector concepts

identified in the literature. The design of the vector tutorials focuses on student understanding of three introductory vector properties. Upon completion of the teaching and learning material, the students should be able to:

1. Differentiate between the magnitude and direction of a vector (Nguyen and Meltzer, 2003; Ivanov, 2011).
2. Demonstrate vector addition using the parallelogram and/or "tip to tail" constructions (Nguyen and Meltzer, 2003; Hewitt, 2009).
3. Consider vector components when adding vectors that are non-colinear nor perpendicular (Flores, *et al.*, 2008).

The inquiry approach consisted of a pre-test, a tutorial lesson, a homework and a post-test. This intervention ran over three weeks. A timeline for the implementation of this part of the study, including the target concepts for the intervention, is shown in Table 4.2. The vectors pre-test completed by the students was the first pre-test for the research, and the first experienced by the students. It was explained to them that the function of the pre-test was to allow for an indication of what they did and did not know about a given topic, that could later be used with the end of topic test (post-test) for comparative purposes.

Section 4.2.1 presents the pre-test results, looking at the difficulties the students have with representing vector magnitude, use of the parallelogram and "tip to tail" constructions and their prior understanding of horizontal and vertical components. Section 4.2.2 presents a narrative of the development of the student's understanding of vector concepts during the tutorial lesson. Section 4.2.3 presents an analysis of the homework assignment, which was developed to allow the students to practice the skills and apply the understanding they developed in the tutorial. Section 4.2.4 presents an analysis of the post-test results which, like the pre-test, focused on student's understanding of representing vector magnitude, the use of the parallelogram and "tip to tail" constructions and their understanding of vector components. Section 4.2.5 presents a comparison of the pre-test and post-test results, and a commentary of the student's progress during the tutorials. Examples of student progression and difficulties that persisted throughout the tutorial and post-test are discussed, and instances in which the conditions for conceptual change (Posner, *et al.*, 1982) were apparent during the tutorial lessons are discussed.

Time		Research Implementation	Target Concepts
Week 1	Class 1.	Pre-test	Relating magnitude to length of vector.
	Class 2.	Tutorial Lesson Homework	Parallelogram / Tip to tail construction of vector addition. Horizontal and Vertical components.
	Class 3		Lesson reserved for addressing difficulties observed during tutorial and homework, and extra time to practice using vector constructions.
Week 2		*N/a*	*Topics unrelated to project: Inclined planes, Momentum and Forces*
Week 3	Class 1	Post-test	Relating magnitude to length of vector. Parallelogram / Tip to tail construction of vector addition. Horizontal and Vertical components.

Table 4.2. Timeline of the implementation of the vector concepts study.

4.2.1. Pre-test: Vector Concepts

The pre-test on vector concepts was completed by the students before any instruction was delivered for vector concepts. As 12 of the 14 students study Applied Mathematics, they were familiar with some concepts related to vectors. Therefore, the pre-test could be used to identify what concepts the students understood before the tutorial, and what conceptual gains observed could be attributed to them completing the tutorial lesson. The students were given 15 minutes to complete the pre-test. The target concepts were magnitude of a vector and vector addition.

The first question, shown in Figure 4.2, looked at student's understanding of vector magnitude. The students were asked to rank the magnitude of five vectors represented by horizontal arrows from weakest to largest. Student's responses are presented in Table 4.3. Results highlighted in bold indicate a correct response or reasoning.

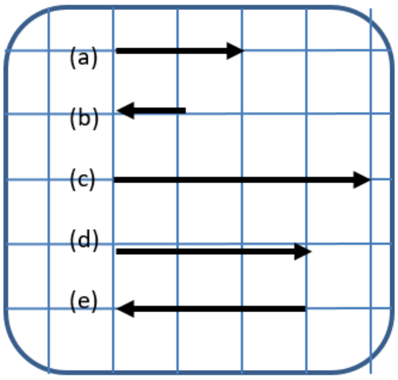


Figure 4.2. Pre-test vector magnitude ranking question.

A common difficulty (11/14) was that students thought the vectors that pointed to the left had a smaller magnitude than vectors pointing to the right. Students explain their reasoning as the vectors pointing to the left having a negative magnitude and the vectors pointing to the right having a positive magnitude. This was not addressed in their physics course, but introductory vectors using $\vec{i}$ and $\vec{j}$ notation is covered in the Leaving Certificate Applied Mathematics course (NCCA, 2006), which all but students 4A and 4J completed. However, as Table 4.3 shows, student 4A produced the correct outcome but gave no reason to suggest they related the vector length to magnitude based on their understanding, and could have been the result of a reasonable guess. For the 11 students that did not produce the correct outcome, this indicates that students do not separate out the mathematical signs from the magnitude of the vectors, in this case, assigning negative to left and positive to right, and reasoning that negative integers have lesser value than positive integers. Nguyen and Meltzer (2003) also showed that when students were given scaled diagrams of various vector arrows, and asked to identify vectors of equal magnitude, their responses were influenced by not just the length of the arrows, but also the direction in which they pointed.

Responses	Students.
b < a < d = e < c	**4A**
c > e = d > a > b **(ranking from highest to lowest)**	**4C, 4M**
e < b < a < d < c	4B, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4N

Table 4.3. Summary of responses to the Vector magnitude ranking pre-test question.

In the second pre-test question students were asked to draw the resultant vector when two vectors at an acute angle are added (see Figure 4.3). Finding resultant vectors using the superposition principle is outlined as a mandatory skill in the Leaving Certificate Physics syllabus (NCCA, 1999). This pre-test question was designed only to determine if the students could combine vectors

diagrammatically, and if so, determine what vector construction they would use. The responses to this question are summarized in Table 4.4.

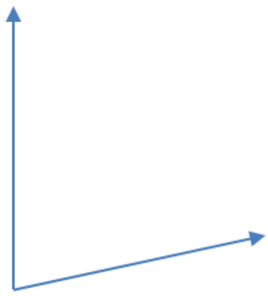


Figure 4.3. Pre-test vector construction of resultant question.

Construction Method Used	***Students***
"Tip to tail" construction (Nguyen and Meltzer, 2003)	**No responses**
"Parallelogram" construction (Hewitt, 2009)	**4B, 4H, 4I**
"Split the angle" construction	4C, 4D, 4G, 4M
Connects tails	4J
No attempt	4A, 4E, 4F, 4K, 4L, 4N

Table 4.4. Student's construction methods to find resultant of two vectors.

The results show us that only three of the students used a parallelogram construction, one of whom (4H) made a minor error in positioning the arrow head of the vector. Four of the students attempted to "split the difference" in which they bisected the angle but did not use any construction to determine the relevant magnitude (Nguygen and Meltzer, 2003). While these vectors may initially appear to be correct, there is no indication by the student's responses that they could determine any information about the resultant vector, other than its approximate direction. One student (4J) joined the resultant vectors with a curve but provided no reasoning, which indicates they made a guess to attempt to answer the question, without an understanding of what was being asked. The remainder of the students did not attempt the question. A sample of the student's constructions is presented in Figure 4.4.

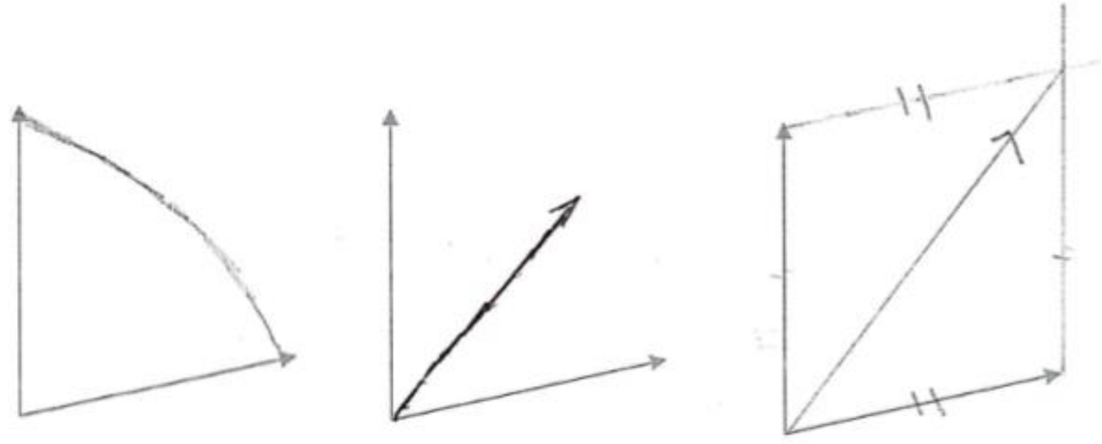


Figure 4.4. Examples of student responses, from student 4J, 4M and 4H respectively.

The last set of questions of the pre-test were used to determine student's understanding of vector addition in a conceptual vector question. Nguyen and Meltzer (2003) discussed common problems in vector addition, such as forcing results to align with the horizontal / vertical axis, or combining the magnitude of the horizontal components inaccurately, suggesting some attempt at the triangle / parallelogram rule. The students completed two questions looking to elicit their understanding of vectors. Both questions were published in Tutorials in Introductory Physics (McDermott and Shaffer, 2003). The first question, shown in Figure 4.5, asked students to determine which body will experience the highest net force, based on the vectors shown.

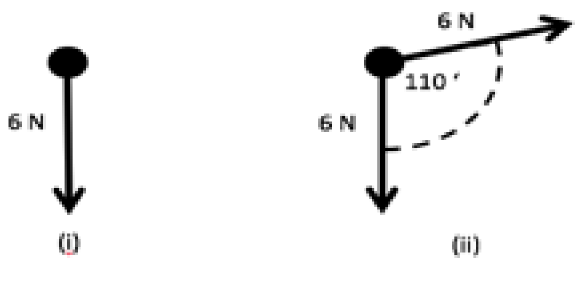


Figure 4.5. Vector pre-test question to elicit student understanding about vector addition.

The student's reasoning could be used to identity difficulties with vector addition. Table 4.5 shows the results from this question, with the students who used the correct concepts highlighted in bold. Where a student used the correct concept but made an error in applying it, their code is in bold and italics.

Concepts Used	Student Responses
Vector Addition	***4H,***
Parallelogram rule (Hewitt, 2009)	***4H***
Split the angle	4E
More forces lower the magnitude	4I
More forces increase magnitude	4D, 4G, 4K, 4M, 4N, 4L
Scalar addition	4C, 4B, 4F
No Reasoning Submitted	4A, 4E, 4J

Table 4.5. Student reasoning used in vector addition pre-test question.

While all the students identified that the net force for the vector diagram in Figure 4.5 (i) would have the highest resultant magnitude, only one gave an answer that explicitly referenced the use of vector addition. As the students were asked to submit their reasoning, the question allows for the identification of the reasoning used by the students. This enables and allows for differentiation between students that guessed the correct outcome, and those that determined the correct outcome. Table 4.5 shows a range of reasons submitted by the students; with the most common being that the more forces that act on the charge, the higher the magnitude of the resultant vector. In some cases, the reasoning given suggested vector addition, in other cases, it was suggestive of scalar addition, and in some cases the reasoning submitted was not explicit enough to include responses under either category, as seen in the following examples.

Student 4D: (ii) experiences the most force, because it experiences 2 forces in the diagram. (Not explicit to put into either category)

Student 4G: (ii) experiences the most force, because it experiences 2 forces in different directions, that when combined, are stronger than the force in (i). (Suggestive of vector addition)

In the case of the three students (4B, 4C and 4F) who used scalar addition, they explicitly stated the final vector would sum to 12 N, in effect disregarding the direction of the vectors. 4I stated that the angle between the vectors would reduce the resulting magnitude but gave no indication as to why. 4E gave a similar response, suggesting the resultant was "split between the angle" between the vector. When asked to explain what this reason meant, 4E volunteered that the combined force would spread across the 110°, so it'd be weaker as it went over a large angle, such as how intensity quantities get weaker as they are spread over larger areas. These responses show that the students not only used incorrect reasoning to attempt the question, but their reasoning was inconsistent with their choice of which of the two vector diagrams would result in the highest net force and did not recognise the inconsistency in their responses.

A fuller picture of student's understanding of 2D vector addition emerged from the second pre-test question This question specifically looks at their conceptual understanding of the addition of horizontal and vertical components. This question was adapted from Tutorials in Introductory

Physics (McDermott and Shaffer, 2003), in which the students were told the vectors represented forces acting on a body; they were asked to state which body experiences the most force, and to justify their choice. The vector diagrams are shown in Figure 4.6, and the student responses are summarised in Table 4.6.

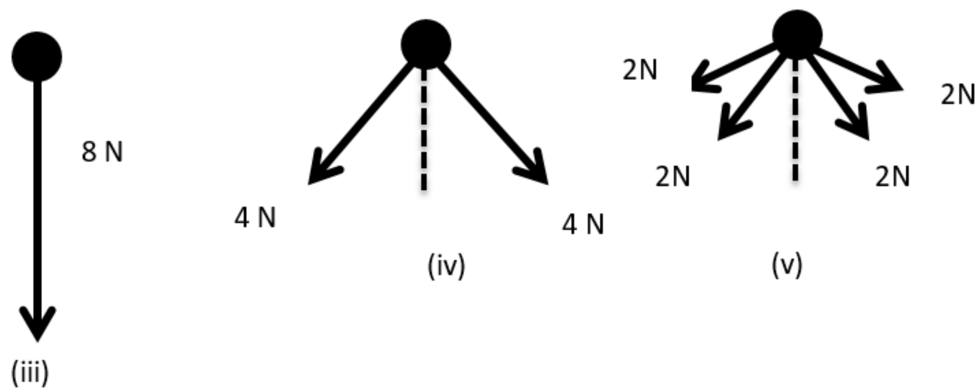


Figure 4.6. Vector pre-test question, to elicit student understanding of vector components

This question gave more clear results about student thinking. As can be seen in Table 4.6, eight of the 14 students added the vectors as if they were scalars, not accounting for the direction of vectors, nor the horizontal or vertical vector components. Only one student, 4H, attempted to use vector addition to complete this question, but errors in their application were apparent, as seen in Figure 4.7.

Concepts Used	Student Responses
Vector Addition	**4H**
Scalar Addition	4A, 4B, 4C, 4F, 4G, 4I, 4K, 4M
More angles mean more force	4D
No Reasoning Submitted / Reasoning unclear	4E, 4J, 4L, 4N
Correct Outcome for Setup	**4E**

Table 4.6. Student reasoning used in vector addition pre-test question, related to vector components.

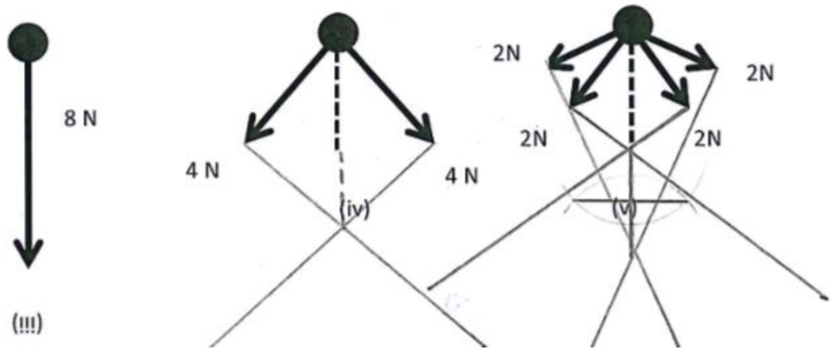


Figure 4.7. Student 4H's response to the vector component pre-test questions (iii)–(v).

Student 4H attempted to use the parallelogram rule to determine the resultant vectors, and in the case for (iv), their sketch is accurate. However, in (v), their attempt to resolve four vectors proved to be difficult and instead of using the parallelogram rule for the two most horizontal vectors, they translated them long an axis through the tips of the other two vectors, when attempting to find the resultant. This is a creative attempt for a student who never attempted a question with 4 vectors, but ultimately highlights an incomplete understanding of the contribution the horizontal and vertical components have in determining the net force.

Two students who used scalar addition were consistent in their responses, in which they found the sum of the magnitudes, as shown Table 4.7.

Student 4B:	Student 4C:
I think all the forces are the same because: *8 = 8* *4 + 4 = 8* *2 + 2 + 2 + 2 = 8.*	*They're all the same* *8N = 4N + 4N = 2N + 2N + 2N + 2N.*

Table 4.7. Student reasoning used in vector addition pre-test question, related to vector components.

Doughty (2013) showed students who completed introductory physics course in a third level setting struggled with vector addition, with vector components being a notable difficulty. It is not surprising to see the second level students in this study to have difficulties with vector addition questions, and due to their lack of experience with vector operations, it is not unreasonable to observe students disregarding the direction of vectors.

The pre-test results indicate that the student's understanding of vector concepts is undeveloped, even though 12 of the students were introduced to vector concepts in their Applied Maths course (NCCA, 2006). This is consistent with the work of Flores, *et al.*, (2008) and Nguyen and Meltzer (2003). The students did not consider direction and magnitude of a vector separately, had difficulty

in determining the superposition of two vectors, and appeared to lack conceptual understanding of the difference between adding vectors and scalars, and in their use and understanding of horizontal and vertical components to combine vectors.

4.2.2. Tutorial lesson: Vector Concepts

Vector concepts were introduced to the students during a lesson which consisted of a 15-minute teacher led class discussion and a 65-minute tutorial. The fourteen students were placed into two groups of four and two groups of three. During the class discussion, students were introduced to vectors by showing them a short clip of a plane flying in crosswinds, on a digital projector. A discussion between the students and teacher on the behaviour of the plane in the crosswinds led to informally discussing the difference between vector and scalar quantities. Formal definitions for each were then presented and examples of each were discussed on a PowerPoint presentation. On the presentation, students were presented with a grid with horizontal and vertical vectors. Students had to attribute positive and negative signs to represent the direction of a series of vector arrows (with reference to the right being positive, and up being positive). They then had to identify the longest vectors. This led to a discussion between students and teacher in which the magnitude of the vector was separated from its direction. An analogy of a strongman pushing a crate with 1000 N of force in various directions was used to aid the discussion. Students were also shown the use of the tip to tail method for vector addition and the parallelogram rule. Due to time constraints, the students were not afforded the opportunity to practice these constructions multiple times during the tutorial, and it was decided to complete them in the following class period.

The tutorial lesson was developed to allow the students to build up an understanding of how two-dimensional vector addition operates differently to scalar addition, explicitly exploring the use of vector components. Initially, students were guided through the labelling and drawing two vectors, $\vec{a}$ and $\vec{b}$, on a graph and sketching the vector components for each vector on the graph. Students were asked to match and draw vectors on graphs from coordinates. Upon sketching the components and producing two right angled triangles, the students had to apply Pythagoras' theorem using the components to determine the magnitude of the two vectors, $\vec{a}$ and $\vec{b}$. The students were then guided through the addition of vectors, using the tip to tail method, using the vectors from Figure 4.8, (i) and (ii).

The students showed an ability to apply the "tip to tail" method on the page to determine the resultant vector, $\vec{c} + \vec{d}$. However, all the students encountered difficulties when attempting to produce the horizontal and vertical components and use them to produce the resultant vector, $\vec{e} + \vec{f}$. Instead of adding the horizontal and vertical components, two groups of students attempted to add

the magnitudes of $\vec{e}$ and $\vec{f}$ directly to produce the resultant, while the other groups directly asked for assistance in completing this section. In all cases, the teacher requested the students sketch the horizontal and vertical components for $\vec{e}$, and then $\vec{f}$. Students were asked what the resulting horizontal magnitude would be if they were to combine the horizontal components only, and then apply the same reasoning to the vertical components. At this point, the student groups were comfortable to combine the resultant horizontal and vertical components to construct the vector $\vec{e} + \vec{f}$.

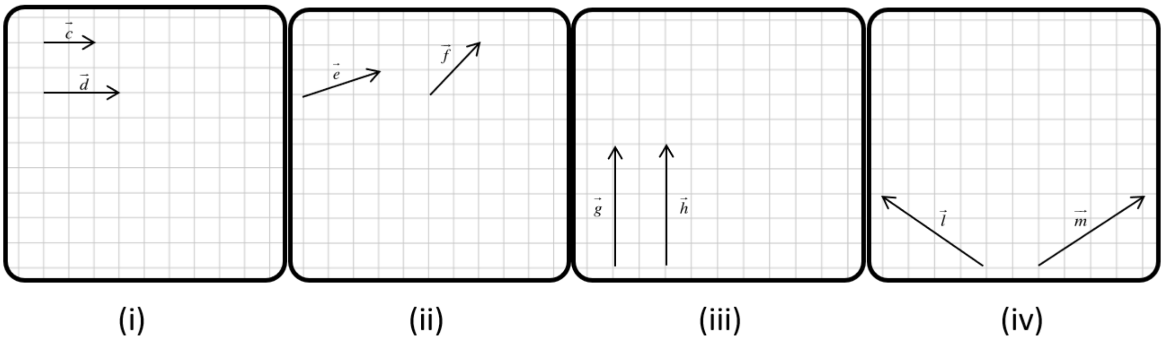


Figure 4.8. Vector addition diagrams from the vectors tutorial.

The students were then required to add the pairs of vectors shown in Figure 4.8 (iii) and (iv), using both the tip to tail method and by adding the horizontal and vertical coordinates. Upon completing this part of the exercise, students were asked to explain, in their own words, why the magnitudes in setup (iii) could be added directly and why they could not be added directly in (iv). In some cases, this proved to be the most challenging task on the whole worksheet, but ultimately, the only guidance needed for the students was to highlight the keywords required to give a full answer (horizontal and vertical components, magnitude, resultant), and they were able to construct their own valid meaning to this question. The following transcript, which was recorded on a dicta-phone during the tutorial lesson, illustrates this type of intervention.

Teacher (reading student's work) "Because they are not going in the same direction so the"… ok, so the direction is important?... Ok I'm going to show you some keywords for your answer. (Teacher highlights the words horizontal and vertical components on the tutorial page).

Teacher allowed students to work on this section for 5 minutes before returning.

Student 4K: Is it cause the horizontal components are going in different directions?

Student 4L: *Yeah*

Student 4J: Well, I said, that way one goes up and then it returns, so then it is equal to zero. Like four plus six would be equal to ten, but we're taking them away, so I said it is zero here. This is why there is like, no change there.

Teacher: I didn't hear you, what were you saying?

Student 4J: Nothing (brief laughter)

Teacher:	No, go with it. It sounded like you were right, I just came in at the end.
Student 4K:	[4J] was saying that these parts are going in a different direction.
Student 4J:	And it is returning, so there is no change…
Teacher	There's no change in what?
Student 4L:	In the horizontal components.
Student 4J:	Yeah… (brief laughter) I came up with that!

This transcript of the dialogue between the teacher and the students showed that initially the students were not necessarily dissatisfied with their initial answer, but they were unable to reason through the task. Upon being prompted to consider the necessary keywords and being given the appropriate time, the students developed reasoning that was plausible and intelligible to them (Posner, *et al.,* 1982) and allowed them to obtain the correct answer of the task.

Upon reviewing the tutorial solutions, there is evidence that the basic skills required in understanding vectors, such as drawing vectors and combining them using the tip to tail method, were relatively straightforward for the students. The difficulties they encountered when using horizontal and vertical components to combine vectors indicates that this was a more challenging task for the students. The study showed that, unless guided, the students did not consider using the components to combine vectors and needed prompting questions to lead them through the process. At the end of the class the students demonstrated the ability to apply this process to vector pairs, as shown in Figure 4.8 (iii) and (iv) and explain how vectors of equal magnitude can produce resultants of various magnitude in terms of the combination of the vector components.

4.2.3. Homework: Vector Concepts

The homework assignment for vectors followed the format of the tutorial lesson. It allowed the students the opportunity to practice the skills developed in the tutorial and class discussion. The questions required students to (i) draw horizontal vectors of various magnitude (ii) combine non-collinear vectors arrows (iii) combine vectors in terms of their components mathematically and (iv) rank the net force acting on bodies with 2 forces acting on them, at different angles.

The first question on the pre-test, as shown in Figure 4.9, presented students with a vector, $\vec{b}$, and asked them to sketch the vectors defined as follows: $[2\vec{b}, 4\vec{b}, -\vec{b}, -3\vec{b}, 2\vec{b} - 3\vec{b}\ ]$. The students showed good ability to correctly represent the vectors, with only some minor errors shown by 4 students. Both student 4F and 4N included multiple arrow heads in their vectors, but otherwise produced valid answers to represent the required vectors. Students 4C and 4K did not use the scale

of $\vec{b}$ having a magnitude that corresponded to two boxes, as seen in Figure 4.9. Both student's vectors were relatively correct, for example, their representations of $4\vec{b}$ were longer than $2\vec{b}$, which were longer than $\vec{b}$, and $-\vec{b}$ pointed to the left with roughly the same magnitude as $\vec{b}$. These difficulties were briefly discussed with the whole class, where they commented on the errors produced by the four students and suggestions to how they may have represented the vectors more accurately were noted by the four students.

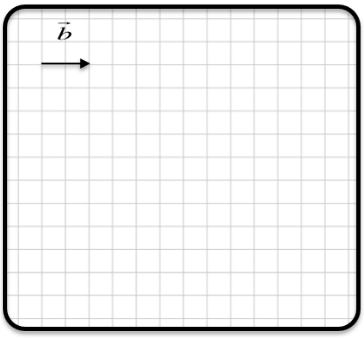


Figure 4.9. Homework question in which students sketch vectors.

In the second homework question, students were given 3 sets of vector pairs to add, as seen in Figure 4.10. In general, it was seen that students answered this section adequately; with all attempting to use the methods they learned in the tutorial, or class exercises, to find a resultant vector. Their results are summarized in Table 4.8.

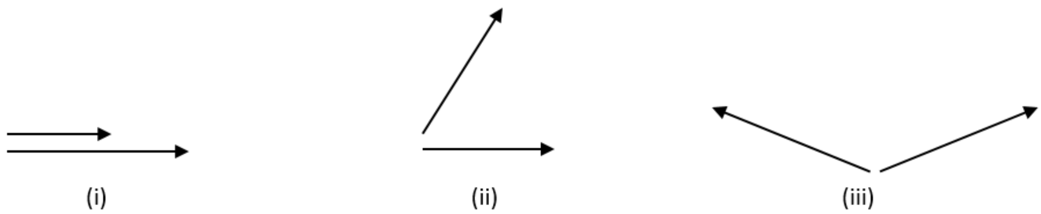


Figure 4.10. Homework question seeking to determine what construction students employ.

Concepts Used	Student Responses
Tip to Tail construction.	**4C, 4D, 4E, 4F, 4I, 4L, 4M, 4N**
Parallelogram construction.	**4A, 4B, 4G, 4H, 4K**
Errors in application of either construction	4E, 4F, 4I, 4K, 4L, 4N

Table 4.8. Constructions used by students to find the result of 2 vectors in homework exercise.

While all students attempted to use a vector addition construction to find the resultants, some did not draw their vectors to scale, and some did not complete arrow heads. In a same number of cases, the tip to tail / parallelogram was sketched but the resultant vector itself was not drawn, leading to an incomplete diagram. As shown in Table 4.2, the students were provided with an extra lesson to practise the vector constructions further. In this lesson, I set the students to work in pairs, to complete a series of exercises from their textbook over the course of 15 minutes, allowing them to attempt to overcome the issues observed in the homework assignment. Examples of student errors are presented in Figure 4.11.

(a) (b)

Figure 4.11. Examples of errors and incomplete diagrams from students 4L (a) and 4E (b).

The third homework question showed students the same vectors as seen in the second question but gave the students the vectors in terms of their horizontal and vertical components, as shown in Figure 4.12.

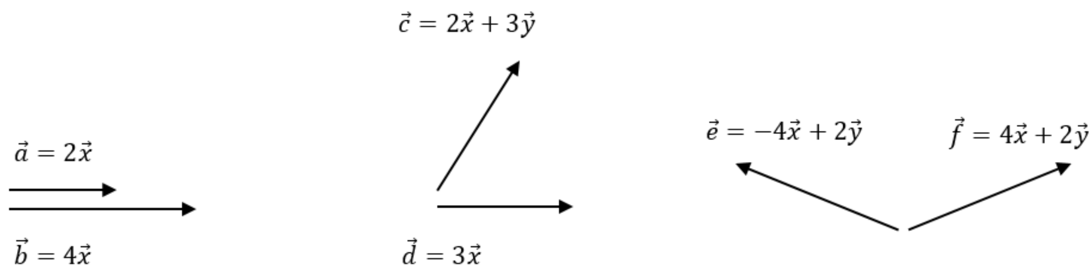


Figure 4.12. Homework question for students to add vectors using components.

The students were required to find the resultant vectors for $\vec{a} + \vec{b}$, $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$. All students demonstrated they could add the vector components in each combination, as this addition is similar to scalar addition in one dimension. Difficulties were observed in the addition of $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$, which were designed to resemble the vector addition pre-test questions, where the student had to consider the vector components in more depth. 8/14 of the students correctly used Pythagoras' theorem to calculate the magnitude of $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$. One student (4C) attempted to use the theorem but made errors in their work. For instance, instead of applying Pythagoras' theorem to $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$, they applied the theorem to $\vec{c}, \vec{d}$, $\vec{e}$ and $\vec{f}$ individually. The remaining students (4A, 4E, 4F, 4L and 4N) appeared unable to apply the theorem correctly. Difficulties seen in these students work showed error in picking appropriate values for the x and y component to use in Pythagoras' theorem (4A, 4F and 4L) or the student did not attempt the question (4E and 4N).

In the next section of the question, the students were asked the following question:

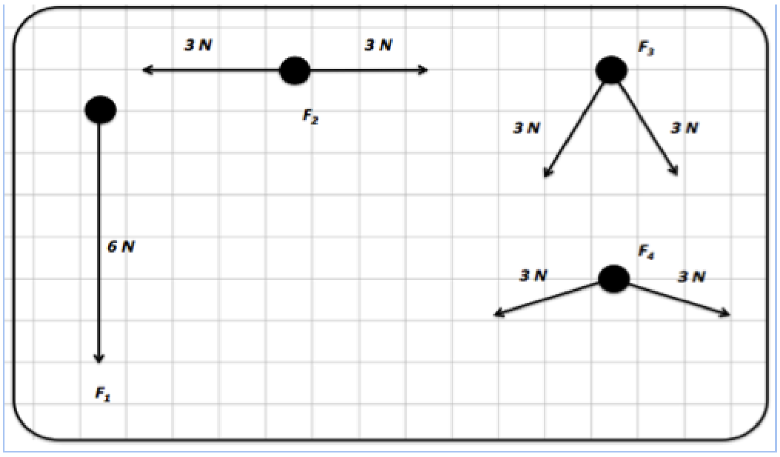
Explain, referring to the addition of horizontal and vertical components, explain why the magnitude of $\vec{c} + \vec{d}$ *is greater than* $\vec{c}$ *and* $\vec{d}$*, individually, but the magnitude of* $\vec{e} + \vec{f}$ *is less than* $\vec{e}$ *and* $\vec{f}$ *individually*

6/14 of the students referenced how the components of the vectors would affect the resultant, where horizontally cancelling components was observed in the $\vec{e} + \vec{f}$ combination. 3/14 of the students referenced the general direction of the vectors being opposite, and this would lead to a reduction in the overall magnitude but did not explicitly refer to the components. The remaining students did not attempt, and were likely unable to give any explanation. This highlights student difficulties to reason qualitatively, even when they are presented with information to help form their reasoning. This suggests the students would require more practise to develop the ability to reason scientifically, and as described by Hewitt (2011b), think in terms of concepts. As the tutorials progress, as shown in chapters 5 and 6, the students are given multiple opportunities to develop their qualitative reasoning skills and apply concepts qualitatively, as well as quantitatively.

In the last homework question, students were given a setup like the final pre-test question, with only a maximum of two vectors acting on a given body at any one time. The questions are shown in Figure 4.13, and the student's results are summarized in Table 4.9. 7/14 students gave the correct outcomes for the ranking. Two of them (4G and 4M) referenced the horizontal components cancelling out, while also using the parallelogram rule. A further three students relied solely on the parallelogram construction, without referring to the addition of either the vertical or horizontal components.

Concepts Used	Student Responses
Correct Outcome for Setup	**4B, 4E, 4G, 4H, 4J, 4K, 4M**
Horizontal / Vertical vectors referenced.	**4G, 4M**
Parallelogram / tip to tail	**4B, 4E, 4G, 4H, 4M**
Scalar Addition	4A, 4C, 4F, 4N
Incorrect ranking with no reasoning	4D
Not attempted	4I, 4L.

Table 4.9. Summary of student responses for homework vector addition conceptual question.



A small ball has a force or numerous forces acting on it as shown in the diagram to the right. The net force (resultant force) is labelled as F_1, F_2 and F_3.

(i) *Rank the net forces acting on the ball, from highest to lowest. Explain your ranking.*

Figure 4.13. Extract from vector homework.

Figure 4.14 presents a homework response from student 4E, which illustrates the use of the "tip to tail" construction to complete this task. The student applied the parallelogram rule to construct the resultant and presented 4 bullet points that commented on the resultant of the vectors. However, in the absence of a vector construction, there is no indication that the students could reason conceptually the correct ranking for this question. As discussed with the previous question, this illustrates the student's lack of practise in using qualitative reasoning in physics questions, or thinking in terms of concepts when approach qualitative exercises.

Students 4A, 4C, 4F and 4N submitted that all forces would be the same in all cases, contrary to the reasoning submitted in their tutorials. For all vector diagrams, they added the magnitudes of the vectors, ignoring the directions of the vectors. In a class review of the material, students 4F and 4N stated they did not think it would be the same as what they did in the tutorial, as in the tutorial they were dealing with magnitudes of vectors, while this was dealing with force. This can be explained by one any of the following three difficulties:

(i) The students could not connect the meaning of the term magnitude and its application to vector quantities.

(ii) The students could not connect the mathematical understanding to a physics concept.

(iii) The students did not consider force to be a vector quantity (4F and 4N explicitly stated this difficulty).

As part of the class discussion to review the homework, other students voiced their understanding of the terms magnitude, direction and vectors, and explained how the magnitude was represented by the length and how the length is a diagrammatic representation of the numerical measurement of the vector quantity. During this discussion section, students 4D, 4I and 4L were encouraged to communicate with the people beside them. Horizontal and vertical vectors were sketched on the board and these students were requested to use the diagrams to determine a ranking for the outcomes. Student 4D quickly realized that the horizontal vectors would sum to zero, whilst the other students came to this reasoning with the aid of their partners.

The homework assignment showed that the students were competent in representing vectors of various magnitudes and using the parallelogram and tip-to-tail constructions to combine vectors. Difficulties persisted in the conceptual understanding of combining vectors when considering the components of the vectors, and instead, students preferred to rely on the constructions to solve problems related to vector addition, for non-collinear vectors.

Student 4E: *F_1 – Longest vertical components*

F_3 – Second longest vertical components

F_2 – Third longest vertical components.

F_4 – That's just nil: $+3N+(-)3N = 0N$

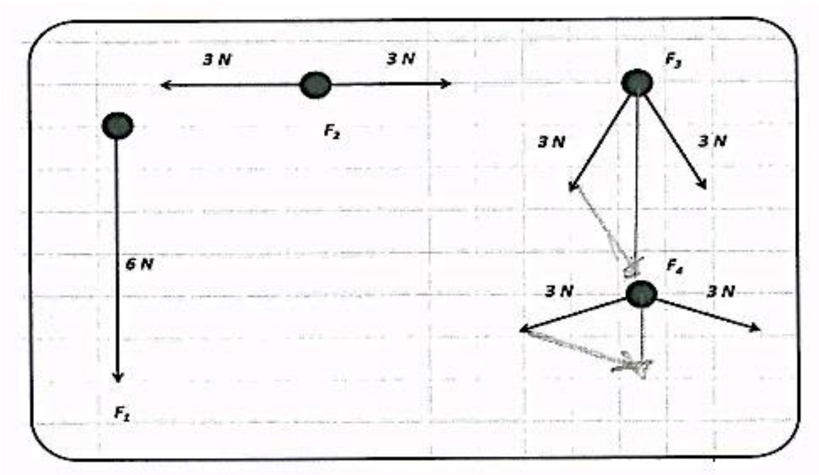


Figure 4.14. Student 4E's homework response, showing their work using the tip to tail to construct their ranking.

4.2.4. Post-test: Vector Concepts

The vectors post-test was written in a manner like both the tutorial lesson and the homework assignment. It tested the student's understanding of vector magnitude, by means of a ranking question, their ability to combine vectors using the "tip to tail," or parallelogram construction, and their understanding of vector components. The post-test took place approximately two weeks after the tutorial lesson. The large time duration was not due to research purpose, but due to a break in the tuition term.

In the first question on the post-test, the students were given a set of vector arrows and asked to rank them, from lowest to highest, based on their magnitude. This question was similar in nature to the first question they saw on their pre-test, to allow for a direct comparison, while also using result from their homework assignment to provide commentary on other difficulties that student may have, that were not seen in the post-test. The vectors are presented in Figure 4.15, and the results are shown Table 4.10.

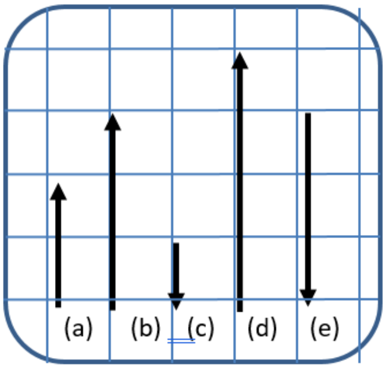


Figure 4.15. Post-test vector magnitude ranking question.

Vector Ranking	Student Responses.
C, A, B=E, D	**4A, 4B, 4E, 4F, 4G, 4H, 4J, 4K, 4L, 4M, 4N**
D, E=B, A, C	4C, 4D
E, C, A, B, D	4I

Table 4.10. Student responses from the post-test vector magnitude ranking question.

From Table 4.10, it is clear to see that all but one of the students ranked the vectors based on the lengths of the arrows, whilst acknowledging the direction of the arrow was not relevant to the magnitude. Students 4C and 4D may have submitted the incorrect ranking, but this is likely based on

misreading the question, as their submissions are correctly ranked from highest to lowest. Student 4I was the only student who based their ranking both the direction and the length of the vectors, as opposed to the length of the arrows alone when determining their magnitude. This shows evidence that most students overcame the difficulty that was apparent in the student's pre-test results.

In the second post-test question, students were required to construct resultant vectors by combining two vectors using the tip to tail, or parallelogram construction. The question is shown in Figure 4.16 and the student results are shown in Table 4.11.

All the students appropriately used one of the vector constructions to find the resultant vector. Some minor issues were observed in so far as some of the constructions were free hand sketches and slightly inaccurate in terms of scale (students 4E, 4F, 4I and 4J). Student 4A used the tip to tail method but extended the resultant beyond where it should have been. This was not expected as in the homework exercises, student 4A used the parallelogram construction and correctly represented the vectors using that representation.

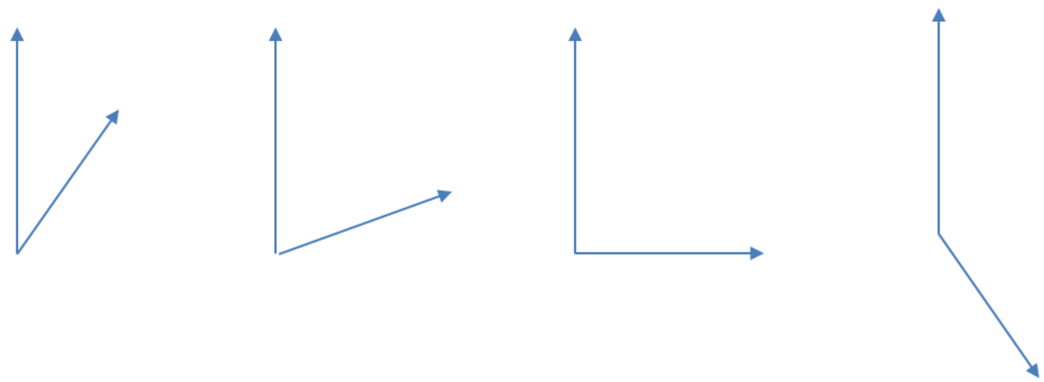


Figure 4.16. Post-test vector construction question

Construction Used	Student Responses.
Parallelogram construction	**4B 4I 4J 4K 4L**
Tip to tail construction.	**4A 4C 4D 4E 4F 4H 4M 4N**

Table 4.11. student's construction methods used in post-test to find resultant of two vectors.

Overall the responses indicate that students are aware of when to apply the vector constructions, but some students did not appear to consider the importance of using rulers to ensure they conserved the magnitude of their vectors, or the slopes of the vectors, when translating them to a new position, when completing the constructions. This was consistently seen in both the homework and post-test by students 4F and 4J, and in the post-test (but not the homework) by student 4E. All other students

consistently took care to draw the vectors to scale, using a ruler, in both the homework and post-test, or in the post-test only.

The final question on the post-test was developed to determine what conceptual understanding students had of adding vectors at angles. Students were given a scenario where an object was being pulled by two ropes, all with the same magnitude. Students were required to determine which of the three scenarios showed the strongest net force acting on the object and the weakest net force acting on the object. All the student's responses, except for 4N, showed they could determine the strongest and weakest force from the three diagrams. Student 4N reversed their answers, in which they incorrectly stated that the setup in which the force vectors were parallel would produce the weakest force, and the force vectors which were diverging with the largest angle would produce the strongest force. The questions are depicted in Figure 4.17, and the reasoning used by the students is presented in Table 4.12.

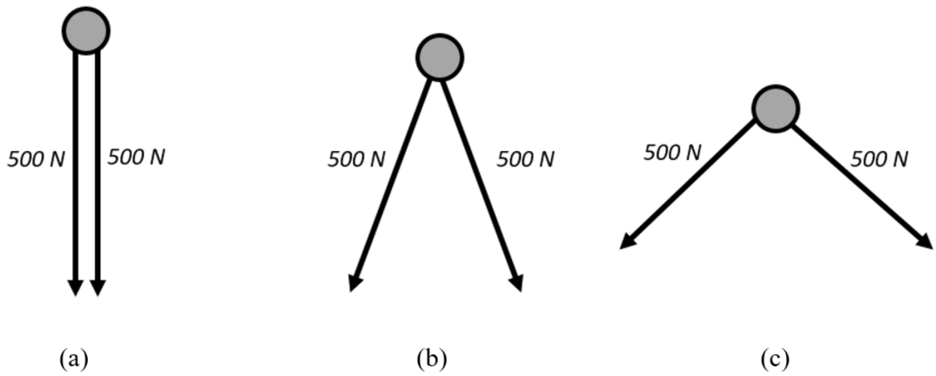


Figure 4.17. Vector post-test question to elicit student understanding of vector components.

Concepts Used	Student Responses
Parallelogram / tip to tail	**4E, 4F, 4G,**
Horizontal and Vertical components explicitly referenced.	**4B, 4C, 4D, 4E, 4G, 4I, 4J, 4K, 4M, 4N**
Horizontal and vertical components suggested / answer incomplete.	4A, 4F, 4L
Angle affect magnitude	4N

Table 4.12. Student reasoning used in vector addition post-test question, related to vector components.

The post-test results show that all but one of the students correctly determined the correct outcome. Of all the correct answers, the use of horizontal and vertical vectors was the primary reason chosen by the students, while a small number of students also included the use of the parallelogram / tip to tail constructions as evidence to support their answers, as shown in Figures 4.18 and 4.19.

Two of the students gave answers that did not fully articulate complete reasoning but were suggestive of the student's understanding about the horizontal components of the vectors cancelling out, while one student incorrectly related the magnitude to the angle and gave an incorrect ranking as a result.

In the cases where the students provided both justifications for their answers, it was seen that students tended to refer to both their diagrams and the components in their reasoning. This would suggest to us that students are not completing the constructions as a matter of rote – learned procedure but indicate that the students can determine the utility in both answering the question using vector constructions and in terms of the combination of vector components.

Student 4H:

The resultant for (i) is the strongest. Plus, it has no horizontal components. The resultant for (iii) is the weakest magnitude. The horizontal components cancel out.

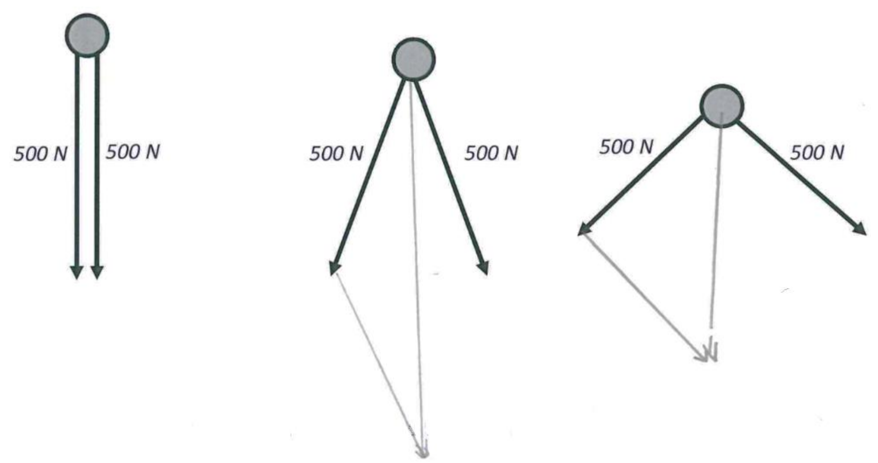


Figure 4.18. Student 4H's response for conceptual vector post-test question.

Other students chose to directly reference the horizontal vectors alone and did not attempt to generate evidence for their reasoning using the parallelogram / tip to tail method.

Student 4B: [(a)]. Both forces are acting the same direction, meaning there are no opposite forces cancelling out.

Diagram [(c)] shows the weakest force as there are large horizontal component forces cancelling out meaning there is less vertical component forces.

Student 4K: The third one, [(c)]. Even though the second one has a vertical and horizontal component, the third one has a wider horizontal component, so the force has to act on two difference horizontal.

Other students submitted the correct answer but gave incomplete reasoning that suggested the use of horizontal and vertical components, but lacked clarity and the use of keywords in the explanation:

Student 4E: A would allow a pulling power of 1000 N, as the combination of the two forces pulling in the same direction would be greater than that of two pulling in different directions, and cancelling each other out to a degree. [Diagram] C [is the strongest]. The forces are pulling in moderately different directions, which causes them to weaken the pulling power, by cancelling most of what the other is doing.

Student 4G:

1 only has vertical components and so is stronger than the others because the x components are 0, therefore all the force are acting the one direction making it the strongest. 3 has larger x values than 1 and 2, there the force acting on the x components are larger and cancel out. When they cancel out, the leave the vertical vector (the resultant vectors) to be weaker than 1 and 2.

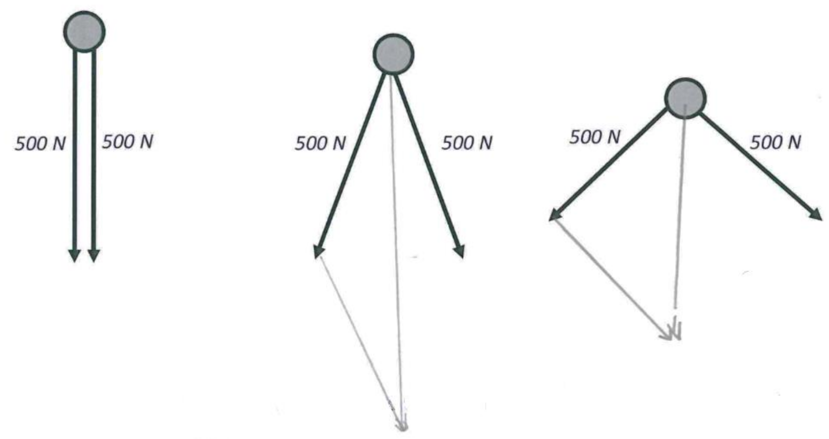


Figure 4.19. Student 4G's response for conceptual vector post-test question.

Student 4N, who did not give correct reasoning, associated the angle between the vectors with the magnitude of the resultant vector. Note that this reasoning was used by students 4D and 4J in the last two pre-test questions, as seen in section 4.2.2. Upon completion of the tutorial lesson, these students no longer produced work that indicated this type of reasoning. However, student 4N (who did not give a response to the pre-test question) was under the impression that a larger magnitude would result from a larger angle between the vectors. From discussion with the student, it was apparent that the student did not grasp the meaning of the term magnitude after completing both the tutorial and general class exercises. While they understood that numerically the magnitude was the number in any given quantity, they did not associate it with the length of the arrows explicitly and instead, associated it with any variable that could be quantified, such as in this case, the angle between

the vectors. Therefore, this reasoning produced a line of thought that a bigger angle had a bigger numerical measurement which gave the largest magnitude in the three setups.

The post-test results indicate that the students developed an understanding of vector magnitude and overcame the difficulty of not separating out the direction and magnitude of a vector (Nguyen and Meltzer, 2003). All the students also showed competency in combining vectors using a vector construction. The students were also able to apply horizontal and vertical component reasoning to the final conceptual question.

4.2.5. Discussion

By comparing the pre-test and post-test results, there is evidence to suggest that the students responded favourably to developing their understanding magnitude by using vector representations in a classroom discussion. Figure 4.20 presents a comparison of the pre-test and post-test results.

During the pre-test, it was observed the students had difficulties ranking vectors based on magnitude, a difficulty based on associating an influence of the direction of a vector on its magnitude (Nguyen and Meltzer, 2003). The occurrence of this difficulty in the class identifies the need for conceptual change during the lessons and tutorial (Hewson, 1992). During the class discussion, the students were presented with a similar ranking question as seen in the pre-test. The students submitted their rankings and reconsidered their rankings in terms of the definitions of vectors and scalar quantities. Through discussion with the teacher, it was seen that most of the students were easily able to identify that vectors associated with a direction that is denoted with a negative sign were equal in magnitude to vectors associated with a direction denoted with a positive sign, when the vectors are of equal length. The prompt to students to review their understanding of the term magnitude allowed become dissatisfied with ranking. They were then able to develop a correct ranking, that was in line with the correct understanding of the term magnitude. Students discussed their rankings in terms of their understanding of the word magnitude, and engaged in conceptual exchange, from which they ranked vector magnitude in terms of length and direction to just length in this task (Hewson, 1992). Based on the increase of correct reasoning used by the students in Figure 4.20, moderate conceptual change was recorded for this concept.

The gains in student understanding were apparent in both the homework and the post-test questions, in which it was seen that most of students submitted correct sketches and the correct rankings with valid reasoning. This classroom discussion appears to have been sufficient for students to engage with the visual representation and overcome the misconception that a negative sign on a vector is associated with a lower magnitude. The post-test results indicate that for most students there

are no persistent misconceptions for the student's understanding of how magnitude is represented by the length of a vector.

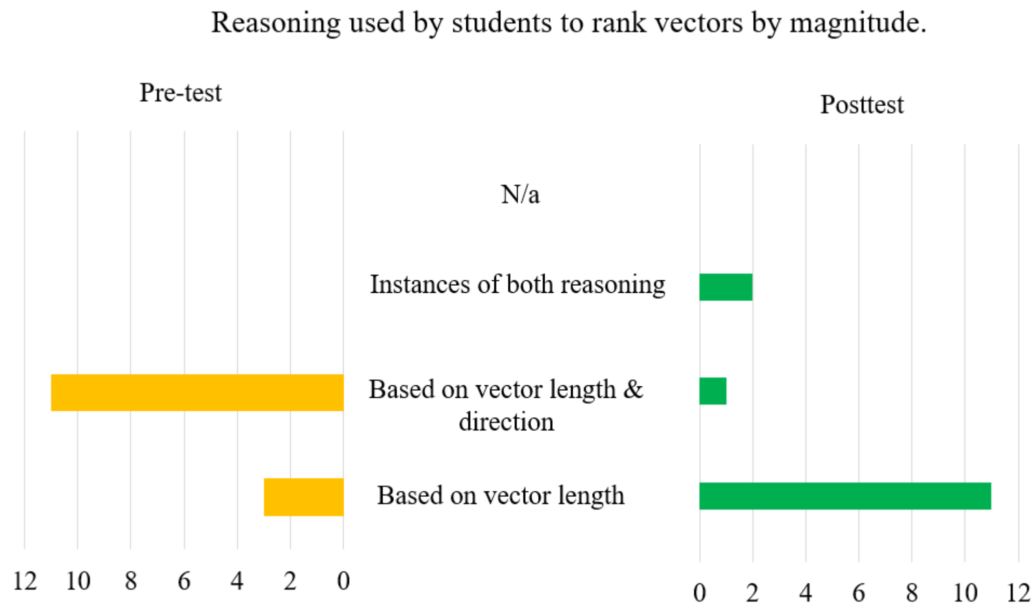


Figure 4.20. Comparison of reasoning used by students to rank vectors.

The next section of the discussion focuses on the students use of vector constructions to combine vectors graphically. Figure 4.21 presents a comparison of the constructions used by the students in both the pre-test and the post-test. During the pre-test, students had difficulties in combining vectors graphically. The most common error seen was students attempting to "split the angle" followed by connecting the tails of the vectors. These difficulties were encountered in literature (Nugyen and Meltzer, 2003) and were identified for conceptual change. In the homework assignment, discussed in section 4.2.3, that followed a tutorial lesson, all the students attempted to use either a tip-to-tail or a parallelogram construction to find the resultant between two vectors, as opposed to only3 students attempting the construction in the pre-test. However, numerous errors were observed in the student's application of the constructions in the homework. These were mainly, but not exclusively due to the lack of use of a scale or correct use of a ruler in completing the constructions. Some of the incorrect vectors also appeared to show some of the errors as seen in Nugyen and Meltzer (2003) such as "split the difference." As a response to the persistent difficulties that were observed in the homework assignment, extra time was additionally given to the students in a separate lesson. The students worked in pairs on completing more vector constructions and allowed for instances of peer tuition to occur. This allowed for an extended response to the student's initial difficulties to correctly implement the vector constructions.

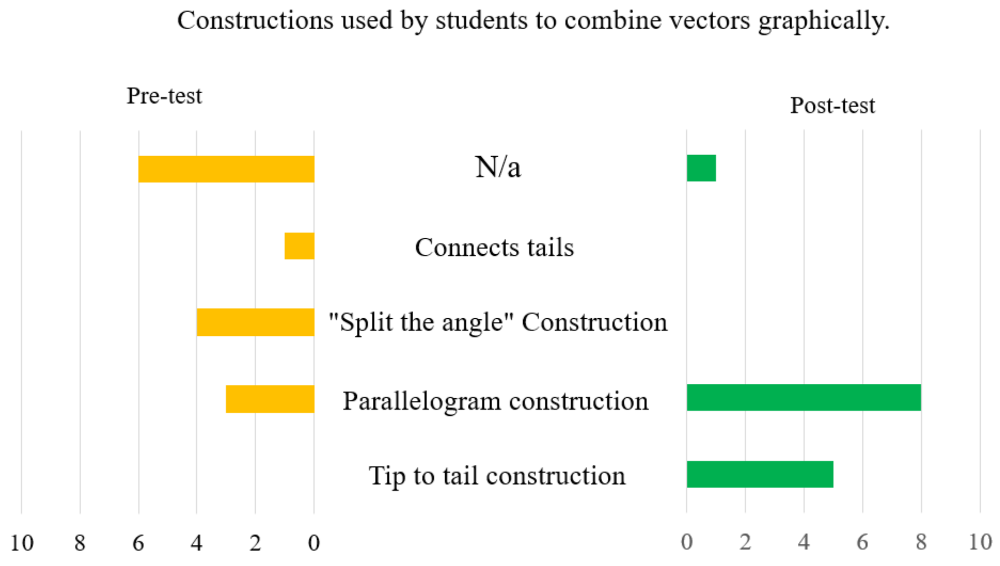


Figure 4.21. Comparison of vector constructions used by students

Upon completion of both the tutorial and additional set of exercises, the post-test showed all students using the constructions in their submissions, with most students having overcome the errors made in the homework. The post-test results showed that all, but one student effectively used one of the two constructions. This suggests that the students practise of the constructions during the tutorial and the extra 20 minutes after the homework activity allowed the students to see the constructions as intelligible and appropriate for adding vectors diagrammatically. The shift in results observed in Figure 4.21, indicates that conceptual exchange occurred, as the students were no longer producing the errors of connecting tail, or "splitting the angle" during the post-test. The shift in results also indicates that moderate conceptual change occurred over the course of this tutorial.

The last section of the discussion focuses on the student's reasoning and understanding of horizontal and vertical components. Figure 4.22 presents a comparison of the reasoning used by the students in vector addition questions, in both the pre-test and post-test. The most prominent gains were seen in the student's responses to ranking the net force acting on bodies when the forces acting are at obtuse angles to each other. Most students used scalar addition when completing the pre-test conceptual question, and this was targeted for conceptual exchange from their over-reliance on scalar addition (Hewson, 1992) to being able to identify when vector addition and scalar addition are appropriate.

In the tutorial lesson, it was observed that the students developed an understanding of components of the vectors, and they considered how these components affect the resultant of a vector. The discussion quoted in section 4.2.3 shows how the students required time to consider and discuss how all the components affected the resultant, and how the horizontal components summed to zero. In highlighting the reasoning used by the students wasn't complete, the teacher provided a source of dissatisfaction in the initial reasoning (Posner, *et al.*, 1982), as students would realise that the

reasoning they provided would not provide an accurate outcome to a tutorial task. They then discussed alternatives amongst themselves. In a previous question, they had mathematically shown that the horizontal component vectors can sum to zero, leaving a vertical vector as the resultant, but the students did not initially consider this as evidence to support their reasoning in the final question of the tutorial. When students were prompted to review all their previous answers, and to consider what their mathematical answers could tell them about the resultant vectors, they developed reasoning that was plausible and intelligible (Posner, *et al.*, 1982) to provide an accurate and well thought out reason to the task.

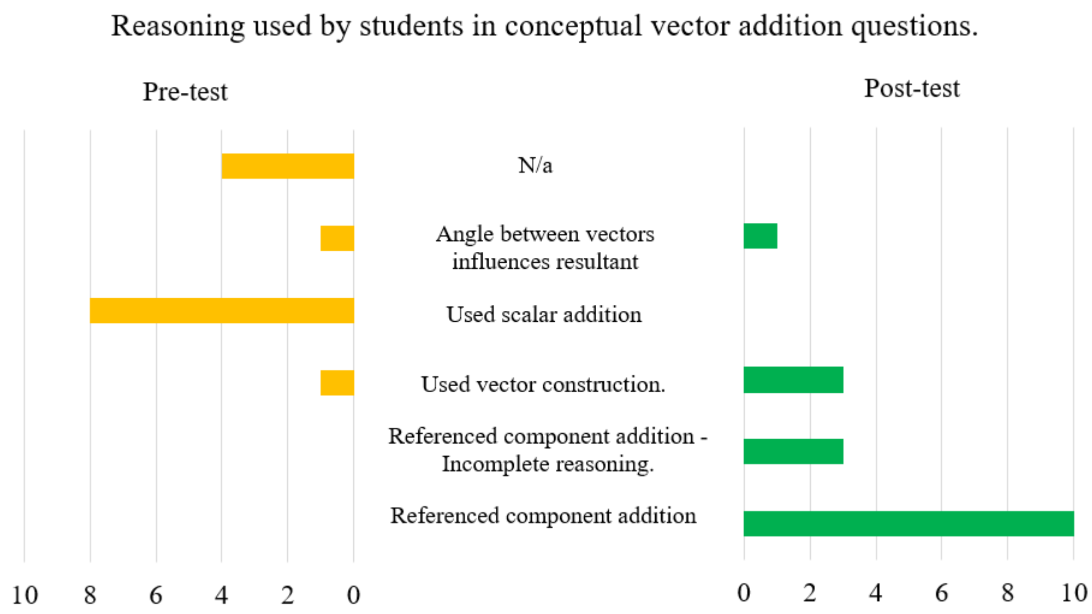


Figure 4.22. Comparison of reasoning used by student in conceptual vector questions.

In section 4.2.4, during a homework assignment it was seen that 7/14 of the students correctly ranked the net forcesacting on a body, at angles ranging from 0° to 180°. Students who submitted correct reasoning either referenced the vector components or used a vector construction. The remaining students reasoned incorrectly about vector components, or used scalar addition without applying the reasoning developed during the tutorial (see also Doughty, 2013). An open class discussion was held, in which the teacher sketched the horizontal and vertical component vectors, gave them the correct ranking and encouraged students to determine why the ranking was correct, referencing the vector components. As the students had the correct outcome presented, this allowed the student to focus on why the outcome was correct and review the reasoning they developed in groups in the previous tutorial lesson. The students worked in pairs in this discussion. As the groups were prompted to consider the components of the vectors, the groups were able to construct a comprehensive, concise and useful explanation that justified the correct ranking (Posner, *et al.*, 1982). The post-test results indicate this teaching approach was effective, as essentially all students gave the correct answers to the final conceptual vector question, with most of the students referencing the horizontal and vertical component summation in their answers. The increase in students

explaining vector addition in terms of vector components and reduction of students applying scalar addition, as seen in Figure 4.22, suggests the students engaged in conceptual exchange (Hewson, 1992) over the course of this tutorial. The increase in correct student responses and reasoning is indicative of moderate conceptual change having occurred. Sections 4.2.1 and 4.2.4 detailed the reasoning produced by the students and the jump from zero to ten students, in the pre-test and post-test, producing reasoning based on vector addition is evidence to support this.

In the homework assignment, the students who submitted a correct ranking tended to favour using the parallelogram or tip-to-tail method to produce their ranking, while only two students considered the vertical and horizontal components. After a discussion of the solution in the next lesson, which primarily used horizontal and vertical components, it was observed that most of the students acknowledged that considering the components provided an efficient method to solving the conceptual vector problem. This discussion could have heavily influenced the student's choice of reasoning and considering the students may have given both styles of reasoning in the post-test had the discussion being balanced to explore both methods to justify the correct ranking. Additionally, even though the students completed mathematical calculations relating to the summation of the horizontal and vertical vectors, in both the lesson and the homework, no students considered it's use in the conceptual questions at the end of either the homework or the post-test. This was unexpected, considering they would be familiar with the tools required to complete these calculations, and they draw a parallel to the reasoning the students developed and explored in their first and second year of second level mathematics, in which they covered the topic of coordinate geometry.

This chapter of the research presents evidence to suggest that conceptual change occurred to various extents between the three target concepts. On completion of the teaching sequence, most students showed they associated vector magnitude with the length of an arrow, regardless of direction. This can transfer to Coulomb's law, to represent the relative strengths of force acting on charged particles, and electric field to represent the field around a charge, and the superposition of two charges, at various points. The use of the constructions can also be applied to electric fields, in which students construct electric field of two, or more, charges and explain how the introducing more charges can increase / decrease the magnitude of the electric field at various points. The student's gains in understanding of components can be utilised by them to give a deeper explanation of the variation of the electric field strength at points between multiple charges, and develop an understanding of positive, negative and zero work and how it applied to potential difference in uniform electric fields.

4.3. Inverse square law

This section presents a narrative and analysis of the development of student's understanding of the inverse square law. The inverse square law applies to many contexts in physics, such as light intensity, sound intensity, Newton's gravitational law, Coulomb's law , electric fields and radiation. The participants in this study are only required to study three of these examples: Newton's gravitation, Coulomb's law and electric field and sound intensity (NCCA, 1999). Bardini, *et al.*, (2004) showed students learning to graph linear equations using contextual problems can help develop their understanding of the properties of an equation. There are numerous analogous practical activities that can be employed in teaching the inverse square law, subject to the time and equipment constraints of a secondary school environment. Hestenes and Wells (2006) showed that presenting the students with data can be sufficient. For the student to explore the inverse square law using a relatively non-abstract context, the tutorials employed a context of spray paint droplets spraying over various areas, which was adapted from Hewitt (2009).

Section 2.1.3.2 detailed difficulties encountered by learners in their understanding of the inverse square law. These difficulties informed the design of the tutorials, so students could recognise, explain and apply the inverse square law using multiple external representations. These are presented in the following learning objectives, as upon completion of the teaching and learning material, the students would be able to:

1. Accurately sketch and switch between graphical and algebraic representations of the inverse square law (Bardini, *et al.,* 2004; Hestenes and Wells, 2006; Bohacek and Gobel, 2011).
2. Apply a diagrammatic model utilising intensity to explain the behaviour of the inverse square law, and make predictions based on the model (Hewitt, 2009).
3. Demonstrate proportional reasoning using the inverse square law (Arons, 1999, Maloney, *et al.*, 2000; Marzec, 2012).

The inquiry approach developed for promoting student understanding of inverse square law consisted of a pre-test, a tutorial lesson and a post-test. This intervention ran over three weeks. The materials focused on student's ability to graph an inverse square law relationship, and modelling the uniform spreading of paint droplets over an increasing area to explain the behaviour of quantities that obey an inverse square law. A timeline for the implementation of the inverse square study, including the target concepts for the intervention, is shown in Table 4.13. As the field lines tutorial was completed in the same three weeks as the inverse square law tutorial, the field lines classes are also presented but bold font is applied to the classes which only applied to the inverse square law.

Section 4.3.1 presents the pre-test results, looking at the difficulties the students showed in representing an inverse square function on a graph, explaining the increase in the area covered by a

bulb illuminating a wall when the bulb is moved away from the wall, and a calculation based on the inverse square law. Section 4.2.2 presents a narrative of the development of the student's understanding of the inverse square law, by guiding them to develop an understanding of intensity applied to a context of using spray paint, model how the spray paint intensity varies as the distance from the source to the surface changes and present this model on a graph. Section 4.3.3 presents an analysis of the post-test results which, like the pre-test, focused on students representing an inverse square function on a graph, explaining the variation of area covered when a source is moved from a surface, and a calculation based on the inverse square law. Section 4.3.4 presents a comparison of the pre-test and post-test results, and a commentary of the student's progress during the tutorials. Examples of student progression and difficulties that persisted throughout the tutorial and post-test are discussed.

Time		Research Implementation	Target Concepts
Week 4	Class 1.	**Pre-test.**	**Representing an inverse square function on a graph.**
	Class 2.	**Tutorial Lesson**	**Understanding of the increase / decrease in area model that follows an inverse square law.** **Inverse square law ratios calculation.**
	Class 3.		*Topics unrelated to project: Newton's gravitational law.*
Week 5	Class 1.	Pre-test.	*Topics unrelated to project: Newton's gravitational law.* Field lines pre-test.
	Class 2.	Tutorial Lesson Homework.	Field lines tutorial Field lines homework.
	Class 3.		Topic summary.
Week 6.	Class 1.	**Post-test**	**Representing an inverse square function on a graph.** **Understanding of the increase / decrease in area model that follows an inverse square law.** **Inverse square law ratios calculation.**

Table 4.13. Timeline of the implementation of the vector concepts study.

4.3.1. Pre-test: Inverse Square Law

The inverse square law pre-test was designed to elicit student's understanding in various representations. The students were required to show their ability to (1) graph an equation of the form $y = k\frac{1}{x^2}$, (2) explain how the area on a wall, illuminated by a torch, varies as the torch is moved closer to or further from the wall, and (3) answer a mathematical question based on proportional reasoning involving the inverse square law. The use of different representations provides the opportunity to use many ways for students to show and for me to gauge their understanding.

The inverse square law pre-test took place after an introduction to Newton's universal law of gravitation. In a class discussion, which lasted 15 minutes, the students were guided through a series of slides outlining the Cavendish experiment, in which a diagram of the setup was shown, and a summary of the observations were presented. The slides also presented data they could use to qualitatively discuss, as a class group, the effect of varying the product of two masses, and the distance between the masses. The students determined the direct proportional relationship between the gravitational force and the product of the masses, but were unable to determine the relationship between the gravitational force and the distance. The students were informed that this was a relationship not covered in their mathematics courses and they would explore this relationship in the next lesson. The class discussion concluded with the students completing the pre-test, with a time limit of 15 minutes.

In the first question, the students were presented with blank *x*-*y* axes. They were asked to draw a pattern to match an equation of the general form $y = k\frac{1}{x^2}$. This allowed the students to demonstrate if they were aware how to transfer from algebraic to graphical representation, using the correct general characteristic curve for the function. A summary of the results is presented in Table 4.14.

Only three of the students correctly represented the relationship on a graph. The most common responses were quadratic curves. The responses from 4E, 4G and 4H were U-shaped curves, with an arbitrarily chosen intercept, while the responses from 4B, 4I and 4J were increasing quadratic curves with an intercept of zero. These students referenced the exponent on the *x*-variable in the equation to determine the function is quadratic. Student 4D presented a linear pattern with a positive slope, while student 4A sketched two linear patterns, one positive and one negative, overlapping. Both student's reasoning suggested they guessed the desired pattern. Of the correct graphs submitted, students 4C and 4M presented mathematics-based reasoning. Both students referenced that the function took the form of a fraction, and as the *x*-variable increased, the *y* value would approach zero, without reaching it, effectively defining an asymptotic function.

Response	***Students***
Decreasing asymptotic curve	**4C, 4F, 4M.**
Increasing curve	4B, 4I, 4J.
Quadratic curve	4E, 4G, 4H.
Decreasing straight line	4A.
Increasing straight line	4A, 4D.
N/a	4K, 4N.

Table 4.14. Student responses from the pre-test inverse square law graphing question.

The second pre-test question used a diagrammatic model for light intensity to probe student's understanding of the inverse square law. The students were presented with an 8 x 8 grid, with a torch illuminating 4 squares on the grid when it is 1 m away, as shown in Figure 4.23 (i). The students were asked to sketch, on the grid shown in Figure 4.23 (ii), the pattern observed if the torch was moved to 3 m away. This test was designed to elicit the student's understanding about scaling, as mentioned by Marzec (2012). The student's responses are summarized in Table 4.15.

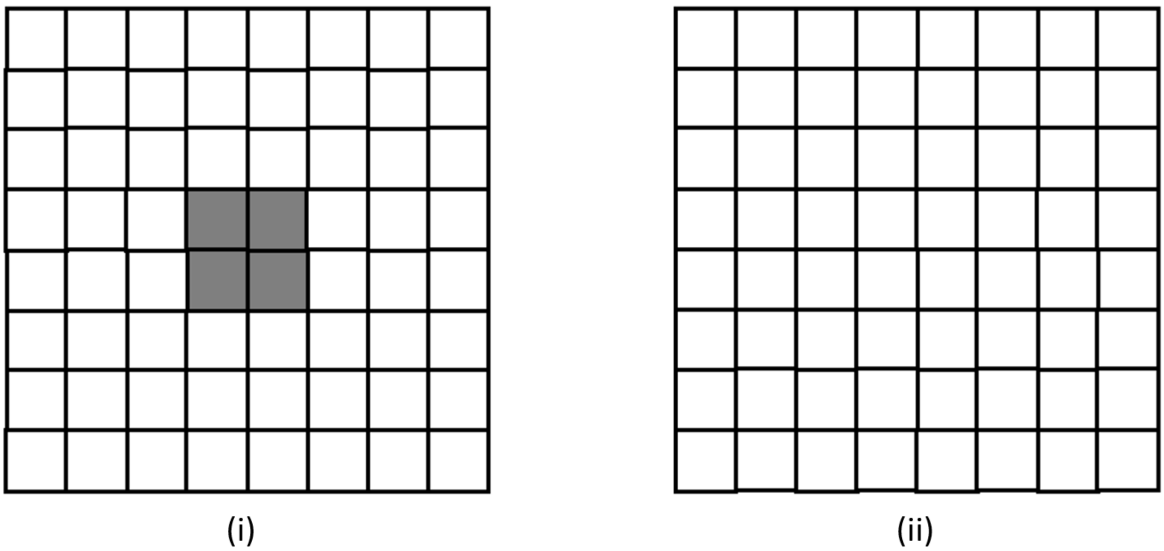


Figure 4.23. Pre-test inverse area question involving scaling.

Response	Students
9 times bigger	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4N**
3 times bigger	N/a
4 times bigger	4M

Table 4.15. Responses for pre-test question seeking to elicit student's understanding of area scaling.

It was observed that all but one of the students were able to correctly represent the area covered when light source distance from the grid was tripled. However, not all student's reasoning directly

reflected this, and in some cases, student's reasoning contradicted their sketches. It was found that there were four lines of reasoning presented by the students:

- Students 4A and 4E submitted that they observed an increase in the "diameter" of the area by 3 times.
- Students 4B and 4H submitted that the area would increase 3 times, which is indicative of an inverse relationship. However, this is not reflected in their diagram, which shows an area that is 9 times bigger. Students 4D, 4F, 4G, 4I, 4J, 4K and 4N did not justify the quantity of their increase in area, but just qualitatively explained why the area of illumination increased, referring to how the light spreads across a larger area.
- Student 4C determined that for every metre the torch was moved back, the area given in Figure 4.23 (i) would increase by the power of the value of the distance from the wall: i.e., if the initial area covered is 4 units and the torch is moved to 3 meters from the wall, the final area would have a value of 4^3 squares on the grid. However, this student shaded 42 boxes, which does not correlate to the reasoning the submitted.
- Student 4M reasoned that in increasing from 1 m to 3 m, for every meter the torch was moved back, the area doubled. In going from 1 m to 2 m, the area increased from 4 boxes to 8 boxes, and in going from 2 m to 3 m, the area increased from 8 boxes to 16 boxes. This reasoning indicates a use of exponential proportional reasoning.

The last pre-test question looked at student's proportional reasoning, involving the inverse square law. Students were given a scenario in which a light sensor was placed 2 m from a bulb, and gave a reading 100 W m^{-2}, and were asked to determine what reading would be given on the sensor, if it was placed 4 m from the light bulb.

A summary of the results is provided in Table 4.16. 11/14 of the students determined that the light intensity would be half the original value, i.e., 50 W. The students did not submit reasoning, but instead produced a numerical value. These answers suggest that students did not consider the inverse square relationship, nor did they consider the patterns they drew in the second question. However, given that most of student's reasoning was incorrect for the second question, this is unsurprising. This indicates that the students had a mental model to help explain inverse relationships but had not developed an extension to the inverse square law.

The pre-test provides evidence to suggest that the students were generally unfamiliar with the inverse square law. 11/14 of the students did not relate the characteristic asymptotic curve associated with an equation of the form $y = k\frac{1}{x^2}$, instead producing a variety of linear and quadratic curves. 13/14 of the students were generally able to determine how the area covered by a source that follows an inverse square law changes, but showed reasoning inconsistent with their diagrams or no reasoning. This suggests some students may have guessed the correct outcomes, based on an

understanding that the area would increase in some manner. None of the students applied the necessary proportional reasoning to a mathematical exercise that required the understanding of the inverse square law. These difficulties in the pre-test agree with the findings of Marzec (2012).

Responses	Students
4 times smaller	**N/a**
2 times smaller	4A, 4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4M, 4N
Other reduction	4F, 4K
N/A	4L

Table 4.16. Responses for pre-test question probing student's proportional reasoning of intensity.

4.3.2. Tutorial lesson: Inverse square law

The tutorial lesson opened with a brief class discussion for 10 minutes, in which the previous presentation of the Cavendish experiment was reviewed. A formal definition for Newton's Gravitational Law was introduced and the discussion highlighted the similarities when comparing the equation $F = G\frac{m_1 m_2}{d^2}$ and functions of the form $y = k\frac{1}{x^2}$. Various physical phenomena that follow inverse square laws were listed by the teacher, such as light intensity, sound intensity, gravitational, static electrical force and the emission of particles from radioactive sources. This provided context for the importance of the mathematical relationship, which the students would explore in the tutorial lesson, which took up the remaining 70 minutes of the lesson.

The tutorial on the inverse square law was designed using an analogy of paint being sprayed over an increasing area, in which the students determined the number of particles of paint spraying over individual segments. In this way, spray paint "intensity" is used to conceptually model the inverse square law. This educational model was developed, and expanded upon, from Conceptual Physics, Practice Book (Hewitt, 2009).

Initially the students were presented with a scenario of a spray can emitting 100 drops of paint per second over a given area and were required to calculate the amount of paint droplets landing each second on a uniform area of 1 m^2. The students were then required to expand their model to the increase of area covered when distance from the can to the area is increased, as depicted in the Figure 4.24. This section of the tutorial was developed to promote the student's conceptual understanding of scaling (Arons, 1999), in which they were guided to reason why the area of the frames increases quadratically, instead of linearly, as the spray moves from left to right in the diagram.

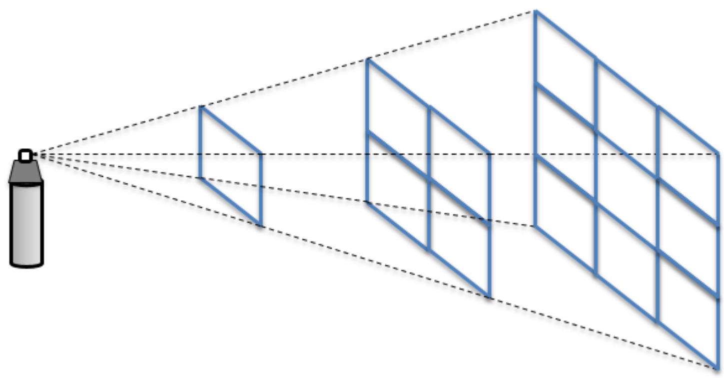


Figure 4.24. Diagram representing spray paint droplets passing through frames.

The students were required to explain, in the case of the second frame which was twice the distance from the can as the first, why the area of the second frame was four times the size of the area of the first frame. From this, they were required to determine how many droplets of paint pass through the individual smaller frames of the second setup, if 100 droplets passed through the first in 1 second. By determining that there were 25 drops passing per second in an individual frame in the second setup, the students were required to determine how this showed the spray paint "intensity" was following an inverse square law. Many discussions took place within groups to develop this reasoning, generally taking somewhere in the region of 15 minutes for the student groups to develop the reasoning to explain how the intensity drops. In the cases where students struggled to progress, the teacher asked the student groups to consider the increase in the length of the overall frame, the width of the overall frame, and discuss how both these increases affected the area of the overall frame.

Upon completion of this question, students had to consider the frame that was 3 times as far from the paint can as the first frame. Again, they were asked to determine why the area was 9 times the area of the first frame and use this to determine the spray paint "intensity" for one frame in one second, on the third setup. The students determined that there were 11 droplets per second, rounded to nearest whole number, and again used this to demonstrate the inverse square relationship. In completing this, the students demonstrated how the growing distance from the can decreases the number of droplets passing through an individual frame. To illustrate these points, the following quotes from student 4A, 4D and 4I were obtained by scanning the student artefacts.

Student 4A: Doubling the distance and the height, fits 4 plates in.
Distance triples and height triples, fits 9 plates in.
The drops are being divided (through the frames) as it grows

Student 4D: The farther away from the can, the bigger the area is because the lines are expanding, meaning more boxes (frames) can be filled in from each side. As the distance from the pain can increases the drops per second decreases because the same number of drops pass through each part but distributed equally into each frame.

Student 4I:	Because you can fit 3 more square in horizontally and vertically, as it gets further away. The droplets have to spread between the area. The more area there is, the less droplets of paint passing through 1 m^2.

This section of the tutorial gave the students a conceptual grounding in the inverse square law, using a tangible model which they could easily picture. The initial difficulty encountered by students suggested students recognized the limits of their understanding and became dissatisfied with it. Whilst some students required prompting to consider the length and width of the frames individually, the groups managed to discuss and construct sound reasoning that allowed them to develop explanations of what was described in the presented model.

The tutorial then turned to a graphical treatment, in which the students graphed the data for the paint "intensity" at various distances between the can and the frame, to show the inverse pattern. From this, they chose data points on the graph and use the data points to show the reduction ratio, when the distance from the can is increased by a factor of 2, and then again by a factor of 5. This enabled the students to confirm, using their graph, that an inverse square law is observed. To illustrate, work from student 4I is reproduced in Figure 4.25.

Student 4I:	It is a quadratic graph, that is decreasing, a slope that gets smaller. 1 m, it is 100. At 2 m, it is 25. If you move 2 m away, it will cause a decrease of 4 times the intensity. 5 m, it is 4. If you move 5 m away, it will cause a decrease of 25 times the intensity.

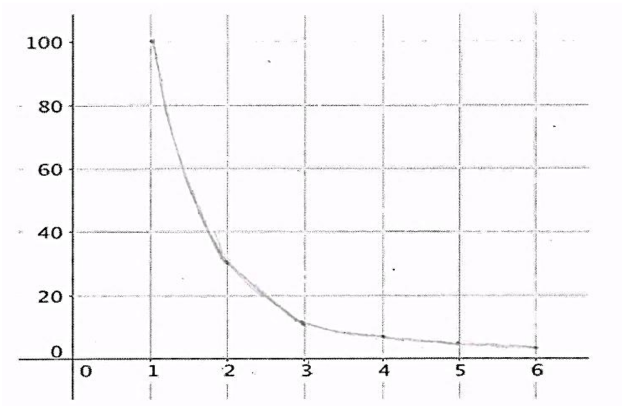


Figure 4.25. Student 4I's Graphical representation of the inverse square law.

There was not enough time for the tutorial to address student's complete quantitative problems the involved the inverse square law. However, in completing quantitative problems in Newton's Gravitational law in subsequent lessons unrelated to this project, the students were afforded the opportunity to practice the mathematical operations involving the inverse square law.

4.3.3. Post-test: Inverse square law

The inverse square law post-test was completed by the students two weeks after completing the tutorial lesson on the inverse square law. It consisted of questions that probed student's understanding of the inverse square law using graphical, diagrammatic and mathematical means. All questions revolved around the context of a spray-paint can spraying on a wall with a grid, which was used to model intensity.

In the first question, the students were presented with a formula for spray paint intensity and asked to produce the shape of an intensity vs distance graph and suggest reasoning as to why they chose the shape that they did. This question was similar in nature to the first pre-test question and would allow for direct comparison. It would allow us to determine if students learned to recognize and transfer the algebraic characteristics of an inverse square function to a graphical one and articulate their justifications. The question is shown in Figure 4.26, and the student's responses are presented in Table 4.17.

While 9/14 of the students answered correctly, only two (4H and 4I) of these students referenced the mathematical relationship between the variables referenced in the questions. These students directly referenced the exponent of the distance variable given in the provided equation, justifying not only the shape of their graph, but the type of inverse relationship in the equation.

Student 4H: As the distance from the nozzle increases, the intensity decreases by a square of the distance.

Student 4I: The intensity decreases by a square factor.

One of the students (4B) referenced the area model used in the tutorial to justify the shape of their graph but stopped short of relating this to the equation given. Another two students (4C and 4E) briefly explained the shape of the graph and how it relates the intensity to the distance from the can. This suggests memorization of the graph shape, but an inability to interpret it correctly in the real-world context.

Student 4B: As the distance from the nozzle increases, the paint is spread over a wider area, meaning the intensity decreases proportionally.

Student 4C: It shows that intensity is affect by distance.

Student 4E: As the distance from the nozzle increases, the intensity decreases.

Furthermore, one student (4K) produced a linearly increasing graph but referenced the inverse square law for the intensity of the spray paint. This indicates a lack of understanding and an inability to link the graphical representation with the relationship they encountered in the inverse square law.

This suggests the student resorted to the use of rote memorization of the law but employed a familiar graph shape that they do not realise does not represent the law they stated. The remaining students explained that a linear pattern produces a line on a graph or submitted no reasoning at all.

A can of spray paint emits 200 droplets of paint per second from the nozzle. The amount of droplet from a can of spray paint that fall on a given area (intensity – I) is given by the formula:

$$I = \frac{200}{0.125\pi r^2}$$

Draw a sketch of the graph to show the relationship between the spray paint intensity (I) and the distance from the nozzle (r) and explain how it shows the relationship.

How it shows the relationship:

Figure 4.26. Post-test question asking students to represent inverse square equation on a graph.

Responses	Students
Decreasing asymptotic graph	**4B, 4C, 4E, 4G, 4H, 4I, 4J, 4M, 4N.**
Decreasing linear graph	4L.
Increasing linear graph	4A, 4D, 4F, 4K.

Table 4.17. Student responses from the post-test inverse square law graphing question.

The second question referred to viewing the wall itself, in which they were presented with the grid in which the paint was only landing a small section of it, when placed two meters from the grid. The question layout was identical in nature to the second pre-test question, as discussed in section 4.3.2, but with a different number of boxes shaded in, and different distances referenced in the question. The students told the spray paint was 2 m from the wall, and the paint spray covered 4 boxes. They were asked to sketch the shape of the painted sections on the grid if the distance were to be increased to four meters. This gave us an opportunity to determine if they could apply the

inverse square law to the area covered by the paint. The diagrams from this question are presented in Figure 4.27 and the student results are shown in Table 4.18.

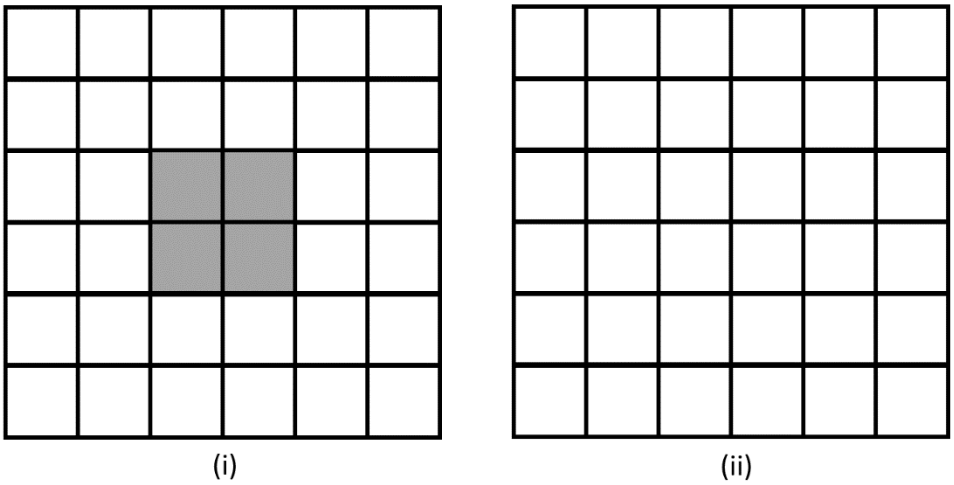


Figure 4.27. Area covered by the spray paint when (i) held 2 m from the wall and (ii) the blank grid.

Responses	Students
Doubling the distance, quadruples the area.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**

Table 4.18. Responses for post-test question seeking to elicit student's understanding of area scaling.

As can be seen from the responses, all students showed that the increasing the distance from the wall increases the area of the spray of paint quadruples. While it its noted that doubling the distance and squaring the distance can result in the same result, if students were to apply this to the grid, it would give an incorrect response (doubling 4 square results in 8 squares being shaded, as opposed to squaring 4 to result in 16 squares being shaded). The students were also asked to determine what the distance the paint spray would be if 36 squares were covered. The diagram presented in the post-test is shown in Figure 4.28, and the student's results are summarized as shown in Table 4.19.

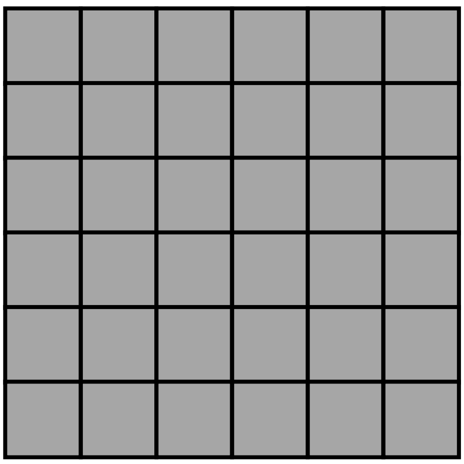


Figure 4.28. Post-test question where students apply proportional reasoning to scaling.

Responses	Students
36 boxes are produced by a radius of 6 m.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4M, 4N**
N/a	4L

Table 4.19. Responses for post-test question in which students determine the distance from the spray paint can to the wall.

Only one student was unable determine the distance required to produce 36 boxes covered in paint. The remaining students produced four lines of reasoning to determine the distance from the can to the wall.

- Mathematical: Students calculated the square root of the area of the shaded boxes to find the length / width of the square. They reasoned, and generalised, in this question that the length / width of the shaded area was equal to the distance from the paint can to the wall. This was valid for this particular question, and is a limitation of it is design. The question was intentionally designed as such, so the ratios would not be difficult for the students to work through when exploring this concept initially. A similar question was also completed by the students during the electric field post-test, in which the numbers are more difficult, and this manner does not directly produce the correct answer. The comparison of these two questions is discussed later in section 6.3.3. As an example, student 4B's reasoning is shown in Figure. 4.29.

Student 4B

$$x^2 = 36$$

$$\sqrt{x^2} = \sqrt{36}$$

$$x = 6$$

Figure 4.29. Sample of reasoning presented by Student 4B.

- Graphical: Students overlaid the paint patterns from the previous two questions. By superimposing these diagrams for 2 m and 4 m, it is easy for the students to extrapolate a pattern across the diagram to determine the distance from the can to the wall. Figure 4.30 shows this solution used by student 4D

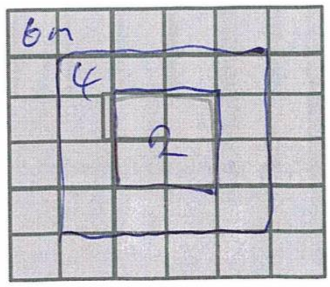


Figure 4.30. Student 4D's graphical reduction used to determine distance.

- Tabular: One of the students, 4E, used a table to determine how many boxes would be shaded at different distances. This acts as a combination of the previous two methods. The student could probably determine how many boxes would be covered for any value of *r*. Note that this student did not employ this method when looking at the electric field, as discussed in section 6.3.3.

Student 4E:

r	*1*	*2*	*3*	*4*	*5*	*6*
r^2	*1*	*4*	*9*	*16*	*25*	*36*

Table 4.20. Data produced by student 4E to demonstrate quadratic change.

- Changing length and width: Only one student (4H) determined the area of the wall when the can was 6 m from the wall using reasoning related to the change in the lengths / widths of the boxes. As the area of 36 boxes has a length of 6 boxes, and the initial diagram showed an area of 4 boxes with a length of 2 boxes, the student divided the lengths to determine that each side of the area grew by a factor of 3. The student then used this factor to multiply the original distance for the area of 4 boxes (2 m) by 3 to determine the correct radius. In their submitted post-test, it was evident that the student struggled to clearly articulate this in their answer.

Student 4H: *If you divide 6* [width and length of shade in question] *by 2* [the original length and width of the shade], *you get 3. So 2 m x 3 = 6 m.* [2m is the original distance]

In the final question presented on the post-test, the students were asked to use the formula presented on the first question to determine the spray paint intensity at both five meters and ten meters from the can. This would afford them the opportunity to use the data to verify that spray paint intensity followed an inverse square law, as developed from their comments in the previous questions in the post-test and use a method that would confirm their reasoning mathematically. The results of this section are summarized in Table 4.21 and Table 4.22.

Responses	Students
Correctly determined both intensity values.	**4B, 4C, 4E, 4F, 4G, 4H, 4J, 4K, 4M**
Correctly determined one value	4A, 4I, 4D
Inverted the distance only.	4N
N/a	4L

Table 4.21. responses for post-test question probing student's mathematical proportional reasoning of intensity.

Values used to verify inverse square law	**4C, 4E, 4G, 4H, 4K**
Incomplete reasoning	4B, 4J, 4M, 4N
Misconception not relating intensity to area	4A, 4D, 4F,
No reasoning given	4I,
N/a	4L

Table 4.22. Students that calculated values to verify the intensity as an inverse square law.

It was seen that 9/14 of the students completed the substitution and evaluation required to determine the spray paint intensity, but only 5 of these used the values to show an inverse square relationship was observed. They used the formula, developed a ratio and commented on how the change in distance from the can to the wall affects the intensity. They could all calculate the intensities and demonstrate that the doubling the distance produces one quarter the intensity.

Student 4C:	Student 4E:
$\frac{200}{0.125\pi(5)^2} = 20.37$ $\frac{200}{0.125\pi(10)^2} = 5.09$ *Distance x 2 = 2^2 = 4.* *20.37 ÷ 4 = 5.09* *The intensity is dependent on the distance. If the distance is doubled, it is 4 times less intense.*	$I = \frac{200}{0.125\pi(5)^2} = 20.37\frac{W}{m^2}$ $I = \frac{200}{0.125\pi(10)^2} = 5.092958$ $\frac{20.37\ W/m^2}{4} = 5.092958$

Table 4.23. Post-test calculations presented by students 4C and 4E.

Four students could qualitatively explain the effect of increasing distance on intensity but struggled to completely justify their understanding quantitatively, as seen in the following submissions:

Student 4B: As the distance increases, the paint intensity decreases proportionally (exponentially).

Student 4J: It is an example of an inverse square law because 5.0929 is more than half the intensity for another 5 m away.

Student 4N: The intensity decreases the further out you go.

A misunderstanding that arose in the reasoning in the post-test was seen in the submissions from students 4A, 4D and 4F, in which they appeared to indicate their understanding of intensity was the amount of paint drops being emitted from the can, as opposed to the paint droplets passing through a defined area

Student 4A: The intensity is the same, only dispersed over a larger area.

Student 4D: It is the same, just displaced over a larger area.

Student 4F: The intensity is spread over a larger area.

However, even accounting for this misunderstanding, these students did not demonstrate an inverse square law mathematically. The remaining students who completed the post-test did not submit any reasoning for their answers or did not attempt this question.

The post-test results indicate that the tutorial lesson had a positive effect on aspects of the student's understanding of the inverse. 9/12 of the students correctly represented a pattern that follows the inverse square law on a graph with an asymptotic curve. Some students demonstrated they could correctly determine the increase in area covered by the spray paint droplets and used reasoning indicative of the increase of the lengths and widths, either graphically or in written form, or analysed the pattern to extrapolate the correct answer. In some cases, the students showed they were applying inverse square proportional reasoning in mathematical questions, although these were not addressed in the tutorial lesson. This can be attributed a combination of the students applying the reasoning developed in the tutorial to mathematical questions and applying the skills they developed in solving problems involving Newton's gravitational law.

4.3.4. Discussion

This section discusses the student's understanding of the inverse square law, by comparing the pre-test and post-test results and referencing development shown by the students during the tutorial lessons. It will then discuss the student's conceptual understanding of the inverse square law using graphical representations, diagrammatic representations and algebraic representations.

Initially, the pre-test results indicated that the students were unaware of the shape of the graph and numerical patterns of an inverse square law. This immediately highlighted an issue for conceptual change to be addressed. During the tutorial lesson, the students were guided in mapping an inverse square law graphically. The post-test results show an increase in the number of students who could transfer the mathematical formula to a graph, represent the function using the correct shape, and provide justifications for the graph choice. This is presented in the Figure 4.31, showing the frequency of different responses in the pre-test and post-test.

The gains seen in the student's responses are in line with the findings of Bardini, *et al.,* (2004), in which guiding students through a function in context can help them develop an understanding of the equation and its transfer to a graph. Several difficulties were seen in the student's pre-test submissions, there were no difficulties that trumped any others, and thus, all difficulties were considered for conceptual change. A lack of clear concise reasoning for the graphs drawn in the pre-test would indicate that the students were not satisfied with the reasoning they were using to construct their graphs (Posner, *et al.,* 1982). In the tutorial, it was observed that the students could be guided to represent data that follows an inverse square law on a graph. They then used the data from the graph to develop ratios to determine that it shows an inverse square law.

They also clearly demonstrated that a decreasing asymptotic graph can be used to represent how the intensity decreases as the distance from the can to the wall increased. This evidence indicates that conceptual exchange and extension occurred, as the students demonstrated they could transfer the inverse square law from one representation to another, and then extended their understanding to develop an intelligible method to analyse the data on the graph (Posner, *et al.,* 1982; Hewson, 1992). Comparing the pre-test and post-test results directly from Figure 4.31, the results indicate that partial conceptual change occurred over the course of this tutorial.

Upon completing the lesson centred on the inverse square law, it was observed that students began to associate the general shape of an inverse square graph to any inverse pattern. Some difficulties persisted as five of the students demonstrated that they were still unaware how to either recognise or transfer an inverse square relationship from a mathematical symbolic representation to a graphical representation, in which four of the students sketched a linearly decreasing pattern, and one student sketched a linearly increasing pattern. This difficulty was later re-addressed in the Coulomb's law tutorial lesson, in which the inverse square is highlighted to the students in the

mathematical notation and the student then completing a graphing exercise with force and distance data, to display and analyse the graphical pattern. This is discussed in section 5.5.2.

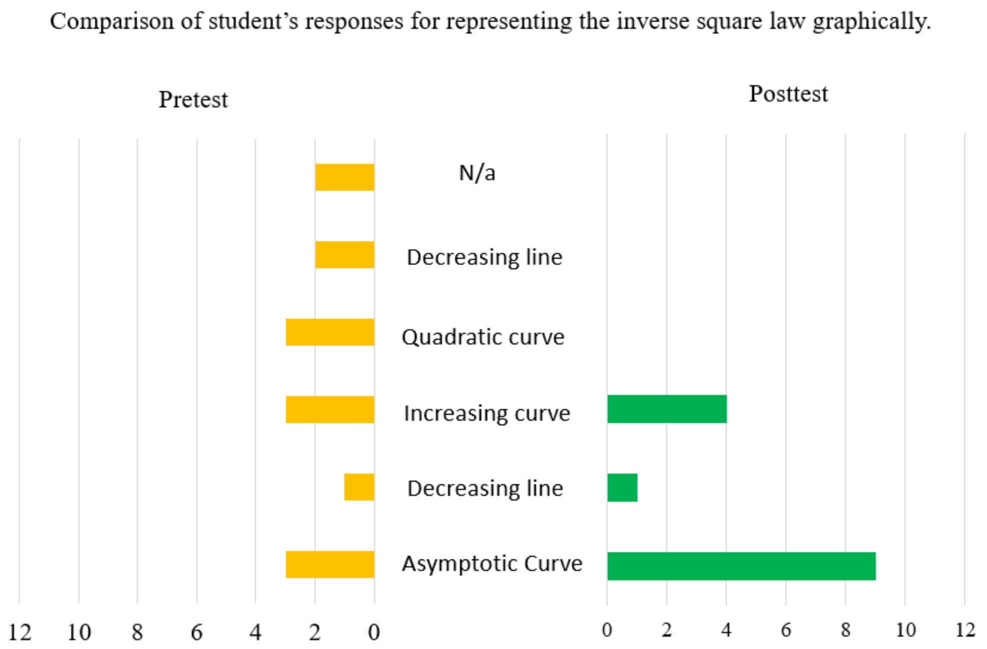


Figure 4.31. Comparison showing for student's graphs of inverse square law.

The development of student's understanding of the area change due to scaling is presented in Figure 4.32. In both pre-test and post-test, it was observed that students could correctly determine the increase in the area illuminated when a light is moved back from a wall. While it was a positive outcome where it was observed that the students could predict the change of the area in the given exercise, it was later observed that this increase did not correlate to student's understanding of a concept like intensity, in which a quantity is spread out evenly over this area. This would indicate that the difficulties to be targeted for conceptual change was not just dimensional scaling, but also applying the scaling area to other quantities and explaining how it applying to concepts like intensity, in which paint / energy is "spread evenly" over the increasing / decreasing area.

During the tutorial lesson, it was found that the model adopted (Hewitt, 2009) of using spray paint passing through square frames helps students visualise the inverse square law, in a relatively simple tangible context. When developing the student's understanding of scaling in the inverse square law they struggled to articulate why the area of the frames grew quadratically with the increase in distance between the nozzle of the paint can and the frame. The students were asked to consider both the increase in the width and height of the frames, and to consider how both these increases could explain the quadratic growth observed in the diagram. Difficulties were also encountered when the students needed to determine the amount of paint droplets passing through the frames when they were presented with 4 and 9 frames. The students tended to consider the total area of all the frames, instead of looking at them individually. The teacher was required to provide initial dissatisfaction to

reasons that focused on the overall area of the frames only. The students were required to extend their thinking to the amount of paint passing through all the frames, and then using their answer to determine how much paint was passing through the individual frame and provide reasoning to scale the "paint intensity" from the larger frames to the smaller individual ones. In the post-test, a small number of the students still considered the overall area, while the remaining students to focused on the change in dimensions of the shape, indicating that conceptual exchange occurred the understanding of these students (Hewson, 1992). This would indicate that minimal conceptual exchange occurred, but an increase in the students focusing on the dimensional scaling is an optimistic finding.

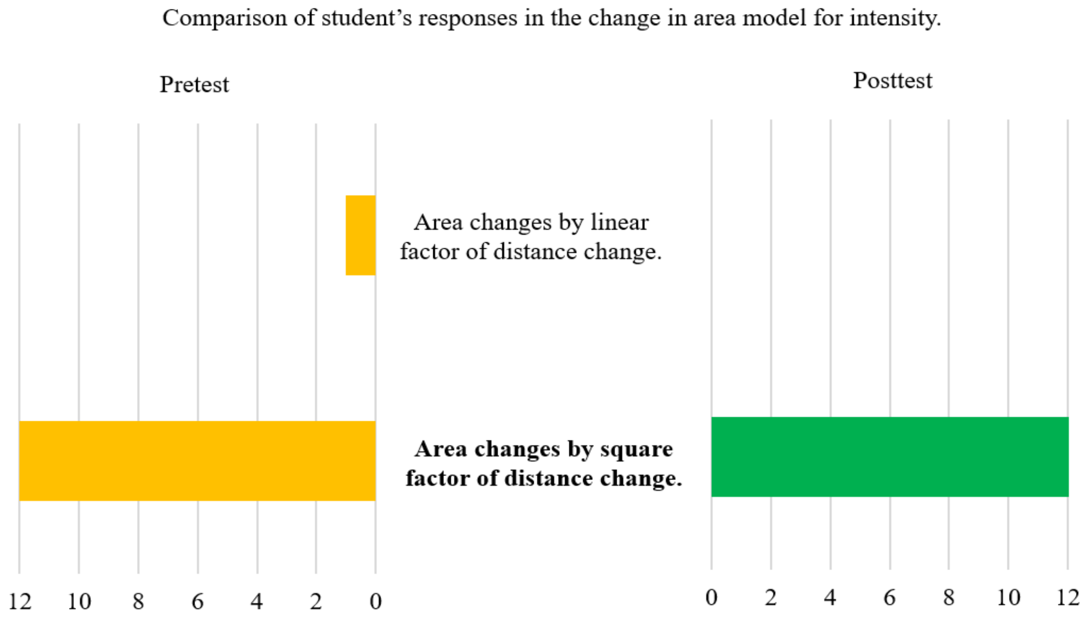


Figure 4.32. Comparison for student's responses using area model.

The last section of this discussion looks at the student's mathematical understanding of the inverse square law. A comparison of the pre-test and post-test results is presented in Figure 4.33. The mathematical pre-test question, in which students needed to apply the appropriate proportional reasoning consistent with the inverse square law, they used either proportional reasoning consistent with a linearly inverse law or qualitatively stated that a reduction would occur. The overwhelming occurrence of students applying linear proportionality indicated a difficulty to be targeted for conceptual extension (Hewson, 1992), as the students need to be aware when to use linear proportionality, and non-linear proportionality. In the post-test, the students were presented with a formula for intensity and were asked to prove that intensity followed an inverse square law, using the same skills developed in using ratios as completed in the tutorial when completing the graphing exercise.

It was observed that 12/14 of the students used the formula to produce at least one correct set of results, but only 5/14 students calculated a ratio, as they were directly instructed to do. These five

students demonstrated transfer of understanding between representations, and their consideration of the overall inverse square law in a task unseen from the tutorial. This suggests conceptual exchange (Hewson, 1992) occurred as the students demonstrated conceptual understanding in an unknown context (Konicek – Moran and Keeley, 2015), and that the students have develop intelligible reasoning that is applicable to further contexts (Posner, *et al.*, 1982). Based on these results, the extent of conceptual change observed was partial. The results also suggests that the difficulty for the remaining students was not the mechanics of using the mathematical operations, but how to apply their calculations to demonstrate an inverse square law as it was observed that nine students could produce values using an equation that involved an inverse square law, but only five could use their calculations to demonstrate the relationship. This is indicative of a gap between their mathematical ability and their ability to understand and apply it in a physics context. The Coulomb's law tutorial was augmented to account for this, as discussed in section 5.4.2.

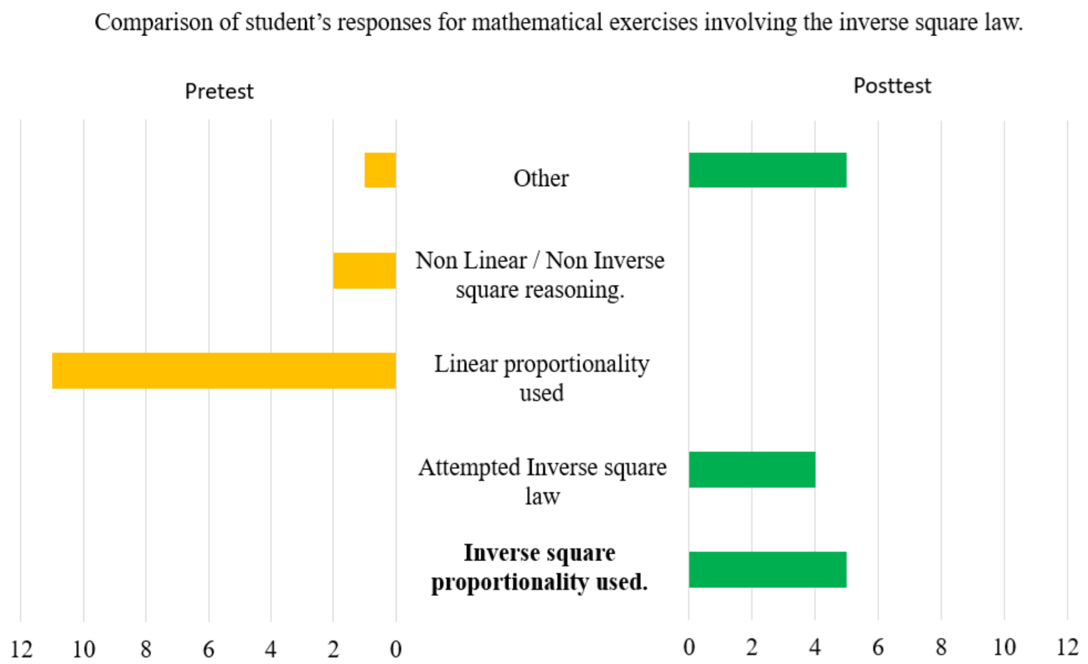


Figure 4.33. Comparison of student's responses for mathematical exercises using the inverse square law.

As discussed in section 4.3.2, the students plotted a graph of data using the inverse square law and completed mathematical calculations based on their graph to demonstrate an inverse square relationship, which they demonstrated. The tutorial itself did not address exploring the inverse square law mathematically. However, the students practised qualitative problems involving calculations involving Newton's gravitational law between the tutorial lesson and post-test. Therefore, we can attribute the increase in understanding of the inverse square law demonstrated mathematically to be a combination of representational transfer utilised in parallel to solving qualitative problems.

This section has shown evidence of gains in the student's understanding of the inverse square law. The discussions show indications that conceptual change occurred, but some difficulties persist with the students. This was to be expected as Arons (1997) and Marzec (2012) showed difficulties can persist beyond initial instruction. The approach adopted allowed for students to progress their understanding of the inverse square law. The reasoning developed by the student can be transferred to Coulomb's law, to explain the variation of the force felt between two charges as the distance between them is varied. This reasoning can also be utilised with students developing their understanding of electric fields, in which the students can explain the variation in the electric field strength at varying distances from a single charge. As discussed in section 5.5.3, one of the electric field homework assignments is written in a format similar in nature to the inverse square law tutorial, where field lines are substituted in lieu of paint droplets, so the students could use the frame model to explain the behaviour of electric field lines, and thus model the variation of field strength with increasing distance.

4.4. Field line concepts

This section presents a narrative of the teaching sequence the students experienced to develop their understanding of field line representations. Field is a key concept in physics, providing a model to explain "action at a distance" for non-contact forces, such as gravitational, electric and magnetism. Greca and Moreira (2006) noted that field theory is rarely covered in high school physics and students are mainly introduced to the theory in the study of electromagnetism third level. When students apply field theory to electromagnetism, the emphasis is placed on mathematical representations. This can lead to student difficulties in their understanding, application and interpretation of field lines as discussed in section 2.1.3.3.

The difficulties identified were used to construct the following learning objectives. Upon completion of the field line tutorial lessons, the students would be able to:

1. Distinguish between force and field (Furio and Guisasola, 1998).
2. Sketch field lines diagrams from vector fields, and vice-versa. (Törnkvist, *et al.*, 1993).
3. Recognise field lines are representational tools and not tangible objects (Galili, 1993).
4. Reasonably determine the trajectory of a body under the influence of a field. (Galili, 1993; Törnkvist, *et al.*, 1993).

The inquiry approach developed for promoting student understanding of field lines consisted of a pre-test, a tutorial lesson, homework, and a post-test. This intervention ran over three weeks. The intervention aimed to promote student's understanding of (1) field strength using field line density,

(2) the direction of force acting on a body in a field and (3) the path taken by a body moving through a field. A timeline for the implementation of the inverse square study, including the target concepts for the intervention, are shown in Table 4.24. As mentioned in section 4.3, this intervention took place in the same period as the inverse square law materials. The sections relevant to field lines are presented in bold.

Section 4.4.1 presents the pre-test results on how students represent field strength, sketch vectors to show the direction of force at different points in a field, and show the path taken by a body in a field. Section 4.4.2 presents a narrative of the development of the student's understanding of the field line conventions, initially looking at uniform fields, then varying fields and finally looking at bodies interacting with a field. Section 4.4.3 presents an analysis of the homework assignment, which was developed to allow the students to practice the skills and apply the understanding they developed in the tutorial. Section 4.4.4 presents an analysis of the post-test results which, like the pre-test, focused on student's understanding of field strength using field line representations, sketching the direction of force at different points in a field and showing the path taken by a body in a field. Section 4.4.6 presents a comparison of the pre-test and post-test results, and a commentary of the student's progress during the tutorials. Examples of student progression and difficulties that persisted throughout the tutorial and post-test are discussed.

Time		Research Implementation	Target Concepts.
Week 4	Class 1.	Pre-test.	Inverse square law pre-test.
	Class 2.	Tutorial Lesson	Inverse square law tutorial lesson.
	Class 3.		*Topics unrelated to project: Newton's gravitational law.*
Week 5	Class 1.	**Pre-test.**	**Field strength.** **Direction of force related to field line.**
	Class 2.	**Tutorial Lesson** **Homework.**	**Path taken by body in field.** **Topic summary.**
	Class 3.		
Week 6.	Class 1.	**Post-test**	**Field strength.** **Direction of force related to field line.** **Path taken by body in field.** **Inverse square law post-test.**

Table 4.24. Timeline of the implementation of the field lines study.

4.4.1. Pre-test: Field line Concepts

The pre-test for field lines was based on gravitational fields, as this is the first topic in Leaving Certificate Physics that students experience in which field line representation can be used. At Junior Certificate level, the would have used field lines to represent the magnetic field of a bar magnet. During the initial presentation of Newton's universal law of gravitation, the students were introduced to field lines as an alternative manner to visually represent the gravitational field of a large mass, as opposed to using vectors.

The students then completed the pre-test, in which the results could be used to determine how much understanding the students develop at Junior Certificate level about field line convention. The results were also used for comparison with the post-test results, to determine any conceptual gains that can be attributed to the students completing the tutorial lessons. The students were given 15 minutes to complete the pre-test and it was administered before the class discussion to introduce the topic. The tutorial mainly revolves around scenarios of bodies moving through space and feeling the gravitational force between themselves and planets. The pre-test probes student's understanding of the following concepts:

- The field strength is represented by the field line density. (Furio and Guisasola, 1998)
- The direction of the field is tangential to the field lines. (Törnkvist, *et al.,* 1993)
- The field lines represent the direction of the force acting on a body, not the path taken by a body. (Galili, 1993; Törnkvist, *et al.,* 1993)

The field line patterns used in probing the student's understanding of both field strength and the direction of the field at various points are shown in Figure 4.34.

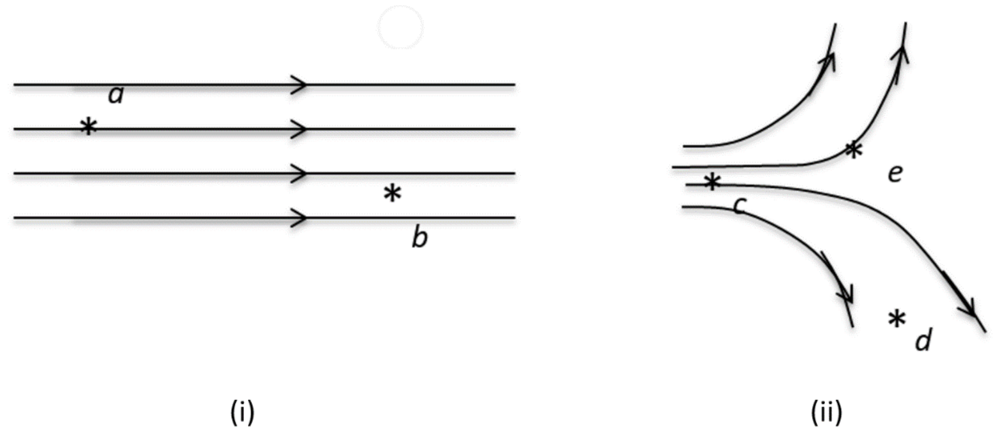


Figure 4.34. Pre-test field line question.

The student's rankings of field strength, from highest to lowest, is shown in Table 4.25. The correct ranking is denoted in bold. The pre-test results indicate that no student had a full understanding that the field line density represented the relative field strengths of the field. The most prominent rankings submitted by students were "d > e> c > a = b" and the converse ranking, "a = b > c > e > d." These rankings could suggest some indication that the field line density represents strength, but some of the reasoning provided by these students indicate otherwise. Reasoning provided included that the straighter the lines, the stronger the force (4B); the more the lines turn, the stronger the force (4H); the more the direction faced downwards aligned with gravity, the stronger the force (4J). The remaining students who chose these ranking did not submit reasoning.

Ranking	Students
c > a = b > e > d	n/a
d > e> c > a = b	4A, 4E, 4F, 4H, 4J.
a = b > c > e > d	4B, 4I, 4N.
e > d > a = b = c	4C
d > e > a > b > c	4G, 4L
d > a > b > c > e	4K
a > c > e > d > b	4M
N/a	4D

Table 4.25. Student's pre-test rankings of field strength, from highest to lowest

The other students who completed the pre-test and gave alternative rankings submitted reasoning such as the angle of the lines determines strength (4K), the apparent (or lack of) movement of the lines as you travel from left to right (4C) and the distance of the points to the field lines themselves (4M). Overviewing these results, it shows that the students were familiar with field line representation, and that when faced with a question on this, approximately half of the students were unable to correctly interpret how field lines representations represent field strength.

The second pre-test question asked the students to determine the direction of the field at the points marked with an asterisk (*) in the gravitational field, and to use vector arrows to represent them. The results are summarized in Table 4.26, with the correct response denoted in bold writing.

Response	Students
Force is tangential	**4M, 4N**
Force follows field line	4B
Force from (a) to (e)	4C, 4G
N/A	4A, 4D, 4E, 4F, 4H, 4I, 4J, 4K, 4L

Table 4.26. Student pre-test responses to representing the field using vector arrows.

Only two students correctly determined the direction of the force at the points labelled, 4M and 4N. Their results showed the force acting in the direction of the uniform field, tangential to the curved field lines. One student, 4B, showed the force following the field lines, not only showing a misunderstanding about the force direction, but also that the vector arrows curve, as opposed to show directions at a single point. Two students appeared to misread the question and drew a vector arrow from the point (a) directly to the point (e), 4C and 4G. The remaining students did not answer the question, suggesting they likely did not possess the understanding required to complete the question. Overall, this suggests that students were unaware of how field lines represent the direction of the force

The final question on the pre-test determined whether students can predict a reasonable path taken by a stationary body when it moves under the influence of a gravitational field of two nearby planets. The field is shown in Figure 4.35, and a summary of the student's responses is shown in Table 4.27.

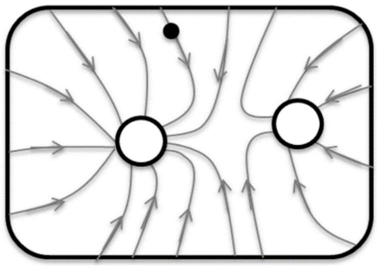


Figure 4.35. Pre-test question in which students were required to draw the path taken by a stationary body under the influence of the gravitational field of two nearby planets

Responses	Students
Path does not follow field line	**N/A**
Path follows field line	4B, 4E, 4G, 4M, 4N.
Directly to planet	4C
N/A	4A, 4D, 4F, 4H, 4I, 4J, 4K, 4L.

Table 4.27. Student's pre-test paths taken by small body in a gravitational field.

Students typically did not understand what information can be gleaned from the field line pattern. No students determined how the field lines represent the direction of the force at the point. Five students appeared to think that the field line represents the path. One student, 4C, indicated that the body would fall directly towards the leftmost planet, ignoring the effect of the gravitational field generated by the rightmost planet, and any acceleration it may have experienced due to it. The remaining students were unable to formulate any reasoning to allow them to attempt this question, and left it blank on the pre-test.

The pre-test results present evidence that students were unaware of the conventions of using field line representations. When presented with a field line pattern, the students were unable to accurately determine the field line strength, using the field line density as a guide (Furio and Guisasola, 1998). 12/14 students showed difficulties in using vectors to represent the direction of the force at various points in the field (Törnkvist, *et al.,* 1993). When required to draw the path taken by a body in a gravitational field, 6/14 students thought the field lines present the path taken by the body, or ignored the patterns of the field lines and directed the body directly to the nearest mass (Galili, 1993; Törnkvist, *et al.,* 1993)

4.4.2. Tutorial lesson: Field line Concepts

The students were briefly introduced to field lines in a class discussion, looking at the use of vectors to demonstrate the inverse square law. They were presented with a planet and vectors shown at points around the planet, getting shorter as the points were further from the planet. The students were asked to explain how the vectors represent that the field was getting weaker as the points were getting further from the planet. In pairs, they discussed this, and all pairs volunteered that the shorter arrows demonstrated a weaker field. They were then informed that drawing vectors in the manner shown can be cumbersome and they were presented with the same planet with eight field lines converging towards the planet. I explained, using diagrams to demonstrate, how field lines and vector arrows could be used to demonstrate the same thing. The conventions of relative field strength being represented by field lines being close together was discussed, and highlighted that they represented the direction of force, as opposed to the trajectory of bodies under the influence of the field lines. I presented three diagrams of field line patterns and the students in pairs practised interpreting the field line patterns, in light of the conventions shown, and verbally gave feedback. Eight of the students initially confused the field strength convention, in which they reasoned field lines further apart were stronger but over the three, they appeared to rectify this error. A depiction of a satellite in orbit was used to illustrate how field lines represent the direction of force experience by a mass at different points, but not the path taken by the satellite, as they clearly were different in the diagram. The students were then informed in the tutorial lesson, the use of field lines would be explored, which they then commenced.

In the first section of the tutorial, the students were presented with a body falling off a cliff, in the path taken as shown in Figure 4.36. The students were guided through a series of questions asking about the force acting on the body as it fell, to guide them to understand that the ball was under a constant force due to gravitational attraction. It was expected that the students would identify that the acceleration due to gravity on the ball was constant and could represent it as such using vector arrows.

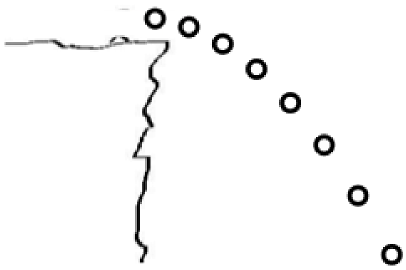


Figure 4.36. Motion diagram of a body falling from a cliff, from the field lines tutorial.

Some of the students highlighted the negligible change in acceleration due to gravity due to the ball being closer to the centre of gravity of the earth as it falls. This was discussed, and they concluded that the change would be extremely small, and practically immeasurable.

|Student 4B:	It (gravitational force) changes very slightly. No (acceleration is not constant), as it changes as it gets closer to the centre of gravity.
Student 4D:	Yes (gravitational force is constant), gravity doesn't change. Yes (acceleration due to gravity is constant), as the force of gravity doesn't change.
Student 4K:	It (gravitational force) remains constant. Yes (acceleration remains constant) because the acceleration due to gravity will be 9.8 m/s^2.

All students presented identical vector arrows to show the acceleration to due to gravity was constant, including students who mentioned the negligible change due to the decreased distance as the body fell. They were then introduced to field lines, and explicitly told that the field line density showed relative strength and the direction of field was in a direction tangential to the field lines. They were asked to apply these conventions to the field line patterns they were shown in the pre-test, as previously shown in Figure 4.34.

As the convention was explicitly presented to the students, there was little difficulty encountered by them to apply the convention correctly to determine the correct ranking and the direction of force acting on a body placed at the points shown. The students were then invited to represent the gravitational field on the body falling from the cliff. It was observed that when representing the uniform field, the following two difficulties were shown by the students, as seen in the sample of their work presented in Figure 4.37.

- These were that the field lines began at the ball when it was falling.
- The field lines terminated before running down beyond the cliff.

These errors were seen in approximately half of the student's responses. This error was mentioned to students, and is explicitly addressed in a later tutorial involving electric fields.

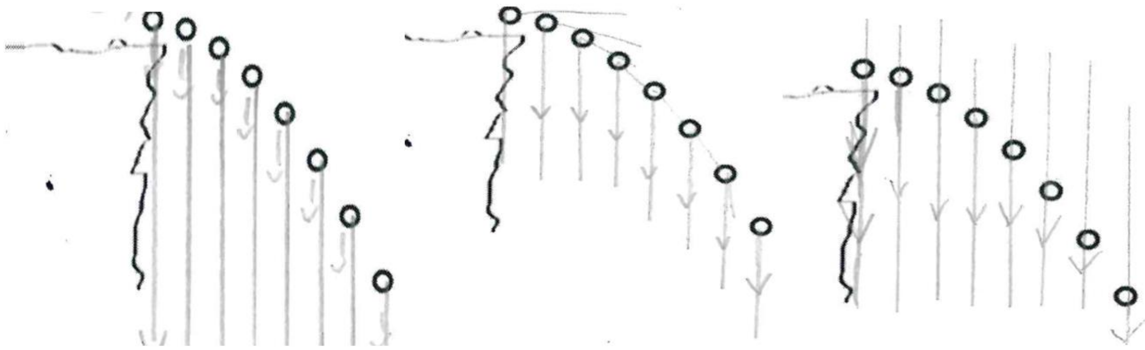


Figure 4.37. Examples of responses, in which field lines begin in body, field lines begin in body and terminate, and an accurate depiction of field lines.

The students were then presented with the diagram shown in Figure 4.38, showing a small meteor moving with an initial velocity and the earth. They were required to draw in the path taken by the meteor and explain how the field shows the variation in field strength, with a suggestion to compare it to the uniform field they previously encountered.

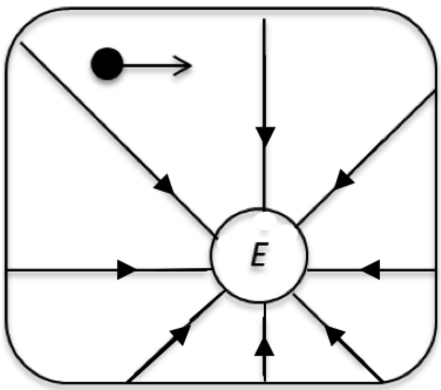


Figure 4.38. Tutorial diagram for difference between the direction of a field line and the path taken by a body.

All the students identified that the field gets stronger at points closer to Earth. Six students referenced that the strength decreases with the square of the distance from the planet referencing the inverse square relationship they encountered in the previous tutorial lesson. However, they could not explain how the field line pattern itself could be used to attributed this relationship, and were instead considering Newton's universal law of gravitation in justifying their reasoning. The remaining students used the field line representation for their justification, as shown in the following quotes taken from scans of student's tutorial worksheets.

Student 4A: No. The field lines are not all equal… the further away they are, the weaker the field.

Student 4E: The lines are far apart at a and b, but c and d are close, so the force is stronger. (Points drawn on diagram by student)

Student 4M: No. The strength decreases. The further from earth you go because the distance between the lines increases, weakening the strength.

The students then had to determine the path taken by the meteor under the influence of the gravitational field. It was expected that students draw their paths so that they would follow one of the field lines, as expected from errors seen in literature where leaners consider the field lines to represent the path taken by a body (Galili, 1993; Törnkvist, *et al.,* 1993). There errors were also observed in earlier versions of this tutorial trailed with pilot groups. However, during the tutorial lesson, the students drew individual paths, and then in their groups, all the students engaged in discussions to determine which of their paths they considered to be valid. While no group said that the meteor would follow a line, there were differences of opinion as to whether the path would be a linear path, a circular path or a curved path due to the initial velocity. When students suggested a deviated linear path for the meteor, the teacher suggested that their chosen path may not be accurate. This allowed other members of their group to explain why a circular or curved path would be an appropriate choice. Although the most accurate paths to represent the paths would have been hyperbolic or elliptical, these types of paths would require a level of depth of understanding the students would not have developed at this point in their education of Leaving Certificate Physics or Leaving Certificate Applied Mathematics. Therefore, both a circular and curved path were considered valid, given the nature of the question, the concept being taught, the lack of numerical details for the necessary calculations to determine the exact path and the prior learning of the students. Each group was asked to explain this in detail to the teacher, to which all the explanations are summarized in the following bullet points:

- The meteor will try to move in the direction it is going with the initial velocity, but the gravitational attraction between the meteor and the earth will cause it to turn.
- This force will cause the meteor to deviate from its original path.
- The field lines will show the direction the meteor will attempt to turn instead of the path.

In the last section of the tutorial, the students were once again presented with the scenario depicted in Figure 4.35. In this case, the meteor has no initial velocity. First, to deepen their understanding of the field line representation, the students were asked to identify which of the two planets had a larger mass, based on the field line density around the planets presented. The students were then required to determine what path would be taken by the meteor. To help them with this process, the tutorial provided a dialogue between two hypothetical students discussing the path taken:

S_1: The field lines indicated the direction of the force, so the meteor will be forced along the line until it hits the left planet

S_2: As the small meteor begins to accelerate, its gained velocity will make it move away from the field line that it was on originally, so we can be sure it'll hit either planet.

Two groups of students inquired into the meaning of the term "gained velocity," as seen in the reasoning seen by S_2. It was explained that this was shorthand for "the meteor would experience a force and accelerate". The students were asked to determine if the velocity would be zero or non-zero, due to the acceleration, after a small amount of time passed.

Initially, students 4F, 4H and 4M agreed with S_1, explaining that the meteor has no initial velocity in this case, leading to the force felt by the particle at all points to turn in tandem with the field line. The teacher then suggested they consider the velocity a small instant after the particle starts to accelerate. The students acknowledged that there would be an non-zero velocity after a small instant in time. The students were then directed to review their reasoning from the previous section covered and discuss with their groups.

Upon review, the students agreed with S_2, and sketched a path consistent with this reasoning. All other students initially considered that the force would cause an acceleration that would take the path off the field line, into a curved path towards the negative charge. Two examples of student work are presented in Figure 4.39.

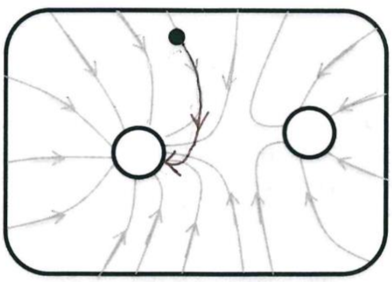
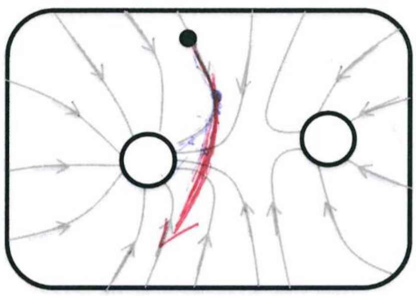
Student 4C:	Student 4H:
S_2, because it has gained velocity, it will move off the field line, to hit the planet. 	I agree with S_2, because as the meteor begins to accelerate, the gained velocity will make it move away from the field line that it was on originally. 

Figure 4.39. Paths depicted by students 4C and 4H.

In summary, the tutorial discussion illustrates how the students developed their understanding of field line conventions. The students were guided to represent a uniform field using vectors and then

transferred the vectors into field line representation. The students were presented with both uniform and non – uniform fields and guided through the reasoning to accurately determine the relative field strength using field line density as an indicator. With assistance from the teacher, the students developed reasoning to explain why a mass does not follow a field line, referencing force, acceleration, velocity and time, to construct a path taken by a body in a field, under the influence of that field.

4.4.3. Homework: Field line Concepts

The homework assignment was developed to reinforce the target concepts developed in the lesson. The homework assignment questioned student's ability to draw field lines, determine field strength based on field line density, and depict the path taken by bodies in various gravitational field.

In the first question, the students again used the context of meteor travelling past planets, in slightly different scenarios to those shown in the tutorial. The first scenario is presented in Figure 4.40.

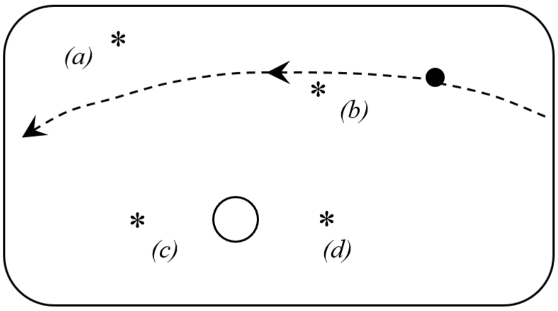


Figure 4.40. Homework question of comet passing planet.

The students asked two questions, in which they were required to draw the gravitational field lines caused by the planet and the second question required them to rank the field strength, from lowest to highest, at the points labelled (a) to (d). The latter question explicitly required the students to reference the field line pattern they sketched for the former question. Their results are summarized in Table 4.28.

Eight of the 14 students sketched a correct field line pattern for the planet's gravitational field, in which straight lines extended from the surface of the planet to the boundary of the diagram, as depicted in Figure 4.38 in the tutorial lesson. Student 4D sketched field lines that terminated in space, before the boundary of the diagram, unlikely realising this depiction suggests the gravitational field terminates at a distance away from the planet. Students 4E and 4F made a similar error, but also sketched their field lines with minor curves, likely not realising that this suggests other masses would be required to be nearby to cause the minor curves they sketched. However, outside of these error in

understanding by the students, they provided an accurate depiction of the field. One student, 4N drew a magnetic field of the earth, but did not provide any reasoning as to why they did this. However, magnetic field patterns are the first field line pattern the students are exposed to (at Junior Certificate level) and it is possible that the student was recalling a pattern they came across two years previously.

Responses	Students
Correct Field line pattern	**4A, 4B, 4G, 4H, 4I, 4J, 4K, 4M**
Used vectors instead of field lines	4C
Field lines terminate	4D
Field lines curve and terminate	4E, 4F
Sketched Earth's B-Field.	4N

Table 4.28. Student's representations of the gravitational field of the planet.

The students were also required to rank the field strength, from highest to lowest, at points marked (a) – (d). Their rankings are summarized in Table 4.29. Seven students provided an accurate ranking. Six students did not explicitly define the ranking between (c) and (d). It is reasonable suspect that they considered the field strength at (c) is greater than (d) in these cases. This would be consistent with reasoning submitted by these students.

Student 4F: C is closest, so it gets affect by the gravitational field the most.

Student 4H: Because as you increase the distance from a planet, you decrease the gravity.

Responses	Students
c = d > b > a	**4A, 4C, 4D, 4E, 4J, 4K, 4M**
c, d, b, a	4B, 4F, 4G, 4H, 4I, 4N
N/A	4L

Table 4.29. Student's rankings of the gravitational field of the planet.

This reasoning was also seen in some students who provided the correct ranking, whilst one student used the field line density to justify their choice.

In the final question, students were asked to sketch the field line pattern of two planets of equal mass and from this, determine a reasonable path the meteor would take, when starting from rest, as depicted in Figure 4.41. There were some difficulties observed in the student's representation of the field lines, such as terminating field lines (4C and 4D), the use of vectors instead of lines (4E) and failing to show the superposition of the two fields (4A, 4B, 4G, 4J and 4K), where lines overlapped instead and two magnetic fields with no superposition (4N). Despite these errors, most of the students

produced a reasonable path to be taken by the meteor, where the path did not follow either the field lines or vectors when used by the students. However, this may have been due to recall of the paths taken in the tutorial questions and applied to this question.

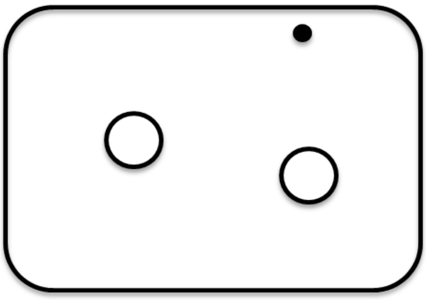


Figure 4.41. Homework question of comet with no initial velocity near two planets.

4D, 4J and 4N submitted paths that were considered unreasonable due to the meteor moving in a direction which does not align with the direction of the net force acting on it (4D), associating a higher gravitational pull to one of the planets at a point equidistant between them (4J) and a path that ignores one of the planets and collides into the other planet (4N).

The homework assignment indicated that the tutorial produced positive gains in some student's understanding. 8/14 students could correctly represent a gravitational field of a planet, with persistent difficulties present such as using vectors instead of field lines to represent the field, field lines terminating inconsistently, field lines being curved when unnecessary or representing the field as the pattern for a bar magnet. 7/14 students could correctly rank the field strength at various points, but in this case, typically used the distance of the points from the planet to justify their ranking. A further 6/14 students submitted answers indicating the correct ranking, also typically referencing the distance from the planet to the points. When representing the field of two planets, student difficulties were more commonly observe, such as representing terminating field lines, drawing vector instead of field lines, not applying the principle of superposition, and drawing bar magnet patterns. Despite these errors, only three students drew paths considered to be unreasonable between the two planets. This indicates that the they were considering the influence of the force of gravity from both planets, the acceleration of the mass, the changing velocity and how these affect the trajectory of the mass.

4.4.4. Post-test: Field line Concepts

The post-test for field lines was undertaken by the students approximately one week after the they completed the field lines tutorial, along with the post-test for the inverse square law. The post-test was designed to elicit student understanding the following three conventions:

- The field strength is represented by the field line density. (Furio and Guisasola, 1998)
- The direction of the field is tangential to the field lines. (Törnkvist, *et al.*, 1993)
- The field lines represent the direction of the force acting on a body, not the path taken by a body. (Galili, 1993; Törnkvist, *et al.*, 1993)

In the first section of the first question, the students were presented with a section of a gravitational field, as shown in Figure 4.42. The students were asked to trace their finger along the lines, starting from where the lines are closest together, so that it travels against the direction of field line, and explain how the field strength varied as they did so.

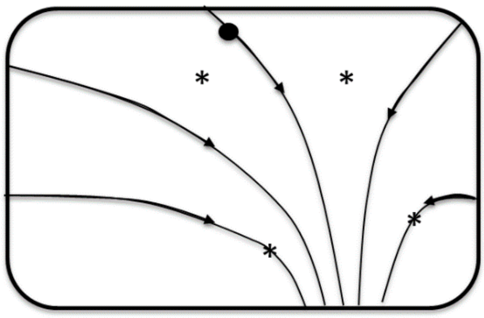


Figure 4.42. Post-test question field lines question.

All the students, except 4I and 4L, explained that the field strength decreases, and this was represented as the field line spreading further apart. Student 4I did articulate that the lines spread out as the field lines were traced but did not reference how this affects the strength of the field itself. Student 4L was not present on the day to complete the post-test.

In the second section of the first question, the students were required to determine the direction of force at the points. The results are summarized in Table 4.30.

Most of the students appeared to have grasped how the direction of the force is represented by field lines. Students 4I and 4N represented the force as acting vertically downwards, which is symptomatic of students who do not consider the field line as a representation of force, and instead assume beyond the bottom of the diagram is a planet in which the gravity uniformly is directed down towards. Student 4G's diagram also suggests a similar reasoning, albeit with the planet just below where the lines appear to converge and the gravitational field acting towards it in a radial pattern.

In the last section of the first post-test question, the students were asked to determine what path a small body would take when it was released from the position marked with a black dot on the diagram. The student's responses are presented in Table 4.31.

Most of the students explained that the meteor's trajectory would not follow the field line, as the bodies inertia would prevent it from directly following the line. They explained that the body would move in the direction of the force acting upon on it and produce a path that is not represented by the

pattern of the field lines in Figure 4.43. The students articulated this by using simpler terms reasoning. The students themselves articulated this as velocity gained by the meteor would cause that path that would carry it from the sketch of the field line. The response summary presented in Table 4.31 indicates that eleven of the students did not think of field lines as a path, and ten of these students could interpret the diagram to draw a reasoning path taken in which the trajectory was influence by, but not identical to, any of the field lines shown.

Responses	Students
Force is tangential to the field lines.	**4A, 4B, 4C, 4D, 4E, 4F, 4H, 4J, 4K, 4M**
All forces point downwards.	4I, 4N
All vectors point to where the line appear to converge	4G
N/A	4L

Table 4.30. Student post-test responses to representing a field using vector arrows.

Responses	Students
Path trajectory sketch diverges from field line pattern in a reasonable path	**4A, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4K, 4M.**
Path trajectory sketch diverges from field line pattern but is an unreasonable path.	4B
Path taken follows the field line.	4N
No path was determined.	4J

Table 4.31. Student's post-test paths drawn taken by a body under the influence of a gravitational field.

However, some students gave additional information about the interaction between the body and the field lines that showed misconceptions. Student 4F correctly reasoned the path and referenced the acceleration, but then also volunteered reasoning that the mass of the body must compete with, and overcome, the force generated by the field lines. Another student, 4H, also gave the field lines a tangible property, in which case the path was a result of the acceleration off the line, the gravitational pull towards the bottom of the diagram and the other field lines pushing the body away to stop the path from intersecting with other field line. This tangible property of field lines has been observed in other research (Galili, 1993)

Student 4F: As the body accelerates, it moves off the field line towards the gravity centre, because its mass is greater that the force of the gravity field.

Student 4H: The body will go off the field line as it accelerates and gains its own velocity. However, the other field lines will push it downwards.

Student 4B reasoned that the area where the gravitational field was strongest would have the most pull, therefore it would pull the body off the field line as shown in Figure 4.43. This suggests confusion to attributing a gravitational force to the field line themselves, as opposed to a representation of which way a body would experience a force. Student 4N acknowledged the body would accelerate, and as it got closer to the bottom, this acceleration would increase. They did not however consider that this acceleration would generate a velocity that would carry it from the field line.

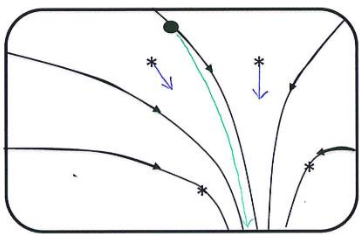
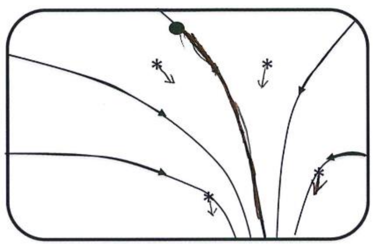
Student 4B:	Student 4N:
The small body is pulled towards the field lines that are closer together, because the strength of the gravity is stronger there.	*The body accelerates faster when the lines are closer together and the body is being pulled down by the force of gravity.*
	

Figure 4.43. Path taken by the body from rest from student 4B and 4N.

The post-test showed that student gains in understanding of how the electric field lines represent the relative strength of the field. 12/14 of the students correctly explained the variation of the field line patterns, with another student providing reasoning alluding to the convention. 10/14 students accurately represented the force vectors as tangential to the field lines, which persistent difficulties in the remaining students observed such as directing the vectors to the point where the field line converge or ignoring the field pattern entirely. The most notable student difficulties were students representing the path taken by a stationary body in a gravitational field. 10/14 of the students sketch the path taken as a pattern that did not follow the field lines, but there were examples of students associating a tangible nature to the field lines themselves, in which they described the field lines are having a gravitational pull of their own. This reasoning appears to treat gravity as a contact force, instead of a non-contact force, and shows the approach did not address the difficulty for these students.

4.4.5. Discussion

From comparing the pre-tests, lessons, homework and post-tests, it can be determined that most of the students made progress in their understanding of field line conventions. The first concept addressed in this discussion is the student understanding of field line density and its representation of relative field strength. Figure 4.44 compares the pre-test and post-test results for this concept.

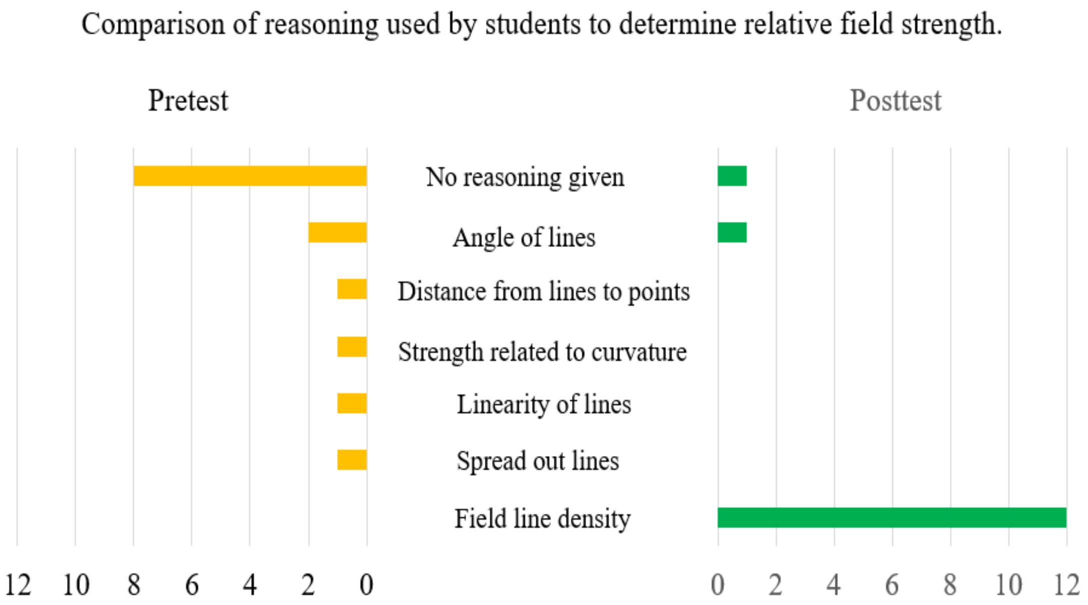


Figure 4.44. Comparison of reasoning used by students to determine relative field strength.

As seen in Figure 4.44, a shift in student reasoning occurred from the pre-test to post-test. In the pre-test, six of the students used incorrect reasoning such for stronger field strength, such as (i) the further spread the field lines, the stronger the field strength, (ii) straight lines being stronger, (iii) the more pronounced the curvature of the line, the stronger the field (iv) the further a point is from a line, the weaker it is and (v) the higher the angle the lines make with the horizontal / vertical, the stronger the field. Eight of them submitted no reasoning for their rankings. The eight submissions with no responses suggest the students had no explanations they were satisfied with to justify their rankings (Posner, *et al.*, 1982), while difficulties the remaining students had were identified. The students were introduced to the convention for field line density representing strength during the tutorial lesson, and section 4.4.3 outlines multiple instances that were sufficient for the students to explore and adopt the convention of field line density to represent relative field strength.

Figure 4.44 indicates that the tutorial lesson was effective in promoting the student's conceptual development, as a clear shift in reasoning submitted by the students was observed in the post-test results. All but two of the students correctly used the field line density to produce the correct rankings, and except for one student, there were no references to the misconceptions observed in the

pre-test. This indicates that conceptual extinction occurred (Hewson, 1992), with the shift in the pre-test and post-test results demonstrating that ideal conceptual change occurred.

The next section of this discussion presents a comparison of the student's transfer from field line representation to vector arrow representation. Figure 4.45 presents a comparison of the pre-test and post-test results for this concept.

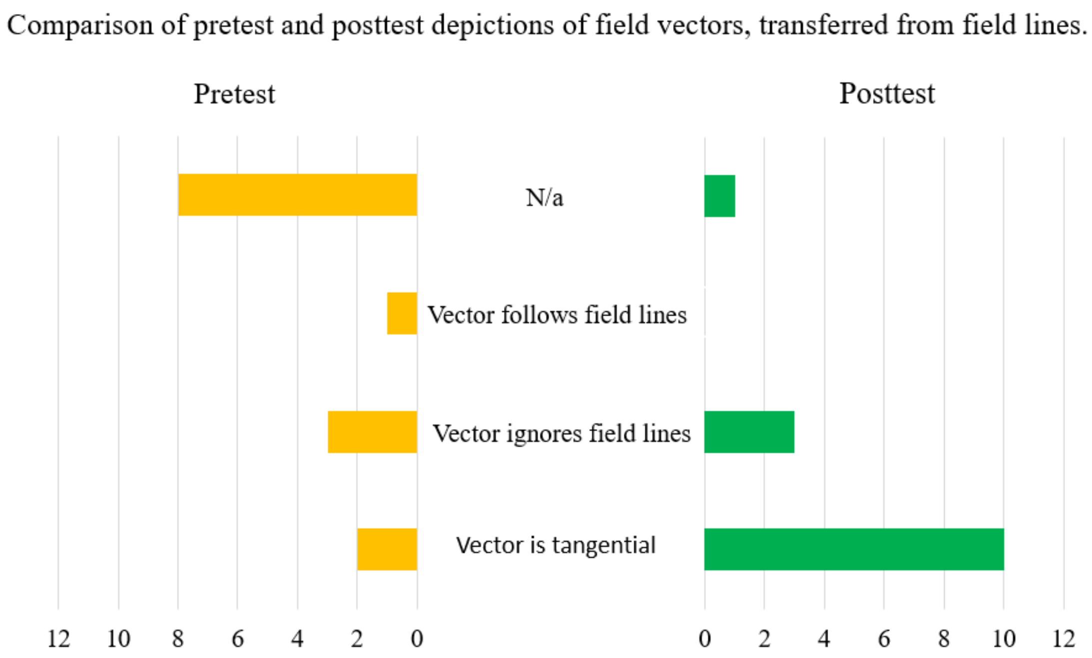


Figure 4.45. Comparison of depictions of field vectors, transferred from field lines.

Figure 4.45 shows that the tutorial lesson also produced gains in student's understanding that the electric field vector at a point is tangential to the field lines. Initially, most of the students were unable to attempt this question in the pre-test and did not have enough understanding to approach the question. The student's inability to reason this question suggests dissatisfaction with their prior understanding (Posner, *et al.*, 1982), while the difficulties observed by the three students who ignored or followed the field lines were identified for conceptual extinction. The students were guided to transfer from field lines to vectors during the tutorial lesson, provided with opportunities to practise the transfer between the representation in the tutorial and homework assignment. During the tutorial lesson, the students used rulers to practise drawing the arrows and repeated this for the homework activity. Figure 4.45 indicates that there was a shift in the number of students that could accurately apply vector diagrams to an electric field context, from the pre-test to the post-test. As the participants demonstrated proficiency of using vectors in section 4.2, this shift indicates conceptual extension occurred (Hewson, 1992), with the shift from two to ten students producing tangential vectors indicating that moderate conceptual change occurred. However, no students attempted an accurate scale, in which the vectors were longer where the field strength was greater, ignoring the magnitude

of the vectors, and instead, focusing on the direction only. This indicates conceptual change occurred, but issues of transfer between the vector representation and the field line representation persisted.

Additionally, in the tutorial lesson, the students built up a model of the gravitational field of the earth and transitioned it to a field line model, but half of the students demonstrated errors such as field lines beginning at the object in the field and terminating before reaching the planet generating the gravitational field. These errors are further discussed in sections 5.6 and 5.7 where the students were given the opportunity to address them.

The last section of this discussion focuses on the student's predictions of the path taken by a body under the influence of a field. Figure 4.46 presents a comparison of the pre-test and post-test responses from the students for this concept. As discussed in section 4.4.1 and 4.4.4, these results are for scenario's in which the body under the influence of the field has an initial velocity of zero.

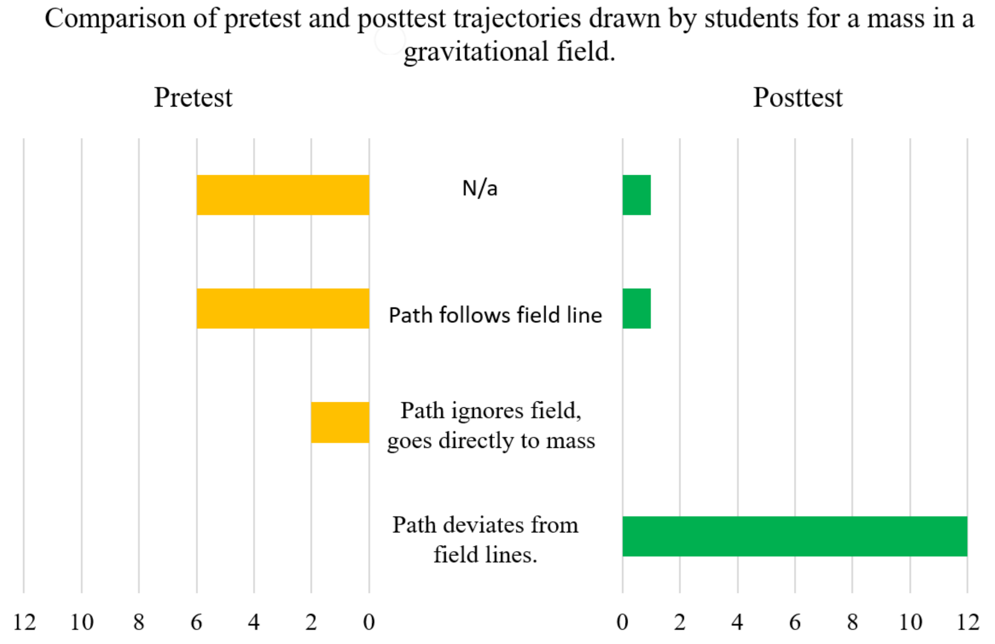


Figure 4.46. Comparison of trajectories drawn by students for a mass in a gravitational field.

The most prominent error by students in the pre-test was that the object would follow the field lines or ignore the field lines and go directly to the mass generating the gravitational field. Both difficulties were predicted from literature (Galili, 1993; Törnkvist, *et al.*, 1993). As the reasoning that produces these difficulties is erroneous, these difficulties were targeted for conceptual exchange (Hewson, 1982). Section 4.4.2 illustrated many examples of the students developing their reasoning for this concept. The tutorial was written with three examples of a body moving with a trajectory influenced by, but not identical to, a gravitational field. The students explicitly looked at this concept on two questions during the tutorial and referenced the first question during discussions with the teacher. As the students were familiar with in the initial scenarios presented in the tutorial, they became dissatisfied with erroneous reasoning that did not explain the observations accurately (Posner

et al., 1982). The students were given ample opportunity to develop and apply the field line representation to explain the observations in the tutorial, and they used it to develop intelligible reasoning to predict the behaviour of objects under the influence of a field, both in contexts they were familiar with and in contexts unseen to them (Posner *et al.*, 1982). In the post-test, all but two of the students depicted a path that diverges from field lines. This shift in student responses from the pre-test to post-test indicates that the tutorial lesson was effective in promoting conceptual exchange in the student's understanding (Hewson, 1982), with the results indicating that ideal conceptual change occurred.

In the absence of presenting students with a field line pattern, such as those seen in the homework activity, it was observed that the students used different reasoning to justify their claims. In the first homework activity given to the students, they were not explicitly presented with a field line pattern, as previously seen in Figure 4.42, and were asked to rank the field strengths. In this case, the students justified the ranking in various manners. 8/14 students based their ranking on using the distance to indicate the gravitational field strength, and all but one of the students ignored their sketches of the field lines and used these to justify their ranking. The students could have also referenced the inverse square law for gravitation to justify their rankings, but this was not seen in any student's responses. Only one student (4B) explicitly based their ranking on the field line density. However, when shown a field line pattern and asked to explain how the field strength varies, as seen in the post-test, 12/14 of the students used the convention for field strength with the representation. This indicates that while the students were comfortable with the representation, they did not always consider it a valuable tool to use, unless directly required to do so.

In the final homework question, the students attempted to use the reasoning they developed in the tutorial lesson, but many errors were observed in the field lines patterns they produced. One tool not observed in the homework responses by the students was the additional use of vectors or motion diagrams to justify the path chosen. When students ran into difficulties in the tutorial lesson, the teacher would ask the students in which direction the body would move and represent this with a vector and sketch the body again at the end of this vector, indicating that this was the body's new position after a short interval of time. The teacher would repeat this process two more times, using the student's answers to define the direction of the movement of the body and its new position after a short interval of time, thus generating a motion diagram with the aid of the teacher. This line of reasoning was not considered by the students in their homework activity. An exercise designed to look which representations students prefer to use is discussed in chapter 5 in which students plot the path taken by a negative charge when placed between two positive charges. In this question, the students are not asked to represent the field in any way, so it will give us an insight as to which representation, or combination they choose to use.

This section has shown evidence of student's gains in understanding of field line conventions. While there are still some difficulties that persist with the students, they appeared in a small number

of the student's responses. Instances of Posner, *et al.*, (1982) conditions for conceptual change were indicated and when possible, the manner of conceptual change that occurred was identified (Hewson, 1992). The reasoning developed by the student can be transferred to electrostatics, to represent the fields of one or two charges. They can also use the representation to help explain the behaviour charged particles in an electric field. Field lines are also utilised in an electric field tutorial to aid students in associating the inverse square law to electric field, using a model of field line passing through a unit frame, similar in nature to the tutorial discussed in section 4.3.3. Students will also employ the use of electric field lines to develop an understanding of positive, negative and zero work in an electric field, to develop their understanding of potential difference.

4.5. Conclusions

The results of the students presented in this chapter show that the student's understanding of vectors, inverse square law and field lines improved by the employing tutorial lesson. Evidence provided supports that conceptual change occurred in some of the student's understanding of these topics. The tutorials both introduced students to the topics, and specifically addressed difficulties typically encountered by students, as seen in literature. This approach has been shown to be effective to address student difficulties, over using traditional instructional methods. (Dykstra, *et al*., 1992; McDermott and Shaffer, 1992).

The results indicate that the approach adopted promotes conceptual understanding of vector magnitude, vector addition and the implications of adding horizontal and vertical vector components. While some of the students preferred to use vector constructions over reasoning based on vector components, it was observed that students engaged with, and overcame, difficulties in the vector concepts such as linking the magnitude of a vector to its direction (Nguyen and Meltzer, 2003), incorrectly combining vector arrows (Nguyen and Meltzer, 2003) treating vector addition as scalar addition, with no consideration for either the directions of the vectors or the summation of the vector components (Doughty, 2013). Regarding these difficulties, the evidence presented in this chapter indicates conceptual exchange (Hewson, 1992) occurred in the student's models, although some difficulties persisted. As the students became more proficient in these vector concepts, the representation could then be utilised by the students to explain the direction and variation of electric field strength between two charged sources, and the forces acting on the charges.

When the students completed the inverse square law tutorial, it was observed that the students could recognize the general shape of a graphical pattern that follows an inverse square function. However, it was noted that students upon completion of this topic, the students could not differentiate between data that followed an inverse square pattern, and an inverse pattern when they complete

investigations into Boyles' law and the focal length of a concave lens. This indicates that students had an over-reliance on the general shape of the pattern and did not consider to mathematically analyse the data presented on a graph. Mathematically, it was seen that the students could evaluate an equation for intensity, but also struggled to analyse their produced values to show an inverse square law. This lack of consideration to find meaning in the numbers produced echoes student's experience of solving quantitative problems that is typically found in traditional methods of instruction in physics education. The students were given more opportunity to consider the inverse square law and practice their ability to analyse the data, both on a graph and using algebraic evaluation, when they completed the Coulomb's law tutorial lesson, as these skills are employed in that lesson, as discussed in section 5.5.2.

Conceptually, it was observed that students could explain the variation of area covered in intensity contexts, when the distance between an object and the frames were varied. Section 4.3.3 narrates how students could reason that area change of frame was a quadratic factor of the change in the distance between the objects due to a change in bother the length and width of the frame, demonstrating their understanding of quadratic pattern observed when scaling up the area of a surface. Difficulties were seen when applying this to intensity when considering how much of a given phenomenon, in this case droplets of paint, passing through the frame. This highlights confusion between their understanding of intensity of a phenomenon and the phenomenon itself. This can also lead to confusion about which factors follow a quadratic increase, in the case of the tutorial; area, and which follow an inverse square law, in the case of the tutorial; paint intensity.

From the student's exploration of field lines, it was shown that the students are reasonably proficient in the use of the representation and can interpret information from a field lines pattern. It was demonstrated, that upon completion of the materials, they could determine the field strength based on field line density, determine the direction of force acting on a body at a point in a field and reasonably plot a path taken by a body in a field, based on this information. However, in the absence of a field line pattern, when asked to construct the field for two masses, the students produced errors in their field lines patterns. The most persistent difficulty seen was the students not applying the principle of superposition of the two fields, field lines overlapping and using vector arrows in lieu of field lines. These persistent difficulties are addressed in a later tutorial, discussed in sections 5.6 and section 5.7.

In the cases of all three of the topics, there is evidence that conceptual change occurred. The pre-test, tutorials and post-test, student - student discussions and teacher-student discussions presented in this chapter provided many instances of the 4 conditions presented by Posner, *et al.*, (1982) required for conceptual change and when the evidence was presented, the type of conceptual change; extinction, exchange and/or extension (Hewson, 1992) was identified. The extent of conceptual change that occurred varied from minimal to ideal, and the instance of each descriptor is presented a

line plot, shown in Figure 4.47. A legend of the codes used in Figure 4.47 can be found in Appendix F.

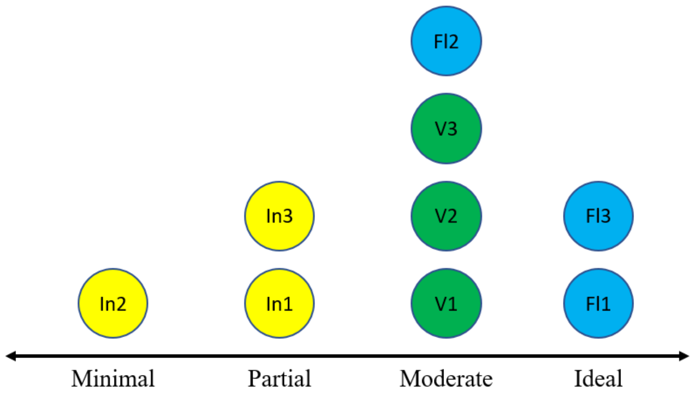


Figure 4.47. Line plot of extent of conceptual change for vectors, inverse square law and field lines.

The extent to which the student's engaged in conceptual change over the course of the tutorials varies depending on which topic the approach addressed. The tutorials that were most effective in promoting conceptual change for the concepts related to the field lines, as moderate and ideal instances of conceptual change were recorded. There were moderate instances of conceptual change for the vectors concepts and minimal and partial instances of conceptual change for the student's conceptual understanding of the inverse square law. The vectors and field line tutorials generally omitted the requirement for students to use mathematical reasoning, while the inverse square law required both mathematical and scientific reasoning to be employed by the students. This requirement to employ dual reasoning could have overloaded the number of items being processed in the students working memory (Reid, 2009) and halted their ability to fully develop their understanding.

By developing the student's understanding of vectors, the inverse square law and field lines, this sets a foundation for the students to build their understanding of Coulomb's law, electric field and potential difference. The students will be able to apply their understanding to these topics to help develop their understanding of electrostatic force, and field. Vectors and the inverse square law are central concepts underpinning Coulomb's law, in determining the forces acting on charged particles, in collinear and non-collinear settings. Vectors and field lines can be used to represent electric field and the behaviour of charged particles in these field, whilst the inverse square law can be used to mathematically quantify the variation in the electric field strength of a charge. When a charged particle moves in an electric field, the use of vectors and field lines can be used to identify whether positive, negative or zero work occurs, which can be used to discuss the variation of potential or define the potential difference between two points in an electric field.

Chapter 5. Coulomb's law and electric fields.

5.1. Introduction

This chapter discusses the development of the student's understanding of Coulomb's law and electric fields, by using inquiry tutorials that employ a multi-representational approach. The tutorials embed vector concepts, exploration of the inverse square law and field line representations in electrostatics. Combined with their prior learning of mechanics, forces and charge, this allows for the students to generate links and their own understanding of the topics and develop the ability to transfer between all representations. This chapter identifies instances in which the students (a) used their understanding of the three element concepts to develop their understanding of Coulomb's law and electric fields, (b) used their experience in developing their understanding of Coulomb's law and electric fields to better their understanding of the target elements, or both.

The following research question is addressed in this chapter:

- To what extent does the use of a multi-representational structured inquiry approach develop student understanding of electric fields?

The following points were considered when addressing this research question:

1. The student's ability to demonstrate that Coulomb's law is an example of an inverse square law.
2. To what extent the students demonstrate their understanding of electric fields and the interaction of charged objects with fields using vector representations.
3. To what extent the students demonstrate their understanding of electric fields and the interaction of charged objects with fields using the field line representation.
4. To what extent the students demonstrate their ability to transfer a depiction of an electric field from one representation to another representation.

Figure 5.1 depicts how the concepts discussed in chapter 4 and the concepts covered external to the project are prerequisites to learning electrostatics. As before, the colour purple denotes topics completed, at this time, during the project; it shows that the students have studied all prerequisite concepts.

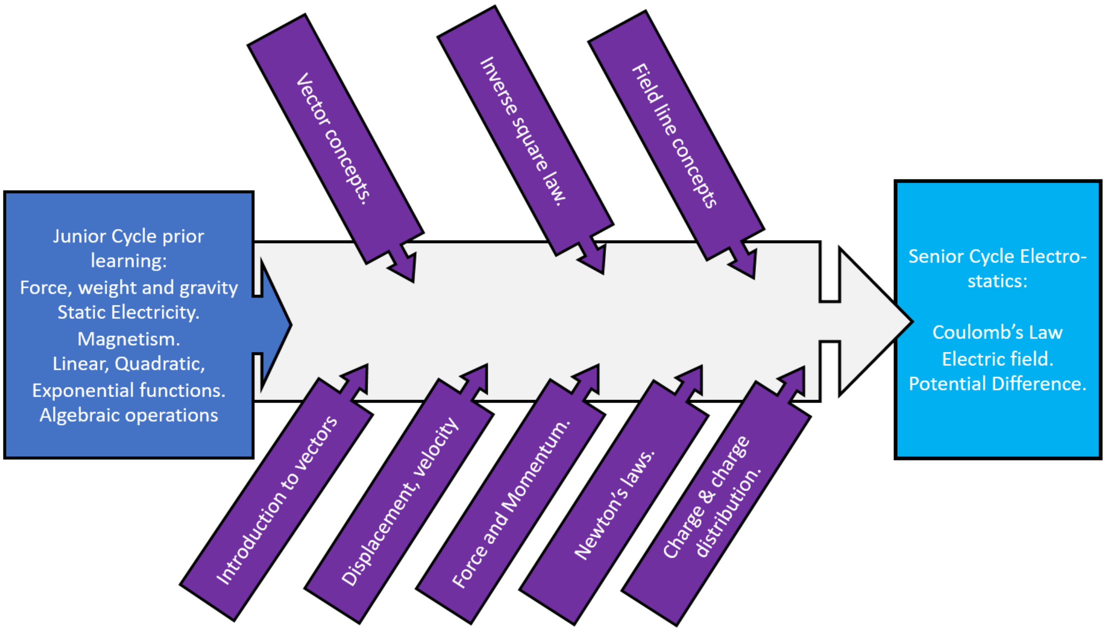


Figure 5.1. Flowchart depicting the topics completed by the students, prior to developing their understanding of Leaving Certificate electrostatics.

The timeline of this section is shown in Table 5.1. Sections in bold refer to materials covered are directly related to the research.

The chapter presents a narrative of the student's use of vectors concepts, the inverse square law and field lines, as they were applied to Coulomb's law and the electric field concept. The student's conceptual development is presented by comparing pre-test and post-test results for the different topics, as well as snapshots of their tutorial worksheets and excerpts of recordings of their conversations during the tutorial sessions.

Section 5.2 reviews difficulties encountered by students in their understanding of vectors, the inverse square relationship and field lines, and their application to Coulomb's law and electric fields, as described in section 2.3.1. Section 5.3 discusses the lessons learned from the research presented in chapter 4, in terms of the difficulties observed in the students understanding after the tutorial lessons, and the implementation of the tutorial lesson format. Section 5.4 discusses how students apply their understanding of vector concepts to the electric field in the context of Coulomb's law. This section focuses on student's use of vector constructions, representing the relationship between magnitude of field strength and distance, and consideration of horizontal and vertical components in vector addition. Section 5.5 looks at the student's application of the inverse square law, and how they apply it to Coulomb's law and electric fields. Students apply the law using tables, graphs, algebra and a scale model, in which they discuss field lines passing through various frames. This approach adapts the model used in Conceptual Physics (Hewitt, 2009). Section 5.6 looks at the student's understanding of the field line representation in electric fields. Here they apply the convections of field line density to represent relative strength, the direction of a field being tangential to a field line. They should understand that the path taken by a body in a field does not follow a field line but the

trajectory of the body is influenced by the field, and the field lines give an indication of the direction and strength of force acting on a body in the field. The students are further introduced to the conventions that field lines do not overlap, and only start and end on electric charges. In the discussions, the results are reviewed to present the impact of the approach on student learning, particularly their ability to transfer representational tools to the electric field context and their reasoning skills.

Week 1, Class 1 (35 mins)	**Presentation to introduce to Coulomb's law.** **Pre-test**
Week 1, Class 2 (80 mins)	**Practice class: Qualitative problems involving Coulomb's Law.**
Week 1, Class 3 (76 mins)	**Research lesson: Coulomb's Law worksheet. Homework assignment given.**
Week 2, Class 1 (35 mins)	**Presentation to introduce to Electric field.** **Pre-test**
Week 2, Class 2 (80 mins)	**Practice class: Qualitative problems involving Electric field.**
Week 2, Class 3 (76 mins)	**Research lesson: Electric field worksheet. Homework assignment given.**
Week 3, Class 1 (35 mins)	**Practice class: More difficult qualitative problems involving Electric field. Homework assignment given**
Week 3, Class 2 (80 mins)	Presentations and demonstrations: Charge distributions and charging by induction.
Week 3, Class 3 (76 mins)	Practice of Leaving Certificate past paper questions.
Week 4, Class 1 (35 mins)	Review of Topics with students.
Week 4, Class 2 (80 mins)	**Post-test.**

Table 5.1. Timeline of the Coulomb's law and electric field tutorial lessons.

5.2. Vectors, inverse square law and field lines in electric fields

This chapter presents a narrative and analysis of the development of the student's understanding of Coulomb's law and the electric field, in terms of their ability to apply vectors, the inverse square law and field lines to this domain. As the students have developed an understanding of these topics, as discussed in chapter 4, the tutorial lessons for this part of the research embed these topics throughout. This gives the students the opportunity to review the material covered in the electric field context, and develop a deeper understanding of the electric field through using multiple representations (Ainsworth, 2006).

The Coulomb's law and Electric field tutorials were designed to provide the students with opportunities to address the difficulties encountered by learners in their understanding of vector concepts, the inverse square law and field line representations detailed in Section 2.1.3. detailed difficulties. The following learning objectives for this section of the research ensued: upon completion of the teaching and learning material, the students would be able to:

- Construct and interpret a uniform and/or non-uniform electric field represented by vector arrows, consider both vector magnitude and direction (Maloney, *et al.*, 2001)
- Apply the principle of superposition to two vector field representations (Maloney, *et al.*, 2001; Nugyen and Meltzer, 2003).
- Students can discuss electric force and field superposition in terms of vector component addition (Furio and Guisasola, 1998; Cao and Brizuela, 2016).
- Demonstrate that Coulomb's law and the electric field follow an inverse square law using a variety of representations (Maloney, *et al.*, 2001; Hewitt, 2009; Moynihan, *et al.*, 2015)
- Apply proportional reasoning involving scaling to inverse square law problems (Arons, 1999, Marzec, 2012).
- Construct and interpret a uniform and / or non – uniform field using field line representations (Törnkvist, *et al.*, 1993; and Galili, 1993; Cao and Brizuela, 2016).
- Apply the principle of superposition to field line diagrams (Törnkvist, *et al.*, 1993; Galili, 1993).
- Accurately predict the behaviour of charged particles under the influence of an electric field (Cao and Brizuela, 2016).

5.3. Lessons learned from previous research

In chapter 4, it was seen that the use of tutorial lessons promoted the development of student understanding of vectors, the inverse square law and field line representations. This approach, patterned after Tutorials in Introductory Physics (McDermott and Shaffer, 2003), breaks concepts

and topics down into a series of lower and higher order questions, designed to elicit student thinking, identify difficulties and provide opportunities to help students overcome difficulties. The emphasis is on students working in groups to think and reason their way through the worksheet. This allows them to apply what they know, organise their thoughts and make judgements and evaluations about physical phenomena. The approach adopted produced various degrees of gains in understanding of vector concepts, though some conceptual difficulties with vector addition remained as it was observed that numerous students did not consider vector addition in terms of component addition. On completion of the inverse square law tutorial most students could recognise and represent an inverse square proportional relationship graphically, explain the variation in the area model using scaling and could apply inverse square proportional reasoning to mathematical questions. Finally, students made some gains in their understanding of field line representations such as relative field strength being represented by field line density, field vectors pointing tangentially to field lines at a point and field lines representing the direction of force experienced by a body, and not the path taken by a body under the influence of a field. However, difficulties related to the superposition of field lines persisted. These difficulties are addressed in the electric field tutorial, discussed in section 5.6.2.

These representations can be used as an aid for students developing conceptually accurate understanding of the electric field, at both second level and during further study at third level. Field theory replaced the preceding model of action at a distance and employs the use of both vector mathematics and mathematics involving the use of calculus. While the application of calculus to electric fields would be beyond the capabilities to the average second level student, an understanding the basic and slightly sophisticated vector concepts can underpin the foundations for future development of further advanced studies into electromagnetism. The inverse square law links to the model of a Gaussian sphere and electric flux density. In Ireland, students typically encounter this model until they have completed second level physics education. If they have not developed a complete understanding of the inverse square law, any developed difficulties would need to be overcome at third level. Field lines are a simple model than can be used to represent a field, and when interpreted correctly, can display a lot of information about a field that would be cumbersome to represent in another format. In developing understanding of these three “pillar” concepts, students can be guided to transfer their reasoning to representations they can struggle with, such as the mathematical formula for Coulomb’s law, or problems that employ the use of unfamiliar contexts.

5.4. Student’s use of vectors in electric fields

This section presents the student’s use of vectors in Coulomb’s law, and electric fields. Section 5.4.1 details the Electric field pre-test results, in which it was observed that students struggled to transfer and apply the vector concepts they developed in chapter 4, such as vector magnitude and

superposition. Section 5.4.2 narrates the use of the electric field tutorial lesson, in which the students review their understanding of magnitude, presented as uniform and varying electric fields. The students then apply the concept of superposition to finding the net electric field at various points using vector addition, and discuss the magnitude of the resultant vector in terms of its horizontal and vertical components. Section 5.4.3 discusses the post-test results, in which positive gains were seen in the student's understanding of vector magnitude. Sections 5.4.4 and 5.4.5 detail research on the student's understanding of horizontal and vertical components through the analysis of a student homework assignment, and a teaching and learning interview involving the concept. Section 5.4.6 presents a comparison of the pre-test/post-test results and discusses student gains observed during the tutorial lesson and teaching and learning interview, as well as illustrating persistent difficulties in student understanding of vector concepts and their transfer to the Coulomb's law and electric fields.

5.4.1. Pre-test: Student's use of vectors in electric fields

In the Electric field pre-test, the students were given a question where they were asked to construct an electric field using vector arrows at various points, based on their relative position around (a) a positive charge, (b) a negative charge, and (c) and positive and negative charge. In this way it was determined to what extent students could depict the direction of the field of the charge correctly, display the relative magnitude at different distances from the charges, and apply vector addition for the fields produced by a positive and negative charge at given points. The diagrams are presented in Figure 5.2, and a summary of student responses is presented in Table 5.2.

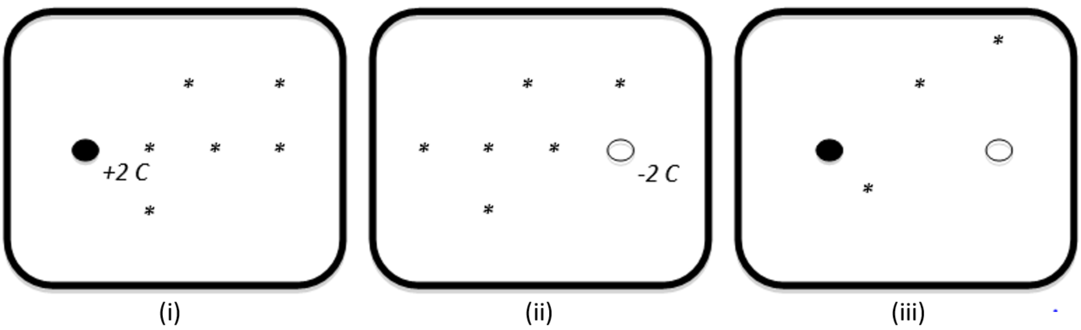


Figure 5.2. Pre-test question applying vectors to electric field context.

The pre-test result shows that a common difficulty was a reversal of the direction in which the vectors should point, (towards positive charge / away from negative charge) to represent the electric field. The students were familiar with using vectors to represent the gravitational field of a planet, so for them to apply the same directions for the vectors in the diagram for the first question was not

unreasonable. It followed that the next question, which asks them to use vectors to represent the field of a charge of an opposing sign, which resulted in the opposite direction being plotted.

Responses	Students
Direction point away from positively charged particle.	**4L.**
Vectors point towards positively charged particle.	4A, 4B, 4D, 4E, 4H.
Attempted field lines.	4F, 4G.
Non-discernable.	4C, 4J.
One vector, originating from charge.	4K, 4M, 4N.
Direction points towards negatively charged particle.	**4A, 4H, 4L, 4G**
Vectors point away from negatively charged particle.	4B. 4C. 4D, 4E. 4F, 4K, 4M, 4N
Vectors originate from charge.	4B, 4C, 4K, 4M, 4N
Attempted field lines.	4D, 4E. 4F, 4G
Non-discernable	4J

Table 5.2. Student responses to vectors and electric field pre-test question.

It was also observed that four students attempted to use field lines instead of vectors to represent the field, and a further five students drew their vectors originating from the charge. The latter suggests that the students have combined elements of the two representations or have confused the field vector for a displacement vector, representing the perceived path to be taken by a charge at a given point.

Table 5.3. presents summary of the vector representational errors made by the students in representing the magnitude of the electric field at the various points. The pre-test showed the students also had difficulties in representing magnitude of the electric field at various points. Only two students drew vectors in which the strength decreased as the distance from the charge increased. Two other students drew vectors that suggested the field strength increased at distances further from the charge, and while one student did not show any variation in field strength. The remaining students results showed difficulties that were not typical of those found in literature, such as vectors of varying magnitudes with no discernible patterns, or displacement vectors between randomly chosen points.

Responses	Students
Magnitude of vectors decreases with distance from charge.	**4H, 4L**
Magnitude of vectors increases with distance from charge.	4A, 4G
No variation in vector magnitude	4M
Magnitude variation follows no discernable pattern.	4C, 4D, 4E, 4F
Vectors magnitude is relative to distance to next point.	4B, 4J
N/a	4K, 4N

Table 5.3. Students responses to variation of field strength with distance.

In the final diagram (Figure 5.2, (iii)), students were required to draw the resultant electric field at three points. A summary of the results to this question is presented in Table 5.4.

Reasonable superposition of vectors demonstrated.	**N/a**
No superposition of vector demonstrated.	4A, 4B, 4C, 4D, 4F, 4H, 4J, 4L, 4M, 4N.
Superposition of field lines attempted.	4E, 4G,
Not attempted	4K

Table 5.4. Students use of superposition with electric fields.

This pre-test question showed that many students did not consider the superposition of their vector arrows from the previous two parts of the question to be appropriate, when drawing the electric field at the points highlighted. Figure 5.3 illustrates some of these difficulties. Ten of the students represented the field but showed no indication of vector addition. In some cases, students drew two vectors acting at the points (4A – Figure 5.3, i), displacement vectors from both charges (4C, 4F), vectors pointing to one charge only (4H, 4J, 4L – Figure 5.3, ii) or force vectors representing attraction between the charged bodies (4B, 4D, 4M, 4N – Figure 5.3, iii).

Two of the students attempted the use of field line representations instead and produced patterns consistent with their observations of the field between two planets (4E), as seen in the field lines tutorial lesson or a pattern consistent with the field lines of a bar magnet (4G).

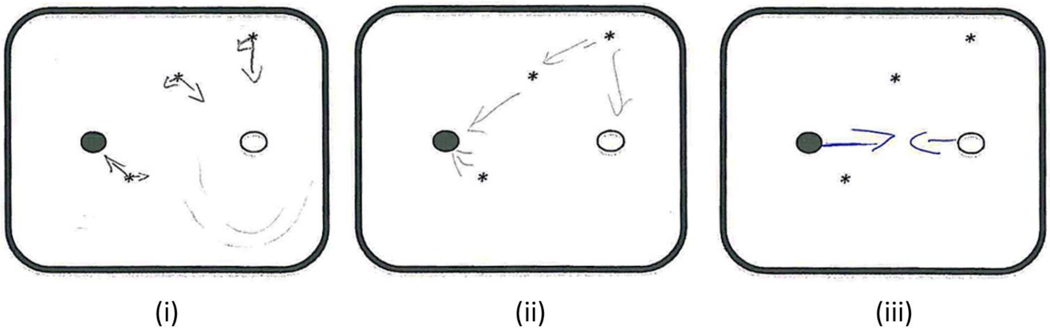


Figure 5.3. Student responses to applying the superposition of vectors to an electric field.

This pre-test showed poor performances by the students to apply their understanding to the electrostatic field context despite previous gains in their understanding of vector magnitude and vector addition. This inability to extend their understanding of vectors to apply it to an electrostatic context indicates that the approach in the tutorial lesson wherein the students are guided through the use of vectors to describe electric fields is justified. This can help promote their ability to transfer their understanding between these two domains.

5.4.2. Tutorial lesson: Student's use of vectors in electric fields

The electric field tutorial lesson was designed for students to transfer the concepts they developed in the tutorials outlined in chapter 4 to electrostatic fields, and further their understanding of electric fields. The students were presented with opportunities to represent electric field of 2 charge systems using both methods. This section will discuss the first half of the tutorial lesson, in which the students explored the use of vectors to show the superposition of an electric field, for a two charged-particle system. The latter half of the tutorial lesson, which looks at the use of field lines, is discussed in section 5.4.3.

The tutorial introduced students to a uniform electric field represented as vectors. The tutorial worksheet defined a uniform field as a field the is equal in magnitude and direction at all points. The students were then required to observe a uniform field and were asked to explain how the vector representation indicated the presence of a uniform field, as shown in Figure 5.4.

The students were required to justify that the vectors are of equal length, and all point in the same direction. Additionally, they were required to rank the electric field strength from strongest to weakest at various given points. While an understanding of vector magnitude should produce a ranking of A=B=C=D, there were discussions within group as to whether D>C>B>A was correct. The reasoning initially used to justify this was that as the vectors move from left to right, more vectors pass through each point. When asked to clarify what was moving, the students would reply that the

field was moving, as indicated by the direction of the arrows. However, when asked to recall a demonstration experiment for lower second level, in which they sprinkled iron filings over a bar magnet to show the field. They recalled that the field pattern they demonstrated did not move from North to South and determined the field. These students were then invited to revisit their notes from the presentation about how the electric field represented the force at an individual point, and with the aid of other students in their groups, they generally volunteered that neither the field or vectors move. The students generally came to the correct ranking during these discussions. In the questions of this initial section, the students were required to represent an increasing electric field, and decreasing electric field, which all students successfully completed.

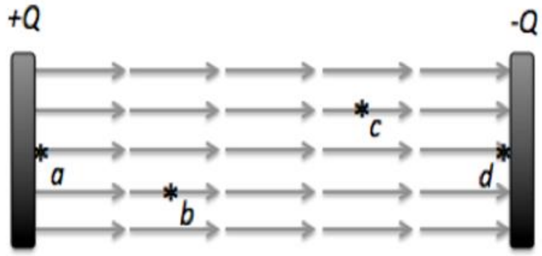


Figure 5.4. Uniform electric represented using vector arrows.

The tutorial then presented a two-dimensional vector addition exercise, in which the students were presented with the steps required to determine the combined field at a point, by utilising a vector construction to determine the superposition of two electric field vectors at a point, as shown in Figure 5.5.

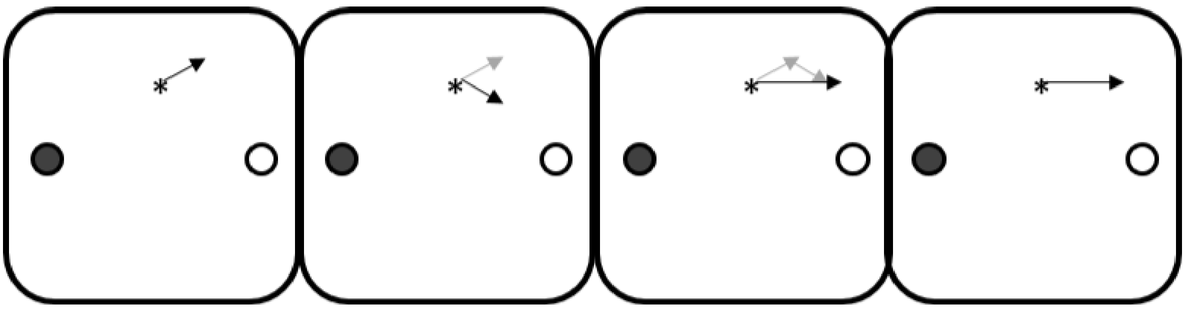


Figure 5.5. Demonstration of the superposition of two vectors representing an electric field.

The students were required to explain each of the steps, demonstrating and applying their understanding of adding vector components. By presenting the steps, and getting them to justify them, it was expected that the students could explain why the electric field is horizontal, and be able to explain the cause of the change in the resultant electric field magnitude, compared to the individual electric field vectors initially presented.

Student 4D: The vertical component cancel out, leaving only horizontal vectors. The vertical components cancel out, because they go in opposite directions, leaving only horizontal components to add to get the magnitude.

The students then were presented with another diagram like that shown in Figure 5.5, in this case where the two particles were positively charged, as opposed to positively and negatively charged. The students were required to use the either the "tip to tail" or parallelogram construction to represent resultant vector. The students were also asked to explain why and how the direction and magnitude of the resultant vector was affected in terms of the horizontal and vertical components. Again, the student groups produced reasoning in which the horizontal vectors cancelled each-other out, leaving only the vertical vector components to combine to produce the resultant electric field vector in a vertical direction.

In the last section of the tutorial lesson that addressed student's understanding of vectors in an electric field, students were presented with the a positively and negatively charged pair of particles, and various positions were highlight for the students, in which they were required to construct the superposition of the electric field at various points. Errors were caused by students initially not considering the magnitude of their vectors to be different, regardless of their distance from the charges. When prompted to consider the field strength based on their distances, the students were quick to realise their errors, and redrew their vectors accordingly. An example of student 4L applying the parallelogram rule to the electric field is presented in Figure 5.6.

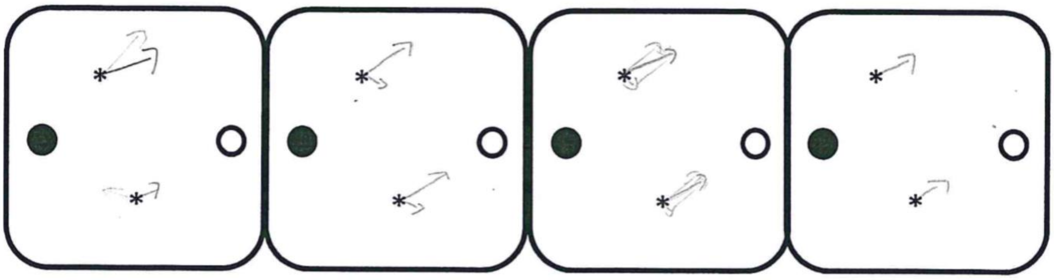


Figure 5.6. Student 4L applying the principle of superposition to represent the electric field.

This section narrated the tutorial guidance used to encourage the students to transfer their understanding of vector concepts to representing electric fields. The student's ability to consider direction and magnitude of vectors was discussed in the initial section of the tutorial, student's understanding of vector addition and how component addition affects the direction and magnitude is illustrated and the student's ability to combine vectors using vector constructions was shown.

5.4.3. Post-test: Student's use of vectors in electric fields

In an Electric field post-test question, the students were presented with a diagram in which they were given the electric field surrounding a charged particle of unknown sign, as shown in Figure 5.7. The students were asked to use the vector representation to identify the sign of the charge and explain why the length of the vectors decreases as the distance from the charge increases. A summary of the student responses is shown in Table 5.5.

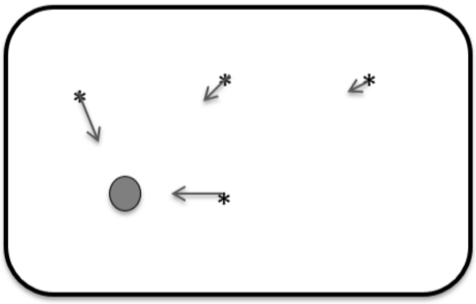


Figure 5.7. Diagram used in Electric field vector post-test question.

Response	Students
Charge is negative	**4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4L, 4M, 4N.**
Charge is positive	4A, 4F, 4K.
Associates direction with negative charge	**4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4L, 4M, 4N.**
Associates vectors with electrons.	4A.
Positive pulls in all arrows	4F.
Associates colour with charge	4K.

Table 5.5. Student responses to Electric field vector post-test question.

Upon completing the post-test, it was clear that most of the students could identify the sign of the charge based on the information in the diagram. Students 4A and 4F associated a negative charge to the arrows and treated them as electrons. This was a persistent difficulty for 4F, as discussed in section 5.4.5, in which they associated a gravitation force to field lines. This would suggest they consider the field to be a tangible construct (Galili, 1993), regardless of whether it is represented with field lines or vectors. Another student, 4K, associated a positive charge to diagrams in which the body is represented as a dark circle, as was done in the tutorial. In the tutorial, this was done to

help students differentiate the two charges with ease but was never intended to set a convention to indicate charge.

The students were then asked to construct the resultant vectors when particle with opposite charge was placed in close vicinity to the original charge, as shown in Figure 5.8.

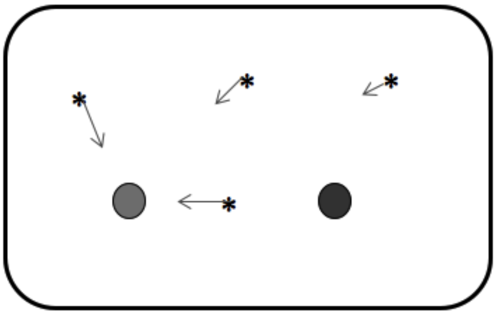


Figure 5.8. Post-test electric field question in which student sketch arrows to represent field components due to positive charge

This would allow me to determine if the students could represent the vectors using the correct direction, display reasonable relative magnitudes and use of the principle of superposition to determine the resultant field vectors. The results are presented in Table 5.6.

All students drew a second set of vectors representing the electric field due to the second charge. Twelve students drew these vectors with a length that varied reasonably with distance from the charge; ten of these correctly drew vector pointing away from the second charge, while two drew vectors pointing towards it, .

The answers of students 4J and 4M are shown in Figure 5.9. Student 4J drew vectors of equal magnitude perpendicular to the original vectors Student 4M drew field lines resembling a dipole field. This strategy could have worked, but the student used superposition to add a vector derived from the dipole field line pattern they drew (i.e., the correct answer) to the original vectors.

Seven students correctly used either the superposition principle, including two students (4A and 4F) who had difficulties in identifying and representing the direction for electric field for the different charges. Additionally, three students (4B, 4C and 4D) correctly applied the principle but for reasons unclear did not apply it to all the points. The remaining students did not apply the superposition principle at all.

Across the three questions in the post-test, it is observed that only two students, 4G and 4I, produced answers which showed they could completely transfer their understanding of drawing and adding vectors to the domain of electric fields. The other students showed errors in some aspect in representing the resultant electric field at the various points, in which case eight students only made one error (4B, 4C, 4D, 4E, 4F, 4H, 4K and 4N) in using vector representation for the electric field of the two charges. The remaining students made two or more errors in the representational transfer.

Responses	Students
Second vector arrow point away from positive charge.	**4B, 4C, 4D, 4E, 4G, 4H, 4I, 4K, 4L, 4N.**
Second vectors point towards positive charge.	4A, 4F.
Vectors are forced to be 90° to previous field vectors.	4J.
No pattern to the vectors	4M.
Reasonable variation of vector length with distance	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4K, 4L, 4N.**
Unreasonable variation of vector length with distance.	4J.
Field line representation used	4M.
Appropriate use of superposition	**4A, 4F, 4G, 4I, 4J, 4L, 4N**
Appropriate use of superposition for some / one of the positions.	4B, 4C, 4D
No superposition of vectors found.	4E, 4H, 4K, 4M.

Table 5.6. Student's application of vector concepts to electric field context.

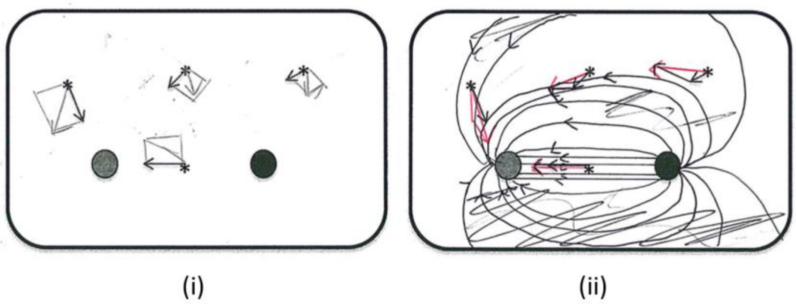


Figure 5.9. Errors in electric field vectors by (i) student 4J and (ii) student 4M.

5.4.4. Homework: Student's use of vectors in Coulomb's law

The students also completed a homework question, in which they could apply their understanding of vector components to a conceptual force question, presented in Figure 5.10. The students were asked to compare the net force acting on the -1 C charged body in (a) with that of (b), and then with that of (c). The question invited the students to use whatever reasoning they deemed appropriate and suggested vector reasoning, calculations or any other reasoning deemed fit by the students. While the vector nature of Coulomb's law was discussed in the class discussion before the tutorial, the tutorial itself did not directly look at the vector nature of electrostatic forces. This question tested if students could transfer their reasoning of vector component directly to the electrostatic context without explicitly exploring it in the tutorial. A summary of the student's responses is shown in Table 5.7.

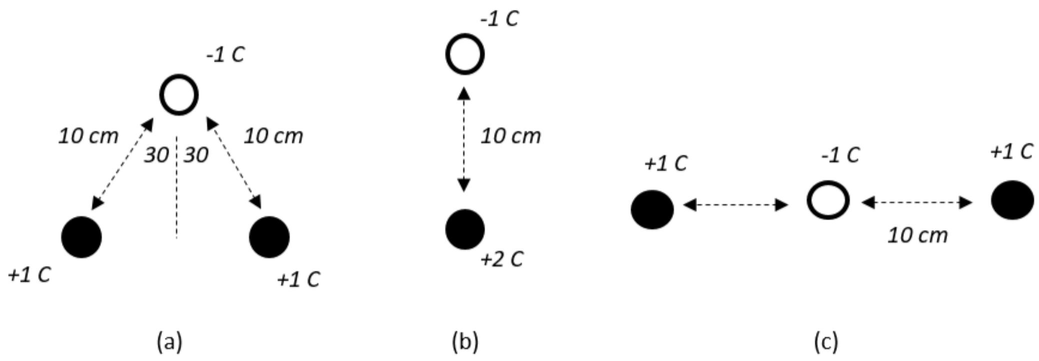


Figure 5.10. Coulomb's law vector concept question

Student Reasoning	**Students.**
Applied vector component reasoning	**N/a.**
Applied incomplete vector component reasoning	4K
Applied scalar reasoning.	4G, 4M
Described forces in terms of attraction and repulsion.	4C
N/A	4A, 4B, 4D, 4E, 4F, 4H, 4I, 4J, 4L, 4N

Table 5.7. Student's application of vector components to electric field context.

None of the students gave a complete answer: the horizontal components in (a) cancel, resulting in a net magnitude less than (b), and that the horizontal forces in (c) would sum to zero. Student 4K, when describing the forces in diagram (a), acknowledged that there is a combination of horizontal

and vertical vector components acting on the negatively charged particle, while only vertical and horizontal vectors act on negatively charged particle in (b) and (c). However, they stated that the net force is stronger in both cases (b) and (c), and did not consider that the horizontal net force is zero in (c).

Both students 4G and 4M used the equation $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$ in all the setups, and in setup (c) treated the two positively charged bodies as one charged body with a magnitude of +2 C. Student 4M also mentioned that the force would be slightly reduced due to the repulsion between both positive charges, but did not explain this in any detail. Student 4C interpreted the question to explain whether attraction or repulsion existed between the different combinations of charged particles. The remaining students, bar three who were absent and unable to complete the homework, did not make headway with the question. They stated that they were unaware how to approach the question. This indicated that the suggestion to use vector reasoning or calculations did not prompt them to use the understanding they had previously developed in the context of electric fields.

5.4.5. Interview: Student's use of vector components in Coulomb's law

There was no post-test question developed to elicit student's thinking about vector components in Coulomb's law, or electric fields. Instead three of the students were interviewed. They were asked to revisit the homework question discussed in section 5.4.4. The students were told they were permitted to ask questions during the interview to help them along, but they were not permitted to ask directly for the solution.

At the beginning of the interview, students 4A, 4B and 4H stated that the forces would be equal in all cases as the -1 C charged particle was being attracted by a net charge of +2 C, in all cases. However, upon being informed that their reasoning was incorrect, and the net force on the particles was not equal in all cases, they considered the use of vectors to analyse the question. The following interview extract illustrates the student's reasoning.

Student 4H:	The distance is there [a] cause that one will be pulled down the centre line. That is just as strong as charge [b], but it is is the most [strongest force], cause it is direct. And that one will cancel out [c], so it'll be zero. That one [b] will be twice as much if that was one [c].
Teacher:	So, C = zero. Why did you say that?
Student 4B:	Cause it cancels out.
Teacher:	What cancels out?
Student 4H:	The horizontals.

Teacher:	Ok… so now we have horizontal vectors. What type of vectors do we have acting here [b]?
Student 4A:	Vertical vectors.
Teacher:	And everything is vertical? [Students nod in agreement] Ok, so let's just say here [one horizontal vector is sketched on c] is 10 N, and this [vector sketched acting in opposite direction] is 10 N, now what's the force acting on this [b]?
Student 4H:	20 N.
Teacher:	Ok, so look here [a] and ignore this [right positive charge]. What force acts on the negative charge?
Student 4B:	10 N.
Teacher:	Now ignore this [left positive charge] What's the force?
Student 4B:	10 N.
Student 4H:	And we can add them tip to tail now.
Teacher:	We can, but also, consider, you mentioned horizontal and vertical components earlier. Keep the idea of components in your head. Do you think the 10 N and 10 N will sum to 20 N?
Student 4B:	The horizontal components in that one [a] will cancel out.
Teacher:	So, you're only left with what?
Student 4H:	Just vertical components.

When the students had to consider alternative reasoning they resorted to vector reasoning without much prompting. The extract shows the teacher did not volunteer any reasoning the students did not mention themselves but guided them to use the reasoning in the three different layouts. The student's reasoning was based on how the component vectors combined. Based on this, they produced an accurate ranking of B > A > C = 0, to represent the net force on the negative charge in each layout.

5.4.6. Discussion

This section presents a discussion of the student's use of vectors in Coulomb's law and the electric field. It presents a comparison of the student's ability to represent the variation in field strength using vectors, and their ability to apply the principle of superposition of electric fields using vectors. Figure 5.11 presents a comparison of the student's responses in the pre-test and post-test, looking at student's representations of the electric field strength at various points in an electric field.

In the pre-test, it was observed that two students accurately used vectors to represent the magnitude of the electric field at points at various distances around a charged particle. As the students

showed good understanding of vector magnitude, as discussed in section 4.2.5, the following reasons could explain the student's difficulties:

- The students consider that a form of proportional relationship exists between electric field strength and the distance from the charged particle.
- They do not consider electric field strength to vary with distance at all.
- They do not consider electric field strength to be a vector quantity.
- The students were unable to differentiate between electric field and other vector quantities, such as displacement.

These reasons are suggested as they are based on the interpretations of conversations with the students during the tutorial lessons and interpretations of discussions overheard between the students combined with interpretations of the student artefacts which were scanned.

Comparison of student's variation of vector magnitude in electric fields.

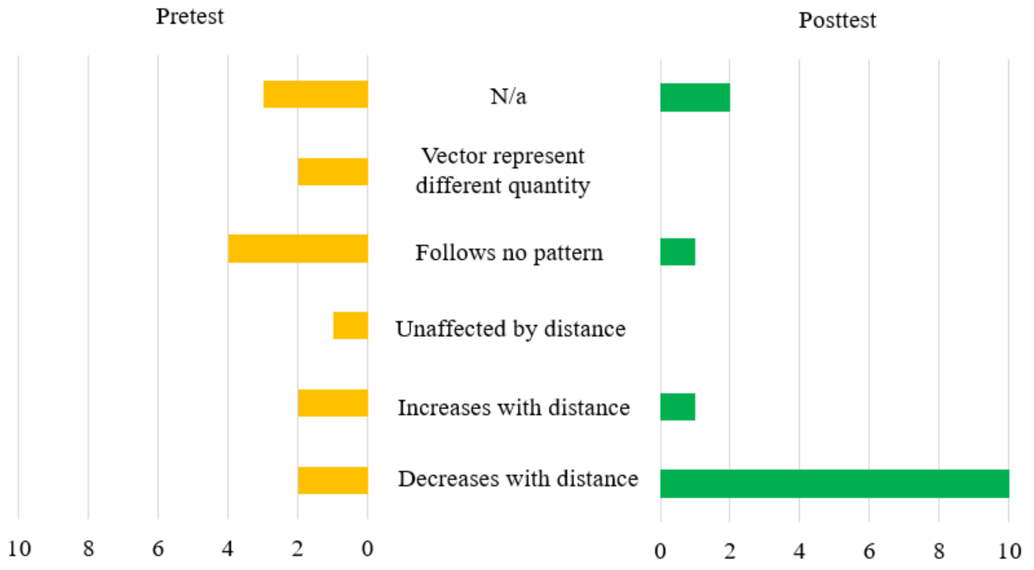


Figure 5.11. Comparison of student's representations of vector magnitude for an electric field.

The post-test showed considerable gains by the students. Most of the students (10/14) correctly represented the change in field strength with distance using vectors of different lengths. The shift in student's abilities to correctly represent the field lines indicates that conceptual exchange occurred (Hewson, 1992), with the comparison of the pre-test and post-test indicating that moderate conceptual change occurred. Ten of the students applied the magnitude convention to their vectors in the post-test, and no longer showed the difficulties seen in the pre-test. Through completing the tutorial lessons, the students applied the reasoning they developed in section 4.2 to a new context, utilising the representation as a useful tool to represent and verbally explain a vector field pattern in an unseen context (Posner, *et al.*, 1982). Not all the difficulties were overcome however, as two of

the student's presented difficulties in which they used field lines and then drew vectors to match the field line pattern, or used vectors previously presented on the diagram as a scale to determine the magnitude of the vectors.

Figure 5.12 presents a comparison of the pre-test and post-test results for the student's use of the superposition principle. In the pre-test, none of the student's applied it to electric fields. This suggests difficulty in transfer to this context, as all the students demonstrated the ability to construct resultant vectors in section 4.3.4. The difficulties reflect those seen in literature when learners struggle to apply vector concepts to the electric field context (Maloney, *et al.*, 2001).

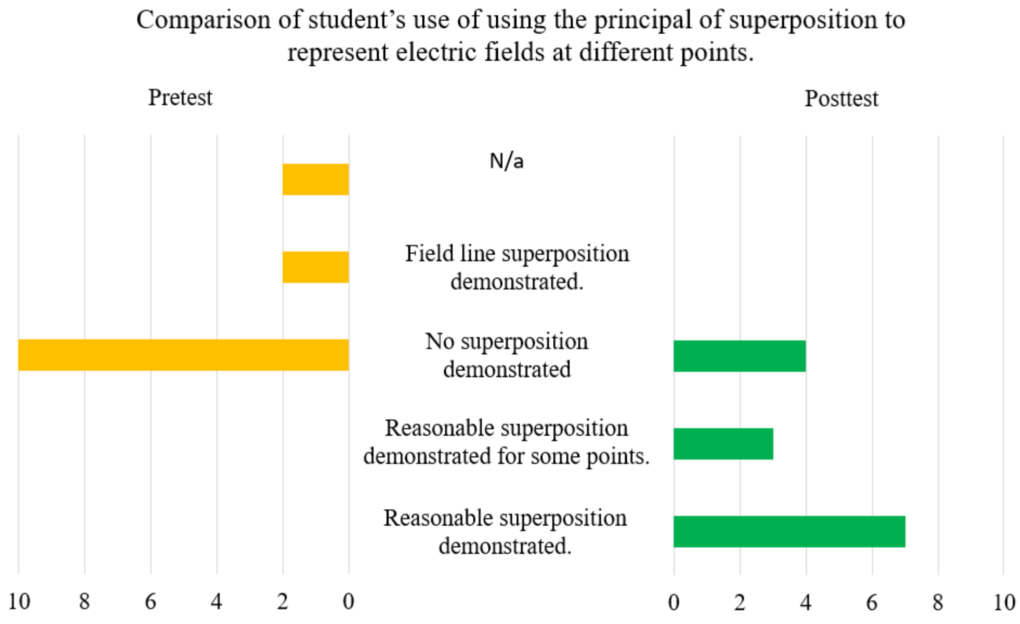


Figure 5.12. Comparison of student's use of superposition to draw an electric field using vectors.

After the tutorial lesson, 10/14 students applied the principle of superposition to the electric field in the post-test. This indicates that students did not initially transfer the skills they developed, as discussed in section 4.2, to the context of electric fields. The students demonstrated this skill in the vectors tutorial but did not transfer this skill to the electric field context until after they had completed the electric field tutorial. This suggests that conceptual extension occurred from completing the tutorial lesson (Hewson, 1992), with a moderate conceptual change observed from comparing the pre-test and post-test results. As discussed in section 5.4.2, the tutorial gave the students the opportunity to practice the vector constructions they previously developed in the context of electric fields, furthering their overall conceptual understanding by applying it to new contexts (Konicek-Moran and Keeley 2015).

Marzec (2012) suggested that learners require multiple opportunities to develop understanding of the inverse square law. The pre-test/post-test comparison and discussion of the tutorial lesson suggest that this could also apply to using vectors in multiple contexts. The students did not

experience any significant difficulties in applying the vector constructions in the tutorial lesson, which indicates that the central difficulty in the pre-test was students not realising the application of their vector understanding. This was also observed in the Coulomb's law homework, in which the students were unable to apply the reasoning they developed in the vectors tutorial question described in section 5.2.4. Apart from one student, the students did not recognise that vector components were applicable to the question given. However, during the electric field tutorial, the students had little difficulty in applying vector component reasoning when drawing the electric field at various points around two charged bodies. The student interview, in section 5.2.5, also indicates that with minimal guidance, the students applied vector reasoning to the homework question. Initially the students based their equal ranking on the net positive charge in each case, instead of the positioning on the charges. This indicates that students value magnitude as the most important aspect of force. This, in turn, suggests that for the transfer of vector concepts to electric fields to be complete, students require the opportunity to develop their reasoning in this context in a classroom setting, such as a tutorial and/or a class discussion, as was the case in this research.

5.5. The inverse square law applied to electric fields and Coulomb's law

The following sections discuss student's understanding of the inverse square law and how they applied it to Coulomb's law and electric fields. Section 5.5.1 discusses results based on student's answering a mathematical problem in a pre-test question, and a problem involving the use of vector representation to show their understanding of the inverse square law. Section 5.5.2 discusses the Coulomb's law tutorial lesson, which focused on the student's understanding of proportionality and the use of tabular data, graphs and mathematical methods to explore the inverse square law. Section 5.5.3 discusses a homework assignment applied the scale model, adapted from Conceptual Physics (Hewitt, 2009) to the electric field. A difficulty in student's understanding of the scaling of area is identified, and a section of a teaching and learning interview is presented to show how students overcame the difficulty. Section 5.5.4 presents the results of a post-test that looks at the student's ability to represent the inverse square law on a graph, how students differentiate between an inverse and inverse square law graphically, student's use of the inverse square law in a mathematical question, and their understanding in change of an area based on a scale model.

5.5.1. Pretest: Coulomb's law and inverse square law

This section discusses the results from the Coulomb's law pre-test, in which the students had to use vector arrows to demonstrate their understanding of a directly proportional relationship, and a

relationship that follows the inverse square law. The latter part of this section presents students graphical representations of a directly proportional relationship, and the inverse square law.

In the first question, the students were asked to state the relationship, between the magnitude of the force, and the distance between them, when presented with the Coulomb's law equation. This allows for gauging the student's ability to recognise relationships in algebraic form and transfer them between tabular, graphical, diagrammatic and mathematical symbolic representations. The students were familiar with the general structure of the Coulomb's law equation, as they had studied Newton's gravitational law and they were formally introduced to Coulomb's law in a presentation and class discussion preceding the tutorial lesson. The equation was presented to the students in the form $F = k\frac{q_1 q_2}{d^2}$. This form presents Coulomb's law as a scalar equation, as the Leaving Certificate Physics course does not employ the use of vector algebra. The students were made aware that this equation can only be used to determine the magnitude of the force between two charges, and when the students are required to determine the directions of the forces acting on the charges, other appropriate methods are employed. A summary of the student's results are presented in Table 5.8.

Inverse square relationship	**4H, 4K, 4M**
Inverse relationship	4G, 4E
Increase distance, decrease the force.	4A, 4C, 4I, 4J
Directly proportional relationship	4B, 4D
N/a	4F, 4L, 4N

Table 5.8. Student pre-test responses to transferring from equation to verbal relationship.

The results show that three of the students could glean the formal relationship between both quantities from the law equation. Two students stated that the force was inversely proportional to the distance, suggesting the students did not observe the index of distance variable, or did not consider its relevance in defining or naming the relationship. A further four students could relate the position of the distance variable as a denominator to determine that increasing the distance from the charges reduces the magnitude of the force between them. Two students were unable to relate the positions of the variables in the equation and determined the variables were directly proportional to each other, while three students did not give an answer.

The students were then presented with a mathematical question, in which they had to apply the inverse square law. The students were not provided with a value for *k* in the Coulomb's law equation, so they would have to employ proportional reasoning to answer the question. The question is provided in Figure 5.13. The student's results are presented in Table 5.9.

Two +8 C charges are held a distance of 10 cm from each other, and the force acting on both charges is 90 N. The charges are moved so the distance between then is now 30 cm. What is the new force acting between the charges? Explain how you know what the change in force is.

Figure 5.13 Pre-test question seeking to elicit student's ability to mathematically apply inverse square law

As the distance between the charges was increased by a factor of 3, the magnitude of the force was reduced by a factor of 9. Only one student, 4C, completed this. It is interesting to note that 4C responded to the previous question by stating the force would decrease without quantifying this decrease, but provided the correct outcome in this question. Three students (4D, 4G and 4K) reduced the force by a factor of 3. Two of these responses were surprising considering student 4D previously stated there was directly proportional relationship and student 4K previously stated there was an inverse square relationship between the variables. This indicates the students did not understand the nature of the relationships or did not apply them in this context. Four students attempted to use the formula to attempt the question, but were unable to complete their calculations since they did not know a value for *k* and could not determine how to tackle the question. This difficulty echoes that shown by Arons (1999). Two students (4E and 4M) respectively stated that the force would increase and decrease, but they did not quantify their answers. This answer was not consistent with 4E's previous question, in which they stated there was an inverse relationship, and 4M did not apply the inverse square relationship they stated in the previous question. The remaining students did not answer this question.

Responses	Students
Reduces force to one ninth original	**4C**
Reduces force to one third original	4D, 4G, 4K
Attempts calculation	4B, 4F, 4L, 4N
Increases force	4E
Decreases force	4M
N/a	4A, 4H, 4I, 4J

Table 5.9. Student's pre-test responses to applying the inverse square law mathematically.

In the final question of this pre-test, the students were asked to use vector arrows to show the change in magnitude of the force with distance. The students were presented with two charges, with vector arrows showing an attractive force between the two charges. They were asked to determine

the effect of doubling the distance between the charges and representing this using vector arrows. The question is presented in Figure 5.14, and the student's results are presented in Table 5.10.

As can be seen in Table 5.10, none of the students correctly represented the vectors by reducing the magnitude by a factor of four. The two more prevalent vector representations showed that the force would reduce by a factor of two or would not be affected at all. This highlights inconsistencies from the four students that previously defined or applied the inverse square law, and the students who applied an inverse relationship, or explained the increasing the distance would decrease the force felt by the charges.

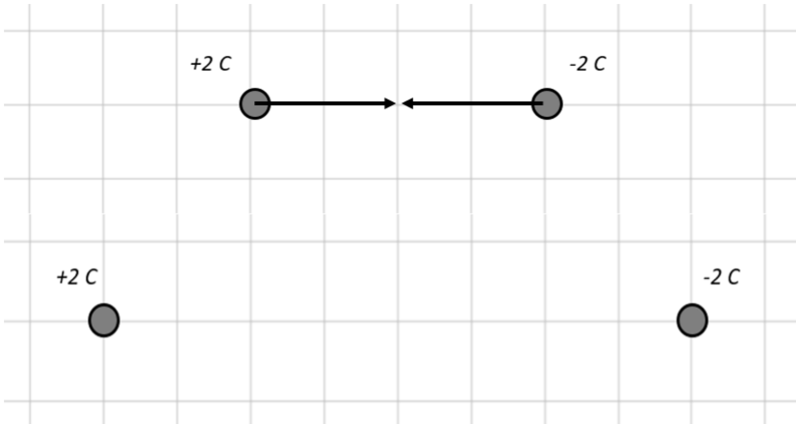


Figure 5.14. Pre-test question utilising the inverse square law and vector representations

Responses	Students
Quarters both vectors.	N/a.
Doubles both vectors	4B
Halves both vectors	4A, 4C, 4D, 4M, 4N,
Increases vectors	4J
No change to vectors.	4E, 4F, 4G, 4H, 4I, 4K, 4L

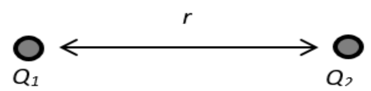
Table 5.10. Student responses to pre-test question that looked at student's application of the inverse square law and vector representations.

These pre-test results indicate that the students encountered difficulties in transferring their understanding of the inverse square law to Coulomb's law. Only a small number of students recognised the inverse square law in the Coulomb's law formula, and only a single student applied the law mathematically. None of the students presented the inverse square law using vector representation, and most of the students displayed reasoning consistent with an inversely proportional relationship.

5.5.2. Tutorial lesson: Coulomb's law and inverse square law

During the Coulomb's law tutorial lesson, the students were introduced to a formal definition of Coulomb's law, in which the magnitude of the force between two point-charges is directly proportional to the product of the magnitude of the charges, and inversely proportional to the square of the distance between them. The lesson aimed to allow students to verify these relationships themselves, using tabulated data, graphs and calculations based on the formula. In the first half of the tutorial, the students were guided through this process for a directly proportional relationship, and in the latter half, they had to apply the same skills to show an inverse square proportional relationship.

The students were presented with tabulated data for the force between two charges, and 3 columns with values for the magnitude of the first charge only, the magnitude second charge only and the values for the product of magnitudes of the two charges, as shown in Figure 5.15.



q_1 (C)	q_2 (C)	$q_1 q_2$ (C^2)	F (N)
1	1	1	2
1	2	2	4
2	1.5	3	6
4	1	4	8
1	5	5	10
2	3	6	12

Figure 5.15. Data set from Coulomb's law tutorial, relating force to product of charges.

The students were to identify which column, when coupled with the force values, provided a familiar pattern, being linear, quadratic, exponential, inverse or inverse square. The students then rewrote these two columns in a separate table and were guided to show that the ratio of force to the product of the charges was constant, by dividing the values for F by the values for q_1q_2. This would allow the students to demonstrate that the pattern followed the general form $y = mx$. The students were then asked to graph their data and explain how the shape of the graph showed a directly proportional relationship. They were required to use their graph to determine the magnitude of the force when the product of the charges was 2 C^2 and 6 C^2. From this, they explained that tripling the product of the charges has the effect of tripling the magnitude of the force.

The students were then required to complete calculations to show the effect of tripling the produce of the charges. They were provided with a sample calculation between a 6 µC charged

particle and a 3 µC charged particle that were placed a distance 1 cm apart, as shown in Figure 5.16. The mathematical operations were completed in the sample and they were required to identify which operations took place. This ensured they were familiar with the nuances of completing the calculations and aimed for students to avoid errors in performing the calculations. They then completed similar calculations, between two charges of magnitudes 3 µC and 9 µC.

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

A

$$F = \frac{1}{4(3.14)(8.9\,x\,10^{-12})}\frac{(6\,x\,10^{-6})(3\,x\,10^{-6})}{(1\,x\,10^{-2})^2}$$

B

$$F = \frac{1}{1.12\,x\,10^{-10}}\frac{(1.8\,x\,10^{-11})}{(1\,x\,10^{-4})}$$

C

$$F = (8.9\,x\,10^{9})(1.8\,x\,10^{-7})$$

D

$$F = 1{,}602N$$

Figure 5.16. Coulomb's law tutorial extract, in which students identify the operations in the calculation.

In the second section of the tutorial lesson, the students were asked to develop the reasoning to show the relationship between the force between two charges and the distance between them was an inverse square relationship. Students were given the table of data shown in Figure 5.17, and they were required to either use the data directly to show it followed an inverse square relationship or place it on a graph and use the graph to do so. During the inverse square law tutorial, as discussed in section 4.3.2, the students can pick x-values from the domain of the graph and read the corresponding y-values for these, they can calculate what factor the values change by and determine if an inverse square law was observed. For instance, choosing the values 1 and 3 from the x-axis would produce a decrease by a factor of 9 in the two y-axis values, if it followed an inverse square law.

To complete the tabular section of the tutorial lesson, the students were presented with the general forms of linear equations, quadratic equations, inverse equations and inverse square equations, in which they were shown what operations between the variables produced a constant. The general forms of $y = \frac{k}{x}$ and $y = \frac{k}{x^2}$ were less familiar to the students, as they generally picked either the quadratic or the inverse relationship initially to use to manipulate the data. However, it was clear that the ratios of force to the square of the distance or the products of force and distance values

did not produce a constant. When the students picked the inverse square proportional equation, the students determined that the product of the square of distance and the force produced a constant. Overall, this section proved to be the most challenging for the students, and took all groups approximately twenty minutes to complete, as they picked the different relationships and explored them.

The students were also required to show the inverse square law on a graph. It was observed that the students plotted the data on the graph, which produced the characteristic curve typical of an inverse square law. However, the students were satisfied that this graph demonstrated an inverse square law, based on its shape, even though it was similar in shape to an inverse graph of the form $y = \frac{k}{x}$. As the students had data points to follow, they did not repeat the errors of drawing quadratic patterns, or linearly decreasing patterns, discussed in section 4.3.4. All groups had to be prompted to pick two points on the graph and show, in this case limited by the domain and range of the data, that double the distance would result in one quarter the force.

Directly proportional general equation: $y = mx, \quad \frac{y}{x} = m, \quad \frac{y}{x} = constant$

Directly proportional to square equation: $y = ax^2, \quad \frac{y}{x^2} = a, \quad \frac{y}{x^2} = constant$

Inverse proportional general equation: $y = \frac{k}{x}, \quad xy = k, \quad xy = constant$

Inverse square proportional equation: $y = \frac{k}{x^2}, \quad x^2y = k, \quad x^2y = constant$

y	F (N)	100	25	11.11	6.25	4	2.78
x	d (m)	1	2	3	4	5	6

Figure 5.17. Coulomb's law tutorial extract, using data to demonstrate the inverse square law.

In the last section of the tutorial, the students were presented with the Coulomb's law equation, with a set of data, and asked to determine the relationship, as shown in Figure 5.18. While this section took time for the students to complete due to the many steps involved in the calculations, no groups found it overly challenging. They did need to be prompted use the values of the two forces to make a ratio at the end of the calculations, in which they could determine the factor decrease observed when the distance between the two charges was doubled. This method is similar in nature to that described on the previous page, involving the use of graphs.

At the end, the students were asked which method they found to be the most effective and simplest to use. Six of the students mentioned that they preferred to use the equation method, as they stated there was a structured approach to follow, in which the ratio was easy to determine. Four students preferred the graphical method, in which the students read the values for the forces directly from the graph and developed the inverse square relationship from their values. They found the method easier than the others, and the values can be easily obtained. Teacher observations from this lesson stated that initially, the students only used the shape of the graph as the reference to the inverse square relationship, and the values were initially ignored. This observation is considered in the post-test question utilising a graph, see section 5.5.4.

This section illustrated that students encounter difficulties in analysing tabular data and graphical representations, when seeking to determine the mathematical relationship displayed. Difficulties seen in using tabular data was determine the steps required to manipulate the data to show that $x^2y = k$, even though they were guided through the process for a linear pattern. This indicates difficulty in both their mathematical ability and the physics context in which the pattern is applied. While it may have been preferable to do this mathematical work during the tutorials completed in chapter 4, time constraints did not allow for this and instead, it was integrated into the electric field tutorial lesson. The student's unfamiliarity with the approach presented in the tutorial is likely to contribute to the difficulty observed, as the method used for analysing data in a table, for graphical analysis in their math course, is not applied to this function (LC Project Maths syllabus, NCCA, 2013). The prevalent difficulty involved in the student's use of analysing the graph was an over-emphasis on the shape of the graph itself. Students tended to ignore the values from the graph and did not analyse the values to show an inverse square pattern, suggesting they would be unable to differentiate between an inverse graph, and an inverse square graph.

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

$\varepsilon = 8.85 \times 10^{-12}\ F\ m^{-1}$, $q_1 = 6 \times 10^{-6}\ C$, $q_2 = 4 \times 10^{-6}\ C$.

$d_1 = 4\ cm$, $d_2 = 8\ cm$

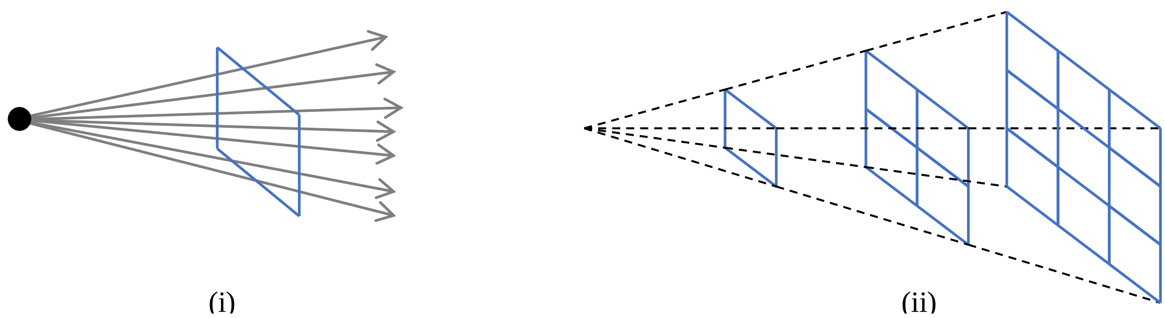
Figure 5.18. Coulomb's law tutorial extract, to demonstrate inverse square relationship mathematically.

5.5.3. Homework: Electric field and inverse square law

This section presents an analysis of the electric field homework tutorial. This was developed to apply the scale model adapted from Conceptual Physics (Hewitt, 2009) and the use of field lines in lieu of spray paint droplets, as discussed in the previous chapter, in section 4.3.2.

The students were presented with the diagrams shown in Figure 5.19. They were told that 100 field lines diverging from a charged particle placed at the left of the diagram pass through the first frame, as shown in Figure 5.19 (i). The model was designed to give students an appreciation of concept of an electric field integrated over the surface area of a sphere, without formally introducing electric flux or Gauss' Law for closed surface contexts. The students were required to explain why the area increased in the pattern shown in the diagram, and to explain what effect this had on the lines passing through each frame as they moved from left to right, as shown in Figure 5.19 (ii).

Five students qualitatively described the increase in area as the frames moved from left to right. This reasoning revolved around the distance from the charge being greater results in a bigger area, without articulating that field lines diverge as the distance increases. One student described the field lines diverging further apart as the distance increase as a reason for the increased area. Three students explained that increasing the distance from the charge increases both the length and width of the frame, increasing the area as shown in the diagram.



(i) (ii)

Figure 5.19. Electric field line homework extract, applying the scale model to electric field and field lines.

The remaining students used mathematical calculations, in which they squared the values of two to get four. It is unknown if this represented squaring the length of the square frames, squaring the factor in which the distance from the charge to the frame, or both. The students consistently repeated applied their reasoning to the 9 frames, and all students determined that the number of field lines passing through each frame would be 25 lines and 11.1 lines respectively. When asked to explain the behaviour of the field lines passing through the frames, eight of the students qualitatively explained that the field lines were spreading out more the distance increased, resulting in a lower number of

field lines passing through the individual frames. Only four of these students added that this models an inverse square law. The remaining students did not answer this question.

Explaining the change in area proved to be difficult for some of the students. During an interview with students 4C and 4E they resorted to relating the change in area to the distance from the charge, whilst not considering the change in the lengths and widths of the frames.

Student 4C: If that distance is 1, [distance to first frame] 1^2 is 1, which is 1 frame. Then this distance is 2 [distance to second frame], so 2^2 is 4, which is four frames.

It was clear that the students could use the model presented to describe the inverse square law but did not consider the source of the change in the area, in terms of scaling up the dimensions of the frames. The teacher engaged the students in a discussion to focus on the length and width of the frames, and to link these back to the increase in distance, to show the overall increase in the area.

Student 4E: The length of the second frame is 2, and the width is also 2.

Student 4C: The distance increases by 2, and so does the length and width.

Student 4E: And therefore, the area by 4.

This suggests that students need to be guided to focus how the change in distance affects the length of the frame and width of the frame separately. They can use the increases in these dimensions to explain the overall increase in the area of the frames.

This section has illustrated the student's application of scale model adapted from Hewitt (2009) to electric field lines. The context used displayed field lines passing through various frames and the students analysed this context to explain the relationship between the distance from a point to a charged body and the magnitude of the electric field strength at the point. Using the scaling model, they demonstrated the electric field follows an inverse square law mathematically. However, some students still struggled to articulate why the area increased in a quadratic pattern, and their final explanations did not reference the inverse square but did reference the divergence of field lines through different frames, showing a conceptual understanding of the behaviour of the model. The interview with two students illustrated the need to be guided to focus how the change in distance effect the length of the frame and width of the frame separately. They can use the increases in these dimensions to explain the overall increase of the area of the frames.

5.5.4. Post-test: Coulomb's law, electric fields and the inverse square law

The first question presented in this section was completed by the students not immediately after completing the Coulomb's law tutorial lesson (which focused on the inverse square law) but as part of the electric field pre-test. As this was given after the Coulomb's law tutorial lesson, the question presented can pragmatically be considered a post-test question.

In the question, the students were presented with the formula that relates the magnitude of the electric field strength at a point to the magnitude of the charge, and the distance of the point to the charged particle; $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2}$. The students were asked to explain the relationship between the electric field strength and distance based on the equation, and to graph the pattern that represents the mathematical relationship. The question is presented in Figure 5.20, and the results are summarised in Table 5.11 and Table 5.12.

The results indicate that 10 of the students associate the asymptotic pattern based on the relative position of the electric field strength and the distance variables in the equation. However, only two students, 4E and 4G, could determine the relationship from the equation. Five students stated an inverse relationship existed between the variables, which would also account for general shape of the graph. Student 4D was consistent in their explanation that doubling the distance would result in one quarter the electric field strength. Student 4F was consistent in graphing a directly proportional relationship, indicating they were unable to determine the relationship from the equation. Both 4J and 4N stated that increasing the distance would increase the field strength, but drew graphs contrary to this reasoning, also indicating they were unable to determine relationships based on the equations. The results suggest that students require more attention in overcoming difficulties in recognising and articulating relationships presented in equations, but progress was made in student's ability to transfer relationships from one representation to another.

4.. The electric field strength around a charge is given by the formula $E = \frac{1}{4\pi\varepsilon}\frac{Q}{d^2}$.

What is the relationship between the electric field strength and (i) the magnitude of the charge causing it, and (ii) the distance from the charge.

(i)

(ii)

Draw these relationships on the graphs below.

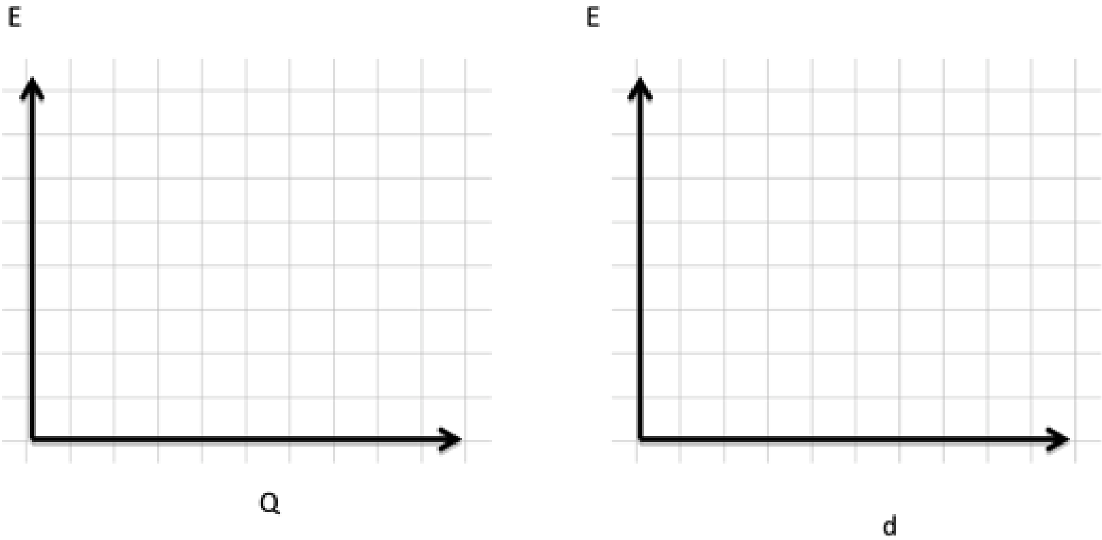


Figure 5.20. Electric field question in which students transfer inverse square law from symbolic to word and graphical representations.

Responses	Students.
Inverse square proportionality.	**4E, 4G,**
Inverse proportionality	4A, 4B, 4C, 4H, 4M
Illustrates inverse square relationship with examples.	4D
Directly proportional	4F, 4N.
Qualitative answer only.	4J, 4L, 4K
N/A	4I

Table 5.11. Student's responses from electric field pre-test, determining student's ability to transfer from equation to verbal relationship.

Responses	Students.
Asymptotic decreasing graph.	**4A, 4B, 4C, 4D, 4E, 4G, 4H, 4J, 4M, 4N.**
Directly proportional relationship.	4F.
Increasing quadratic curve.	4L
N/a.	4I, 4K.

Table 5.12. Student's responses from electric field pre-test, determining student's ability to transfer from equation to graphical representation.

The previous questions indicated that the students related an asymptotic pattern to an inverse and inverse square relationship. However, it did not determine if the students could recognise the difference between the patterns on the graph, or if they had just memorised the general shape of the graph. The first question presented in the Coulomb's law and Electric field post-test tested for this. The students were asked to pick out and justify the correct shape of both a directly proportional relationship and inverse square relationship in the graph shown in Figure 5.21. The students were also asked to determine which patterns showed the relationship between the force between two charges and (i) the product of their magnitude and (ii) the distance between the charges, to see if they could transfer the formal definition to a graphical representation. A summary of the student's responses is presented in Table 5.13.

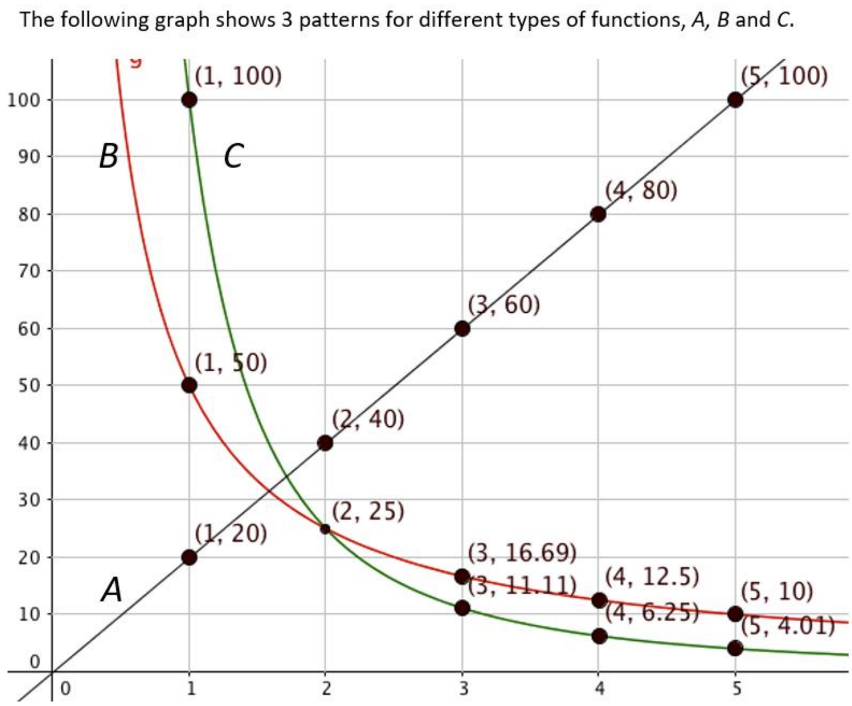


Figure 5.21. Graphical representations of a directly proportional, an inverse and an inverse square relationship.

Responses	Students.
Picks A as directly proportional.	**4A, 4B, 4C, 4E, 4F, 4G, 4H, 4I, 4J, 4L, 4M.**
Picks B as directly proportional.	4N
Picks C as directly proportional.	4D
N/a	4K
Picks C as inverse square.	**4B, 4E, 4G, 4H, 4J, 4I, 4L, 4M**
Picks B as inverse square.	4D, 4N
Picks B and C as inverse square.	4C, 4F
N/a	4A, 4K

Table 5.13. Student's post-test responses in determining relationships based on graphical data.

For both relationships, the results show that eleven and eight students respectively determined the correct graphical pattern for the directly proportional and inverse square relationships in Coulomb's law. The students primarily referenced the shape of the graph, being linear with a y-intercept of zero, to explain the direct proportion relationship. One student, 4E, also employed the use of a table to demonstrate the relationship, although they had apparent issues articulating the final reasoning.

Student 4A: They form a straight line… product (of charges) are proportional.

Student 4E: $y = mx$, is a straight linear function. y is directly proportional to x, it has to get bigger.

Student 4K: A. Because $y = mx$ is directly proportional, which tells us it is a straight line.

However, only 8 students correctly determined which of the graphs represented the inverse square relationship Typically students substituted values from the graphs into the two equations and saw which set of results fit the equation. Some students determined a value for "k" in the manner they were guided to use in the Coulomb law tutorial lesson (section 5.5.2). Mistakes commonly made by the students were that they looked at the pattern and recognised an inverse relationship, but could not differentiate between the inverse and inverse square pattern shown on the graph, in which they picked either B or both B and C.

Student 4B: (1, 50)
$50 = 100\left(\frac{1}{1^2}\right)$
Not curve B.
(2, 25)
$25 = 100\left(\frac{1}{2^2}\right)$
Curve C is of the form $y = k\frac{1}{x^2}$. As the distance increases, the force decreases

Student 4F: C is $y = k\frac{1}{x^2}$, as it does not decrease at a constant rate.

B. It is inversely proportional to the distance squared.

Student 4G: Coulomb's law states that the force between 2 point-charges is directly proportional to the product of the magnitude of their charges and inversely proportional to the distance squared. If x = 1, and y = 100, when the distance is doubled, it is 2 (x) and $100\left(\frac{1}{2^2}\right) = 25$ (y), and that point is on C.

Students 4N and 4D consistently picked B as the graph which represented both $y = k\frac{1}{x^2}$, and the magnitude of the force between two point charges as a function of the distance between them. Student 4C picked both B and C to represent the mathematical equation and the relationship in Coulomb's law, suggesting that they focus on the shape, but not the values produced by the pattern. Student 4F correctly determined that C represented $y = k\frac{1}{x^2}$, but chose pattern B as the representation of the inverse square relationship in Coulomb's law. As they quoted the inverse square law in their response, this would also indicate they focused on the shape over the values produced by the pattern. Students 4A and 4K did not complete this question, which suggests the tutorial was ineffective for these students in aiding their development of transferring the inverse square law to this context.

The results of the last two questions would indicate that 8 of the students were able to graphically represent the inverse square law on a graph and differentiate it from a pattern that represents an inverse relationship. Some students had persistent difficulties in which they ignored the data and based their responses on the recollection of the shape of the graph, focusing on the mathematical implication that as one variable increased, the other decreased. Additionally, while the students were able to apply their understanding of the inverse square law in this graphical form, they struggled to transfer the relationship from an equation into written form.

A later question on the post-test was designed to determine the student's ability to apply the inverse square law in an electric field context. The students were presented with a positive charge held in a fixed position, with 3 points around the charge, as shown in Figure 5.22. The students were required to rank the electric field strength around the electric field, and determine the ratio of the electric field between points A and C. The student's responses are summarised in Table 5.14.

The results from this question suggest that the students are clearly aware that as distance from the charge increases, the electric field strength decreases. Nine of the students showed that the electric field strength at *c* was one quarter the strength at *a*, whilst another two students (4E and 4J) produced a ratio that had a value of 1:4, but not in its simplest form.

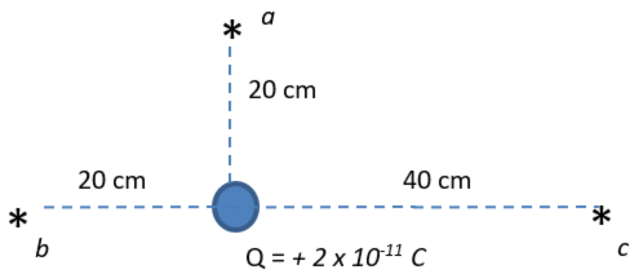


Figure 5.22. Post-test electric field question, testing understanding of inverse square law.

Student's reasoning was predominantly based on calculations (seven students) using $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2}$ or developing reasoning that doubling the distance would produce an electric field strength that was reduced by a factor of four (six students). All students who used the latter reasoning produced the correct ratio. In some cases, they substituted values for the charge and distance variables but ignored the $\frac{1}{4\pi\varepsilon}$ term in the equation. Two of the students made calculator errors, but they did not attempt to revise their work to develop alternative values for the magnitude of the charges. Two of the students simply halved the magnitude of the field strength when comparing *a* to *c*, as the distance was doubled.

Responses	Students
c < a = b	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M.**
Ratio of c to a is 1/4	**4B, 4C, 4D, 4G, 4H, 4I, 4K, 4L, 4M.**
Ratio of c to a is 1/4, not in simplest form	4E, 4J
Incomplete due to error in using calculations to determine ratio.	4F.
Ratio of c to a is 1/2	4A, 4N

Table 5.14. Student's post-test responses applying the inverse square law mathematically.

From looking at the examples of students work in Table 5.15, there were numerous errors in the student's calculations, which could still lead the students to develop the correct ratio. Difficulties that stand out are the use of scientific notation and prefixes, use of the $\frac{1}{4\pi\varepsilon}$ terms in the electric field equation, and misinterpreting electrostatic force for electric field strength.

<table>
<tr><td colspan="3">Student 4L:
The distance between the charge and particle is doubled. This means that the magnitude of “c” is ¼ that of “a”.</td></tr>
<tr>
<td>Student 4C:
$\frac{Q}{d^2}:\frac{Q}{d^2}$
$\frac{2\,x\,10^{-12}}{0.2^2} : \frac{2\,x\,10^{-12}}{0.4^2}$
$5\,x\,10^{-11}$
$: \ 1.25\,x\,10^{-11}$
$4 : 1$</td>
<td>Student 4J:
$E = \frac{Q}{d^2} \qquad E = \frac{Q}{d^2}$
$E = \frac{2\,x\,10^{-6}}{0.2^2} E = \frac{2\,x\,10^{-6}}{0.4^2}$
$E = 1.25\,x\,10^{-5} \ E = 5\,x\,10^{-5}$
$1.25\,x\,10^{-5} \ : \ 5\,x\,10^{-5}$</td>
<td>Student 4A:
$\frac{1}{4\pi\varepsilon}\,x\,\frac{q}{d^2}$
$\frac{1}{4(3.14)(8.89\,x\,10^{-12})}\,x\,\frac{2\,x\,10^{-11}}{0.2^2}$
$1.118\,x\,10^{-12}\,x\,\frac{2\,x\,10^{-11}}{0.04}$
5.59
$5.59 \div 2 = 2.80\,N$</td>
</tr>
</table>

Table 5.15. Examples of responses from student 4L, 4C, 4J and 4A.

This reliance on calculations, while useful for students to generate their own evidence / data to interpret, can lead to a habitual approach where the students unknowingly make careless mistakes, but as they were familiar with what the answer should be, from their tutorial exercises, they did not revise their calculations and are satisfied to submit their answer. For example, in Table 5.15, it was seen that student 4J made errors in their calculation using the electric field formula and also mathematically demonstrated the 1:4 ratio, with non-simplified values. Conversely, six of the students demonstrated the ability to correctly develop the ratio using no formulae, overcoming difficulties presented by Arons (1997) and Maloney, *et al.*, (2000).

In the final question, the students were presented with a shaded grid, similar to the one used in the inverse square law paint can “intensity” homework. They were presented with the diagram in Figure 5.23 (i) and told that 100 field lines pass through the shaded region when the charge is held 10m from the charge. They were then asked to determine at what distance the 100 lines would only pass through the shaded region, as seen in Figure 5.23 (ii).

This question was designed as a manner to engage their conceptual understanding of the area model employed in the homework, as discussed in section 5.5.3. A similar question was presented in section 4.3.3, which the students did well on. However, here the distance values do no related to the area covered in the diagram, as like previous questions, this allowed me to determine to what extent the students were focusing on the area of the shaded squares and/or the distance from the charge. A summary of the student results is presented in Table 5.16.

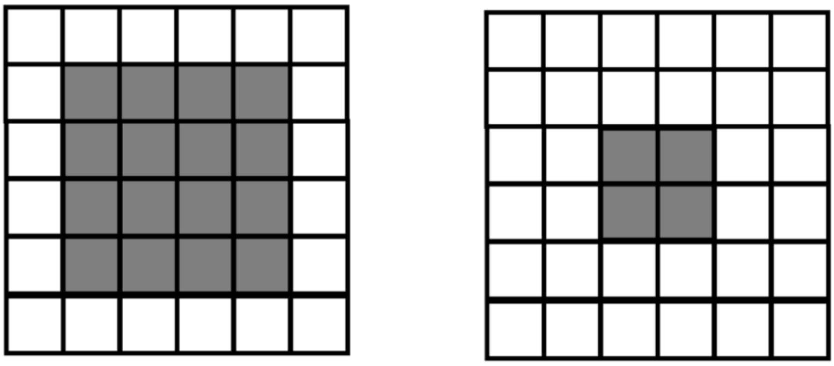


Figure 5.23. Post-test electric field question, utilising the area / scale model.

The results showed that five of the students could correctly identify the distance from which the charge would produce one quarter the field line intensity. There was a variety in responses from students who determined the correct distance that the distance was *5 m*, in which the they sketched an isometric view of the charge and the frames, demonstrated partial or complete reasoning based on reducing the length / widths of the frames, and qualitatively referencing the related increase and decrease of the variables involved. Some examples of this reasoning are shown in Figure 5.24.

It was also seen that students submitted the correct answer but give the wrong reasoning, based on an inversely proportional relationship, such as 4A and 4F. Both these students stated that as the intensity is doubled, the distance is halved. It is not clear how these students determined the intensity doubled, when considering going from sixteen frames to four frames. However, this error allowed them to produce the correct final distance from the charge to the frame.

Responses	Students.
r = 5 m	**4A, 4D, 4E, 4F, 4G, 4L.**
r = 2.5 m	4C, 4K, 4N.
r = 2 m	4B, 4I
r = 20 m	4H, 4M
r = 40 m	4J
References quadratic link between distance and area – Scaling	**4E, 4G, 4H, 4I, 4M.**
References inverse relationship	4A, 4C, 4F, 4K, 4N.
Closer to charge puts lines through less boxes.	4B, 4D
Uses alternative mathematical ratio.	4L.

Table 5.16. Student responses to scaling model, relating distance to area covered by spray can.

<table>
<tr><td colspan="3">Student 4E:

</td></tr>
<tr><td>Student 4D:
The closer to the charge, the less spread out the field lines are.</td><td>Student 4G:
The length of the width / length is half the original $(4 \div 2 = 2)$. This means the distance from the charge must also be half the original $(10\ m \div 2 = 5\ m)$.</td><td>Student 4L:
If $4\ x\ 4\ \rightarrow 10\ m$
Then $2\ x\ 2\ \rightarrow 5\ m$</td></tr>
</table>

Figure 5.24. Inverse square law reasoning provided by students 4E, 4D, 4G and 4L.

Another difficulty was presented by some students determined that the area of the second frame was one quarter the area of the first, and that the distance reduced by the same ratio. Other students determined that the distance reduced by a factor of 2, but instead of charging the 10 m distance by this factor, they suggested new distance was 2 m. One student attempted to use an inverse square law, but instead increased the distance from the charge to the frames, instead of decreasing it, suggesting they did not consider the effect of diverging field lines passing through multiple frames, as displayed in Table 5.17.

The post-test results show that students gained understanding of how the inverse square law applies to Coulomb's law and the electric field. Most students could draw the shape of an inverse graph from the electric field formula. The post-test question showed that eight of the students could differentiate between an inverse and inverse square pattern, primarily using a form of data analysis of values obtained from the graph to verify their choices. Prevalent difficulties in the remaining students involved interpreting Coulomb's law as an example of an inverse law. When analysing graphs, it was observed that without intervention, the students focused solely on the shape of the graph, and did not consider obtaining values from the graph and analyse them, as discussed in section 5.5.2.

Student 4B:	Student 4M:
$$\frac{4}{16} = \frac{1}{4}$$ $$\frac{16}{4} = 4$$ $$\sqrt{4} = 2\,m$$ *If I move the particles closer, the number of field lines that pushes through each box increases. To have 100 field lines pass through the grid, the number of boxes must decrease.*	$$16\ x\ \frac{1}{4} = 4$$ $$\frac{2}{1} \rightarrow \frac{1}{2^2} = \frac{1}{4}$$ *Double the distance. – 20m.*

Table 5.17. Erroneous reasoning produced by students 4B and 4M.

When completing the ranking question, and asked to develop a ratio, it was seen that six of the students applied the inverse square law while seven attempted calculations. Some errors in mathematical operations caused difficulties for some of these students, preventing them from showing the ratio. In two cases, the students produced an accurate ratio, but did not reduce it to its simplest form. A small number of students demonstrated that they still applied inverse proportional reasoning in this question. The final question indicates that just under half of the students could applied the inverse square law correctly using a scale model to determine the distance from a charged particle to the frames presented. Prevalent difficulties presented themselves as students applying inverse proportional reasoning, using qualitative reasoning instead of quantitative, or applying the inverse square law incorrectly.

5.5.5. Discussion

This section presents the discussion on the student's application of the inverse square law to Coulomb's law and the electric field. The student's pre-test and post-test results are compared to indicate instances of conceptual change that occurred and references to the tutorial lesson and homework discussion (section 5.5.2 and 5.5.3) are highlighted as examples of evidence that conceptual change occurred. The discussion mainly focuses on the student's application of the inverse square law mathematically, while issues and examples of understanding of scaling are referenced (Arons, 1999; Marzec, 2012).

The pre-test indicated that students encountered difficulties when required to transfer their understanding of the inverse square law to the Coulomb's law and electric field contexts. As seen in Figure 5.25, in the Coulomb's law pre-test question presented, it was observed that only three students could recognise that Coulomb's law followed contained an inverse proportional relationship. Additionally, only one student could successfully mathematically apply the inverse

square law to the pre-test. The remaining students either qualitatively, randomly attempted some manner of calculations or were unable to attempt the question. This suggests that the students required instruction that would result in conceptual extension, to promote the students to apply the reasoning and understanding they demonstrated in section 4.3, to an electrostatics context (Hewson, 1992; Konicek-Moran and Keeley, 2011).

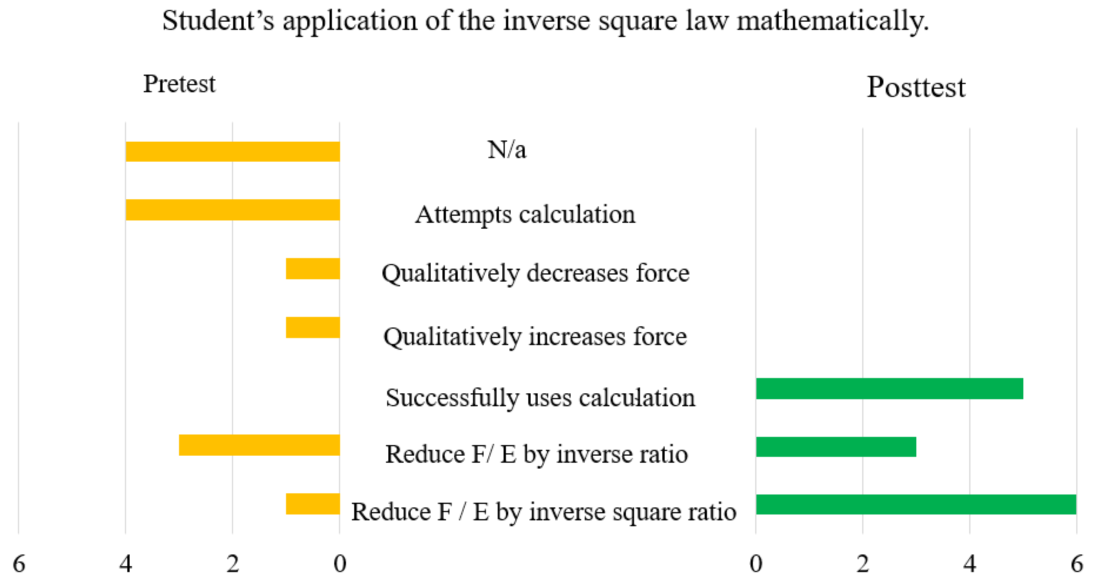


Figure 5.25. Comparison of students use of inverse square law in mathematical problems.

Gains were observed in the student's understanding and application of the inverse square law in the post-test results, as discussed in section 5.5.3. In the post-test six students applied inverse square law reasoning, without relying on formulaic substitution and evaluation, whilst a further five students did rely on the formula. Two students relied on use of the formula but used inverse proportional reasoning, while one student produced a ratio based on the distances presented in the question. This suggests that half of the students are reliant on the use of formulae, whilst the other half consider their understanding of the proportional nature of Coulomb's law and the electric field sufficient to approach the problem. The shift in results from the pre-test to post-test indicate that conceptual exchange (Hewson, 1992) occurred, as more students are observed correctly applying the inverse square law to the post-test contexts than was observed in the pre-test, and the students with persistent errors made more progress before encountering difficulty in the post-test, whereas they were unable to attempt the questions in the pre-test. Based on the shift in student responses shown in Figure 5.25, partial conceptual change was observed over through employing the use of the tutorials in the Coulomb's law and electric field contexts.

The student's reliance on formulae in the post-test is unsurprising. In tutorial the use of formulae was the most preferred method to explore the inverse square law. The students used a multiple representational approach to explore the inverse square law mathematically, and modelled the inverse

square behaviour of electric fields in the homework exercise. This homework activity was intentionally written in the same style as the spray paint model, discussed in section 4.3, as the students were familiar with the model, to allow for ease of transfer. The homework responses indicated that the students were proficient in the use of the frame model to explain the behaviour of the electric field lines. The interview indicated that the approach was unsuccessful in promoting student's consideration of mathematical area scaling, which underpins learner difficulty (Arons, 1999; Marzec, 2012). During the interview, the students demonstrated that they did not think in terms of the concept (Konicek-Moran and Keeley, 2011) of varying dimensions as they used the frame model to explore the variation of intensity through the unit areas at different distances from the charge, as discussed in section 5.5.3. However, the presentation of the frame model itself may reduce the requirement to rely on the understanding of the dimensional scaling, as students can consider the area of the frame directly.

Difficulties were also observed in the final question of the post-test using the area model, as depicted in Figure 5.23. Some students struggled to correctly show that reducing the distance from the can to the frames by a factor of a half would decrease the area by a factor of four. Students referenced the inverse square law in this question, suggesting they did not consider the quadratic nature that links the area of the spray to the distance from the can. Whilst the students attempted to approach the problem without using formulae and utilise their understanding, their application of the inverse square law directly instead of considering the dimensional scaling of the area. It suggests that they incorrectly apply the wrong relationship when considering the scaling itself but can determine the distance related to the given area, as both distance and area are familiar quantities to the students.

The electric field graphing pre-test question asked students to represent the relationship between the magnitude of the electric field at a point, and the distance from the charge. Ten of the students correctly represented the relationship using a characteristic decreasing asymptotic curve, while the remaining students erroneously produced linear of quadratic graphs or did not answer the question. However, in the post-test question, in which the students were presented with both an inverse and inverse square pattern, it was observed that difficulties were prevalent in six student's abilities to determine which pattern followed an inverse square law. Student difficulties related to the shape of the graph, not analysing the data on the graph by mapping the data into equations of the form $y = k\frac{1}{x}$ and $y = k\frac{1}{x^2}$, or analysing the data in terms of the behaviour of the reduction factors. This indicates the difficulties presented in section 4.3.3 were persistent at the end of this study, and that conceptual extinction of them did not occur.

5.6. Student’s use of field lines to represent electric fields

Literature informs us that there are many representational difficulties that students can have when using field lines to represent an electric field (Galili, 1993; Törnkvist, *et al.*,1993; Maloney, *et al*., 2001). The following representational conventions for field lines were identified as target areas for the students to learn during the tutorial lessons, as they are accessible to the student’s level of cognitive capability and could be used in application for other parts of the course, such as explaining the photoelectric effect, thermionic emission, production of x-rays and the use of particles accelerators.

- The closer field lines are, the stronger the field.
- When a field line curves, the direction of the force is tangential to the field lines.
- The field line represents the direction of force acting on a body, not the path taken by a body in the field.
- Field line patterns do not overlap or intersect. Two bodies that cause a field both contribute to one overall field surrounding both objects.
- Field lines only terminate on electric charges. They should extend to infinity / off the page / to the end of the diagram boundary.

Section 5.6.1 discusses the pre-test results, illustrating the student’s understanding of the direction of force on a charged particle in a field, student’s representations of the path taken by a body in a field, student’s determination of relative field strength, and using vectors to represent the field at various points. Section 5.6.2 details the tutorial lesson in which students applied field line conventions to electric fields. Section 5.6.3 reviews the student’s responses from the post-test, in which the student’s gains in their understanding of field line conventions are illustrated.

5.6.1. Pre-test: Electric field

In the pre-test, the students were presented with the field pattern presented in Figure 5.26. While this field pattern shows some errors, due to poor design, from the learning outcomes for fields described in section 5.6, the questions asked of the students generally do not require the reasoning that field lines extent to infinity to correctly answer the pre-test questions. This field pattern is also not representative for a single point charge, before the students commenced the pre-test, that were informed that this diagram was a snapshot of a bigger field diagram with other charges elsewhere affecting the shape of the field lines. They were to only focus on the information they could use from the pattern they observed in the pre-test question.

They were asked a series of questions to determine the student's transfer of conceptual understanding from the field lines tutorial, and gauge what new concepts would be developed when transferring from gravitational fields to electrostatic fields. The students were asked the following questions:

- To draw in the path taken by an electron placed at the white circle.
- To rank the field strength of the point *a, b* and *c*.
- Draw vectors to represent field at the points *a, b* and *c*.

The first two of these three questions are discussed in this section. The last question, which involves the transfer of field lines to vector representation, is discussed in section 5.7.

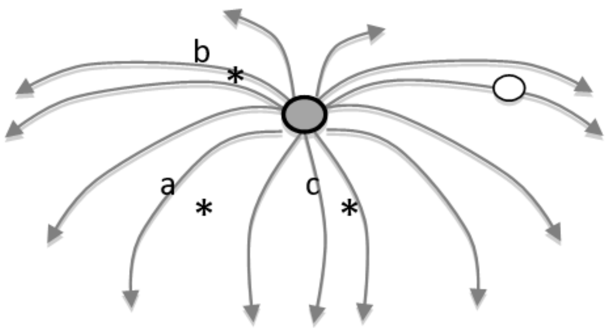


Figure 5.26. Pre-test question electric field pattern.

When representing the path taken by a negatively charged particle in this question, the students needed to consider the direction of the force acting on the charged particle, and how to represent the path under the influence of the field. Expected errors would be that the force on the negatively charged particle would be in the direction of the field line and that trajectory of the particle would follow the field line. A summary of the student's results for this question is given in Table 5.18.

The results show that three of the students determined that the direction the charge would follow was against the field line, while seven of the students determine it would go with the field line. In the class presentation to the students, the direction of the field line indicating the direction of force acting on a positive test charge was emphasized, and the opposite direction for a negatively charged particle was referenced. However, only three students recalled and applied this to their pre-test responses. This is not unreasonable as, except for the class discussion, the students did not spend any time exploring this concept.

Responses	Students
Electron moves against field lines.	***4A, 4J, 4M.***
Electron moves with field lines.	4B, 4C, 4E, 4F, 4G, 4K,4L.

Electron ignores field lines.	4D, 4H, 4N
N/a	
Electron diverges from path.	***4A, 4B, 4D.***
Electron takes linear path	4H.
Electron take unrealistic divergent path.	4E, 4K.
Electron sticks to field line.	4C, 4F, 4G, 4M.
Circular path	4N.
Electron ignores field and moves towards charge directly.	4D, 4J.

Table 5.18. Students pre-test results in determining the path taken by a negatively charged particle in an electric field.

Two of the remaining students (4D and 4J) drew paths in which the particle ignored the field and moved directly to the positively charged body. This is not considered a valid answer for this concept, even though the direction of the force acts against the direction of the field, due to the interpretation that the students did not consider the shape of the field lines when determining the path taken by the negatively charged particle. The final student (4N) ignored the field lines and drew a circular path for the electron around the positively charged particle. As there was no initial velocity for the charged particle, it is reasonable to speculate that student 4N is recalling aspects of the field line tutorial involving gravity. The production of a circular path would reflect the curved orbital paths referenced in the earlier tutorial.

Table 5.18 also shows the students results for using the field line as a guide for the path taken by the electron. Four of the students drew paths that reasonable diverged from the field lines. Errors were seen in one student (4H) drawing a path in the correct direction, but linear instead of curved. This indicates the student was not considering the force acting on the charge as it moves, and instead considered it a "once only" interaction between the field and the charged particle. Four students (4C, 4F, 4J and 4M) draw paths along the field line, indicating that the field line represented not on the force acting on charged particle, but the trajectory taken by the path. Two students (4E and 4K) drew an unreasonable diverging path, in which the electron moved away from field line, in a direction that does not correspond to direction of the force at its initial point. This indicates that while they are aware that field lines do not represent path taken, they are unclear how to apply the concepts of force, acceleration and velocity to determine the path taken. A sample of these paths are depicted in Figure 5.27.

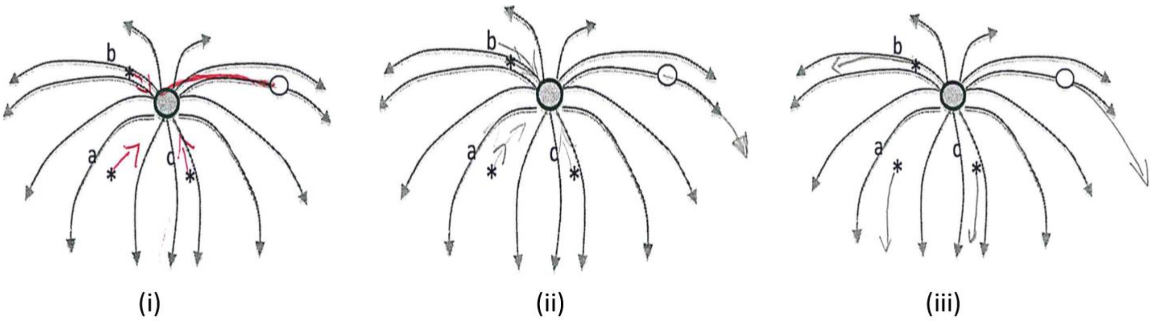


Figure 5.27. Student depictions of path of charged body which reasonably diverges from field lines (i), follows field line (ii) and diverges unreasonably (iii).

The next question on the pre-test looked at student's ability to rank the field strength at various points in a field, based on the diagram they were given. It was envisaged that successful students would use the field line density to determine the relative field strength, with a small number of students relying on the distance from the positively charged particle. A summary of the student's results is shown in Table 5.19.

Responses	Students
B > C > A.	**4A, 4B, 4D, 4E, 4F, 4G, 4H, 4J, 4K, 4L, 4M, 4N.**
B > A > C.	4C.
Field line density reasoning.	4E, 4G.
Distance from charge reasoning.	4A, 4B, 4C, 4D, 4E, 4F, 4H, 4L, 4N
Distance from field lines	4J, 4K, 4M.

Table 5.19. Summary of student's pre-test ranking of electric field strength and reasons used.

Many students were able rank the field strength surrounding the group of charges accurately. However, the reasoning utilised by nine of the students was based on the distance from the charge, whilst only three of the students attempted to use field line density as the evidence to justify their rankings. Additionally, three students used erroneous reasoning, utilising the distance from the points to the field line to represent the strength, i.e., the closer to a field line, the stronger the field. This suggests that most of students did not transfer this skill from the field line tutorial, or they did not warrant it as important over the reasoning based on distance from the charge. This could be highly likely, as this pre-test followed the tutorial on Coulomb's law, in which the inverse square relationship between force and distance was stressed as an important relationship.

Student 4G: b > c > a.
b is between 2 lines that are closest together and c is between 2 that are slightly further apart and a is between 2 lines that are very far apart.

Student 4D: b, c, a.
Because the closer you are [to the charge], the stronger the strength.

Student 4M: b, c, a.
B is closed to the long field line, c is slightly further away and a is much further away.

In summary, the pre-test results show that the student gains discussed in section 5.4 did not transfer to electric field as much as one would have hoped. A notable number of thought the field lines represent the trajectory of a body under the influence of a field. Most of the students assumed a negatively charged particle would follow a field line, although this is reasonable for the students at this point, as they had not yet explored the behaviour of a negatively charged body in a field. Moreover, in gravitational fields, there is no equivalence for this behaviour. In ranking electric field strength, most of the students relied on distance from the charge to determine strength, as opposed to using the field line density to justify their rankings.

5.6.2. Tutorial lesson: Electric field

As mentioned in section 5.4.2, the electric field tutorial lesson addressed both vector representations and field lines representations. This section looks at the latter half of the tutorial lesson. In this section, the students recapped previously covered concepts in their study of field lines, such as (a) the closer field lines are, the stronger the field, (b) when a field line curves, the direction of the force is tangential to the field lines, and (c) the field line represents the direction of force acting on a body, not the path taken by a body in the field. From the previous tutorial on field lines, as mentioned in section 4.4.5, it was observed that students made errors that the electric field line tutorial addresses. Those errors were as follows:

- Field line patterns do not overlap or intersect. Two bodies that cause a field both contribute to one overall field surrounding both objects.
- Field lines only terminate on charges. They should extend to infinity / off the page / to the end of the diagram boundary.

These rules were presented to students in an introductory paragraph on the tutorial sheet, and students were prompted to refer to them during course of the tutorial lesson. The students utilised these rules to explain the variation of an electric field of a positive charge and construct the electric field for a negative charge, whilst explaining the similarities and differences of both patterns.

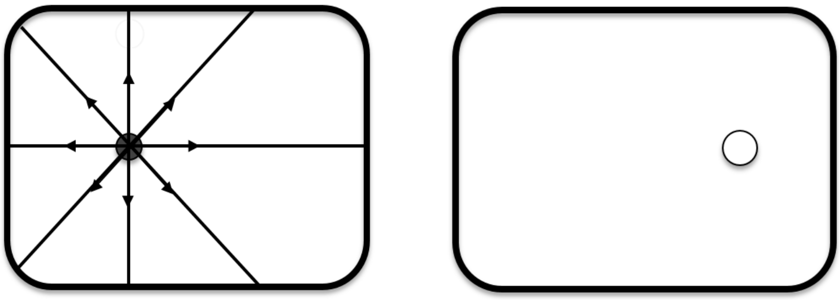


Figure 5.28. Tutorial setting where students represent electric fields using lines.

The students were required to use the field lines seen in Figure 5.28 to identify the sign of the charge as positive, and explain how the variation of field strength was represented. All students were consistent in referring to the direction of the field line pointing away from the charge as the indicator for the positive charge, in some cases referring to the fields moving to infinity. The students were also consistent in using the field line density as an indicator for the field strength. All students successful represented a negatively charged particle using the same field line pattern, with the directions placed towards the negatively charged body, as opposed to away from it, highlighting the differences between the two patterns.

Student 4E: Positively charged because field line are pointing outwards to infinity. [Field strength] decreases, field lines are further apart from each-other. [Differences] D = Arrow changes direction because negatively charged. [Same] S = field line still stretch to infinity.

The next section of the tutorial required students to apply the superposition principle to construct field lines for a system of two charges. In the first case, the students were required to represent the field of two dissimilar charges, and in the second case, the field of similar charges. In both cases, the students were required to draw the path taken by a positively charged particle under the influence of the field at a point. There were no errors shown in the student's representation of field lines for these cases. Some groups represented the path taken by the charge along the field lines. Questioning from the teacher or discussions with other students about the initial acceleration of the particle helped these students confront the errors in their initial responses. The prompt questions asked students to consider the direction of the force at the initial moment and consider the position of the particle at the next moment, having moved under the influence of the force. Student's revised their paths taken, using this approach, but still produced reasoning that was indicative of some confusion about the field line representation. For instance, one group of students, consisting of students 4D, 4E, 4F and 4N submitted the reasoning shown in Figure 5.29.

The use of the term "off track" in this case is still indicative of the error that the field lines represent a path or trajectory of some type, but the forces acting on the charge have caused it to move away from the track, akin to a car sliding off the road on a turn. Students 4G, 4H and 4I also submitted

the correct path, but gave reasoning based on the force interaction of the charges, as opposed to the field. This indicates the students were considering the interaction of the forces between the charges over the interaction of the positive particle and the field. Using this force orientated reasoning, the students appear to use the field to guide the trajectory, that's influenced by the charges generating the field.

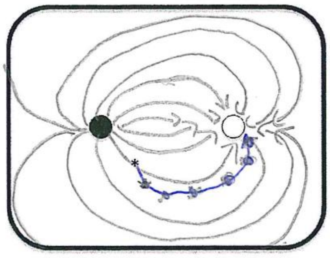
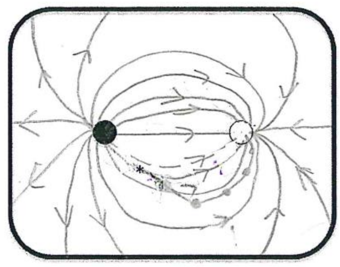
Student 4D:	Student 4G:
No velocity at the start. It accelerates a little and begins to move off track. 	*As the particle is repelled by the positive charge, it will gradually be more affected by the negative charge than the positive charge, as it moves further away from the positive charge.* 

Figure 5.29. Students 4D and 4G's depiction of path taken by charged particle in an electric field.

This tutorial narrative presented evidence that the students applied their understanding of field lines to electric fields, to some degree of success. The students identified and represented the electric fields for single charges and use the field line density to identify the variation of electric field strength for the charges. The students could also represent the superposition of similar and dissimilar charges, using field lines. While further work appears to be required to further develop the student's reasoning for the interactions of charged particles in an electric field, the students responses showed, with prompting, they accurately predicted the behaviour of a charged particle in an electric field and draw its trajectory accurately.

5.6.3. Post-test: electric field

In the post-test, the students were presented with the electric field of two positive charges shown in Figure 5.30. The students were asked a series of questions to determine their understanding of the information provided in the diagram. They were required to:

1. determine the charges on P and Q,
2. determine which of the two charges had the greatest magnitude,

3. explain the variation of field strength as a field line is followed from P to the boundary of the diagram,
4. draw the path a negative charge would take, if placed at R,
5. represent this electric field using vector arrows at different points. The results are shown in the following table.

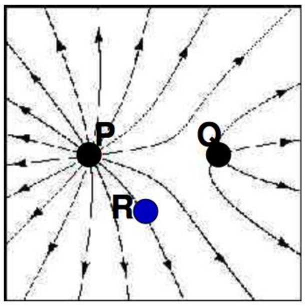


Figure 5.30. Diagram from the electric fields lines post-test question.

The first four of these questions are presented in this section, while the fifth question is addressed in section 5.7, as it specifically deals with transfer between representations. Table 5.20 presents a summary of the student's responses for the first two questions, including the reasoning provided by the students.

The post-test results indicate that the students were successful in applying their understanding of field line representation for positive and negative charges to this question. Except for one student (4F), all the students correctly identified that both bodes were positively charged, using the field line direction as justification for their responses. All the students recognised that the number of field lines coming out of the bodies could be used to determine the relative magnitude of their strength, which is an application of the field line density concept.

Student's 4A and 4G also incorporated the field line density into their responses. Student 4N referred to the number of field lines, but showed an error in their reasoning, alluding to the field lines of P pushing the field lines of Q away, indicating the field lines themselves exhibit their own tangible charge like property, or force-like behaviour.

Student 4E:	[The magnitude of P is] greater. The electric field lines are closer together.
Student 4G:	[The magnitude of P is] greater than [Q], because there are more field lines that are closer together from P, meaning that it is stronger.
Student 4N:	[The magnitude of P is] greater, because it has more field lines, and it pushes the field lines of Q away.

Responses	Students
P and Q are positive	**4A, 4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**
P and Q are negative	4F
Based on field lines leaving the charge.	**4A, 4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**
No reasoning.	4F
Magnitude of P is greater than Q.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**
Answer recognized the number of field lines is greater for P over Q.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**

Table 5.20. Student responses of the charges on P and Q, and relative charge magnitude between the bodies.

The third question asked students to place their finger on the charge P and follow one line to the boundary of the diagram. They were required to explain the variation of field strength as their finger moved and justify their explanation. Student responses are shown in Table 5.21.

The results clearly indicate that the students correctly identified the variation of the field strength using the field line density as justification. Only one student, 4M, used the distance from the charge as an indicator for field strength. Neither of these examples of reasoning are incorrect, but the results show a shift by the class to using the representational conventions of field line density, from relying on the relationship between the field strength and distance from the charge.

Student 4J: It decreases cause the field lines are more separated.

Student 4M: It gets weaker because the further you go from a charged particle, the weaker the electric field strength.

The fourth question required students to draw the path taken by a negatively charge particle in the field, when placed at the point R, depicted in Figure 5.29. This question was like the pre-test question, looking to determine if the students would draw a path that reasonably diverted from the field line, and whether the force acting on the negatively charged particle would go against the direction of the field. A summary of the student's responses is presented in Table 5.22.

While most students were able to show a reasonable path diverging from the field lines in a direction that moved against the field lines, minor errors were still observed in some student responses. Students 4L and 4M both submitted the correct path, but used naïve reasoning based on the attraction of the positive charge P, and the negatively charged particles placed down. These students failed to explicitly articulate their reasoning for the shape of the path. By presenting a reasonable curved path that diverges from the field line, they indicated that they understood the

velocity (or inertia) of the particle moves it from the field line. Student 4B drew a path in which the initial force was in the correct direction and was tangential to the field line. However, as the particle moved, the direction of the net force on the charge would have changed, but 4B did not take this into account and drew a linear path that followed the direction of the initial force. Student 4C also drew a linear path, but in this case, they ignored the shape of the electric field, and draw a path directly to the charge, basing their path on the attraction between the charges.

Responses.	Students.
Field strength decreases.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N**
Field strength increases.	N/a
Field strength remains unchanged.	N/a
Justified by using field line density.	4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4N
Justified by distance from charge.	4M

Table 5.21. Student's post-test responses for the variation of field strength, and their justifications.

Responses.	Students.
Negative charge moves against field line.	**4A, 4B, 4C, 4D, 4G, 4H, 4I, 4K, 4L, 4M, 4N**
Negative charge moves with field lines.	4F
Reasoning based on attraction	4L, 4M
N/a	4E, 4J
Reasonable path off the field lines produced.	**4A, 4D, 4F, 4G, 4H, 4I, 4K, 4L, 4M, 4N**
Paths was linear	4B
Path ignore field lines, direct to P.	4C
N/a	4E, 4J

Table 5.22. Summary of student's post-test responses to drawing a negatively charged particle moving in an electric field.

Student 4F had incorrectly identified that P and Q were negative charges instead of positive charges and drew their path accordingly. While the path did leave the field line, in what is considered a reasonable path, the student did not use information based on the field line direction to determine what direction the force on the charge would act. They used the interaction between the negative charge placed down at R and the force it would experience based on its repulsion from P and Q. Student 4F did however explain that the increasing velocity would move it from the field line, and

drew a reasonable curved path that indicated repulsion on the mobile charged particle from both P and Q. A sample of the students correct, and incorrect paths are displayed in Figure 5.31.

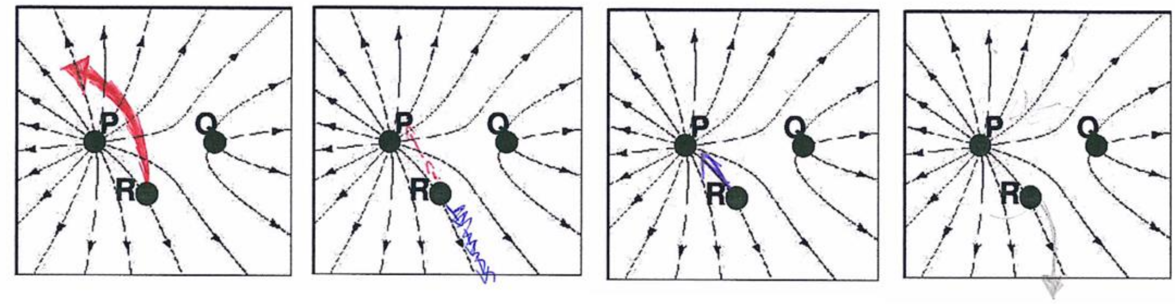


Figure 5.31. Paths taken by negative charge in field, from students 4H, 4B, 4C and 4F.

This section discussed the post-test results. The results indicate that, across the whole group of 14 students, gains occurred in the student's understanding and application of field line concepts. The students demonstrated they could identify a charge, based on the field line pattern, and determine the relative magnitude of a charged body based on the number of field lines going into or out of it. All but one of the students used the field line density to explain the variation in field strength as an indicator for relative field strength. The final question discussed in this section showed that the majority of students could accurately draw the path taken by a negatively charged particle in a field accurately but errors were persistent for a small number of students, such as the direction of the force acting on the negative charge in a field, the continuous nature of the force acting on the negative charge, and students ignoring the field and drawing the path based on attraction interaction between the negative charge and P or Q.

5.6.4. Discussion

This section discusses the student's pre-test and post-test results, focusing on their understanding of the direction of force acting on a charged object, using the field lines as a guide to draw a reasonable trajectory, and the used of field line density as an indicator for field strength. When appropriate, the student's ability to transfer their reasoning from the tutorials in Chapter 4 is also discussed in each section. The first part of this discussion focuses on the student's understanding of the direction of force acting on a charged particle, under the influence of an electric field. Figure 5.32 shows the pre-test – post-test comparison for the student's understanding that the force on a negative charge acts in a direction that goes against the field.

The pre-test results indicate that over half of the students were unaware of how a field line can be used to determine the direction of force acting on a charged negatively charged particle or ignored the field itself and focused on the interaction of charges. Both difficulties are commonly observed in

learners understanding of electric fields (Furio and Guisasola, 1998; Cao and Brizuela, 2016), and both were identified as targets for conceptual extension (Hewson, 1992).

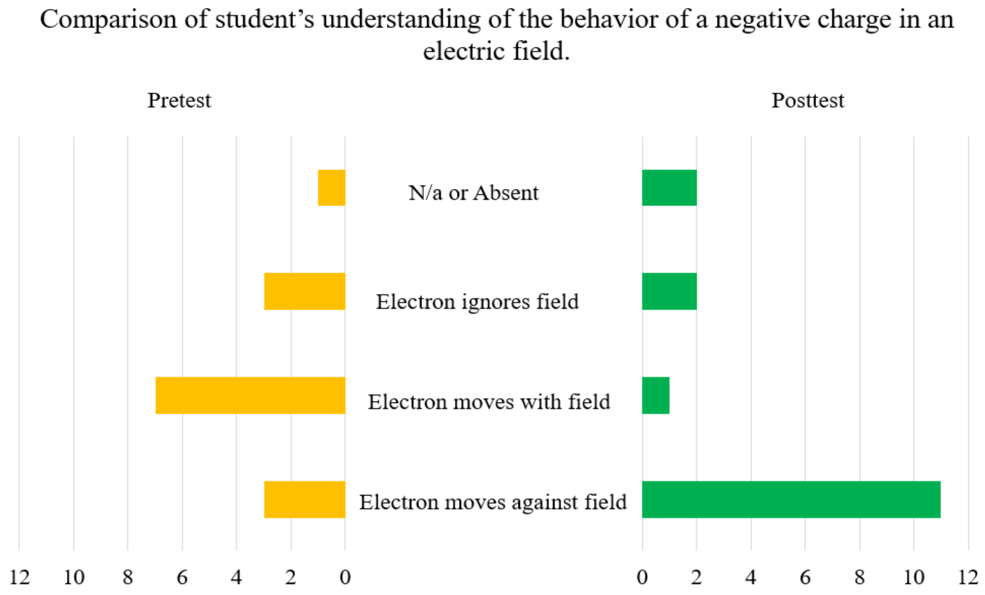


Figure 5.32. Comparison of results regarding the force on a negatively charged particle in an electric field.

Section 5.6.2 narrated the student's development during the tutorial. The latter part indicated that when both a field and charges are presented to the students, the students reasoning is based on the interaction of charges, as opposed to the interaction of the field with the charged particle. However, this was not observed with high frequency in the post-test. The post-test showed a shift in the number of students that determined an electron would be influenced by a force to move against the direction of the field using reasoning based on the charged particle and the field. Figure 5.32 displays the student's responses to this question and indicates that conceptual extension occurred, with a jump of 3 students to 11 students using field-based reasoning. Reasoning based on the interaction between charges was rarer in the post-test with only two students (4L and 4M) using the reasoning. The paths drawn by these two students were correct and suggest that they were aware of how the electron would behave, but they appeared to value the interaction between the charges as a stronger indicator to determine the path than the interaction with the electron and the field. Both types of reasoning, charge interactions and field interactions, were observed in the tutorial and post-test, and this suggests that the students consider the scenarios in terms of both styles of reasoning and applied them as they saw fit to the task presented to them. This suggests conceptual exchange did occur in the student's models, but conceptual extinction did not occur. As the comparison of the results demonstrates an increase in eight students utilising the correct reasoning, the extent to which conceptual change was observed was moderate.

The next section compares the paths drawn by a charged particle under the influence of an electric field. Figure 5.33 shows the pre-test – post-test comparison, in which it is clearly observed that there was a reduction in the number of difficulties encountered by the students.

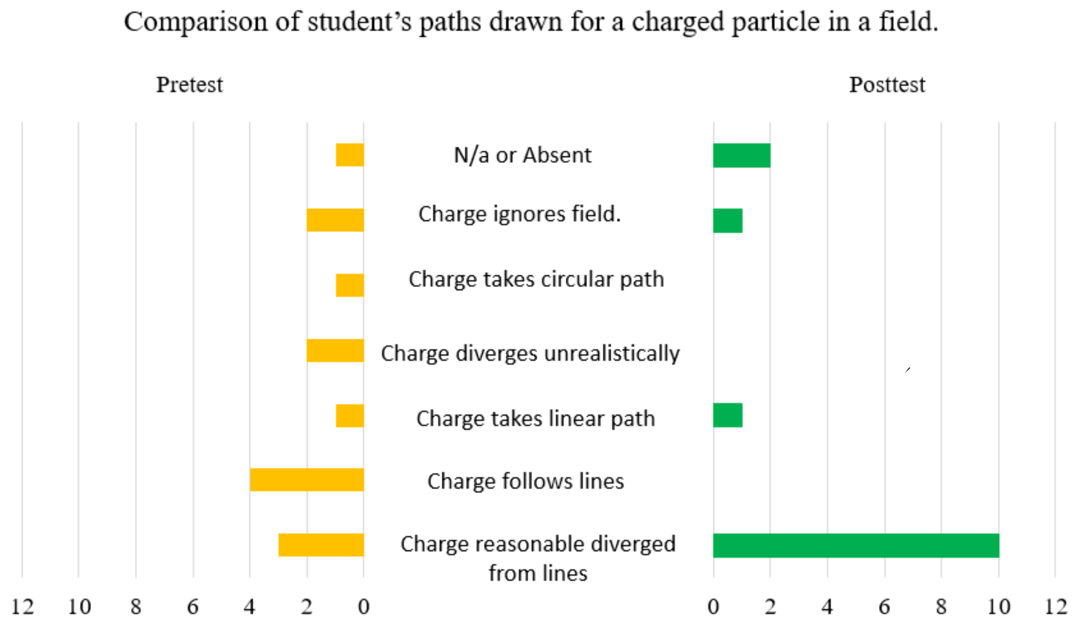


Figure 5.33. Comparison of results regarding the path taken by a charged body in an electric field.

While only a small number of students showed the path following the field line in the pre-test, which was expected to be the most prominent error, the results from Figure 5.33 suggest that students struggled to construct a reasonable path during the pre-test (Törnkvist, *et al.*, 1993; and Galili, 1993). In section 4.4.4, it was seen that 12 of the students constructed a reasonable path taken by a body under the influence of a gravitational field. When this is compared to the electric field pre-test, it is clear the students did not transfer their understanding to the electric field context. An inability to transfer a concept to multiple contexts is an indicator of the limit of the student's conceptual understanding (Konicek-Moran and Keeley, 2011) and the tutorial was developed to aid student extend their understanding to the electric field (Hewson, 1992), to which partial conceptual change was observed.

In the electric field tutorial lesson, it was seen that students required time and guidance to consider how the force acting on charged particles affects their acceleration and velocity to construct a reasonable path. When difficulties occurred, the motion diagrams were utilised as prompts by the teacher. This allowed the students to consider how the charged particles moved from moment to moment, whilst the students explained how the force at every moment would cause the particle to change its motion. The teacher provided the dissatisfaction in the explanatory power of considering the field lines as indicative of the path taken by a charged body and presented a motion diagram which allowed the students to develop intelligible reasoning based on their understanding of forces, acceleration and velocity (Posner, *et al.*, 1982). As the students to focus on applying the outcome of

the interaction and the concept involved, as opposed to focusing solely on the outcome in the electric field context (Konicek-Moran and Keeley, 2011). The post-test results showed that 10 students constructed reasonable field line trajectories, overcoming most misconceptions presented in the pre-test. This suggests an extension in the student's understanding to generate a reasonable path in a gravitational field to an electric field context (Hewson, 1992)

Figure 5.34 illustrates the pre-test post-test comparison for the student's justifications in determining relative field strength in an electric field.

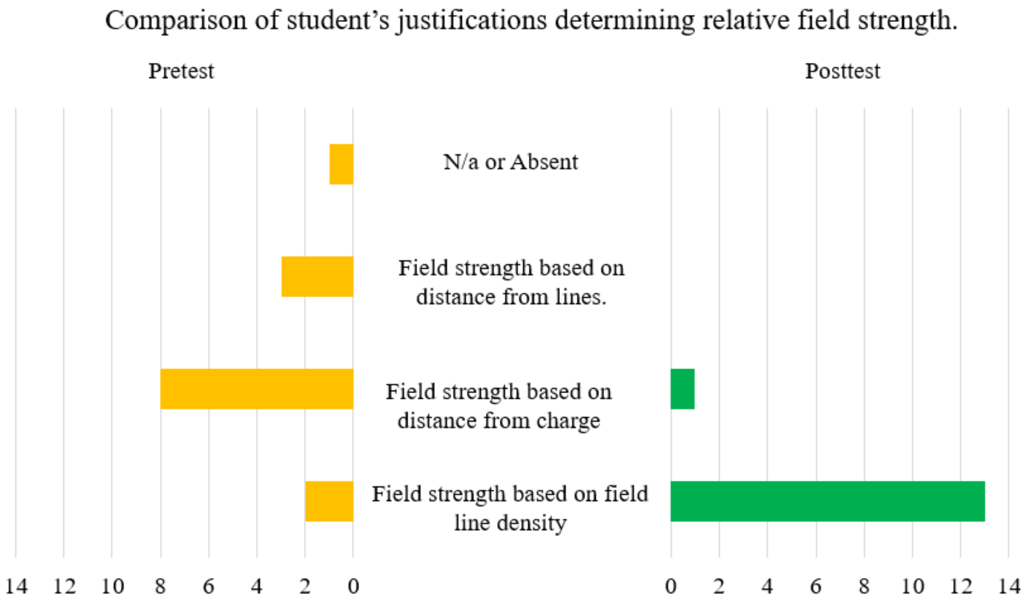


Figure 5.34. Comparison of results regarding the relative field strength in an electric field.

In chapter 4, section 4.4.5, it was observed that 10 of the students could rank field strength based on field line density. In the electric field pre-test, it was seen that only two students extended their understanding field density to determine the relative field strength of different points to the electric field context (Hewson, 1992; Törnkvist, *et al.*, 1993; and Galili, 1993), whilst eight students relied on distance from the charge. It is unclear whether the students were unable to extend and transfer their understanding to from gravitational lines, or whether they did not consider this concept to be more appropriate to employ over using distance from charge to justify their ranking. As the students previously completed the tutorial on the inverse square law and Coulomb's law, it is reasonable they considered the qualitative relationship first as it was recently used in their memory. During the tutorial lesson, the students demonstrated that there was no difficulty in apply the convention and apply it in the electric field context (Konicek-Moran and Keeley, 2011), and in the post-test, it was observed that all but one of the students applied it in the post-test. As this question could be answered using both distance reasoning and field line density reasoning, with both concepts recently in their memory, most students opted to use field line density as their justification. This indicates that the students were able to extend the concept of field line density to represent the relative field strength

between points, but they may opt to using alternative, but valid, reasoning when the task they were completing allowed them to do so. Therefore, the level of conceptual change observed in the student's understanding of relating relative field strength to field line density was moderate.

This section discussed the student's understanding of the force acting on a negative charge and showed the tutorial lesson allowed for conceptual change to occur. Post-instruction, some minor errors were persistent such as students ignoring the field focusing on the interaction between the charged particles, or directing the force acting on negatively charge particle following the field line direction. Ten of the students showed that the path taken by a charge would reasonably diverge from field lines, with only one student considering the force to act on the charged particle at a single instant, and one student ignoring the field. Finally, the students demonstrated that they could utilise the field line density to rank the field strength at different points, instead of relying on the relationship between electric field strength and distance, when the concept was recently employed in their classwork.

5.7. Student's use of vector and field lines representations in electrostatics

Thus far, this chapter has illustrated and discussed the student's use of vectors, the inverse square law and field lines, as they applied them to Coulomb's law and the electric field. This section details how students transferred between field line representations and vector representations (Törnkvist, *et al.*, 1993; and Galili, 1993). The ability to transfer isomorphically, that is transfer between both directions without difficulty either way, is demonstrated as a trait of expert problem solvers (Kozma, 2003). The first half of this section discusses an example of students representing vector arrows from a given electric field, and the latter half discusses an example of students representing a vector field using field lines.

5.7.1. Student transfer from field lines to vectors

In the electric field post-test, the students were presented with an opportunity to construct a vector field plot from a field line diagram. The field line diagram was shown in Figure 5.30, at the beginning of section 5.6.3. The students were presented with a diagram that presented the charges P and Q and six points indicated with asterisks shown in Figure 5.35.

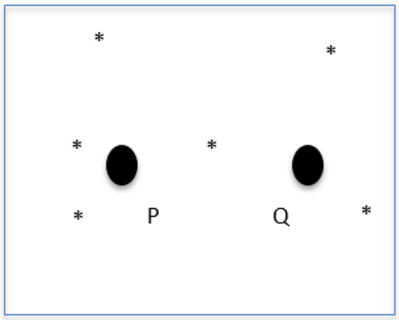


Figure 5.35. Points to represent vector arrows from field lines diagram.

When asked to represent the electric field using vector arrows at the points shown, most of the students encountered difficulties in transferring between the two representations. The student's results are summarized in Table 5.23.

Responses	Students.
Vectors are correct direction and correct relative magnitude	**4B, 4D, 4E, 4G**
Magnitudes are not relatively in scale.	4C, 4H, 4I, 4J, 4L
Directions of vectors are mildly non-tangential.	4M
Tangential, but points towards P and Q.	4C, 4J, 4K
Vectors from both charges shown, no super-position of vectors attempted.	4A, 4F
Attempted superposition shown	4N

Table 5.23. Student's attempts to transfer from field line to vector representation.

Only four of the students could correctly represent the electric field at various points around the two charges using vectors. The most prominent error shown by the students was sketching vectors of the correct orientation but incorrect relative magnitudes. Section 5.6.3 showed that the students were aware of how the field line density represented the field strength, and during section 5.4.2, it was noted in the tutorial lessons that the students tended to forget to represent the magnitude of the field correctly before applying the superposition principle. Although the students were prompted to the error and they rectified it in the tutorial, it was clearly not sufficient to produce a long-term change in their vector representation, when constructing vector fields.

Three students, 4C, 4J and 4K, reversed the direction of the vectors relative to the direction of the field at the point. All these students previously identified the sign of the charge on P and Q, based on the direction of the field lines, so these vectors are not consistent with their previous reasoning. It is possible that these students drew these vectors to represent the force acting on a negative charge at the given points, as the previous question required them to draw the path acting on a negative

charge. The relative magnitudes of these student's vectors were reasonable and accurately represented the field strength at the points shown.

Another difficulty in transferring to vector representation was encountered by students 4A and 4F. This their case, they drew vectors surrounding each individual charge, but attempted no superposition. This is not considered an attempt at transfer between the representations, but an attempt at construction of the vectors from scratch, suggesting that the student's do not comprehend the link between the two representations.

Student 4N had similar difficulties in which they attempted to construct the vector field based on the positions of the charges, without using the direction of the field lines as a guide for the direction and relative magnitudes of the vectors. Student 4N also overlaid the electric field on their diagram and produced reasonable resultant vectors. Whilst their vectors were drawn reasonably, it is clear they did not ground their construction in transferring between field lines, but instead relied on the principle of superposition. Examples of the student's constructions are presented in Figure 5.36.

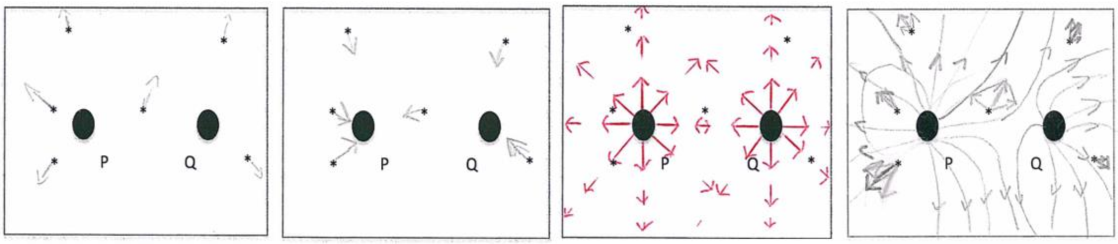


Figure 5.36. Vector fields transferred from field line representations, from students 4D, 4C, 4A and 4N.

This section showed that the approach undertaken in this study did address the student's ability to transfer from field lines to vector representations. Difficulties about representing vector magnitude qualitatively using the length of the arrows were the most persistent difficulty presented by the students, whilst most students appeared to grasp that the direction of the field was tangential to the field lines at the points shown. In some cases, students did not transfer the direction of the arrows correctly, even though they were depicted on the original field lines diagram. This difficulty may have been influenced by the previous question in which the students was asked to consider a negative charge, indicating they do not consider the field independent of the charges it interacts with. Other students resorted to rote learned vector patterns without drawing the superposition and one student was able to represent the field drawing vectors but relied using both superposition between the charges and overlaying the field line pattern to scaffold their construction.

5.7.2. Student transfer from vectors to field lines

In a separate question in the post-test, the students were presented with a vector field, as shown in Figure 5.37 (i), and asked to identify the sign of the charge on the particle. They were told an oppositely charged body was nailed down next to the first charge and they were asked to construct the superposition of the field at the points shown in Figure 5.37 (ii). The student's responses to these questions were discussed in section 5.4.3. As an extension to these questions, the students were required to draw electric field lines to match the vector field they constructed, as seen in Figure 5.37 (iii). A summary of the student's responses is presented in the Table 5.24.

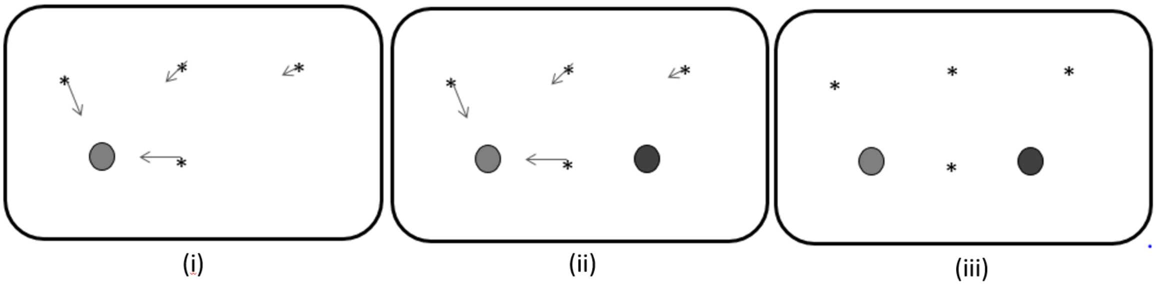


Figure 5.37. Post-test question in in which students apply vectors to an electric field.

Responses.	Students.
Field lines consistently represent vectors	**4D, 4E, 4H, 4J, 4L, 4M.**
Field lines not consistent with vector arrows	4A, 4B, 4F, 4G
Incorrect superposition – Not all vectors consistent with lines.	4C, 4I, 4K,
No transfer demonstrated – used both representations	4N

Table 5.24. Student's attempts to transfer from field line to vector representation.

The results indicate that half of the students could construct a field line representation that was consistent with their vectors, while the other half of the students could not. The six students who correctly transferred their vectors to field lines aligned field lines reasonably well with the directions and magnitudes of the vectors, as shown in the example of student 4H's response in Figure 5.38. In all cases, minor errors were seen, and it was reasonable if the student's diagrams accurately represented 3 of the vectors, and reasonably represented the fourth.

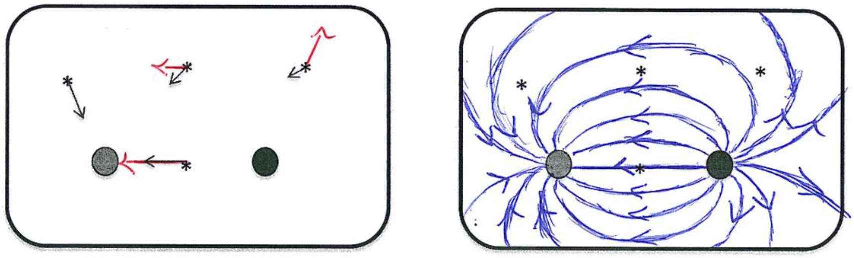


Figure 5.38. Example of Student 4H representing vectors using field line representation.

As both charges were equal in magnitude, the field line pattern should have been symmetrical, but in cases where the student's vectors were inaccurate with magnitude, this error was transferred to their field line diagrams. These errors are considered to be reasonable as the diagrams are qualitative in nature, and the consistency in the error suggests the students were employing representational transfer. For instance, student 4E did not find the superposition of the vectors, and when representing the field lines, skewed the symmetry of the diagram to produce the shape of the field in line with the strongest vectors in their field, except for the top right point, as seen in Figure 5.39.

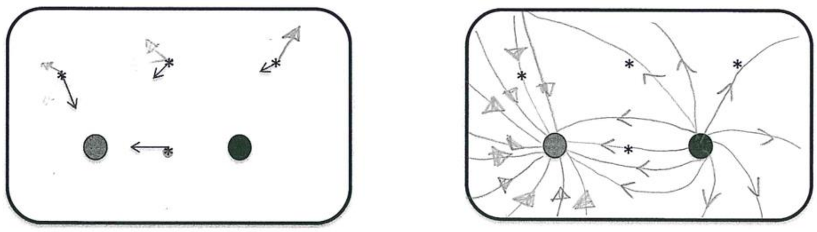


Figure 5.39. Example of Student 4E transferring error consistently to field line representation.

Four students drew field patterns which were correct and symmetrical but did not represent the vectors. This indicates they may have produced a rote learned model but did not consider the direction of the field at all points. For instance, Figure 5.40 shows student 4G showed the superposition at various points, but when drawing the field lines, three of the points did not align to their vectors. Furthermore, their field line diagram suggests only the field in the space between the charges is where the superposition principle applies, ignoring the areas on the left and right of the diagram. This contradicts the vector superposition they represented in the vector field diagram.

This error was also seen in the responses of students 4A and 4B. Student 4F gave incorrect vector directions. They interpreted the question to be two negatively charged particles. However, the field line pattern they drew was indicative of oppositely charged particles. This indicates the student was resorting to rote learned patterns, as their diagrams are inconsistent.

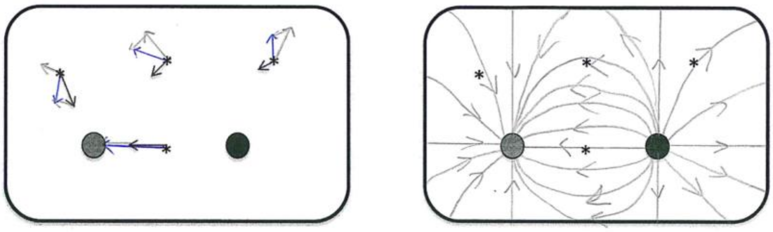


Figure 5.40. Inconsistencies in the vector transfer to field line representation, from student 4G.

Students 4C, 4I and 4K drew patterns that also did not transfer from their vector arrows. As seen in Figure 5.41, the two points in the centre of the diagram are not represented for their field. They drew a pattern that indicates repulsion and is inconsistent with their vectors. However, an example of transfer can be seen on the points shown to the top right and top left of their pattern, in which their patter follows the direction of the vector of highest magnitude, an error also seen in student 4E's submission. Student 4K also displayed this error. Student 4I drew overlapping lines of the field lines pattern for both attraction and repulsion and display the field line direction to be in the opposite direction of that which they drew directions of the vector arrows. This is depicted in Figure 5.42.

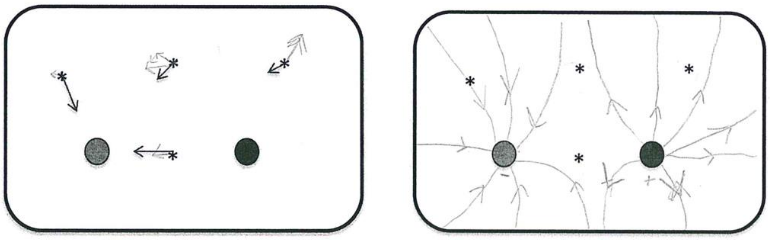


Figure 5.41. Inconsistencies in the vector transfer to field line representation, from student 4C.

This section showed that only a small number of the students represented a field line pattern using field lines. The most persistent difficulty in students was maintaining consistency in the direction in the diagrams drawn from vector arrows to field lines. A likely issue in the responses was students resorting to rote learned patterns for field lines of oppositely charged particles. The students appeared to demonstrate an overreliance on position of the charges to produce their field line pattern, and not on the vector arrows they drew.

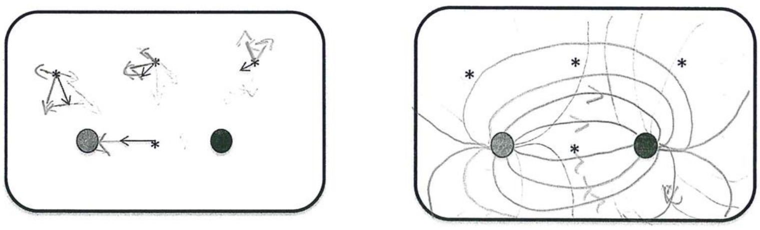


Figure 5.42. Examples of errors in the vector diagram transfer to field line representation, from student 4I.

5.7.3. Discussion

This discussion reviews the student's understanding of concepts related to vector and field lines representations and provides a commentary on student gains observed in the previous two sections. Section 5.4 illustrated the student's progress in extending their understanding of vectors to applying it in an electric field diagrams, when required to represent the field using arrows at various points around a charged particle (Maloney, *et al.*, 2001). Section 5.6 illustrated the students understanding of the relative field strength and the direction of force acting on a charged body at various points in an electric field (Galili, 1993; Cao and Brizuela, 2016). As seen in section 5.7.1, the gains that the students displayed in the post-test for these representational contexts individually were not as frequent when the students were required to transfer between the two representations.

Section 5.7.1 showed that the prevalent difficulty in student's ability to transfer between representations was representing vector magnitude that was consistent with a given electric field line diagram (Törnkvist, *et al.*, 1993). This difficulty was not observed to the same degree in the post-test question, seen in section 5.4, as in this section where the students were required to transfer between field line representation to vector representation. As section 5.4 indicates that the students could apply the vector convention to the electric field context, this suggests that conceptual extension did not occur effective (Hewson, 1992), to enable the students to apply the concept in a task that involved transfer between the two representations. As nine of the students were able to correctly represent the directions of the vectors, this indicates that some level of transfer occurred, in which the students could recognise the tangential nature of the electric field at positions represented along a field line.

Section 5.7.2 detailed the post-test question, which elicited the student's ability to transfer from vector representation to field line representation. It was observed that half of the students were able to construct a field line pattern that was consistent with their own vector constructions (Törnkvist, *et al.*, 1993). The most common difficulties in the post-test were that students drew vector fields that were suggestive of both charges having the same charge, but the fields indicated their charges were opposite or vice-versa. This suggests these students did not extend their understanding to interpret their vector and apply their field line reasoning but constructed their field line patterns from patterns that were memorised, or results to guessing.

From reviewing both section 5.7.1 and 5.7.2, it was observed that only students 4D and 4E consistently answered the post-test questions without error and demonstrated the understanding to allow them to transfer between the representations isomorphically. Students 4B, 4G, 4H, 4L and 4M answered one of the two questions without error, and tended to make minor errors in the other question. This suggests that conceptual extension (Hewson, 1992) occurred for these students during the tutorial, as these students displayed proficiency in the vector and field lines tutorials, as seen in chapter 4, and showed progress in transferring their understanding to the electric field context. The

remaining students demonstrated errors in both transfer questions. The errors displayed by these students are reflective of the errors seen in chapter 4 and/or the difficulties observed when applying these concepts to the electric field, as seen in section 5.4 and 5.6, such as terminating field lines, or lines overlapping. As these students displayed errors in their understanding of both representations in chapter 4, it is reasonable that difficulties in transfer between the representation occurred.

5.8. Conclusions

This section of the chapter presents the conclusions based on the studies presented in this chapter in two separate subsections. They address the impact on student learning, and the student's ability to transfer the representational tool to the electric field context. The conclusion addressed the overall research question that was introduced at the beginning of this chapter:

- To what extent does the use of a multi-representational structured inquiry approach develop student understanding of electric fields?

This question is addressed in the impact of student learning, which addresses the following considerations related to the research question:

1. The student's ability to demonstrate that Coulomb's law, and the electric field, is an example of an inverse square law?
2. To what extent the students demonstrate their understanding electric fields and the interaction of charged objects with fields using vector representation.
3. To what extent the students demonstrate their understanding electric fields and the interaction of charged objects with fields using field line representation.

The student's ability to transfer representational tools to the electric field context addressed the following consideration related to the research question:

4. To what extent the students demonstrate their ability to transfer a depiction of an electric field from one representation to another representation?

5.8.1. Impact on student learning

The approach adopted in this study focused on developing student's ability to transfer and their understanding of vectors, the inverse square law and field lines to Coulomb's law and the electric

field. The vector nature of forces is a fundamental pillar of electrostatics, and further electromagnetism, in which vector mathematics is utilised, along with calculus. The inverse square law is one of the fundamental relationships studied by students, which is seen in optics, sound, gravitational and electrostatic forces, and radiation. The concept of a field underpins the interactions of particles and field lines are an efficient manner to represent them, as they can convey various information about a field, in a simple manner.

The first of the research considerations looks at student's application of the inverse square law to Coulomb's law and electric fields. The approach adopted utilised two methods for students to apply their understanding of the inverse square law to Coulomb's law and the electric field. The first method involved mathematical data analysis in various forms, as suggested by Hestenes and Wells (2006), and the second method was adapted from the spray paint model, presented by Hewitt (2009). The pre-test results for the student's application of the inverse square law to Coulomb's law and the electric field suggested that the students did not incorporate the ability to recognise Coulomb's law as an equation of the form $y = k\frac{1}{x^2}$, and apply mental mathematical ratios to determine a reduction in force, or did not remember how to apply inverse square proportional reasoning. This suggests the students did not transfer what they learned during the inverse square law tutorial lessons, as discussed in section 4.3, where nine students successfully used the law or made reasonable attempts in applying it. However, in the Coulomb's law pre-test, this number dropped to four across different manners of representing the inverse square law. Upon completing the Coulomb's law tutorial and electric field homework exercise, it was seen that approximately half of the students still require formulae to apply the inverse square law, and struggle to fully conceptually grasp the mathematical scaling that applies to the inverse square law. However, they were able to apply the scaling to diverging field lines. The tutorial also showed the students prefer to utilise algebraic substitution and evaluation to apply the inverse square law, due to their perceived ease of use. This is also an interesting observation to note, as there are less mathematical operations involved in using the graphical or tabular analysis that the students completed. This may be due to the student's familiarity with solving quantitative problems, which rely on algorithmic problem-solving strategies. The post-test results showed that repeated exposure to the inverse square law promoted student's ability to perform proportional reasoning operations to show the ratio of reduction for 2 points, but over half the students relied on the use of mathematical formulae with substitution and evaluation to complete the task.

Additionally, the electric field homework interview and post-test scaling model question results indicate that the students do not consider the dimensional change in width and height of the scale model when using area problems, and do not link this to the change in distance. This error in student thinking can lead to them apply directly proportional thinking to the model, which in turn leads to inverse proportionality when applying concepts such as intensity and field strength to the model. Further development of these concepts, in a mathematical or physics context, could employ active learning strategies focusing on constructions of geometrical regular shapes such as squares and

rectangles which involve increases in dimensions of lengths and width, and recording and analysing the patterns between the variables could help students with these concepts.

The second research consideration addresses the student's ability to transfer their understanding of vectors to the electric field context. Section 5.2 detailed the student's progression in learning vector concepts in isolation of any physical context, and the discussions showed that progress was made by the students with regards to representing vector magnitude using the lengths of the arrows, accurately using vector constructions to show the superposition of two vectors and consideration of horizontal and vertical components of vectors. The electric field pre-test suggests that the students struggled to recognise and transfer this representation to the electric field context. Errors were seen in which students did not demonstrate the reduction in magnitude with distance and could not consistently apply the principle of superposition. It is unlikely they were unaware of the relationship between the distance from a point to electric field strength as the preceding Coulomb's law tutorial and class discussion introducing electric field reviewed the relationship. The electric field tutorial lesson afforded students the opportunity to apply vector concepts and vector addition to an electric field context. This explicit application was justified as the students required prompting to consider both the magnitude and direction of the field at different points, and then apply the principle of superposition. The student gains in transfer manifested in the post-test results, in which most of the students demonstrated the relationship between distance and electric field strength using vector arrows, and reasonable apply the vector constructions to find resultant vectors. This indicates that students learning vector concepts in isolation in not effective for contextual transfer, and the concepts require explicit application in a variety of contexts for students to utilise vectors as a tool to explain and represent those contexts.

The third research consideration addresses student's use of field lines in representing the electric field. Section 5.4 showed progression in student's understanding of field line density representative of relative field strength, the force on a body acting tangentially to a point on a field line and reasonable deviations of a body from a field line. The students did not demonstrate transfer of these concepts as much as was expected to the electric field pre-test. Few students could reasonably draw the path taken by a charged body in a field and field strength was ranked by students using qualitative reasoning based on distance instead of the field line density. The tutorial lesson allowed students to apply the field line conventions to field line contexts and gains were seen in the post-test results. As was seen with the vector concepts, explicit lessons in which field lines are applied to electrostatic contexts are required for them to correctly apply it to the context. This is justified as the results indicate the field line tutorial using a gravitational context alone were not sufficient in helping students develop and transfer the understanding of basic field lines to electrostatics. Additionally, the behaviour of negatively charged particles in field lines does not have an equivalent concept in gravitational field lines. Further explicit use of vectors and field lines in context is appropriate in magnetostatics and electromagnetism, to explain the variation of magnetic field strengths, application

and understanding of the right-hand grip rule and Fleming's left-hand rule and determining the direction of induced current in electromagnetic induction.

The extent to which the student's developed their understanding of Coulomb's law and the electric field varied during the tutorials. Figure 5.43 presents a line plot to illustrate and compare the extent of the conceptual change recorded in these studies. A legend of the codes used in Figure 5.43 can be found in Appendix F.

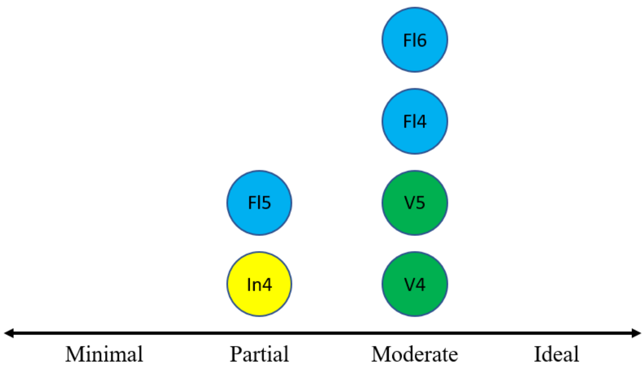


Figure 5.43. Line plot of extent of conceptual change in student's understanding of Coulomb's law and the electric field.

Unlike the results shown in section 4.5, the range to which the extent of conceptual change occurred was not as large between the student's transfer of vectors, the inverse square law and field lines in this section of the research. The most gains were observed in field line and vector concepts, but this is unsurprising as the most gains were observed in with these representations in chapter 4. The student's development and application of the inverse square law in an electro-static context was the most challenging, as seen by only one instance of partial conceptual change observed in Figure 5.43. As discussed in section 4.5, the tutorials in the electrostatic context also employed both mathematical reasoning and scientific reasoning when the students were applying the inverse square law to Coulomb's law, and when using the scale model applied to field lines, as opposed to field lines and vector concepts, when the students were primarily engaging with scientific reasoning. This requirement to employ dual reasoning may once again have overloaded student's working memory resources (Reid, 2009) and impede the student's progress.

5.8.2. Student's transfer between representations in electrostatics

This section addresses the last of the research considerations, in which present the conclusions for the student's ability to transfer. Section 5.7 outlines the student's ability to transfer between vector and field line representations. In transferring from field lines to vectors, persistent difficulties generally related to either students ignoring the magnitude of the vectors, relative to the field strength,

or drawing the direction of the arrows opposing the field lines. The former was a difficulty persistent seen in the students use of vectors, but suggests these students only considered the direction of the field lines when transferring between the representations. The latter difficultly it likely due to a previous question which involved the presence of a negatively charged particles influencing their rationale. This also indicates the students consider the field to be influenced by a charge that interacts with it, instead of being independent of the field.

In transferring from vectors to field lines, it was observed that just under half of the students drew the correct field diagram consistent to their vector representation. The most common difficulty observed was the student's reliance on drawing a field based on the position and types of charged bodies present, instead of being consistent with their vector diagrams. One student utilised the superposition principle to find the resultant vectors are various points, but also sketched field lines, in which their resultant vectors were consistent with their field lines, suggesting the student did not transfer directly from one representation to another, but utilised both representations to accurately show the field.

In comparing both transfers, it was seen that only two students iso-morphically transferred between vectors and field lines, i.e., they could transfer from field lines to vectors accurately, whilst transferring from vectors to field lines. The results suggest that the students were more comfortable with transferring from field line representation to vectors, with minor errors in the transfer present. When transferring from vector to field line representations, slightly more students completed this task effectively, but the errors shown by the remaining students suggested they did not consider the vectors but the setup of the charged particles, which is a major hindrance in their ability to transfer between representations.

One aspect of the difficulties observed in the student's transfer could relate to the mathematical underpinnings required to fully construct a mental model of electric fields. The mathematical courses completed by the students do not contain vector mathematics, nor have the students completed the calculus section of the mathematics course required to completed build a more accurate model of electric fields, in line with the scientific consensus. Issues of transfer between the representation were presented as rules, as opposed to delving into the reasoning underpinning them. For instance, the equation $\vec{E} = \frac{\vec{F}}{q}$ is treated as an algebraic equation as opposed to a vector equation. Reference to the direction of the force being tangential to the field is referenced by the sign of the variable is the same when the charge used is positive, and they're opposite when they're negative. However, without a deeper understanding of vector mathematics, the students unable form coherent models and instances of the students operating with incomplete models were observed in the tutorials sections of this chapter. These issues like have contributed to the relatively low numbers of students being able to transfer between vector and field line representations iso-morphically.

Chapter 6. Work and potential difference

6.1. Introduction

This chapter discusses the development of the student's understanding of the concepts of work and potential difference, as applied to electric fields. The Work tutorial employs the use of vector concepts and field lines to get students thinking about positive, negative and zero work. The potential difference tutorial employed verbal, mathematical and graphical representations, to promote student understanding of the concept, and ability to predict the behaviour of charges acting under the influence of a potential difference. The students also apply their understanding of work and potential difference, along with concepts covered in chapter 5, to explain the behaviour of current and potential difference in a simple circuit.

The following research question is addressed in this chapter:

- To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

The following points are considered when addressing this research question:

- To what extent does the use of vectors and field lines, representing force and displacement, enable students to identify positive, negative and zero work?
- What affect does the use of graphs and diagrams have on students understanding of potential difference?
- What difficulties are encountered by the students during this transfer to a potential difference context?

The timeline of this section of the project is shown in Table 6.1. Sections in bold refer to materials covered as they related to the research. Sections that are not presented in both are required to be covered for completion of the required syllabus for Leaving Certificate Physics. As the Coulomb's law, electric field, work and potential difference tutorial lessons were run concurrently, Table 6.1 starts on Week 5, which chronologically followed Week 4 of the Coulomb's law and electric field tutorial lessons, as was shown in Table 5.1.

The last two subsections of section 2.1.3 detailed difficulties encountered by learners in their understanding of work and potential difference. Based on the difficulties typically encountered by learners, as discussed in these sections, the Work and Potential Difference tutorials were designed

provide the students with opportunities to overcome these common difficulties. These difficulties influence the drafting of the learning objectives for this section, as upon completion of the teaching and learning material, the students would be able to:

1. Identify and explain instances of positive, negative and zero work (Lindsey, *et al.*, 2009; Doughty, 2013)
2. Identify work and displacement, based on electric field line diagrams (Doughy, 2013)
3. Associate relatively high and low potential to positively and negatively charged particles respectively (Hazelton, 2013).
4. Explain the behaviour of charged particles, under the influence of a potential difference (Guisasola, *et al.*, 2002; Maloney, *et al.*, 2003; Hazelton, 2013).

Week 5, Class 1 (35 mins)	Presentation to introduce to review work **Pre-test**
Week 5, Class 2 (80 mins)	**Research lesson: Work worksheet.**
Week 5, Class 3 (76 mins)	Presentation to introduce potential difference. Practise class: qualitative problems.
Week 6, Class 1 (35 mins)	Further qualitative problems involving potential difference, work, potential energy and kinetic energy.
Week 6, Class 2 (80 mins)	Practice class: Qualitative problems involving Electric field.
Week 6, Class 3 (76 mins)	**Research lesson: Potential difference. Homework assignment given.**
Week 7, Class 1 (35 mins)	Review of topics.
Week 7, Class 2 (80 mins)	**Post-test.**

Table 6.1. Timeline of implementation of work and potential difference tutorial lessons.

This chapter presents a narrative of the students use of various representations as they were applied to the context of work and potential difference, whilst targeting the 4 learning objectives. The student's development is presented by comparing pre-test and post-test results for the different topics, as well as display the development of students understanding during the tutorial lessons, with both snapshots of their tutorial worksheets and extracts of recordings of the student's conversations that occurred during the tutorial sessions. Section 6.2 discusses the use of student's application of their understanding of vector and field concepts to develop their knowledge of work, and the use of various representations to develop their understanding of potential difference as a mathematical ratio of work done per charge, their association of relative high and low potential to charged bodies and

how charges move under the influence of a potential difference. Section 6.3 compares student difficulties before and after instruction, and comments on expanded contexts that were used in the post-test, that draw on concepts seen in chapter 5, the pre-test and the tutorial materials. The chapter closes with conclusions, which address the research question and considerations under the headings of the tutorial's impact on student learning, and how the use of various representations helped develop student understanding.

6.2. Work and potential difference tutorials

This section of the chapter discusses the implementation of the work and potential difference tutorials. Section 6.2.1 discusses the pre-test, which focused on student's understanding of work done as a charge is moved from in an electric field, how force and displacement vectors are used to determine several types of work, the behaviour of charged objects in the presence of a potential difference. It also focuses on the association of relative high and low potential to regions surrounding positively and negatively charged particles. This concept is illustrated in Figure 6.1 using a pHet simulation, in which the positive and negative charge are held fixed and equipotential lines are displayed. Any positively charged mobile test charge placed in the space between the charges held fixed would be influenced to move from left to right in this figure, while a negatively charged mobile test charge would move in the opposite manner. This concept would be more commonly applied to the relative potentials of the terminals of a battery, or the plates of a capacitor.

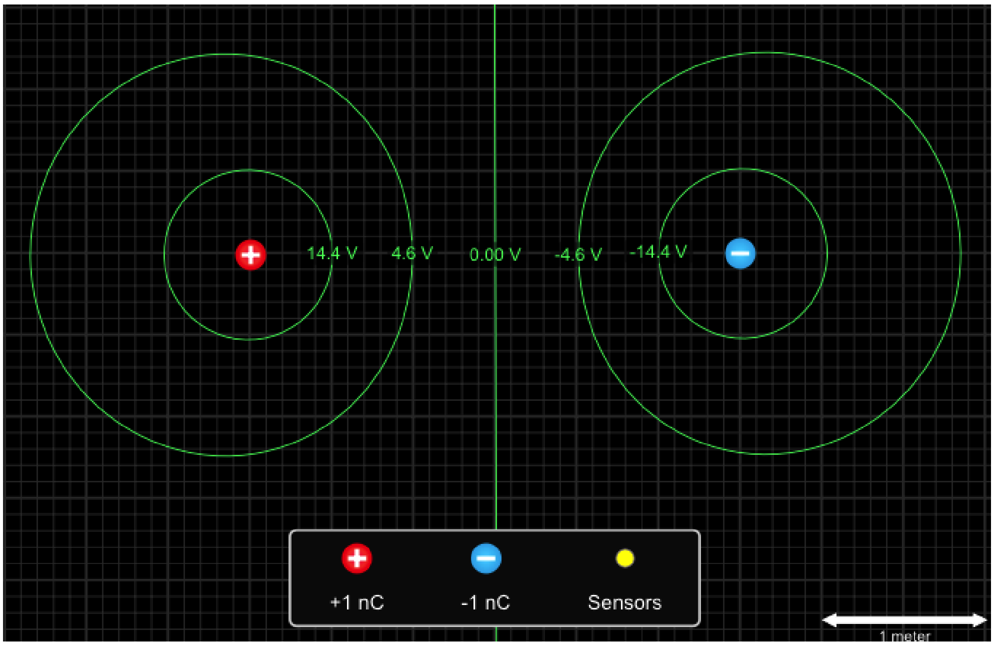


Figure 6.1. pHet simulation displaying the relative high and low equipotential lines due to the presence of positive and negative charges.

Section 6.2.2 and 6.2.3 narrate the Work and Potential Difference tutorial lessons, which identify instances of difficulties encountered by the students. Section 6.2.4 discusses the homework, which advances the students understanding of potential and applies a graphical approach to thinking about the variation of potential due to positive and negative charges. Section 6.2.5 analyses question given to students in the post-test, that either directly relate to questions asked in the pre-test, or present contexts that are extensions from questions seen in the pre-test, tutorials and homework. The tutorials were written in the same style as those seen in Tutorials in Introductory Physics, with some ideas and contexts taken from Conceptual Physics (Hewitt, 2009).

6.2.1. Pre-test: Work and Potential difference

The pre-test question was designed to test student's understanding of work, focusing on their understanding of displacement vs distance travelled when a body moves from one point to another in a field. Figure 6.2 shows the diagram, in which the students were to rank the work done in moving from A to B, along the three paths. Correct reasoning would show that the work done is the same in all cases. This could be argued by the displacement in all cases being the same, or discussing the contribution of positive, negative and zero work along all the paths taken. Expected incorrect reasoning would be the student's relying on the distance travelled along the different paths, instead of displacement. A summary of the student's responses is presented in Table 6.2.

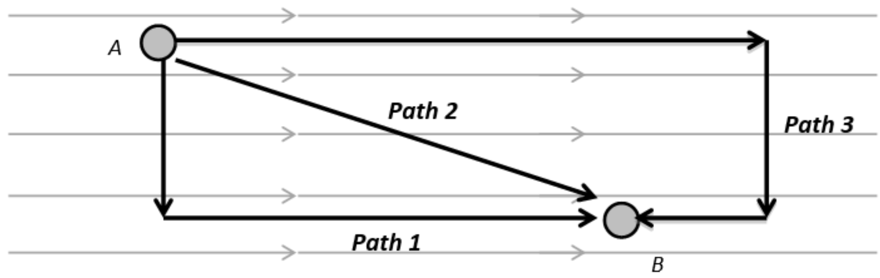


Figure 6.2.Extract from pre-test question in which student's rank work done in 3 paths.

The pre-test results indicate that the students did not consider positive, negative and zero work reasoning to the problem, and predominantly focused on the distance travelled along the paths as the justification of their ranking. Furthermore, five of the students referenced that in path 3, the path moves against the field which leads to more work, with two of the student referencing the ease taken along the other paths and the resistance encountered in going against the field. This suggests these students still conceptualise the field with tangible properties, in this case, as contributing to a resistance that must be overcome, similar in nature to swimming against the current in a river. Similar reasoning was also observed in student 4H's responses, in which they said the path was deflected by the field. They reasoned that path 2 had only one deflection at an acute angle, while path 1 deflected

at ninety degrees leading to more work involved in this path and as there was two perpendicular turns in path 3, there was more work in that path. Another student, 4C, referenced both the distance taken in the paths, but also noted that path 2 was the resultant of the components vectors for path 1, and equated this as contributing to more work in path 2.

Responses.	Students.
$\boldsymbol{W_1 = W_2 = W_3}$	**N/a.**
$W_2 < W_1 < W_3$	4A, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M.
$W_2 < W_1 = W_3$	4B.
$W_1 < W_2 < W_3$	4C.
$W_2 < W_3 < W_1$	4N.
Displacement / Positive, negative, zero work reasoning.	**N/a**
Reasoning based on distance travelled.	4B, 4G, 4I, 4J, 4K, 4L, 4M.
Reasoning based on "resistance" encountered.	4A, 4C, 4F, 4G, 4I.
Reasoning based on vector resultant.	4C.
Reasoning based on field "deflection" of paths	4H.
No clear reasoning	4D, 4E, 4N.

Table 6.2. Student's responses and reasoning to pre-test work ranking question.

In the second pre-test question, the students were required to rank the magnitude of the work done, for various pairs of force – displacement vector pairs. Christensen, *et al.*, (2004) noted ~75% of undergraduate students in their study could mentally apply the dot product to rank vector pairs when the angles were in the range $0^{\circ} \leq \theta \leq 90^{\circ}$, but when angles greater than 90° were introduced, the percentage dropped to ~60%, noting students had difficulties in recognising vector layouts that would produce negative work. Their research specifically looked at vector mathematics, whilst the pre-test question uses a work context and as the magnitude of negative and positive work are equal in this question, it was decided that the sign of the work would not have to be observed to consider a correct answer, if the students referred to the scalar nature of work. Figure 6.3 shows the question presented in the pre-test, and Table 6.3 summarises the student responses.

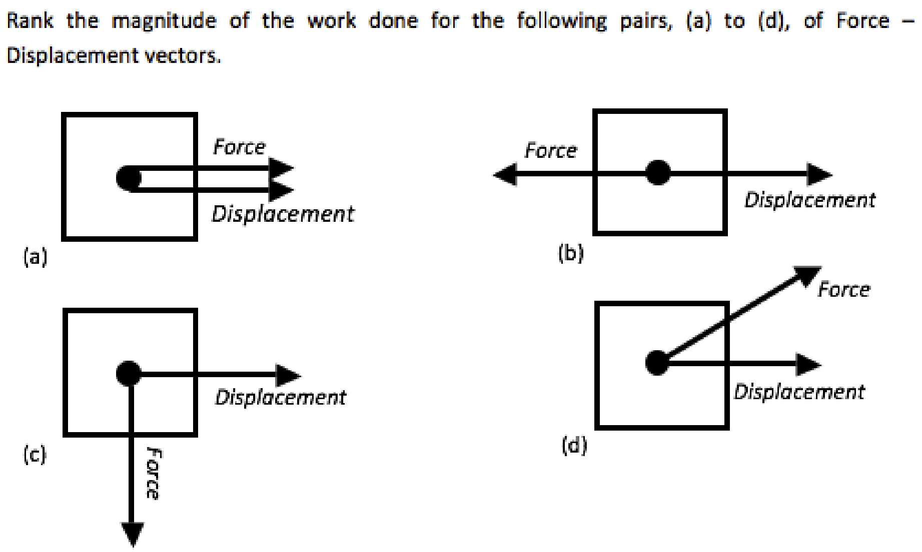


Figure 6.3 Pre-test question in which students use vectors to rank work done.

The results show that none of the students ranked the magnitude of the work done correctly in any of the setups shown. The most common incorrect ranking, produced by 9 of the students, was $W_A > W_D > W_C > W_B$, and the reasoning produced by these students varied, with three students (4C, 4D and 4E) using vector addition reasoning with the force and displacement. Two students (4F and 4M) referenced the force as adding resistance to the displacement when they are anti-parallel. These students may think force adds a thrust to the displacement when they are parallel, and does not affect displacement when perpendicular. Thus, the angle between the force and displacement would determine the work done (student 4G) and the relative displacement between the arrowheads of the force vector and displacement vector indicates the amount of work done in the system. Of the other rankings, only student 4H provided reasoning, suggesting as the displacement in all cases did not change, the force was indicative of the work and used this to produce their ranking.

The next question was designed to determine the student's understanding of the behaviour of charged objects in a potential difference. They were required to predict the movement of the two boxes, shown in Figure 6.4, and justify their predictions. A summary of their responses is shown in Table 6.4.

Responses.	**Students.**
$\boldsymbol{W_A = W_D > W_C = 0 > W_B}$	**N/a**
$\boldsymbol{W_A = W_B = W_D > W_C = 0}$	**N/a.**
$W_A > W_D > W_C > W_B$	4A, 4C, 4D, 4E, 4F, 4G, 4K, 4M, 4N.
$W_A > W_B > W_C > W_D$	4B.
$W_D > W_A = W_B = W_C$	4H.

$W_C > W_B > W_D > W_A$	4I, 4J.
$W_D > W_C > W_A > W_B$	4L.
Force and displacement cancel out.	4C, 4D.
Force decreases with distance.	4A.
Determines resultant of force and displacement.	4E.
References resistance.	4F, 4M.
Wider angle results in more work.	4G.
Force magnitude determines work.	4H.
Distance between arrowheads / vectors indicates work.	4K, 4N.
N/a.	4B, 4I, 4J, 4L.

Table 6.3. Student's responses and reasoning to pre-test work questions, based on force and displacement components.

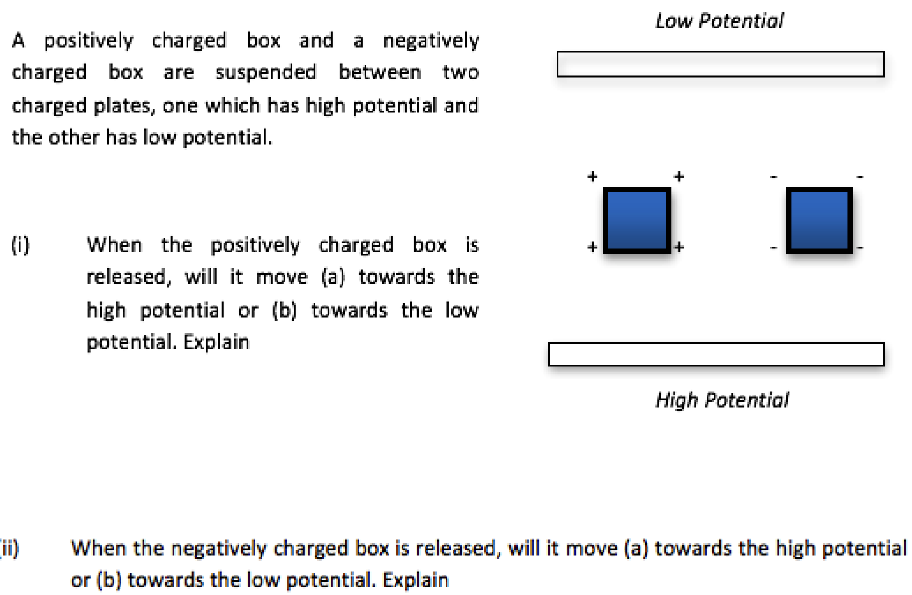


Figure 6.4. Pre-test question about charged objects under the influence of a potential difference.

While the students correctly predicted the behaviour of the charged boxes in a potential difference, they did not apply correct reasoning to do so, based on the information they had. The most common reason used to justify the student's predictions was to associate a positive charge with the plate labelled high potential and negative charge to the plate labelled low potential. Once they had determined the charges on both of the plates, they predicted the motion of the boxes based on the force interactions between the boxes and the charge of the plates. While such reasoning is valid,

the students did not come across this in the tutorials nor the class discussions and may have (a) guessed this was how the potential of the plates can be used as a reference to determine the charge on them or (b) assumed that positive indicates high and negative indicates low due to positive integers having higher values than negative integers, which are relatively lower in value. Two students (4J and 4K) referenced the effect of gravity on the boxes, although student 4K's responses were not consistent with their predictions, in which the bodies would move in anti-parallel directions. Students 4C and 4D referenced that the boxes had less and more protons for their respective charges and moved to the low potential and high potential because of this, without clarifying why they referenced the protons as they did or explain how the behaviour was dictated by protons. Student 4B referenced that the bodies become unstable due to having potential energy. As they stated that both boxes would move from high to low, this may be a reference to gravitational potential energy.

In the final question on the pre-test, the students were given a graphical representation of a profile for potential difference and asked to sketch the positions of charges to produce the low – high – low potential variance, as shown in Figure 6.5. This question was designed to elicit student's thinking about the relative potential associated with positive and negative charges, positive being high potential and negative being low potential. A summary of the student's responses is found in Table 6.5.

Responses.	Students.
Positive moves from high to low.	**4B, 4C, 4D, 4E, 4F, 4G, 4M, 4N**
Positive moves from low to high.	4A, 4K, 4L 4J
N/a	4I, 4J
Negative moves from low to high.	**4A, 4C, 4D, 4E, 4F, 4G, 4H, 4J, 4L, 4M**
Negative moves from high to low.	4B, 4K
N/a	4I
References behaviour due to potential difference.	**N/a.**
Assumes charges on plates.	4E, 4F, 4G, 4M, 4N.
Use gravitational field as for reasoning.	4J, 4K
References protons.	4C, 4D
Instability due to potential energy.	4B
Reasoning unclear / no reasoning submitted.	4A, 4I, 4L

Table 6.4. Responses and reasoning to pre-test question involving the movement of charges bodies acting under the influence of a potential difference.

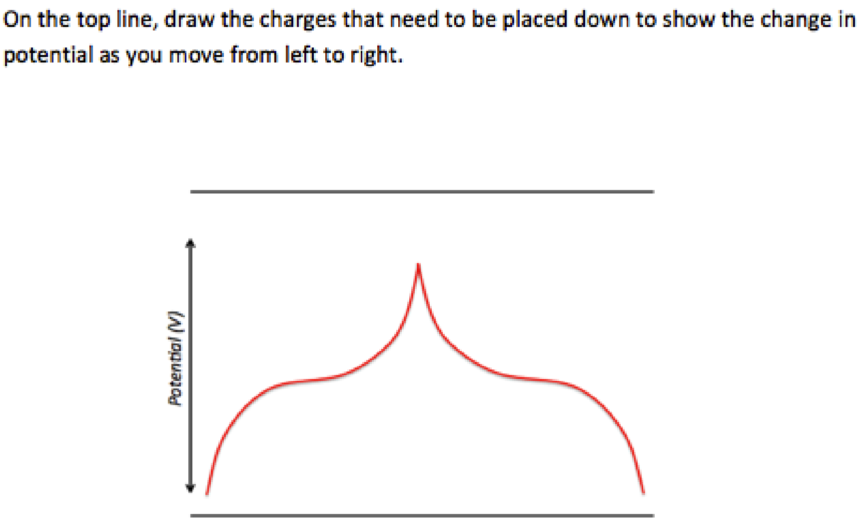


Figure 6.5. Pre-test question eliciting student's association of high and low potential to charges.

Responses	*Students.*
Associate high potential with positive charge.	**4B, 4G, 4I.**
Associate low potential with positive charge.	4E, 4F, 4L, 4M.
Does not associate charge with potential.	4A, 4C, 4D.
N/a.	4H, 4J, 4K, 4N.
Associate low potential with negative charge.	**4G, 4I.**
Associate high potential with negative charge.	4E, 4F, 4L, 4M
Does not associate charge with potential.	4C, 4D.
N/a.	4A, 4B, 4H, 4J, 4K, 4N.

Table 6.5. Responses to pre-test question involving the association of high and low potential of charged bodies.

A correct response to this question would place negatively charged particles at the two positions where the graphs are at the minimal values. A positively charged particle would also be expected to be placed at the position where the graph was at the maximum value. This would show the student applying the association of relative high and low potential to the areas around positively and negative charged particles, as described in section 6.2 and illustrated in Figure 6.1. The pre-test results show that most of the students did not associate relative high potential to regions with positively charged bodies and regions negatively charged bodies with relative low potential. Only two students, 4G and

4I, submitted the correct charges for the high and low potentials, with only student 4G submitting reasoning. Student 4G was consistent with their assumption shown in the last question, in which they associated high potential to positive and low potential to negative, whilst 4I gave no reasoning in either this question or the last question. Surprisingly, students 4E, 4F, 4M and 4N correctly guessed the association in the last question, but the first three of these students reversed their association in this question, while student 4N did not attempt the question. As the remaining students provided no reasoning for their responses, it is unclear as to why they chose the charges as they did, but it is probable that it was due to guessing.

This section detailed the student's initial misunderstandings and limits of their understanding of work and potential. It was seen that when considering work in a field, when different paths can be taken from one point to another, the prevalent difficulties encountered by the students considering the distance travelled over the displacement travelled, or the considered the field to act with a tangible property providing a resistance for the work to overcome in moving a charge against the field. It was seen that the students could not rank the work done in moving a box when the force and displacement vectors were aligned in various directions, due to an inability to identify positive, negative or zero work, or consider the absolute value of the work done, regardless of whether it involved increasing or decreasing the energy in the system. The third question showed the students could predict the movement of positively and negatively charged objects under the influence of a potential difference, but several students associated positive and negative charge with high and low potential, which was not consistently observed in the last pre-test question.

6.2.2. Tutorial lesson: Work

The tutorial lesson on work began with a twenty-minute class discussion and presentation of the concept of work. The initial context used was the same used in the mechanics section of their physics course, in which a car moves from one point to another and the students were asked to explain what effect a force has on the car when pointing (i) in the direction of the displacement, (ii) against the displacement and (iii) directed towards the ground, as shown in Figure 6.6. Using a think-pair-share strategy, the student groups were able to explain that when the force and displacement were parallel, the velocity of the car would increase; when in opposite direction, the velocity would decrease, and when perpendicular the velocity would not change. The teacher reintroduced the terms positive, negative and zero work and applied explained them in terms of the student's responses, as these terms were used when they initially studied mechanics. The students were then presented with a formula for work, $W = Fs\cos\theta$, and required to substitute values in for the three force-displacement diagrams shown on the car. The students were not presented with vector notation due to their unfamiliarity with the notation in their mathematics course, and because the aim to develop their

understanding in the tutorial was primarily qualitative and conceptual in nature. The discussion repeated for contexts such as a ball on the end of a compression spring pushed from its equilibrium position to a compressed state, and a charged particle being pushed towards a positively charge dome. These contexts were taken from Conceptual Physics (Hewitt, 2009).

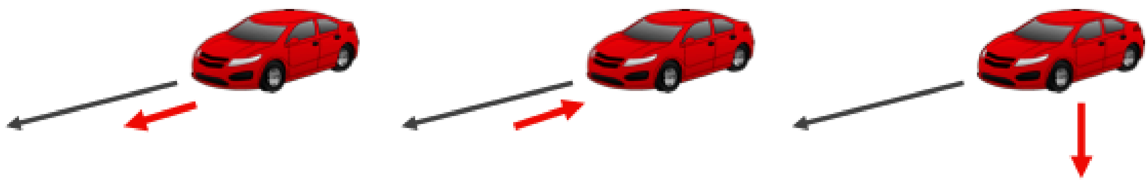


Figure 6.6. Initial context used to illustrate the concept of positive, negative and zero work.

The tutorial worksheet involved the students considering the path taken in getting from two points using different paths, as shown in Figure 6.7 (i). The students were required to explain the different between the distance travelled and the displacement. The students were then presented with a block of weight 100 N, that is pushed a total displacement of 6 m with 50 N of force. They were required to calculate the work done by the person pushing the block, and determine if any work was done by gravity, and if so, how it contributed to the movement of the block. The students were then required to consider the block being pulled with a force of 50 N through the 6 m, with the rope making an angle of 30° to the horizontal, as depicted in Figure 6.7 (iii). The students were required to resolve the force vectors, with acted along the 30° diagonal, its horizontal and vertical components, and determine which components contributed to the positive work and zero work. The students were required to calculate the net work done on the block, and were verbally asked to explain why the net work done, from the scenario presented in Figure 6.7 (iii) did not equal the work done when the force was applied horizontally, as presented in Figure 6.7 (ii), even though both blocks were moved with 50 N through a distance of 6 m.

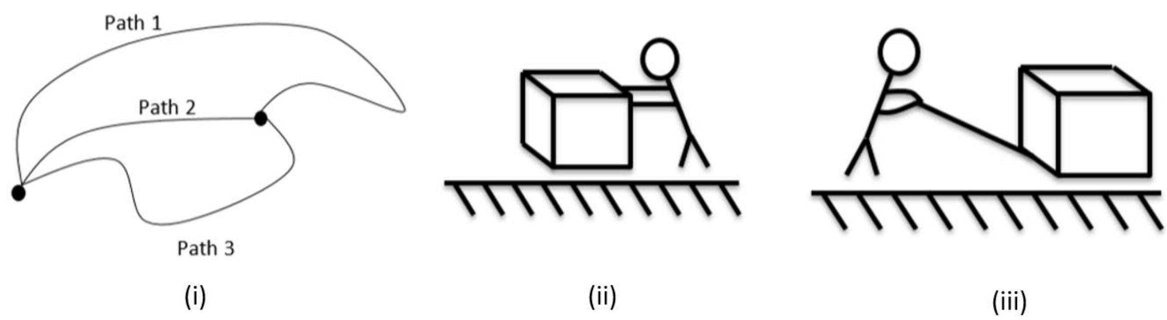


Figure 6.7. Diagram extracts from the work tutorial involving displacement and forces.

The students were then presented with the second pre-test question (see Figure 6.3), in which they had to rank the magnitude of the net work done in all four cases. They were encouraged to use

rulers to record relative magnitudes and, in some cases, resolve the force vectors into their horizontal and vertical components. Having discussed the concepts of positive, negative and zero work, as well as completing the previous questions in the tutorial, the students tended to produced rankings of $W_A = W_D > W_C > W_B$, or $W_A = W_B = W_D > W_C$. The students were encouraged to measure values from the diagrams by using a ruler and applying a scale of 1cm : 1N and 1cm : 1m, to determine the force and displacement respectively. In cases of the first ranking, students explicitly stated that the negative work in diagram B was less than zero, in cases of the last ranking, students stated that although the work was negative, it would have the same value as diagram A and D. One group displayed a unique error, consisting of students 4J, 4K, 4L and 4M, in which they added the force and displacement vector magnitudes, instead of multiplying them. This error is inconsistent with the work they completed in the previous section of the tutorial, in which all students multiplied the force and displacement magnitudes. One possible source of this error is the student's interpretation of the boxes Figure 6.3, in which they considered and applied vector addition when they observed the vector arrow pairs. If this is the case, this error came from the students misinterpreting the representation used, and were thus unable to correctly transfer the information to the mathematical symbolic representation, W = F.s.

Student 4H: 5.29 [J] is greater than 0 [J], which is greater than -5.29 [J]

Student 4K: The [horizontal] components in A, B and D result in 4 [J] because work is 2 + 2 which is 4, but because C has a vertical vector, the magnitude = 0.

In the last section of the work tutorial, the students were presented with the diagram shown in Figure 6.8, in which they were informed that a 1 kg mass was moved between different points.

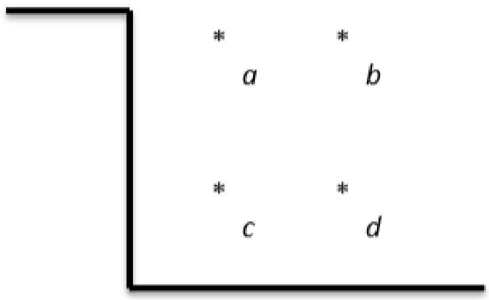
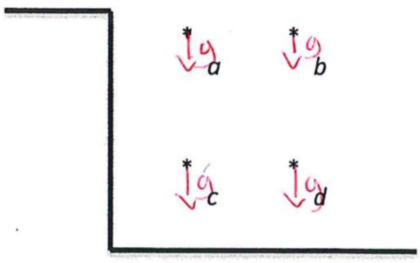


Figure 6.8. Diagram from work tutorial question focusing on positive, negative and zero work done by gravity.

They were required to identify the direction of the gravitational field, and determine if the direction of the displacement from *a* to *c* was parallel, anti-parallel or perpendicular to the gravitational field. This allowed the students to determine if the work done in moving from *a* to *c* was positive, negative or zero. From identifying the sign of the work done, the students had to

comment on the potential energy at the top and the kinetic energy at the bottom. The students then had to identify the work done in moving from *d* to *b* and *a* to *b*, and upon completion, were verbally asked to comment on the potential energy changes for these types of work done. Each group of students stated that negative work, with respect to the gravitational field, occurred when the body moved *d* to *b*, reasoning that the force of gravity was anti-parallel to the displacement travelled. They also reasoned that zero work occurred when a body moved from *a* to *b*, as the vectors were perpendicular. Having identified this, the groups verbally articulated that, with respect to the influence of the gravitational field on bodies between the points, positive work resulted a decrease in gravitational potential energy and an increase in kinetic energy, negative work resulted in an increase in gravitational potential energy and a decrease in kinetic energy and zero work resulted in no change in the gravitational potential energy. An example of a student's written response that displays most of these points is shown in Figure 6.9.

Student 4B:



The displacement is in the direction of the gravitational force. Positive [work moving from a to c] because the force is in the same direction of the displacement. The potential energy increases from c to a. The potential energy at a = to the kinetic energy at c.

Figure 6.9. Student 4B's response for section on work due to gravity in tutorial lesson.

This section discussed the student's progress throughout the work tutorial. It was observed that the students could identify instances of positive, negative and zero work. Having been guided through how to analyse and perform calculations to determine a value for work for instances in which force and displacement vectors diverge at an acute angle, all but one of the groups were able to apply this skill to a similar context. I suggest that the group's error was their interpretation of the information presented on the diagram, in which they decided to apply vector addition to the arrows, instead of multiplication of their magnitudes. The last section displayed that students could identify, in a field setting, positive and negative work, which was targeted so (a) they could apply it the concept of potential difference in an electric field context and (b) they could relate work the maximum potential and kinetic energies so they could apply conservation of energy calculations in electric fields, such as electrons moving in a cathode ray tube or x-ray tube.

6.2.3. Tutorial lesson: Potential difference

This section discusses the potential difference tutorial. The tutorial employs the use of field lines and displacement vectors to identify positive, negative and zero work. The students were then guided through developing the formal definition of potential difference, i.e., the negative of the work done by the field per unit charge when a charge is moved from point to another, through analysing a diagrammatic and mathematical scenario. The students were then given an electric field and were asked to complete calculations involving work, potential difference, and potential and kinetic energy.

Figure 6.10 presents the first electric field with various points marked. The students were required to identify the work done in moving through various combinations of the points, similar in nature to the exercise at the end of the work tutorial.

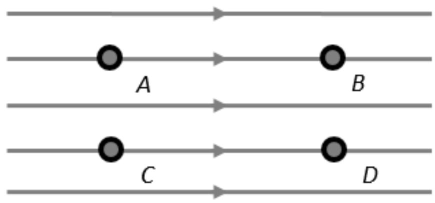


Figure 6.10. Diagram from potential difference tutorial focusing on positive, negative and zero work done on a charged body.

Having identified instances of positive, negative and zero work, the students were then required to consider paths taken in which involved a combination of positive and zero work, through analysing a dialogue between two hypothetical students.

Person 1: When we move the charge from A to D directly, there is less work done than moving it from A to C to D as we add up the work done moving from A to C directly to the work moving from C to D directly.

Person 2: When the charge if brought from A to C and C to D, the displacement has a vertical component which gives zero work. This makes the work done independent of the path taken.

By thinking about this dialogue the students would consider both the displacement between the initial and last point and how combinations of positive and zero work, or negative and zero work, can simultaneously contribute to a mobile test charge moving in a field. When the students were comfortable with applying the concepts of work to the electric field, the tutorial shifted to applying mathematics to moving charges in an electric field to develop their understanding of potential difference. They were presented with Figure 6.11, (i) and (ii), in which they initially considered the potential energy of a body lifted 3 m into the air, dropped to the ground.

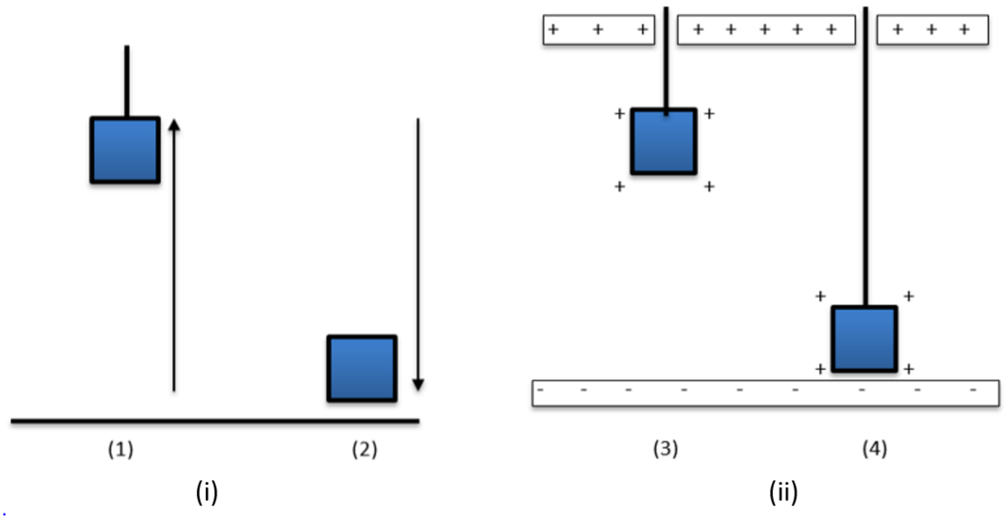


Figure 6.11. Diagram from potential difference tutorial comparing gravitational work with electrostatic work.

The students calculated the work done by gravity when 1 kg, 2 kg and 3 kg were dropped, and determined the ratio of work done per mass of the box. While there is no measured quantity for gravitational potential difference, this allowed the students to observe the fixed quantity nature of potential difference, in a context that is not abstract, in this case, the work done in moving 1 kg of a body. The students then compared the potential energy of the box at position (1) and the kinetic energy of the body at position (2) and expressed this in written form, and mathematically.

This set of activities was repeated using an electric field and a charged body, as shown in Figure 6.11 (ii). The students worked out the work done in moving charge bodies of +1 C, +2 C and +3 C a distance of 3 m. The students calculated the work done per unit charge in each case, observing that potential difference was a fixed quantity in the setup. The students had to summarize their finding by defining the ratio in their own words.

Student 4L: The work done is directly proportional to the charge → the ratios are constant.

Student 4B: Equal and proportional. The work divided by the charge is equal to 6/1, and is always constant.

Student 4I: 6 J of work is needed per +1 C of charge.

In the last section of the tutorial, the students were again presented with an electric field, and two points, A and B, as seen in Figure 6.12. The students were introduced to the formal definition of the potential difference, and the formula, $V = \frac{W}{q}$. The students were required to calculate the potential difference in moving a -1 C charge along path 1, and then path 2, and explain similarities between the two values. Most students said the potential difference was the same in each case, favouring explanations that reference the displacement between the points, instead of commenting on positive,

zero, and negative work reasoning, although one student, 4F, did identify the work moving from B to A would be positive but did not explicitly link it to either of the paths taken.

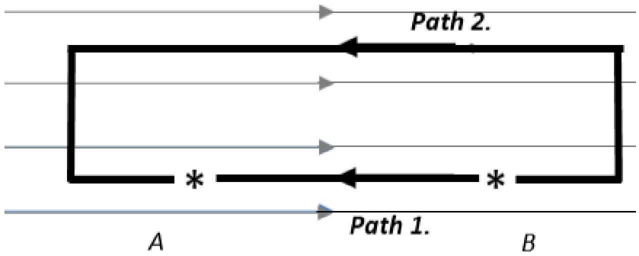


Figure 6.12. Diagram extracted from tutorial in which students apply work, potential difference and energy to different paths.

Using the potential difference value, the students were required to determine the work done in moving a -4 C charge and determine the magnitude of a charge if the field produces 36 J of work in moving the charge. They then applied their understanding of the conservation of energy to calculate the velocity of a body with mass 0.5 g and charge of magnitude -3 C as it moves from B to A. This allowed them to show that a body moving under the influence of a potential difference will convert potential energy, based on its position in a field, into kinetic energy. A sample of calculations is shown in Table 6.6. This demonstrates how the last section of the tutorial guided the students to relate the potential difference between two points in a field to apply the conservation of energy. By completing this, they demonstrate how to complete calculations involving potential and kinetic energy in electric fields. This is typical of the style of calculations the students complete when studying x-ray tubes, cathode ray tubes and particle accelerators, in which they need to utilise the potential difference of the device to calculate the energies and velocities of particles that move in the device.

Student 4H:	
$6 = \frac{w}{3}$ $w = 6\,x\,3 = 18\,J$	$w = \frac{1}{2}mv^2$ $18 = \frac{1}{2}(5\,x\,10^{-4})v^2$ $v^2 = 72{,}000$ $v = 268.33\,m/s$

Table 6.6. Example of calculations produced by student 4H.

6.2.4. Homework: Potential difference

The homework involved students using their understanding of attraction, repulsion and electric fields to build up an understanding of the behaviour of charged particles acting under the influence of a potential difference. As seen in the potential difference tutorial, in section 6.3.3, the students

initially explained the behaviour of a body acting under a gravitational field of high and low potential, and applied the reasoning to an electric field context, as seen in Figure 6.13 (i). They then extended their model, as seen in Figure 6.13 (ii) to incorporate the behaviour of negatively charged bodies in a potential difference, a phenomenon which has no equivalent behaviour in mechanics.

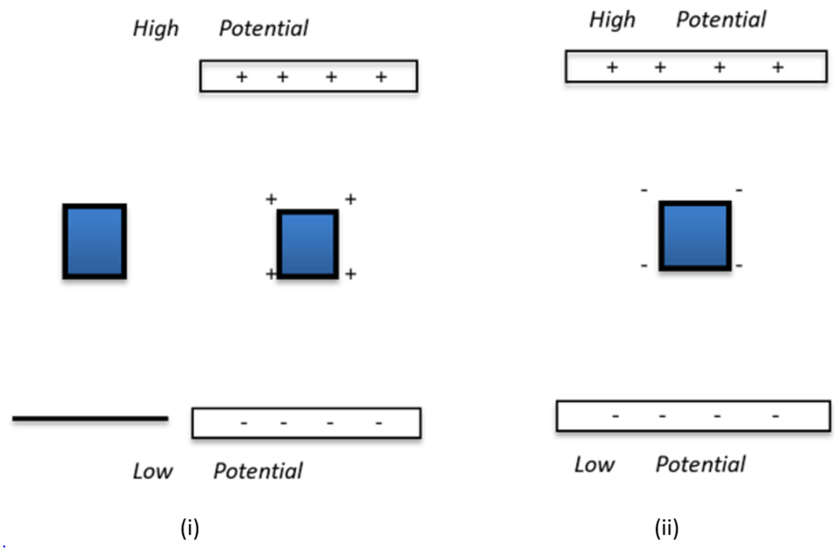


Figure 6.13. Diagram from homework for charged bodies moving under the infleunce of a potential difference.

It was seen that all students who completed the homework assignment explained that the positively charged box moves from high to low potential, generally noting how the box is attracted to the negative charged plate and reflects the behaviour of a body falling under gravity. They did not reference the interaction of charged box and the positive plate, and exclusively based their reasoning on the force of attraction they predicted. All the students acknowledged the negatively charged body would move up towards the positively charged plate and most could use this behaviour to determine that the negatively charged bodies would move from regions of low to high potential. However, student 4I and 4K contradictorily stated that it would move towards areas of low potential. When asked to explain this in feedback, they acknowledge that if the body is moving up towards the positive plate, it must move to high potential, and behave in the opposite fashion to the positively charged box. Student 4F explained that the body would build up potential energy, as it moved under the attraction to the positive plate. When asked to explain this, the student suggested that since the body was higher, it would have more potential energy. The student was asked to consider the phenomena as displaying the opposite behaviour to the positively charged box and asked to consider which of the two mimics gravitational fields and which acts in the opposite manner.

In the last section of the homework, the students were given a set of graphs with a charge layout, as seen in Figure 6.14. These graphs were to help students visualise and associated positively charged regions as areas of high potential and negatively charged regions as areas of low potential. The shapes of the graph were intended to show how relative potential decreases as the distance from a positive

charge increases, and the relative potential decreases as the distance as the distance from a negative charge decreases, as previously illustrated in Figure 6.1. Initially, the student had to explain the shapes of the graph, as the position from moves from left to right.

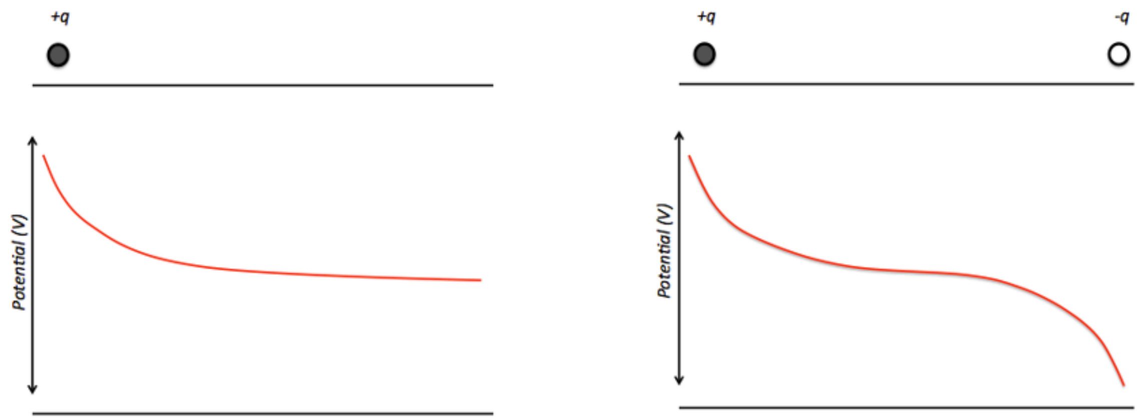


Figure 6.14. Graphs from potential difference homework.

The students had many difficulties with this section, and only two students (4B and 4G) explained that the first diagram showed variation from high to low potential while the second graphed showed variation from high to even lower potential, due to the negatively charged particle. Student 4K also indicated that moving from left to right showed moving from high to low potential but did not qualitatively compare how the presence of the negatively charged particle affected the potential.

A prevalent difficulty observed in the remaining seven student's responses was that they associated the negative particle as repelling the potential downwards or focusing on the shape of the graph and stating that the potential displays an inverse pattern that was stronger near the positive charge and weaker near the negative charge. The initial difficulty indicates a failure to understand potential as a property of an electric field produced by a charge. There is also an indication of attributing a tangible property to potential, which is commonly seen with field lines, but unexpectedly, this difficulty was translated onto a graphical representation. The latter difficulty shows an example of a student mathematically analysing the pattern shown, which accurate models the pattern seen in inverse relationships. The use of the terms weaker and stronger indicates a property of dominance between the two regions, typically seen in the superposition of vector quantities, such as force and motion. In both cases, students are attributing qualities and properties to the potential that the tutorial lesson and homework failed to address.

The students were then asked to graph the potential energy for the first set of charges in Figure 6.15, and then to sketch what types of charges could produce the graph seen the second half to the diagram.

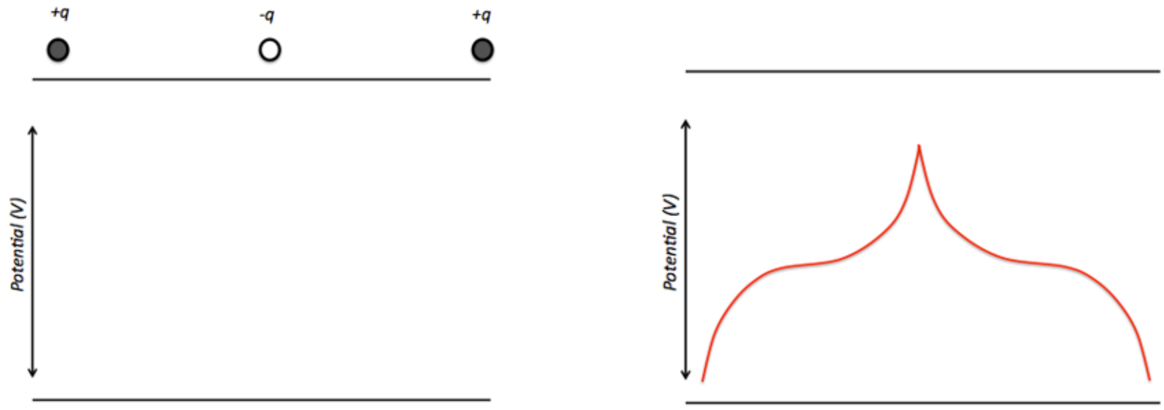


Figure 6.15. Graphs from potential difference homework, representing the variation of potential and charge layout.

All the students drew graphs of the correct shape for the first half of the diagram, with the two positive charges being regions of high potential, and the negative charge as a region of low potential. Additionally, all the students placed negative charges to the ends of the region and a positive charge in the middle, for the second diagram. However, the difficulties of attraction, repulsion and stronger and weaker potentials were also encountered in this section.

This section showed that using diagrammatic representations and drawing comparisons to between a body's behaviour in a gravitational field and a charged body's behaviour in an electric field can help student's construct their own understanding of how charged behave in a potential difference. It was also seen that the use of graphical representations can be employed, but difficulties about the nature of potential were seen and further instructional design would be required to address these difficulties.

6.2.5. Post-test: Work and potential difference

This section outlines the post-test results from the students, in which they apply their understanding of work in an electric field setup. The students also had to apply their understanding of the behaviour of charge under the influence of conducting spheres, connected by a conductor under the influence of an externally charged rod. The students were then required to use their understanding of vectors, forces, electric fields and potential difference to explain the behaviour of current in a closed circuit containing a battery and a wire. Two students were absent during the period this post-test was administered (N=12).

In the first post-test question the students were presented with an electric field, as shown in Figure 6.16, and asked to identify the direction of force acting on a negatively charged body placed at point *O*. A correct response for this question would be the student identifying that the force acts against

the field, due to the field pointing in the direction of a positive test charge. From this the students were asked to determine what type of work was done in moving a negatively charged particle to the points *A*, *B*, *C* and *D*. Correct reasoning would indicate the students were consider where the force and displacement vectors were parallel, anti-parallel or perpendicular to each other. The student results are presented in Table 6.7.

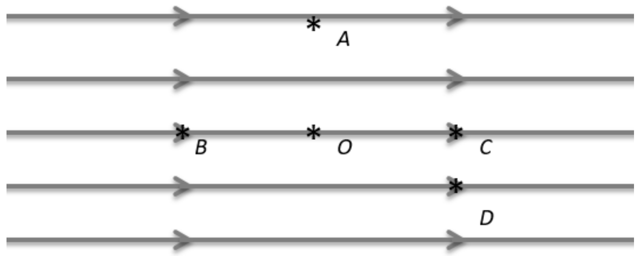


Figure 6.16. Diagram from post-test question involving work done in an electric field between various points.

Four of the students (4F, 4K, 4L, 4N) stated the stated that the negatively charged particle would follow the field line, not considering the convention that the field line points in the direction of force acting on a positively charged body. The remaining students correctly depicted the direction of the force acting on the negatively charged particle was anti-parallel to the field at the point *O*. Accounting for the error presented by the four students, it was observed that all the students consistently identified the work done was positive when the force and displacement vectors were parallel, negative when they were anti-parallel and zero when they were perpendicular. A summary of student's comparisons of the work done in the path (i) *O* to *C*, (ii) *O* to *D* and (iii) *O* to *C* to *D* is presented in Table 6.7.

The results show that only six of the students determined that the work done in moving the designated paths were equal. Four of these students (4D, 4G, 4M and 4N) produced explicit reasoning, in which they referred to the horizontal displacement producing non-zero work and vertical displacement producing zero work, or the horizontal displacement and force acting on the body being the same in all cases. Four students (4C, 4H, 4I and 4L) stated that only horizontal vectors need to be considered in this question, which alludes to their understanding of non-zero and zero work being done without being explicit about it.. Two of these students, 4C and 4H, produced the correct ranking with this reasoning. Students 4I and 4L stated that there is more work done in moving directly from *O* to *D* than there is in moving to *D* via *C*, using the same reasoning as students 4C and 4H. This suggests that these students applied the concept of non-zero and zero work when the vectors are explicitly parallel, anti-parallel and perpendicular, but struggle to apply this when the angle between the displacement and force vector is acute. This was seen in the case of the work done in moving from *O* to *D*, in which the displacement vector can be resolved into horizontal and vertical component, which contribute non-zero and zero work respectively, which was not considered by these students.

Responses.	Students.
$W_C = W_D = W_{CD}$	**4C, 4D, 4G, 4H, 4L, 4M.**
$W_D > W_C = W_{CD}$	4B, 4I, 4K, 4N
$W_D = W_{CD} > W_C$	4F
$W_D = W_{CD}$, W_C not defined.	4J
Perpendicular vectors produce zero work.	**4D, 4M, 4N**
Same force and same displacement in each path	**4G**
Only need to consider horizontal vectors.	4C, 4H, 4I, 4L
Displacement reasoning.	4B, 4F, 4I, 4K, 4N
Other	4J
N/a	4A, 4E

Table 6.7. Responses to post-test question involving ranking the work done in moving between different points in an electric field.

Two other students, 4K and 4N explicitly referenced that the displacement from *O* to *D* was the greatest but equated the other two paths. This suggests they also considered to apply the concept of non-zero and zero work to those paths but did not consider the displacement for *O* to *D* as a combination of horizontal and vertical vectors. Student 4F based their ranking on the absolute displacement for the start and end points of the paths. In this case, they ranked that *O* to *D* and *O* to *C* to *D* as having the same work done, but more than *O* to *C*, due to the smaller displacement in the last path. Student 4J also used the same reasoning for the first two paths but did not explain how the work done in the last path compared to the first two.

Another post-test question looked the student's ability to explain the behaviour of electrons moving under the influence of a potential difference, in a depiction of two metal spheres connected by a wire, with a galvanometer. The students were told that a rod of different charge is placed close to the spheres and that negative charge moved in the direction as shown on the galvanometer, as shown in Figure 6.17.

The students were asked to compare the potential on the two spheres in each of the three cases and explain their rationale. This question only references the initial potentials, just as the charges are being made to move, not the final potential when the charges have stopped moving and the potential is equal in all cases. A summary of the student's answers is presented in Table 6.8.

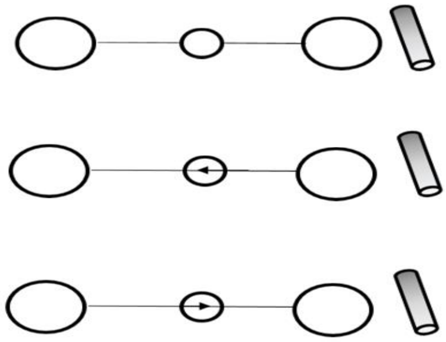


Figure 6.17. Diagram from post-test question eliciting understanding of the movement of charge in a potential difference.

Table 6.8 shows that the seven students stated the right dome had lower potential than the left when the charge moved to the left, and higher potential than the left when the charge moved to the right. This indicates that the students explained the direction of current moving is influenced by areas of low to high potential, in the initial moments of the scenario shown. One of the students (4G) used the movement of charge and considered how the charge built up on each sphere as time went on. For instance, in the middle diagram of Figure 6.17, their reasoning suggested that the right sphere was positively charged as it lost electrons. As it became positively charged, it developed a relative high potential. The opposite was true of the leftmost dome, in which it built up an excess negative charge and developed a relative low potential due to this build up.

Student 4C: The potential on the left is stronger than the potential on the right.

Student 4G: Low potential in the left [sphere] because it is negatively charged. High potential in the right [sphere] because it is positively charged.

Four students (4B, 4D, 4L and 4N) stated the opposite potentials for the spheres in the pictures, with reasoning that indicated they were considering the potential of the spheres are time moved on. For instance, in the last picture, in which the current moves to the right sphere, they indicated that as the charge builds up on the sphere to the right, it develops a negative charge and its potential lowers, whilst the sphere on the left builds up a positive charge and its potential rises. In the second case, they stated the opposite to be true. This suggests the students were considering the effect of potential at the spheres as the charges build up over time, although the question asked them to consider the potential at the start as the current just starts to move.

Student 4L: There is a higher potential at “A” [Left sphere] as the negatives have just travelled to “B.” [Right dome].

Responses	Students.
Both domes have equal potential	**4B, 4C, 4D, 4F, 4G, 4H, 4J, 4I, 4J, 4L, 4M, 4N.**
Right dome has low potential, left dome has high potential.	**4C, 4F, 4G 4H, 4J, 4I, 4J, 4M.**
Right dome has high potential, left dome has low potential.	4B, 4D, 4L, 4N.
Right dome has high potential, left dome has low potential.	**4C, 4F, 4G, 4H, 4I, 4J, 4M.**
Right dome has low potential, left dome has high potential.	4B, 4D, 4L 4N.
N/a	4A, 4E, 4K.

Table 6.8. Responses to post-test question involving the movement of negative charge under the influence of a potential difference.

All students who attempted this question could indicate that the potential of both sphere was equal when no current was observed on the ammeter, when a charged rod was placed beside the spheres for a long time. The reasoning was based on the observation that if no current was flowing, no potential difference exists between the two bodies. None of the students however mentioned how the potential difference observed in the first two setups reduced to zero as time moved on.

Student 4C: The potential is equal. Meaning that the electric charge wo not move.

Student 4L: They're equal, as there is no electricity moving.

The students were also presented with a graphing question, in which they were required to sketch the variance in potential for a series of positive and negative charges in various positions. The two graphs setups the students were presented with are shown in Figure 6.18, and the student results are summarised in Table 6.9.

The results clearly show that the students associated high and low potentials with positive and negatively charged objects, as guided in the homework exercises described in section 6.2.4. However, the general shape of the graphs drawn by the students were crude and did not fully represent the behaviour of potential for point charges, in which the inverse relationship was notable missing or badly represented. Only two students, 4G and 4H, produced at least one graph which were considered to reasonably show the relationships, with 4H producing two graphs which showed the inverse pattern that potential around a point charge displays. The remaining student made errors in

which the potential only changed between positive and negative charges but did not change before or after the charges had moved between the spheres. In these cases, when the potential dropped when close to a negative charge, the potential remained low and did not increase unless a positive charge was close. There were also instances in which this was shown for the positive charge. The third graph in Figure 6.19 illustrates this, where it is clear student 4I sketched a constant value for the potential until their graph moved into the region that represented the space between positive and negative charge, where the shape of their graph showed a decrease. However, as the potential decreases as the distance from a charged particle increases, the constant potential shown should not have been sketched.

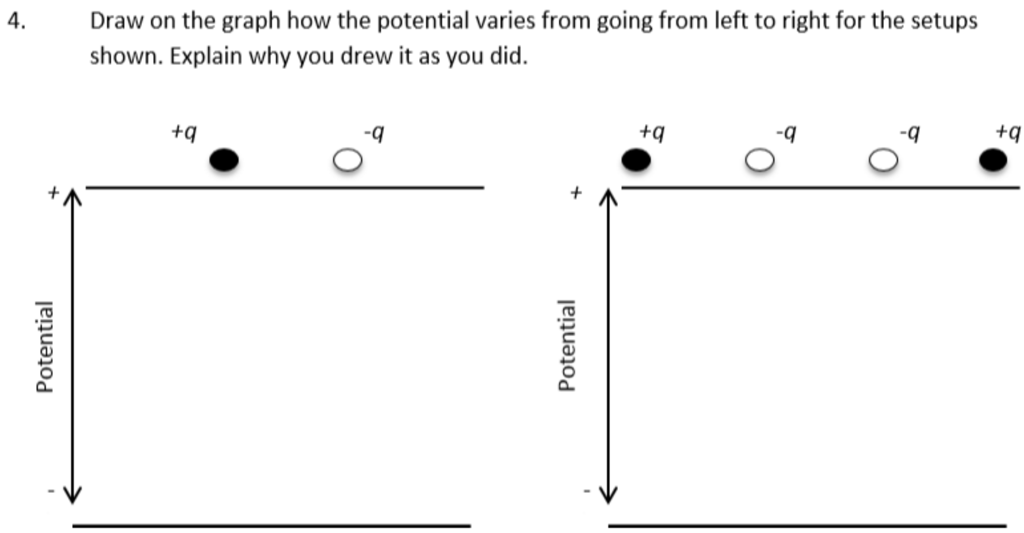


Figure 6.18. Post-test question, utilising graphical representations for potential.

Figure 6.19 shows the graphs produced by students 4G, 4H and 4I. Reasonable graphs can be seen by 4G and 4I, where they display not only the drop in the potential between the positive and negative charges, but also the increase and decrease in potential before and after the charges (i), and the increase in potential between the two negative charges (ii). In the final diagram, (iii), errors can be seen in which the potential was constantly high and drops between the positive and negative charge. In this case, an error is also seen in which the potential continues to drop as the graph moves to the left past the position of the negative charge. These errors were typical of those made by the remaining students. This indicates that students associate the positive and negative charges with high and low potential, but do not correctly consider the variation of potential around point charges.

Responses	Students
Positive is high potential	**4B, 4C, 4D, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N**
Negative is low potential	**4B, 4C, 4D, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N**
Positive is low potential	N/a
Negative is high potential	N/a
Reasonably correct shaped graph	**4G, 4H**
Notable errors in graph shape	4B, 4C, 4D, 4F, 4I, 4J, 4K, 4L, 4M, 4N
N/a	4A, 4E

Table 6.9. Responses to post-test question involving the association of high and low potential with charged bodies.

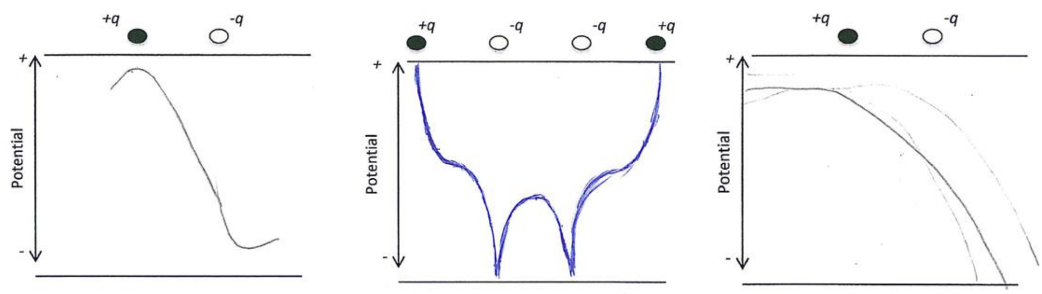


Figure 6.19. Examples of responses from students 4G, 4H and 4I.

In another post-test question the students were presented with the illustration of a battery connected by two wires from the positive to negative terminal shown in Figure 6.20.

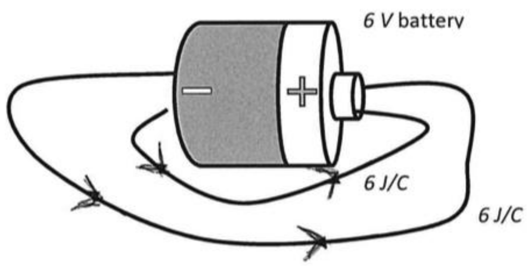


Figure 6.20. Diagram from post-test question requiring students to explain behaviour of current.

The students were required to explain the following two behaviours in the circuit, using the concepts they had learnt in the tutorial lessons:

1. Current (which is moving negative charge) flows from the negative to the positive terminal.

2. The *work done per unit charge* in moving charge from one terminal to the other is constant, regardless of the length / layout of the wire.

The students were given a list of key concepts, and asked to use their understanding of some, or all, of them to explain the two observations. The key concepts were work, potential, electric fields, electric field lines, vectors, the behaviour of negative charges in electric fields and the behaviour of negative charges between a potential difference. The forces of attraction and repulsion were explicitly omitted from this list, as to gauge how the students could apply the concepts mentioned in the list to explain the behaviour of current in a simple circuit. Although reasoning related to the attraction of the electrons in the current to the positive terminal, and repulsion between the electrons and the negative terminal are valid, the aim of the question was to elicit the other manners in which students could explain the behaviour of current. A summary of the student's concepts used in the first question is presented in Table 6.10. Students marked with an asterisk signify they referenced the concept but applied it incorrectly in this context.

Concepts referenced	Students
Attraction / Repulsion.	4B, 4C, 4D, 4G, 4H, 4J, 4M
Electric field.	4G, 4M, 4N*
Potential of plates.	4G, 4H, 4K*
N/a.	4A, 4E, 4F, 4I

Table 6.10. Responses to post-test question explaining the movement of current in a circuit.

The results show that seven of the students used reasoning based on the attraction of the mobile electrons to the positive terminal, and repulsion between the electrons and the negative terminal, of the battery. Six of these responses were exclusively based on these force interactions, without any further reference to the electric field or the relative potential of the plates suggested by the students. Student 4G expanded their explanation to include the behaviour of the negative charges in an electric field, in which they stated the direction of the field points from positive to negative and stating how the charge behaves in a field. They also stated how negative charges moves due to reasoning based on potential difference, stating that they flow from an area of low potential to high potential. Student 4H also submitted this reasoning based this potential difference, in addition to using attraction and repulsions reasoning. However, their explanation included a minor error, in which they referred to "potential" as "potential difference." The source of this error is unclear. Student 4M correctly used both attraction / repulsion reasoning, and reasoning based on the behaviour of the electrons in an electric field.

Student 4G: The negative current is repelled from the negative terminal of the battery and is attached to the positive terminal. Because it is negative, it is also attracted to the

high potential at the positive terminal of the battery. Negative charges always act in the opposite direction to the electric field.

Student 4H: Because the moving negative charge is attracted towards the positive terminal of the battery. Also, the negative charge goes towards the higher potential difference at the positive terminal of the battery.

Student 4M: Current flows from negative to positive because only negative charge moves and is attracted to the positive charge. The electric field lines go from positive to negative, but the charges go against the field lines. The is, the current flows from negative to positive.

Only two students, 4K and 4M, used reasoning that did not involve force. Both these students incorrectly used reasoning based on electric field lines and potential difference. Student 4M incorrectly stated that the field would point from the negative plate to the positive plate, which suggests they believed the electrons would move in a parallel direction to the field lines. Student 4K associated a high potential to the negative plate, and low potential to the positive plate, and indicated that charge would move from high to low potential. In both cases, it is observed the student's errors are rooted in incorrectly reversing the conventions of electric field and potentials, as they are applied to positive and negatively charged objects.

Student 4K: It travels from the high potential energy area to the low potential.

Student 4N: The field lines go from the negative to the positive.

The second part of this question required the students to explain why the work done in moving a unit of charge from one terminal to another in the circuit was constant. Again, the students were encouraged to use the key concepts listed in the question and apply them to the circuit to produce their explanation. A summary of their responses is presented in Table 6.11.

Concepts referenced	Students
References displacement between the plates	**4B, 4C, 4D, 4G, 4H, 4I, 4K, 4M**
References the force exerted on the current.	4G
Uses other reasoning.	4J, 4L, 4N
N/a.	4A, 4E, 4F

Table 6.11. Responses to post-test question explaining why the length of a wire does not affect the potential difference in the circuit.

As seen in the last part of this question, most of the students picked a concept with which they could explain the observation. The most prevalent response was that the terminals have a fixed displacement between them, so the work done in moving between them will be constant. Student 4G added to this by stating that the force acting on the charges would also be constant in both wires, presented in Figure 6.20. Although it is not clear what reasoning the student used to ascertain that

the force would be constant, as the net work done in both cases would be equal, this was considered a correct answer. Student 4J appeared to reference a model of current, referring to a constant flow in the wire, regardless of length. If referring to current by the flow, then the student infers that a constant current would associate constant work between the positive and negative terminal. Student 4L and 4N produced responses that reworded the observation of the direction of the current but provided no reasoning.

Student 4B: The work done is constant, because no matter how long the wire is, the displacement between the positive and negative terminal is constant.

Student 4G: The work done per charge is constant because the charge experiences the same force from the battery and moves a uniform displacement (straight line distance).

Student 4J: The work done is constant no matter the length of wire or layout cause it going to be the same flowing from 1 terminal to another terminal.

This section presented and displayed the results of post-test questions undertaken by the students that help elicit their thinking of work and potential difference. The results indicate that the students can identify instances of positive, negative and zero work using field line and vector representations. However, in instances in which the force and displacement vectors produce acute / obtuse angles in which combinations of zero and non-work are presented, students can encounter difficulties. It was seen that the students can generally compare, and justify, the potential difference between two points based on the movement of current, and display that when electrons have travelled through a system, the potential will equalise. When using their understanding to explain the behaviour of current in a circuit, it was observed that the students generally focused on explanations that involved using only one concept, instead of trying to apply multiple concepts to model their observations.

6.2.6. Discussions

This section compares pre-test and post-test data for student's understanding of work and potential difference. It reveals student's gains in both reasoning and being able to either apply various representations to the different concepts or discern and interpret information from the various representations. The concepts addressed in this section are the ability to identify positive, negative and zero work, identify work with regard to the direction of displacement using an electric field line diagrams (Doughy, 2013), associate higher and lower potential to positively and negatively charged particles respectively (Hazelton, 2013), and explain the behaviour of charged particles under the influence of a potential difference (Guisasola, *et al.*, 2002; Maloney, *et al.*, 2003; Hazelton, 2013).

The first of these concepts is the student's identification of positive, negative or zero work. The first question in both the pre-test and post-test probed the student's understanding of work. A comparison of the student's responses is shown in Figure 6.21.

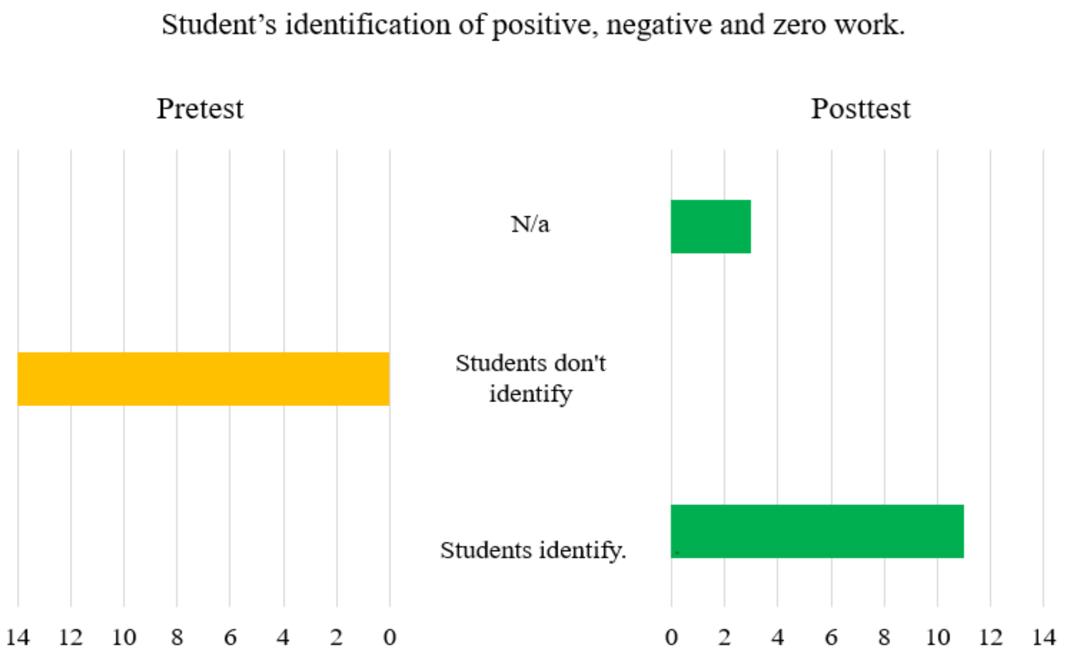


Figure 6.21. Comparison of student's ability to identify positive, negative and zero work.

In the pre-test, it was observed that none of the students identified instances of positive, negative and zero work. These students relied on using distance reasoning to determine the work done in moving from one point to another. In the post-test, all the students in attendance identified instances of positive, negative and zero work. Student reasoning focused on the displacement and force vectors being parallel, anti-parallel, zero or a combination of both. This progress reflects that observed by Doughty (2013). This shift in reasoning suggests that the tutorial was effective in enabling students to link their understanding of force, displacement and vectors and extend them to conceptually identify instances of positive, negative and zero work (Hewson, 1992; Konicek-Moran and Keeley 2015). The gain of 11 students developing their understanding indicates that moderate conceptual change occurred.   Section 6.2.2. discussed the introduction of the concepts of positive, negative and zero work, and how the students applied this concept during the tutorial. The section illustrates an instance in which students were incorrectly applying the concept, but upon realising their error, they readdressed their understanding and figured out the application of work to produce the correct ranking (Posner, *et al.*, 1982). Section 6.2.3 presented the initial section of the potential difference tutorial, in which they applied this concept of work to an electric field context. Whilst there was a notable gain in the student's understanding from pre-test to post-test, it was seen that combinations of two types of work (work that has components that are both zero and positive/negative) caused difficulty for several students.

When students had to consider the absolute value of the work done, student's beliefs about the direction of the displacement of a charge in a field influenced their reasoning. Figure 6.22 shows students who considered work to be based on the total distance travelled by a charged particle in an electric field, and students who considered the work to be based on the net displacement of the charged particle.

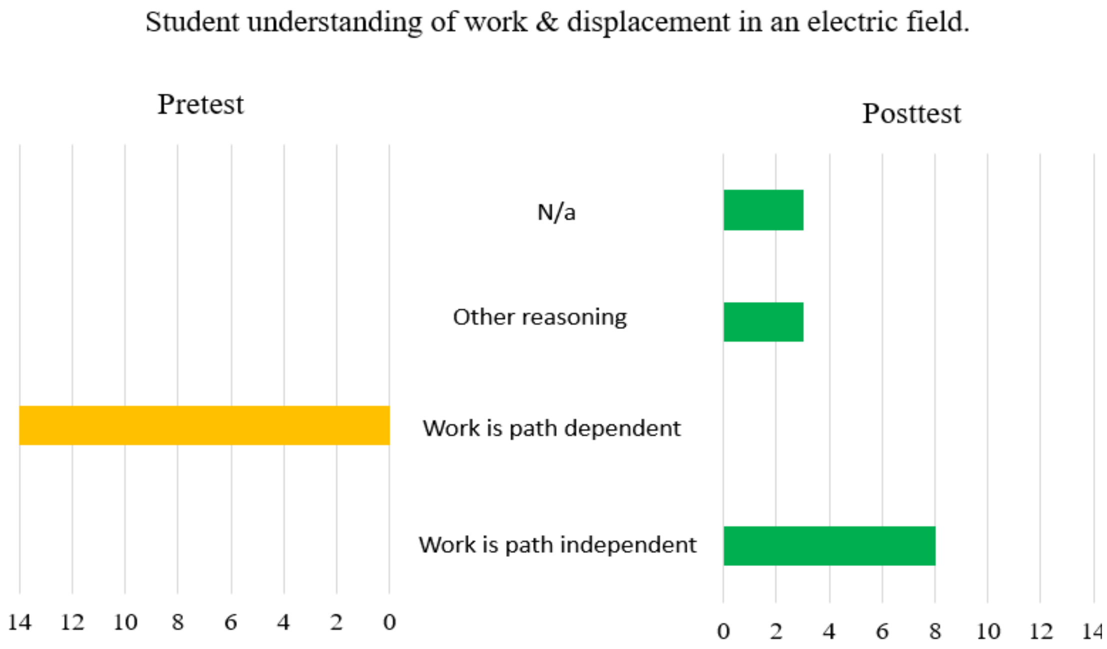


Figure 6.22. Comparison of student's understanding of the use of displacement in determining work done.

Even though the students had previously covered quantitative questions involving force, displacement and work during the mechanics section of their physics course, the pre-test results indicated they considered that the distance travelled affected the work, as opposed to the net displacement. To challenge this difficulty, the work tutorial addressed the quantities of distance and displacement using diagrammatic and verbal reasoning, as discussed in section 6.2.2. This allowed the students to apply both concepts to work, and become dissatisfied with the distance concept, and develop confidence to apply the displacement concept in an intelligible manner (Posner, *et al.*, 1982). Section 6.2.3 then illustrated how these concepts were applied to the potential difference context.

In the last post-test question, as discussed in section 6.2.5, it was seen that eight of the students shifted their thinking to consider the displacement between the start and end-point of a path in an electric field to determine the work done when current flows in an electric circuit. This indicates that conceptual exchange occurred and their conceptual understanding improves, as these students did not reference the errors observed in the pre-test and they applied the concept to an unseen context (Hewson, 1992; Konicek-Moran and Keeley 2015). The gain in eight students developing their understanding indicates that the extent to which conceptual change occurred was moderate. One difficulty that was persistent post-instruction was that some students reasoned that a constant current

in the circuit requires a constant voltage, regardless off the path taken. This reasoning would be erroneously transferred to electric circuits and does not account for variation of current caused by difference resistances in various branches of combinations of parallel and series components in circuits.

In the first question of the post-test, discussed in section 6.3.5, it was also observed that several the students did not consider the displacement vectors that were parallel and perpendicular to the field. Difficulty arose when students were required to consider displacements that were combination of parallel and perpendicular components. There was little difficulty in student identifying a path with two stages, the first being negative work and the second behind zero work. There was also no difficultly in students equating this work to a path in which only the first stage is taken. However, in a path which combined the positive and zero or negative and zero work, it was seen that students suggested the work would be greater than the two stages separately. This indicates that when considering parallel, antiparallel and perpendicular paths, the students analyse the problem in terms of positive, negative and zero work. However, when the paths taken make acute or obtuse angles to the field, the students shift their reasoning to think in terms of absolute displacement. The sources of difficulty with this thinking is the student do not consider parallel, anti-parallel and perpendicular displacement components separately, and relate them to the work concept. This difficulty is not directly reflected in the work done by Lindsey, *et al*., (2009), but in both cases, it was seen that a lackof understanding of whether displacement or distance travelled is the relevant concept when considering work can lead to student confusion.

The next section of the discussion associate relatively high and low potential to positively and negatively charged particles respectively. Figure 6.23 shows how the presents the students associations of potential to bodies of different charge, which was tested for using graphical representations.

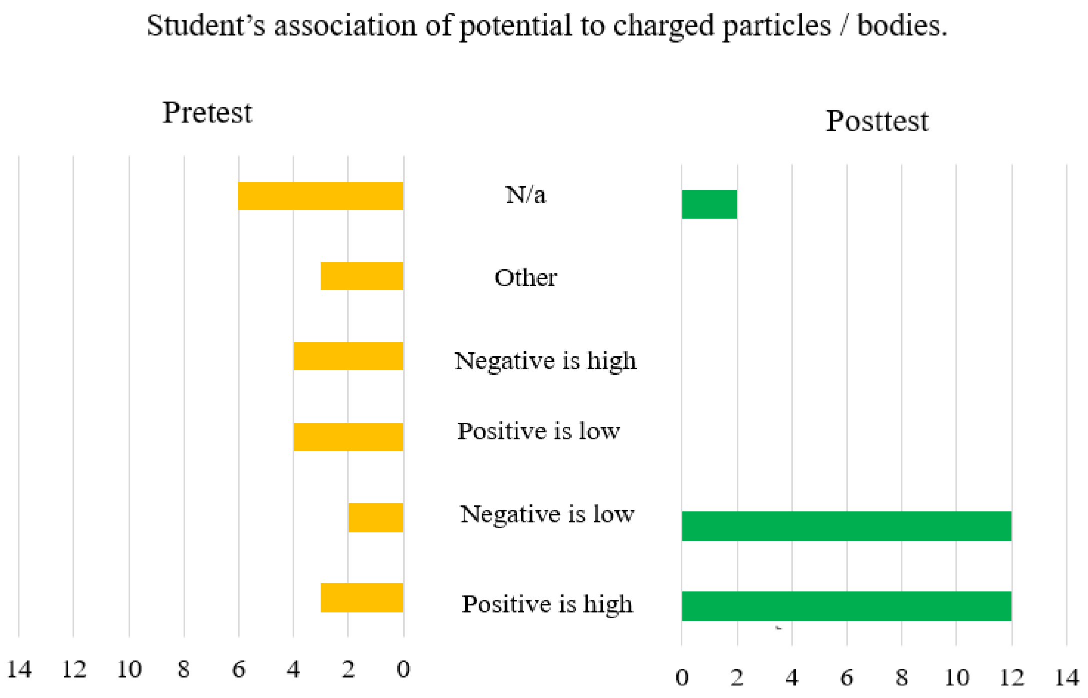


Figure 6.23. Comparison of student's association of potential with charged particles, using graphical representations.

The pre-test results clearly indicate that the students were unaware of the association of positive and negative charges to potential, with only two students successfully answering the question. In the post-test, there was a clear shift in student's associations to correctly apply the association to point charges. These results suggest that moderate conceptual exchange occurred, insofar there was a total gain of nine students correctly associating high potential with a positive charge relative to a low potential with a negative charge (Hewson, 1992). However, errors were observed in both the pre-test and post-test where students struggled to correctly represent the shape of the graphs correctly. However, the shape of the inverse patterns was not a target of this study and was not explored by the students during the tutorial.

One prevalent error in student's responses was that they appeared to represent the potential as constant until another charge increases or decreases it, or that the increase / decrease in potential continues past the point where the charges are placed, as shown in last student response of Figure 6.19. These students may have been considering a uniform electric field, which does not vary with distance, and applying this property to potential. As sections of the tutorials used contexts involving uniform electric fields, using field lines, more so than varying ones, this is not unlikely, but further work would need to be completed to help students separate these two types of thinking.

The last section of this discussion discusses the student understanding of the movement of a charge body under the influence of a potential difference. Figure 6.24 compares the student's pre-

test and homework results for this concept. In this case the homework assignment was used in lieu of a post-test question.

In the pre-test, it was observed that the students could predict the behaviour of charged bodies in a potential difference, and a drop in the student's correct responses was observed in the homework assignment. However, in the pre-test, the predominant strategy employed by the students was to assume the charges of the high and low potential plates. This is in line with difficulties presented by Guisasola, *et al.*, (2002). The homework exercise aimed to address this, by initially asking students to answer in terms in charges, and then answer in terms of potential. This appeared to enable some of the students to develop their understanding, but gravity-like thinking, in which all bodies move from high to low potential, was persistent in some of the student's responses, even when they directly contradicted their previous responses.

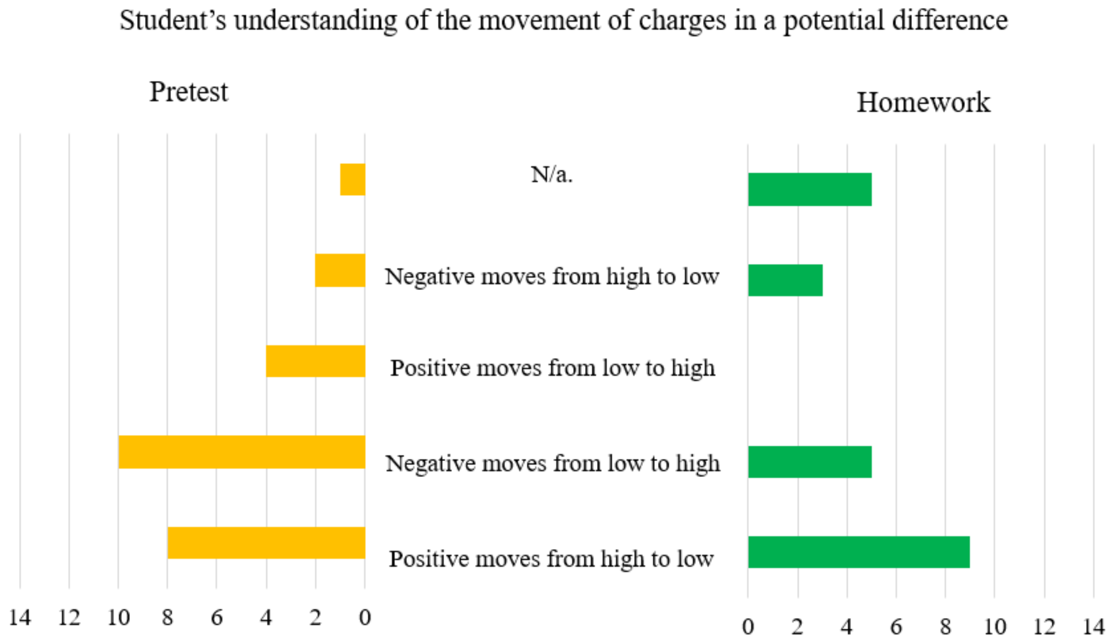


Figure 6.24. Comparison of student's understanding of the movement of charge under the influence a potential difference.

The reasoning used in the homework indicated that more students were thinking in terms of the potential difference of the scenario they were presented with, than the relative location of positive and negative charges in the presented scenario. This suggests that the tutorial helped promote conceptual exchange (Hewson, 1992), as there was a shift in the focus from considering charges to considering the potential difference of the setups presented in the pre-test homework assignments. Further time to develop this concept may be required, using a combination of static and current electricity contexts, to encourage the remaining students to focus on thinking and applying potential difference.

In the final pre-test question shown, the students were asked to explain the behaviour of charge in a field. The most common responses by the students involved the interaction of the charge with the charged plates, referring to the attraction and repulsion force experienced. Guisasola, *et al.*, (2002) stated that student's resort to using charge models to explain interactions when possible, even then other manners such as potential are appropriate or required. Attraction and repulsion was not listed on the key concepts to answer the question with, but the students favoured it. However, other responses in the student's homework and post-test questions suggest they could explain the behaviour under the influence of potential difference, when a question prompts them to directly. The student's familiarity with attraction and repulsion, and their interpretation for the apparent ease in using this reasoning may explain their reliance on using it. As the students demonstrated they could associate high and low potentials in line with the learning expectations of the tutorials, but their application of the reasoning was typically rooted in attraction and repulsion, the extent to which conceptual change occurred was determined to be partial.

This section presented the discussions on student's development of work and potential difference. Whilst the approach adopted in the tutorial helped the students develop understanding, there is still space for development. The use of field lines and vectors showed gains in the student's ability to reason situation of positive, negative and zero work, but difficulties were seen in correctly applying the displacement vector in situations where the force and displacement were neither parallel or perpendicular. The use of graphs and diagrams help students develop understanding of potential and the behaviour of charges in potential difference, but the use of graphs without the context of a mathematical formula produced errors for the student's understanding of potential and led to difficulties indicative of thinking about uniform electric field. The use of diagrams to enable students to develop understanding of the behaviour of charged bodies in potential difference opened the opportunity to use charge-based reasoning to explain the student predictions, and not focus on the potential difference concept.

6.3. Conclusions

This chapter seeks to address the following research question:

- To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

This question is addressed by addressed the following considerations:

- To what extent does the use of vectors and field lines, representing force and displacement, enable students to identify positive, negative and zero work?
- What affect does the use of graphs and diagrams have on students understanding of potential difference
- What difficulties are encountered by the students during this transfer to a potential difference context?

The approach developed used tutorial lessons that employed the use of vector and field lines to help students develop their understanding of work. Mathematically, work is a scalar concept that employs a dot product to produce a scalar from two vector quantities. This mathematics is not covered by the students and instead, the approach adopted looked to develop conceptual understanding. The work tutorial employed the use of vectors to enable the students to consider the relationship between force and displacement and produced an opportunity for them to deconstruct parallel and perpendicular component vectors, so they could be considered to produce non-zero and zero work respectively (Doughty, 2013). This skill was seen in the student's use of work in the potential difference tutorial, in which they could identify the sign of the work done in an electric field. However, component deconstruction was observed to be a difficulty in answering conceptual questions in the post-test and requires focus in future lessons for the students to correctly apply the skill. Additionally, how the work done in electric fields, as completed in the research, did not address the concept of how work increases or decreases the energy of a system.

The use of graphs and diagrams was seen to promote conceptual understanding of potential difference. The graphical method provided ease for students to develop an association of high and low potential to positive and negative charges. By students constructing their own sketches in line with the initial examples in the homework, they developed an intelligible method to apply the association of relative potential to positively and negative charged bodies in similar contexts, to allow them to apply and engage with the concept (Posner, *et al.,* 1982; Konicek-Moran and Keeley 2015). The use of diagrams provided a simple model for students to consider when thinking about the movement of charged bodies in a potential difference. Guiding student's reasoning to consider attractive and repulsive forces employed their prior knowledge and extended it by getting them to consider the potentials involved in the positively and negatively charged regions. This approach also reinforced their association of high and low potential to charged plates.

The representational approach highlighted difficulties in the student's models of potential difference. The use of graphs in the post-test questions elicited that students thought that potential drops remains constant along a path unless another charge increases or decreases it. This model is in error for point charges, although it is useful in explaining and representing potential along conducting wires in circuits with components (Reeves, 2003). The use of diagrammatic representation presents

the opportunity for students to be over-reliant on the use of charge interactions to explain processes that can be explained in terms of potential difference (Guisasola, *et al.*, 2002). In combining the association of potentials to charges, and the movement of charge under a potential difference, this model could be employed to help students understand the behaviour of a capacitor discharging, combining diagrammatic models involving potential and current, and graphing data of potential difference vs. time and current vs. time, showing many processes occurring using the different representations.

The extent to which the student's developed their understanding of work and potential difference is displayed in Figure 6.25. A legend of the codes used can be found in Appendix F.

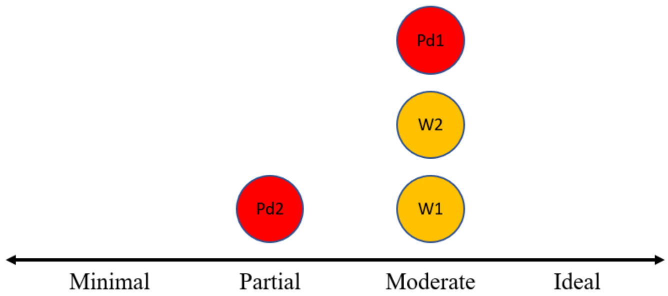


Figure 6.25. Line plot of extent of conceptual change for student's understanding of work and potential difference.

It was seen that the student gains in understanding from completing the tutorials were mostly moderate. The student's understanding of work and potential difference was rooted in enabling them to explain the behaviour of charged bodies under the influence of a potential difference. As the student's mathematical abilities are limited, calculus and vector mathematics was not employed in the tutorials and limited the depth of understanding developing of the students. Therefore, the developed student reasoning was phenomenological in nature, which impeded the mental models constructed by the students.

Overall, the use of multiple representations through structured inquiry tutorials encouraged conceptual change in the student's understanding of work and potential difference. There were persistent student difficulties observed that could be addressed in future research, and extensions to other topics to use multiple representations to explain the behaviour of different processes. Examples of such an extension study student's application of vectors and field lines to various processes in electromagnetism, applying tabular data, graphs, strobe diagrams and mathematical symbolic representations to developing students understanding in mechanics topics such as motion and the conservation of energy.

Chapter 7. Conclusions and implications.

This section summarises the main findings from chapters 4, 5 and 6, and discusses these findings in the light of the research questions presented in the last section of chapter 1. Section 7.1 presents the conclusions for the student's understanding of vector concepts, the inverse square law and field lines. Section 7.2 presents the conclusions related to the use of multiple representations to promote student's understanding of Coulomb's law and the electric field. Section 7.3 presents conclusion related to the use of field lines, vectors, graphs and diagrams in developing student understanding of work and potential difference, section 7.4 discusses implications for learning and section 7.5 presents further conclusions.

This thesis addressed 5 research questions in relation to student development of conceptual understanding in electrostatics. The 5 research questions were as follows:

RQ 1. To what extent does the use of a structured inquiry approach develop student understanding of vector concepts?

RQ 2. To what extent does the use of a structured inquiry approach develop student understanding of the inverse square law relationship, by employing multiple representations?

RQ 3. To what extent does the use of a structured inquiry approach develop student understanding of the field, when utilising the field line representation?

RQ 4. To what extent does the use of a multi-representational structured inquiry approach develop student understanding of Coulomb's law and electric fields?

RQ 5. To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

The approach adopted in these research studies was structured inquiry tutorials. A small number of individual questions were directly taken from Tutorials in Introductory Physics (McDermott and Shaffer, 2003), but otherwise, the suite of pre-tests, tutorials, home-works and post-tests developed in this research are of original design. As discussed in Chapter 2, the literature that references student difficulties primarily revolves around research in third level, typically with undergraduates studying introductory physics. Published research that utilised the approach of Tutorials in Introductory Physics (McDermott and Shaffer, 2003) also revolves around undergraduates, while there is little

research published that looks at adopting the approach to the second level context. One piece of research that uses the approach at second level adopted two tutorials in a current electricity context and reports the gains by students by comparing the pre-test and post-test results (Benegas and Flores, 2014). Their study presents quantitative data in terms of student gains between pre- and post-tests but gives little indication of how the student's developed their understanding or what difficulties they encountered during the tutorials. In contrast, the studies in this research used specifically designed tutorials, along with many methods of data collection. This approach allowed me to probe, develop and assess the student understanding, by analysing and interpreting the qualitative data collected. The tutorials in this research were of original design to ensure the targeted concepts were accessible to the students at an appropriate level for their ability. The use of this approach enabled the collection of evidence to identify instances where conceptual change did/did not occur in the student's understanding of vector concepts, the inverse square law, field lines, Coulomb's law, the electric field, work and potential difference. The findings of these studies allowed for the determination of the type of conceptual change occurred, identifying instances of conceptual extinction, exchange and extension (Hewson, 1982). The approach adopted also allowed the extent to which conceptual change was achieved, ranging from minimal, partial, moderate to ideal.

7.1. Vector concepts, the inverse square law and field lines

When attempting to discern student difficulties when developing their understanding of Coulomb's law, the electric field, work and potential difference, it can be difficult to determine if the difficulties are rooted in the topics themselves or in prerequisite concepts. To address this, the approach that was taken in this research allowed the students to first develop their understanding of vectors, the inverse square law and field lines in a mechanics context, in the first set of tutorial lessons as discussed in chapter 4. Research questions 1, 2 and 3 address the development of the student's understanding for these concepts.

The inquiry tutorial focused on developing student's understanding of vector magnitude, their application of vector constructions and their conceptual understanding of vector addition in terms of superposition of the vector components and how it can affect the resultant magnitude of two, or more, vectors. The results indicated that moderate conceptual exchange was observed over the course of the tutorial lessons. The tutorials focused on students developing a conceptual understanding of vector concepts, without utilising specific vector notation or operations. The student's mathematical ability, or lack thereof, impeded the depth of understanding to which the tutorial lessons could target. Had the students been more familiar with vector mathematics, the opportunity to develop a richer understanding of vectors in a physics context may be been possible. Considering this limitation to construct the boundaries of the vector concepts targeted in this research, the results indicate that the

tutorials approach was effective in promoting conceptual change in the student's understanding. Such an approach would be fruitful for other teachers to adopt into their practise, to develop students understanding of vectors at upper secondary level.

The tutorials which focused on the inverse square law employed a multi-representation approach, which used a diagrammatic model involving scaling, tabular data and graphical analysis. The diagrammatic model which utilised area scaling enabled students to explain the behaviour of phenomena that follow an inverse square law. The use of multiple representations during the tutorials enabled the students to explore the inverse square law relationship both qualitatively and quantitatively. The extent to which conceptual change was observed in the tutorial lessons was in the minimal and partial range. This indicates that the students encountered difficulties in transferring between the representations and relating their qualitative reasoning to their quantitative findings. Two possible explanations and implications from this study are presented here. The first implication is that the student's unfamiliarity with inverse quadratic functions in their mathematical education could have hampered their learning. In lower secondary level mathematics, the student's encounter linear, quadratic and exponential functions and briefly explore inverse proportionality. Joint planning between mathematics and physics teachers in a second level setting could provide a better opportunity for students to develop a more consistent understanding of inverse relationship and mathematical operations related to them. This cooperative planning could also be extended to include understanding of inverse square relationships. The second implication relates to the use of the number of representations and the dual reasoning between quantitative and qualitative required during the tutorials. As there is a lot of information transferred between representations, it is plausible that the student's working memory was saturated with information and therefore they were unable to develop a coherent understanding of the inverse square relationship (Reid, 2009). Marzec (2003) states that multiple exposures to contexts that involve the inverse square law may be required before learners can develop an intelligible understanding of the relationship. The implementation of tutorials that allow students to encounter this relationship in various contexts would support upper secondary students develop their understanding. Over time, this could allow the students to cluster the information from the multiple representations and would allow them to free up their working memory capacity. This would enable students to incrementally form a coherent understanding of the inverse square law.

The findings from the studies involving field lines showed moderate and ideal gains in the student's conceptual understanding and their ability to use field line representation. The tutorials facilitated students to associate the field line density with relative field strength. Students were able to determine the direction of the field at various points, and to show that the path taken by a body in a field is influenced by, but not identical to, the pattern of the field lines (Törnkvist, *et al.*, 1993; Galili, 1993 and Cao and Brizuela, 2016). The tutorials focused on students transferring from vector to field line representations and explore the behaviour of bodies acting under the influence of the

field. As the students have limited knowledge of vector mathematics, transferring between vectors to field lines involved students primarily applying rules, such as the force is tangential to the field lines, without developing an appreciation for why this is based on mathematical equations using vector notation. In line with this constraint, moderate and ideal change was observed in the student's understanding of the field line representation. The students developed clear reasoning to explain the behaviour of bodies under the influence of a field, linking the representation to relevant vector concepts such as velocity, momentum, acceleration and force (Konicek-Moran and Keeley, 2015). Overall, the approach adopted in these tutorials was effective in promoting the students understanding of the basic behaviour that bodies display under the influence of fields, in a manner that is appropriate for teaching and learning at upper secondary level.

7.2. Coulomb's law and electric fields

Research question 4 addressed the development of the student's understanding of Coulomb's law and the electric field, by employing different representational tools for the topics. This section presents the conclusions of how the students revisited vectors, the inverse square law and field lines as they applied them to Coulomb's law and the electric field. The use of structured inquiry tutorials not only enabled me to gather evidence of the students as they developed their understanding of these electrostatic topics, but also to gather evidence of successful transfer of vectors, the inverse square law and field lines to the electrostatics context.

The prior learning of vector concepts developed in completing the tutorials presented in chapter 4 was not initially demonstrated in the electrostatic context, based on the pre-test results presented in chapter 5. This suggests that the students struggled to apply their understanding in a new context and/or the status they associated with the conflicting concepts was not in line with their initial understanding of these concepts. This is not surprising, as the students working memory must process concepts relating to vectors and simultaneously interpret information about electrostatics. If their initial understanding of vector concepts, discussed in chapter 4, were not clustered together, it is plausible the students working memory was full and therefore they were unable to apply the vector reasoning to new contexts (Reid, 2009). Revisiting these concepts in the tutorial lessons gave the students an opportunity to develop clusters in their working memory. Over the course of the Coulomb's law and electric field tutorials, the students revisited the concepts from the initial vector tutorial and transferred them to this context. Moderate instances of conceptual change were observed in the student's application of vector concepts to Coulomb's law and the electric field.

When students were applying the inverse square law to Coulomb's law and the electric field, partial conceptual change was observed in the students understanding. Using both qualitative and

quantitative reasoning in the application of the inverse square law in the electrostatics context, it was observed that there was a lack of coherence between the student's understanding and their ability to use the inverse square law mathematically. This suggests the students were operating without complete mental models but were effective with using the inverses square law to approach and solve quantitative mathematical problems. As discussed in section 7.1, multiple exposures to the inverse square law in context in tutorial classes could allow the students to cluster information regarding the inverse square law, freeing up working memory capacity to develop a deeper understanding of the relationship. The reasoning displayed by the students during teaching and learning interviews, discussed in section 5.5.3, indicated that when the students were prompted to focus on the change in dimensions in the area model, they displayed deeper conceptually coherent reasoning than was previously recorded in the initial tutorial, as discussed in chapter 4. This indicates that exposure to multiple explorations of the inverse square law across various contexts can incrementally develop student's understanding.

Difficulties were recorded with the student's transfer of their understanding of field line concepts to the electrostatics context. Gains recorded in the field lines tutorials and post-tests in the initial section of the research, as discussed in chapter 4, were not observed in the electrostatic field lines pre-test with similar frequency. Having completed two tutorials revolving around these representational tools, moderate conceptual change was observed to have occurred, which allowed them to effectively apply the concepts to electrostatic contexts. The Coulomb's law and electric field post-test results indicated that the students were proficient in interpreting field lines and predicting the behaviour of charged bodies in electric fields. As mentioned in section 7.1, the students lack understanding of using vector mathematics and this hindered the depth to which the students could explain the conventions they applied to using field lines. The direction of force being tangential to the field was referenced considering the definition of electric field strength, but the student's application of this was primarily phenomenological. Taking this into account, the tutorial lessons were effective in enabling the students to use field line representations to describe simple systems of charge particles, and could be adopted into teacher practise, not only for electrostatics, but also electromagnetism.

The difficulties targeted for conceptual change were chosen from reviewing literature, as discussed in section 2.3.1. The findings of the research indicate that the tutorial lessons were effective in addressing these difficulties, although persistent difficulties did remain in some cases. While not generalizable, the findings of this section of the research could be used by teachers who wish to develop the structured tutorial approach in their own practise. The findings presented could be used to inform the development in teaching and learning materials for other teachers who would wish to address these difficulties with their own students.

7.3. Work and potential difference

Structured inquiry tutorials were used in the teaching and learning of work and potential difference. In these lessons, the structured tutorial approach employed field lines, vectors, symbolic and graphical representations. This allowed for multiple collections of evidence to gauge the student's conceptual understanding across the different representations. The results can also be used to illustrate instances of conceptual extension, evidenced by the successful transfer of vectors concepts and field lines to work and potential difference.

The pre-test and tutorials on Work and potential difference highlighted that initially the students had conceptual difficulties with the implications of the dot product and did not consider the relative parallel / perpendicular components of the displacement and force vectors. The post-test results discussed in section 6.2 indicate that conceptual change occurred in the student's understanding of the relationship between force, displacement and work, but some difficulties persisted post-instruction. Conceptual extension occurred in which the students applied vectors and field lines to a mechanical work context, and exchange occurred as the tutorial was effective at developing student's understanding of positive, negative and zero work. However, scenarios which involve combinations of zero and non-zero work by parallel and perpendicular components of forces proved difficult for the students. Further development of student's understanding of displacement in terms of components, and its application to work could help alleviate this difficulty in students understanding.

The use of multiple representations in the context of potential difference allowed students to develop the concepts targeted in the research. The use of diagrammatic models allowed for a relatively easy manner for the students to determine the behaviour of charged bodies acting under the influence of a potential difference. However, some students continued to rely on reasoning based on the force of attraction and repulsion between a charged body and oppositely charged stationary plates. This suggests that conceptual extension did occur, but not in a manner that was envisioned during the design of the potential difference tutorial, as the majority of the student's relied on reasoning based on force interactions, as opposed to reasoning potential difference which the students only referenced if explicitly asked.

The employment of graphical representations was successful in helping students associate the high and low potential to point charges. In this manner, the graphical representation enables the students to engage in conceptual exchange, through their interpretation of the graphs. The sketches of the student's graphs provided evidence to indicate the student's association of the relative high and low potential of areas surrounding positively and negatively charged particles. However, some difficulties persisted, as a number of the students could not accurately represent of the variation of potential with distance for simple systems of two, or more, point charges and suggests further revision in this area would be warranted to complete the student's conceptual exchange. Their

understanding could be further developed by employing tabular data for various arrangement of charges and guiding the students to explain the variation of potential, before constructing graphs of their own design. This approach has proved fruitful in the inverse square law and Coulomb's law tutorials.

Prior to, and during, the carrying out of these studies, the participating students had not completed calculus mathematics - which is a necessary tool required to develop a complete understanding of potential and potential difference in an electrostatic context. Electrostatic potential is an abstract topic, with very little accessible examples that upper secondary physics students can relate to. Mathematics is an important tool in helping students develop an understanding of such abstract physics topics. Therefore, this study was limited to students developing a phenomenological understanding of work and potential difference in the electrostatic context. Partial and moderate conceptual change was observed in the students understanding of work and potential difference, but it is acknowledged that their mental models are likely incomplete due to the lack of possession of the required mathematical tools to fully explore these concepts.

As the difficulties targeted for work and potential difference were recorded in research literature, the findings of this research could be used to aid in the design of teaching and learning sequences in other classrooms. This could allow other students to address the common difficulties when learning about work and potential difference.

7.4. Implications for classroom teaching and education policy.

The use of the inquiry-based learning approach in this work enabled the students the opportunity to overcome difficulties related to electrostatics. As discussed in section 2.1.2, Mestre (1991) and Roth (1990) suggest this opportunity would likely not have been provided in traditional lesson sequences, and it is likely the difficulties in student understanding would have been unknown to both the students and teacher as the students progressed through their physics course. The approach adopted allows for the specific identification of student difficulties in upper secondary level electrostatics topics, and difficulties in student's ability to transfer conceptual understanding between representations. When these difficulties were addressed, there were gains in student understanding evidenced, that varied from conceptual extinction to conceptual exchange and conceptual extension.

Having covered the various representational tools in lesson before electrostatics, it allowed for discussions involving these concepts and tools, to help students develop deeper understanding than it typically afforded in traditional approaches. The use of multiple representations in inquiry learning, as presented in this research, allowed multiples dimensions to collect evidence of student understanding of electrostatics. Comparisons of the student's responses in the different

representations can gauge if they were consistent in their reasoning when addressing the same concept in different ways, or if the representation influenced the reasoning used by the students. This also has implications for assessing student's understanding in other topics in physics, and some aspects of mathematics.

The use of inquiry-based learning that employ multiple representations is not limited to developing student understanding of electrostatics concepts. The student's understanding of vectors, field lines and the inverse square law and utilisation of graphs, tabular data, diagrams and mathematics can be applied to other domains of secondary school physics, such as mechanics, optics, radioactivity, sound and electromagnetism. By employing inquiry-based learning in these contexts, a teacher may gather evidence of student learning, such as pre-test and post-test data, as well as insights from the tutorials themselves, to make a judgement as to the efficacy of the teaching and learning that occurred. By determining the overall impact that the approach had on student learning, they could adjust their lesson planning for future iterations to tackle difficulties they notice with their own students. The tutorials not only allow for the teacher to address difficulties in student learning, but also to explore how students to engage in tasks, in which they develop reasoning to address difficulties and engage in conceptual change.

In addition to the use of this approach in teaching upper secondary physics, this research highlighted other issues with student's development of their understanding of Coulomb's law, electric fields, work and potential difference. There were several instances in which students lacked the appropriate mathematical knowledge and understanding to develop a complete understanding of the electrostatic concepts targeted and this impeded their ability to form complete mental models, as discussed in the conclusions of chapters 5 and 6. The student's ability to perform calculations based on the inverse square law, vector and calculus mathematics and understand the mathematical concepts underpinning these is necessary to develop a coherent model for understanding electric fields, work and potential difference. Numerous actions could be considered to address this. Educational Policy on curriculum and assessment could align the learning outcomes of mathematics with the outcomes of science, technology and engineering subjects to ensure that the students also explore these concepts in their mathematics courses, enabling them to develop and apply their understanding. This coherence in policy would enable consistency in teaching and learning in classrooms, and promote students developing links across different subjects. Another action could be to require physics teachers to dedicate lessons to focus on the development of mathematical understanding and continuously build on student's understanding over the implementation of the physics course. A final proposed action would be for education policy makers to review the level of coherence between the mathematics and the physics syllabi. For example, in Ireland there is a shift towards learning outcomes-based curricula and this provides an opportunity to align the two emergent specifications. In the cases where this cannot occur, there is an argument to make that certain topics in physics may not be appropriate for secondary level and it would be more appropriate

to be taught in third level. At this level, the students would have acquired the necessary mathematics to develop conceptually accurate mental models of these concepts. While these actions have their advantages and drawbacks, they are worth considering by policy makers and educational stakeholders.

Another implication arising from this research addresses the role of assessment in physics education. Assessment can be a strong influencer on what pedagogical approach teachers use in their classrooms. As shown in Figure 1.1, the assessment of upper secondary physics in Ireland relies heavily on content that can be memorised and uses qualitative problems that can be solved using algorithmic procedures (SEC, 2015). The inclusion of a broader range of questions, such as qualitative conceptual questions, two-tier ranking questions or diagnostic questions in physics could enhance the assessment of student understanding and influence teacher's classroom practice. This approach could provide evidence of the student's development of mental models and problem-solving strategies.

7.5. Implications for research.

Possible extensions to this work would be to conduct the research with a bigger sample size. As these research studies were completed with a small group of students, the findings of this research would not be generalizable to the wider population of students learning physics at upper secondary level. However, a series of research studies in which gathers data from a larger group would be informed by both the student gains and student difficulties presented in this thesis and could be used to reliably determine how frequent each of the gains and difficulties occur with different groups of students. If multiple teachers were to adopt the approach, the research could also gauge teacher attitudes to the use of structured inquiry tutorials and multiple representations in their own practise, and gauge how teacher implementation of the approach affect's student's understanding. Another extension would be to gauge the efficacy of adopting structured inquiry tutorials and multiple representations in other domains of Physics at second level. The topics covered in this research would allow for extensions into electromagnetism and mechanics topics.

Research regarding the use of tutorial lessons primarily focuses on the development of conceptual understanding in third level, but it is less studied at secondary level. One example of such research is Benegas and Flores (2014), who implemented tutorial lessons with upper second level students in Argentina, in which they presented quantitative analyses of pre-test and post-test results, with little insight into the student's conceptual development. The research presented in this thesis presents qualitative findings with upper second level students, developed using the tutorials approach, which is relatively unreported in the literature. This research represents a novel approach

in employing the use of the structured tutorials at secondary level and highlights the opportunity for further research in this area. A coordinated research study that uses both qualitative and quantitative findings would validate the efficacy of this approach and illustrate how this approach can enable students to develop their understanding and form coherent mental models of various topics of physics. Tutorials in Introductory Physics (McDermott and Shaffer, 2003) address multiple topics in Physics and they could also be used as a guide to draft and develop tutorials that adopt the tutorial approach in the second level context, as they were for this research. The findings of this research could also be used to guide future research at lower secondary level science and upper secondary chemistry and biology. If the approach were to be used at lower secondary level, it may be advisable to limit the representations to diagrammatic, graphs and tables, to help students understand how a process occurs, but remove any complex symbolic representations, as their mathematical ability to interpret them effectively would likely be underdeveloped. At upper secondary chemistry and biology, a focus for the structure tutorials on visualising complex processes could help students link observations between the atomic-scale, micro-scale and macro-scale.

A conceptual change model was employed as the underpinning theoretical framework employed in this research. This is not the only framework that could have been utilised. A framework revolving around developing and assessing the student's mental models could have been employed. The various types of evidence collection used in this research and use of multiple external representations could be used as indicators as to the mental models the students possess. However, unlike the closely related topics of magnetism (Borges and Gilbert, 1998) and current electricity (Borges and Gilbert, 1999), there is not an abundance of literature of naïve models of electric fields in which to compare to compare the student's mental models to. A future extension to this research could be to probe students understanding and establish descriptors of naïve mental models of electric fields, in a similar manner to that used by Borges and Gilbert (1998, 1999). Upon completion of the tutorial lessons, the analysis of the collected evidence could be used to determine what initial mental models the students were operating with, and how these mental models changed over the course of the lessons. The student's overall conceptual understanding would still be developed and assessed, without any critical changes to the tutorial lessons required to take place. Employing a modelling framework to the approach taken in this work could extend the research presented in this thesis and align it with trends in modelling research currently taking place in the physics education research sphere.

Chapter 8. References

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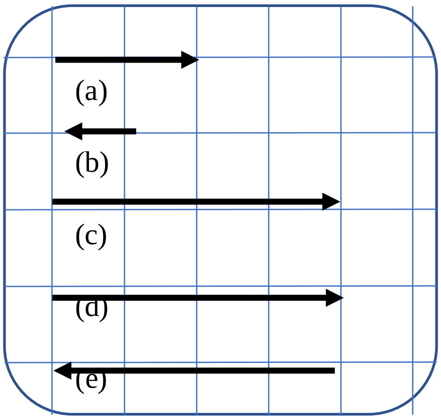
Appendix A

Vectors tutorial materials.

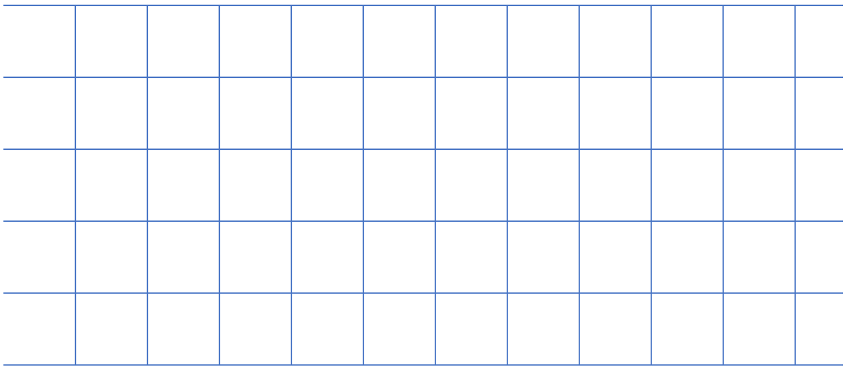
1. Rank the following vector arrows (a – e) from weakest magnitude to strongest magnitude.

Ranking:

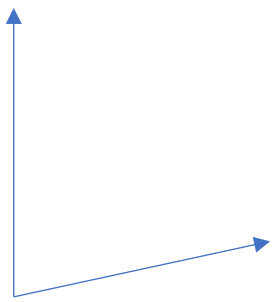
Explanation:



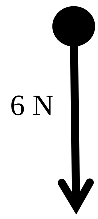
2. In the space below, Construct the resultant vector of (a) and (c) from question 1.



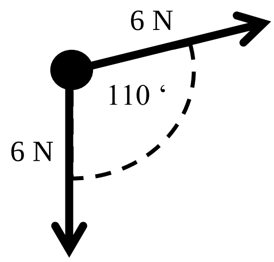
3. Construct the resultant vector of the following two vectors.



4. A charge experiences a force as shown in diagram (i). An equal charge experiences 2 forces as shown in diagram (ii).



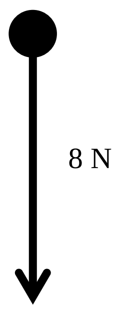
(i)



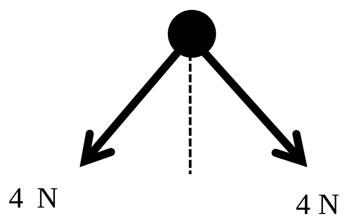
(ii)

Which charge experiences the most force? Explain your choice. You may draw on the diagrams if necessary.

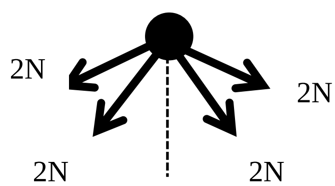
5. The following set of diagrams shows charges experiencing multiple forces. Which, charge, if any, experiences the most force? Explain. If they are the same, state so explicitly and explain why. (Angles shown are in (iv) are 45’ and in (v) are 45’ and 75’)



(iii)



(iv)

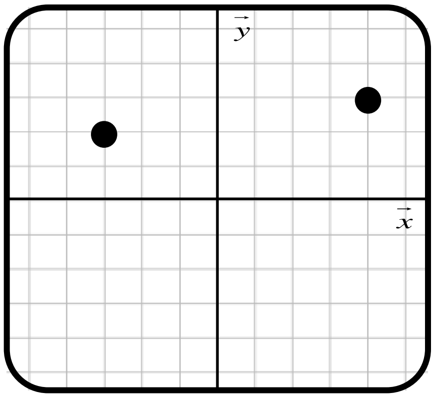


(v)

Writing vectors in terms of components using vector notation.

(i) Starting from the origin, explain to get to point "a" as shown.

(ii) Write how to get to "a" in terms of steps in the x direction and y directions. (e.g. 2 steps in the x and 1 step in the y is written as "$2\vec{x} + \vec{y}$")



(iii) Circle which of the following correctly maps how to get from the origin to the point labelled "b."

(a) $3\vec{x} + 2\vec{y}$ (b) $2\vec{x} - 3\vec{y}$ (c) -$3\vec{x} + 2\vec{y}$ (d) $2\vec{x} + 3\vec{y}$

Explain why you picked the answer you did. (It may help to explain why the other answers are incorrect)

(iv) On the diagram above, draw arrows from the origin to the points "a" and "b." We will call these vector arrows $\vec{a}$ and $\vec{b}$ respectively.

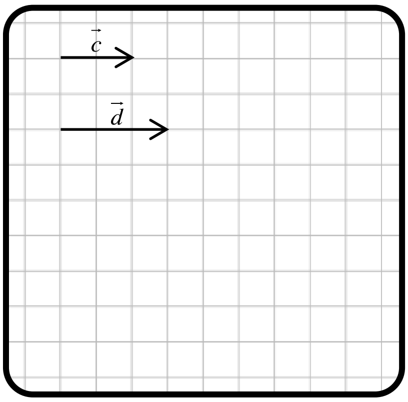
(v) On the diagram above, draw $\vec{a}$ as a combination of arrows along the $\vec{x}$ and $\vec{y}$ axis. Do the same for $\vec{b}$.

(vi) Using either trigonometry or co-ordinate geometry, explain how you could find the magnitude (length) of the vectors $\vec{a}$ and $\vec{b}$.

(vii) Find the length of $\vec{a}$ and $\vec{b}$.

II. Adding vectors.

Two vectors, $\vec{c}$ and $\vec{d}$, are shown to the right.

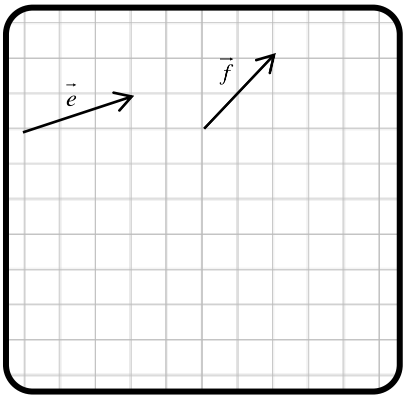


(i) Write both vectors in terms of their horizontal and vertical components. (eg, $\vec{a} = 4\vec{x} + 3\vec{y}$)

(ii) By connecting the tail of one vector to the tail of the other vector, show the resultant vector of $\vec{c} + \vec{d}$, on the diagram.

(iii) Add the horizontal components of $\vec{c}$ and $\vec{d}$ together. Add the vertical components of $\vec{c}$ and $\vec{d}$ together. Do the horizontal and vertical components define the resultant vector you drew in part (ii). Explain.

Two vectors, $\vec{e}$ and $\vec{f}$, are shown to the right.



(iv) Write both vectors in terms of their horizontal and vertical components. (eg, $\vec{a} = 4\vec{x} + 3\vec{y}$)

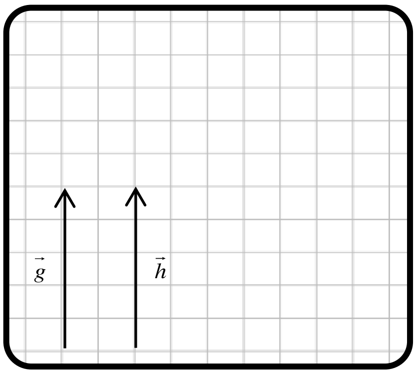
(v) By connecting the tail of one vector to the tail of the other vector, show the resultant vector of $\vec{e} +$ →

(vi) Add the horizontal components of $\vec{c}$ and $\vec{d}$ together. Add the vertical components of $\vec{c}$ and $\vec{d}$ together. Does the horizontal and vertical components define the resultant vector you drew in part (ii). Explain.

(vii) Explain how adding vectors head to tail is the same as adding vectors by adding their components.

III. Determining how horizontal and vertical vectors affect the resultant vectors.

Two vectors, $\vec{g}$ and $\vec{h}$, are shown to the right.

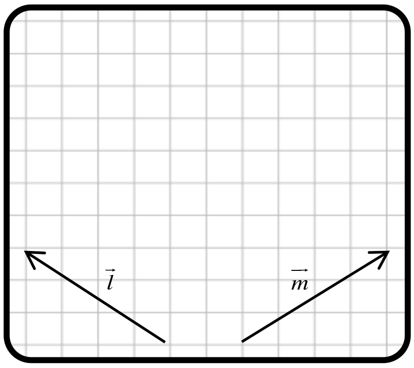


(i) What is the magnitude of $\vec{g}$ and $\vec{h}$?

(ii) Draw in the resultant vector of the $\vec{g} + \vec{h}$. What is the magnitude of the resultant vector you drew?

→

Two vectors, $\vec{l}$ and $\vec{m}$, are shown to the right.



(iv) Show the magnitude of $\vec{l}$ and $\vec{m}$ is 5 units for each vector.

(v) Draw in the resultant vector of the $\vec{l} + \vec{m}$. What is the magnitude of the resultant vector you

(vi) Write both vectors in terms of their horizontal and vertical components.

(vii) What is the result when the horizontal components are added together?

(viii) What is the result when the vertical components are added together?

(ix) From your results in (vii) and (viii), explain why the magnitude for the resultant you got in (v) is less than 10 units. (i.e. directly adding the magnitude of both vectors; 5 + 5 = 10)

I. **Drawing vectors.**

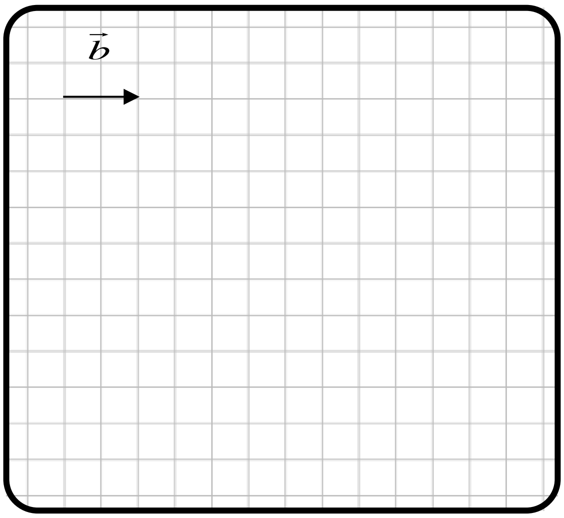
The arrow in the diagram to the right shows a vector, $\vec{b}$. In the extra space, draw the following vectors.

(i) $2\vec{b}$

(ii) $4\vec{b}$

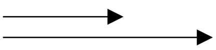
(iii) $-\vec{b}$

(iv) $-3\vec{b}$

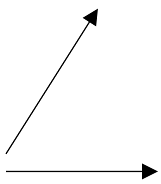


I. **Adding vectors.**

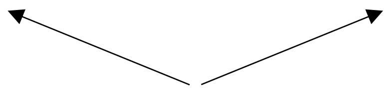
Use the tip to tail, or the parallelogram rule to add the following pairs vectors. Clearly show any construction lines you draw.



(i)

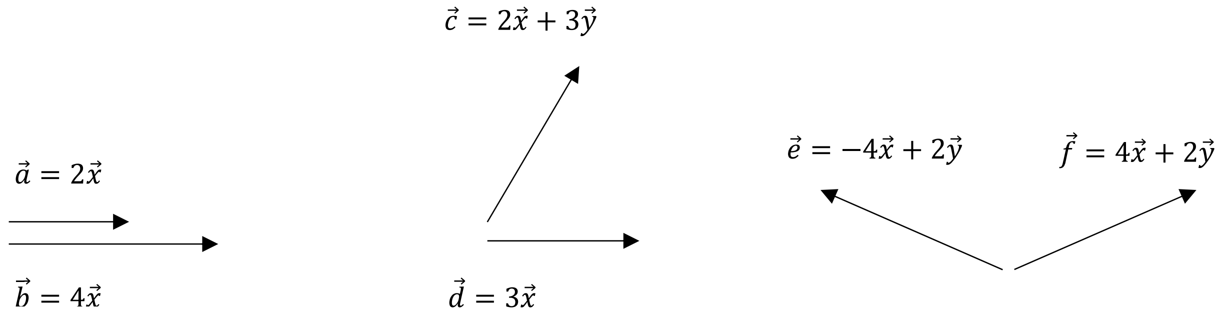


(ii)



(iii)

II. Adding vectors using components.



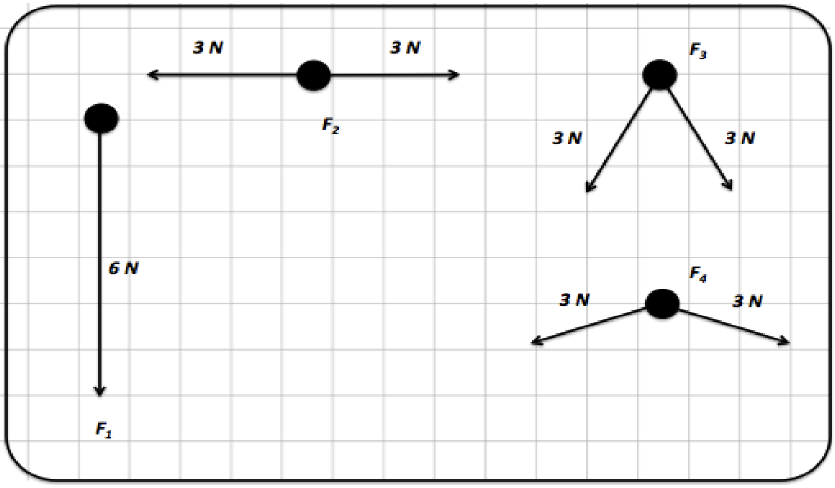
(i) Using the vector notation given, add the following vector pairs:

$\vec{a} + \vec{b}$, $\quad \vec{c} + \vec{d}$, $\quad \vec{e} + \vec{f}$

(ii) Use Pythagoras' theorem to find the magnitude of $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$.

(iii) Explain, referring to the addition of horizontal and vertical components, explain why the magnitude of $\vec{c} + \vec{d}$ is greater than $\vec{c}$ and $\vec{d}$, individually, but the magnitude of $\vec{e} + \vec{f}$ is less than $\vec{e}$ and $\vec{f}$ individually.

III. Looking at how horizontal and vertical components affect the resultant.



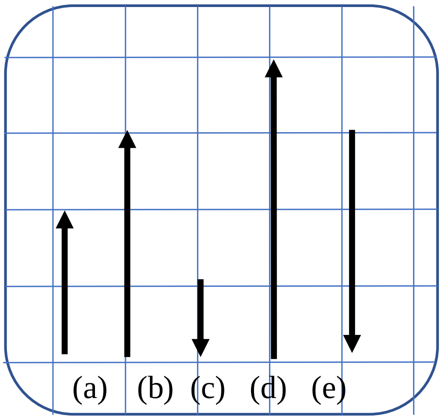
A small ball has a force or numerous forces acting on it as shown in the diagram to the right. The net force (resultant force) is labelled as F_1, F_2, F_3 and F_4.

(i) Rank the net forces acting on the ball, from highest to lowest. Explain your ranking. (Refer to either the tip to tail / parallelogram rule or refer to horizontal or vertical components, or both to give a full answer)

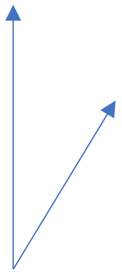
1. Rank the following vector arrows (a – e) from weakest magnitude to strongest magnitude.

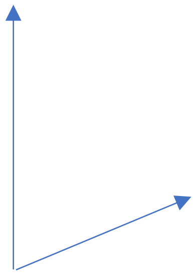
Ranking:

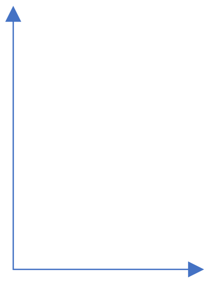
Explanation:



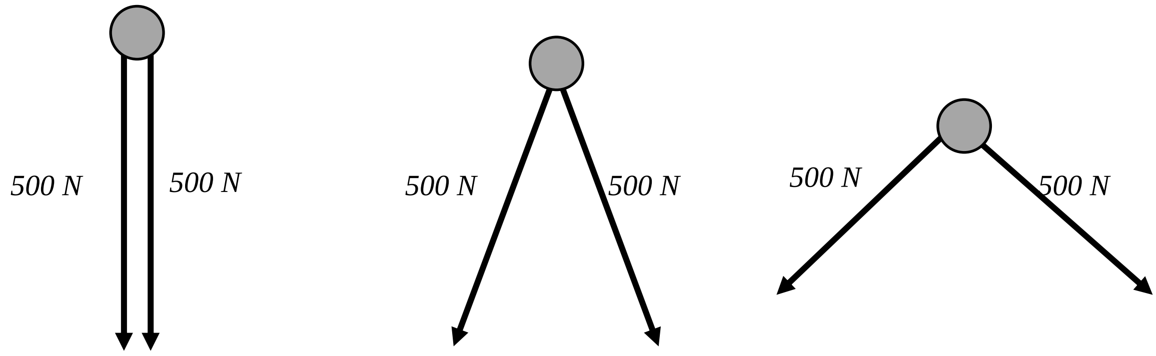
2. Show how to construct resultant of the following vectors.







3. A truck is broken down and needs to be pulled. A number of different cars can be used to pull the truck to a safe spot. All cars have the same pulling strength as shown buy the force vectors in the 3 diagrams below. The circle represents the centre of mass of the truck.



(i) Which diagram represents the vectors that will result in the strongest magnitude of force pulling the truck?

(ii) Explanation for part (i).

(iii) Which diagram represents the vectors that will result in the weakest magnitude of force pulling the truck?

(iv) Explanation for part (iii).

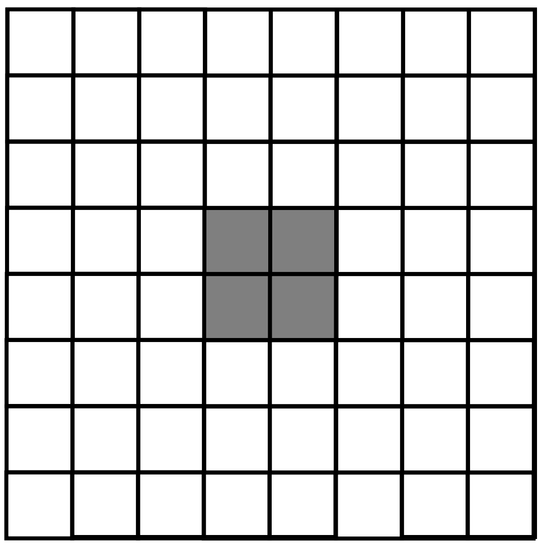
Appendix B

Inverse square law tutorial materials.

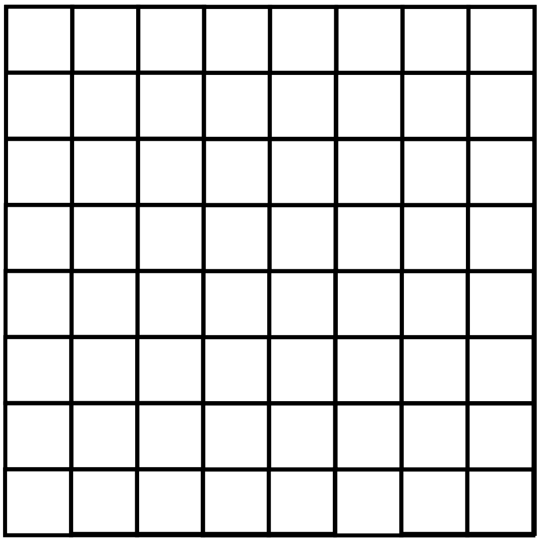
I. On the graph, sketch a graph of the pattern seen when you graph the function $y = k\frac{1}{x^2}$

Explain why you drew the pattern as you did.

II. A bulb shines on a wall from 1 m. The wall has an 8 x 8 grid on it



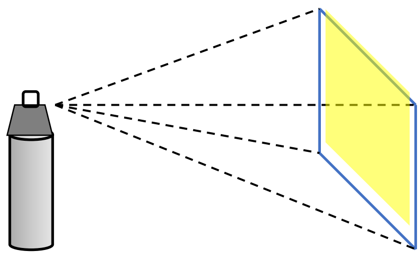
(i) If the bulb were moved to 3 m, shade in the shape would look like on the *8 x 8* grid below. (next page)



(ii) Explain why you drew it at you did.

III. A bulb is held 2 m from a light sensor. The sensor record the light intensity to be 100 Wm^{-2}. If the bulb is moved so that it is 4 m from the light sensor, what reading will the light sensor read?

I. Spray paint "Intensity."



A can of spray emits 100 drops of paint in 1 second.

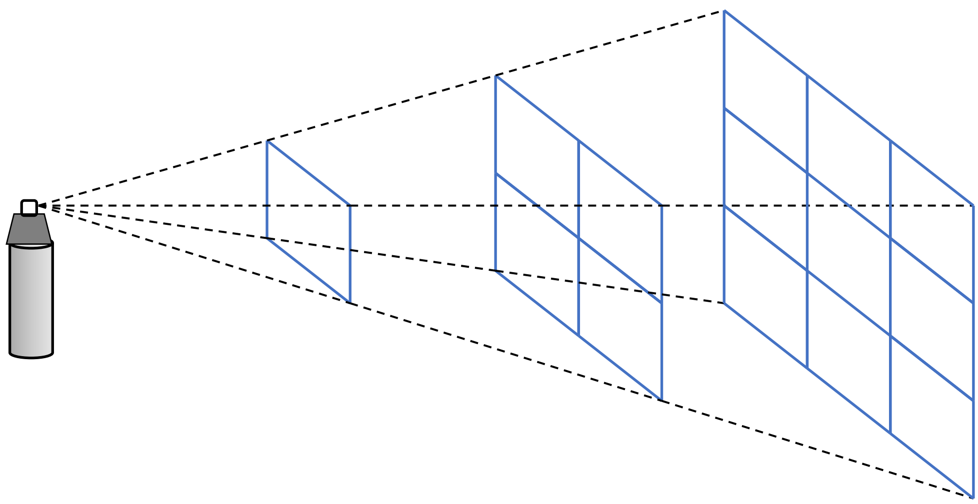
A metal frame, in the shape of a square, is placed in path of the paint spray so that all of the droplets of paint passes through it is area.

The square frame has a width of 10 cm and each of the

(i) Calculate the area, in m^2, of the metal frame

(iii) Determine how many paint droplets pass through the $1m^2$ frame in 1 second. Choose an appropriate unit for your answer.

II. How distance affects spray paint "intensity."



A second metal frame is placed away from the can, so that all it is outer corners are 10 cm from the can of paint. This metal frame is made up of smaller frames that are identical to the frame discussed at the top of the page.

(i) Explain why this second frame has an area that is four times bigger than the original frame used on the last page.

(ii) What is the value of the area of the overall frame that is 10 cm from the can.

(iii) Determine the how many droplets of paint pass through $1m^2$ frame in 1 second, for this overall frame.

A third metal frame is placed away from the source, so that all it is outer corners are 15 cm from the can of paint. This metal frame is made up of smaller frames that are identical to the frame discussed at the top of the previous page.

(iv) Explain why this third frame has an area that is nine times bigger than the original frame used on the last page.

(v) What is the value of the area of the overall frame that is 15 cm from the can.

(vi) Determine the how many droplets of paint pass through $1m^2$ frame in 1 second, for this overall frame.

(vii) As the distance from the paint can increases, the number of droplets of paint passing through a 1 m^2 area in 1 second decreases. Using your answers from the previous questions, explain why this occurs.

The relationship between distance from source and spray paint "intensity" is an example of an inverse square relationship. Inverse square relationships have the general equation:

$$y = k\frac{1}{x^2}$$

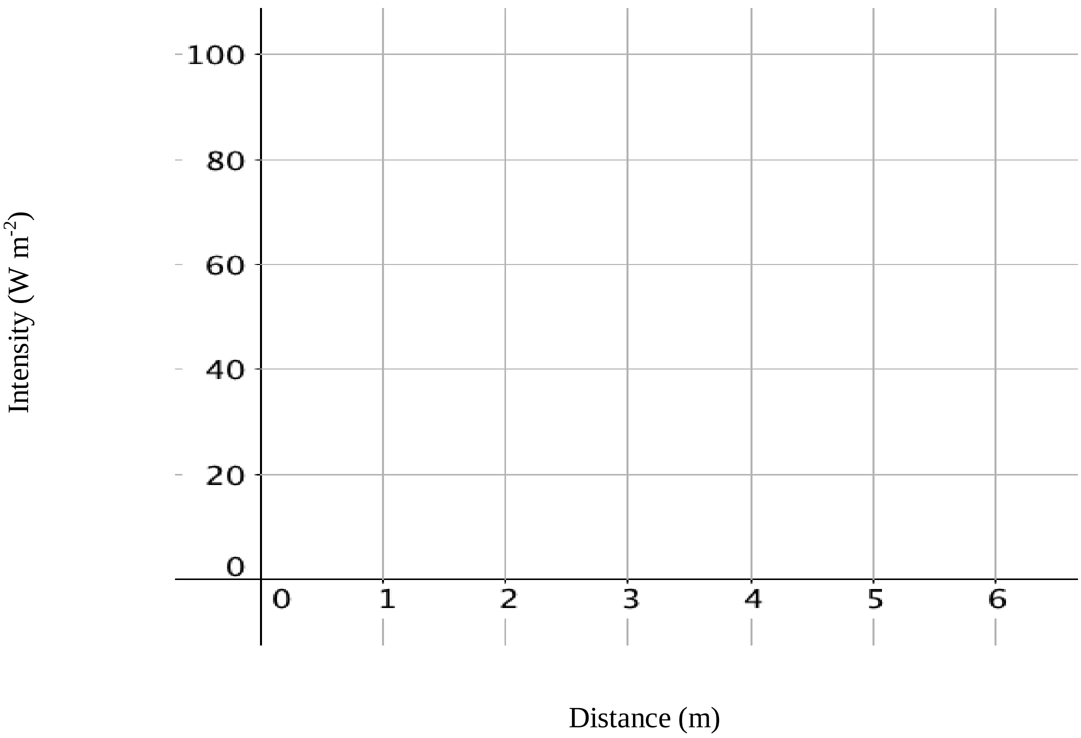
Note that as the *x* values increase, the *y* values decrease. This is due to the position of the x as a denominator on the right hand side of the equation. We also can note that the power on the *x* is x^2, which separates the inverse square relationship from an inverse relationship. We will now explore this mathematically.

III. Exploring the inverse square relationship.

The following table shows spray paint intensity given by a the can that emits 100 droplets of paint per second.

Plot this data on the graph below.

Distance (d)	Intensity (I)
1	100.0
2	25.0
3	11.1
4	6.3
5	4.0
6	2.8



(i) Describe the pattern shown using the following criteria: shape, increasing / decreasing, change in the slope. Include others if you think of them.

(ii) As the distance from the bulb increases, does the intensity increase, decrease or stay the same? Explain with reference to the shape of the graph.

(iii) Use your graph to determine the intensity of the light at a distance of 1 m from the bulb, and 2 m from the bulb

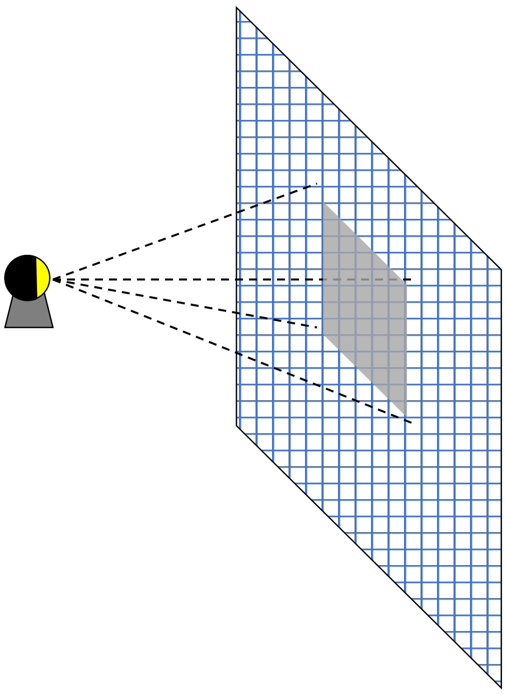
(iv) By what factor is the intensity at 2 m smaller than the smaller/bigger than the intensity at 1 m?

(v) Use your graph to determine the intensity of the light at a distance of 1 m from the bulb, and 5 m from the bulb

(vi) By what factor is the intensity at 5 m smaller than the smaller/bigger than the intensity at 1 m?

(vii) Using your answers, and the reasoning you developed on the first two pages, explain why this is an example of an inverse square law.

I. Light Intensity.

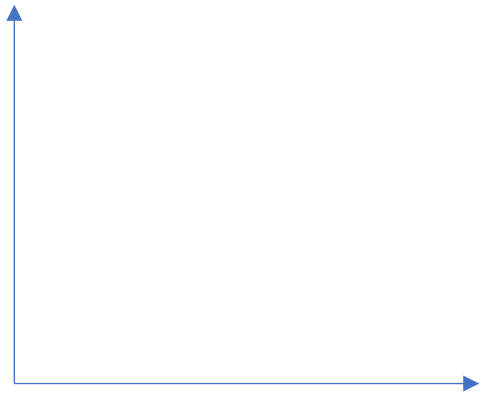


(i) Calculate the light intensity at a point 2 m from the bulb. (Formulae: $I = \frac{P}{A}$, $A = \pi r^2$)

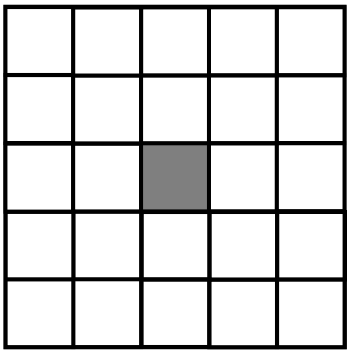
(ii) If you increased the distance from the wall to to the bulb, would the light intensity increase, decrease or stay the same? Explain your reasoning. (review what you covered in the worksheet for spray paint "intensity" if you need to)

(iii) Sketch a graph to show the relationship you explained in part (ii) and explain how it accurately shows the relationship from the formula you used in part (i)

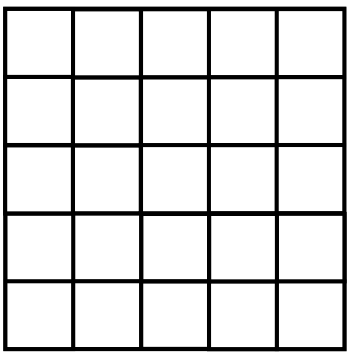
How it shows the relationship:



The photograph of the wall shows the following shape of the light on the wall. This is the shape of the light when the *6 x 6* grid is looked at head on.



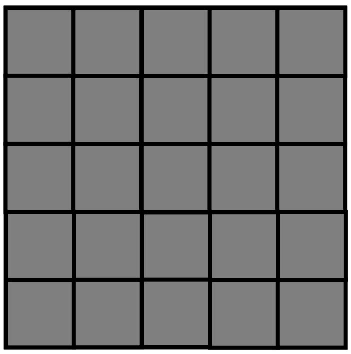
(iv) If the bulb were moved to 4 m, shade in the shape would look like on the *6 x 6* grid below.



(v) Explain why you drew it at you did.

(vi) Has the light intensity on the grid increased, decreased or remained the same? Explain your reasoning (consider the effect of moving the bulb back, in conjunction with your sketch in (iv)).

(vii) The bulb is moved to a distance where the shape of the light is shown on the grid on the next page. Determine how far the bulb is from the wall. Ensure you show how you figured it out.



III. Calculations. (Formulae: $I = \frac{P}{A}$, $A = \pi r^2$)

(i) A 200 W bulb is placed 5 m from a light sensor. Calculate the light intensity that the light sensor.

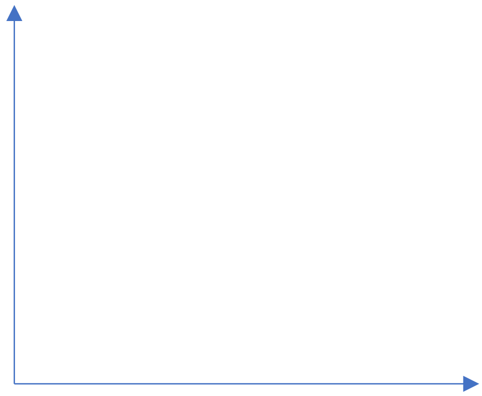
(ii) The 200W bulb is moved to 10m from the light sensor. Calculate the light intensity that the light sensor.

(iii) Use your answers from (i) and (ii) to show that light intensity follows an inverse square law.

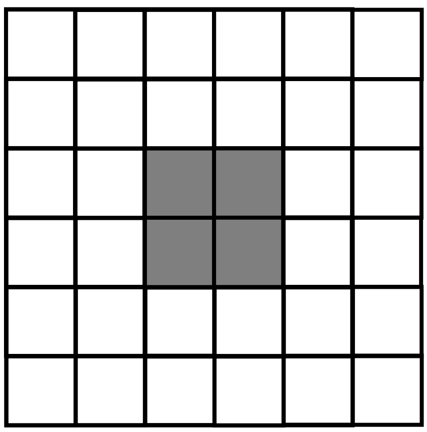
I. A can of spray paint emits 200 droplets of paint per second from the nozzle. The amount of droplets from a can of spray paint that fall on a given area (intensity – I) is given by the formula: $I = \frac{200}{0.125\,\pi r^2}$.

Draw a sketch of the graph that represents the relationship between spray paint intensity (I) and the distance from the nozzle (r), and explain how it shows the relationship.

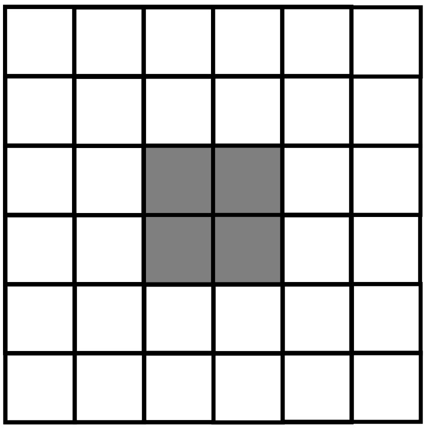
How it shows the relationship:



II. The can is held 2 metres from a wall that has squares marked on it like a grid. The can is then sprayed for 1 second. This is the shape of the paint landing on grid is looked at head on, is shown in the diagram below.



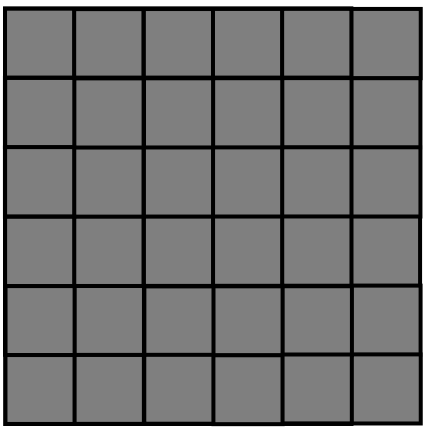
(iii) If the can was moved to 4 m, shade in the shape would look like on the *6 x 6* grid below.



(iv) Explain why you drew it at you did.

(v) Has the intensity of the droplets per square on the grid increased, decreased or remained the same? Explain your reasoning

(vi) The can is moved to a distance where the shape of the paint is shown on the grid below. Determine how far the can is from the wall. Ensure you show how you figured it out.



III. Calculations.

(iv) The can is placed 5 m from a wall. Calculate the paint intensity, using the formula given on the first page.

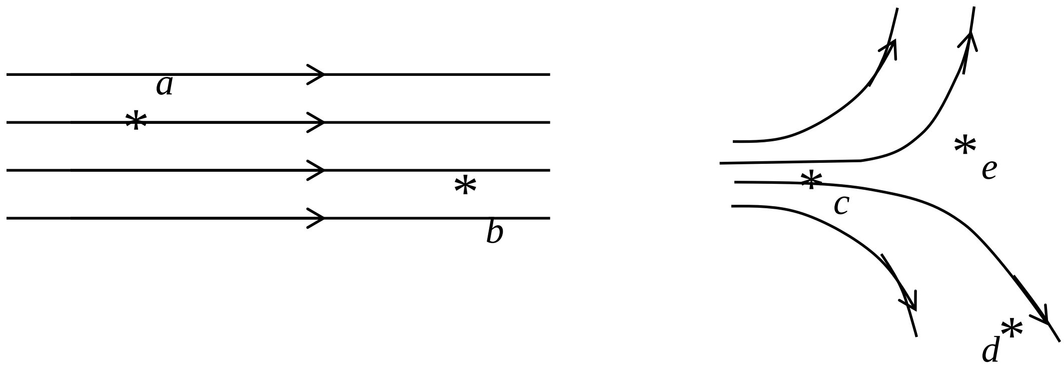
(v) If the distance from the can to the wall is doubled, what is the new “paint intensity?” Explain how you got your answer?

(vi) Explain how your answer from (ii) shows this is an example of an inverse square law.

Appendix C

Field lines tutorial materials.

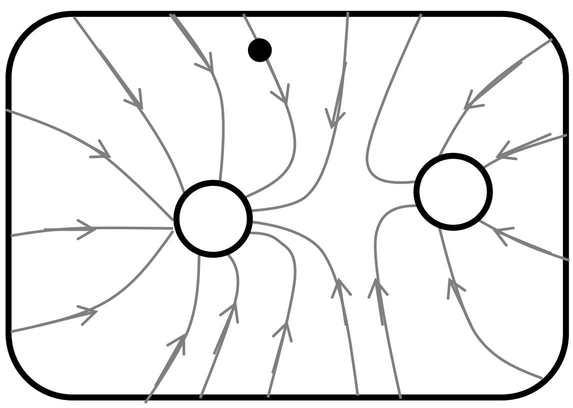
1. Here are two examples of field lines.



   (i) Rank the field strength, from highest to lowest, for the points (a) to (e). Explain your reasoning.

   (ii) Use vector arrows to show the direction of force acting on an object if it were to be placed at (a) to (e).

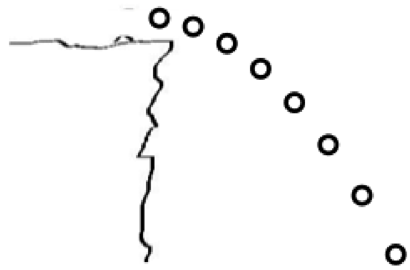
2. A meteor is placed at rest between two planets, that are close to each other. The gravitational field of both planets is shown in the diagram using field lines.



On the diagram, show the path taken by the meteor as it accelerates under the influence of the gravitational field. Explain why you drew the path as you did.

I. Gravitational field (Acceleration due to gravity).

A ball is thrown off a cliff, with an initially velocity *10 ms⁻¹*. A camera takes a photo every quarter of a second and all the pictures of the ball are combined into one picture as shown.



(i) Can you tell from the diagram whether the magnitude of the velocity of the ball is changing? Explain.

(ii) Can you tell from the diagram whether the direction of the velocity of the ball is changing? Explain.

(iii) Can you tell from the diagram alone, whether the object accelerates in the horizontal direction or vertical direction. Explain.

The ball has a mass 0.5 kg.

(i) Calculate the force of gravity acting on the ball. (Take mass of earth as 6×10^{24} *kg*, radius of earth as 6.4×10^{6} *m* and the gravitational constant as 6.7×10^{-11} $N\ m^2\ kg^{-2}$.

(ii) Is the force of gravity acting on the ball constant during the duration of the fall? Explain.

(iii) Using $F = ma$, determine the acceleration, due to gravity, acting on the ball.

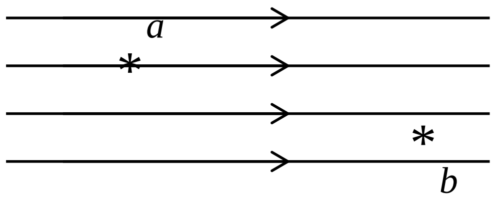
(iv) Is the acceleration, due to gravity, acting on the ball constant during the duration of the fall? Explain.

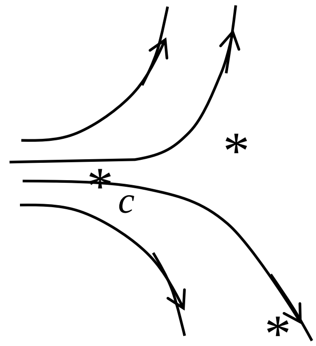
(v) Draw vector arrows on the all the balls to represent your answer from (iv). (8 arrows in total, one from each ball).

II. Gravitational field (lines).

In the last question of section II, you were asked to draw 8 vector arrows to represent the acceleration acting on the ball. Drawing vectors arrows in some cases can be clumber-some, and it is sometimes easier to use field line representations instead. Field lines represent the direction and strength of the force felt by objects that interact with those fields.

These are continuous lines that, when a point in picked, a tangent to the line at that point denotes the direction of force at that point. The closer field lines are together, the stronger the force is and a body does not have to be on a field line to feel a force. Here are two examples of field lines.





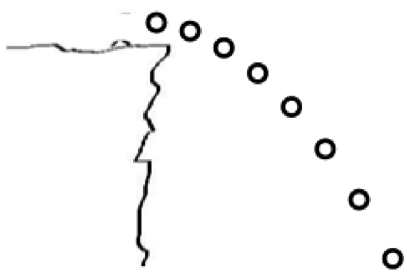
(i) Rank the field strength, from highest to lowest, for the points (a) to (e). Explain how you used the field lines to justify your ranking.

(ii) Use vector arrows to show the direction of force acting on an object if it were to be placed at (a), (d) and (e) (use a ruler).

A uniform field is described as a field that always points in the same direction, and has a constant field strength.

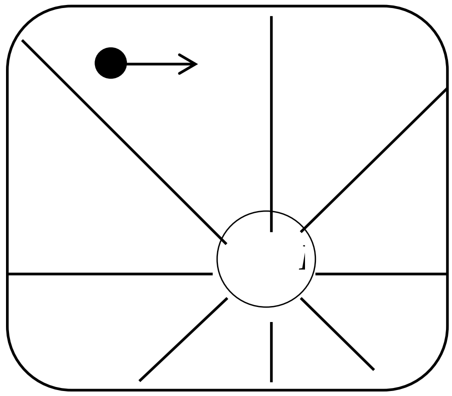
(iii) Which field (left or right) shows a uniform field? Explain how the field shows it is uniform.

(iv) Use field lines to sketch the field that causes the ball to fall. Explain if the field is uniform or not. (refer back to section I)



III. Gravitational field of the earth.

The diagram shown contains a small meteor that is passing close to the earth, shown as a small black circle. It has a velocity of 10 *km/s*, shown by the vector arrow. The earth is shown as a big circle, and it is gravitational field is sketched. At no point, does the meteor collide with the planet.



(i) Is the strength of the gravitation field caused by the ***earth*** the same everywhere? Explain.

(ii) Using $F = mg$, and $F = G\frac{Mm}{r^2}$, show that the acceleration due to gravity is given by $a = G\frac{M}{r^2}$

(iii) Explain how the field lines represent that gravity follow an inverse square law.

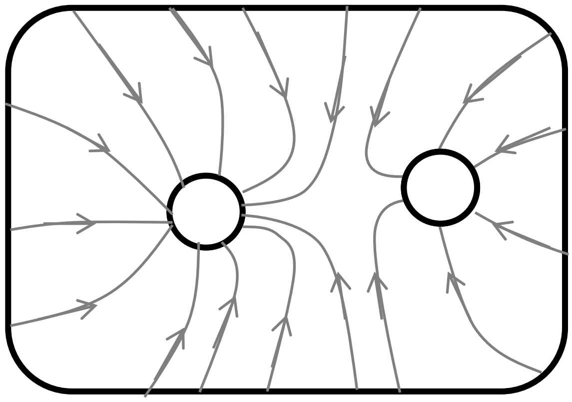
(iv) Use arrowheads to show the direction the field-lines point. Explain why you drew them as you did.

(iii) Draw the path followed by the meteor, under the influence of the gravitational field. Explain your reasoning for drawing it as you did. **(Remember, it has an initial velocity as shown with the vector arrow in the diagram) (Explain why your path follows / does not follow the field lines)**

IV. Gravitational field between two planets, very close together.

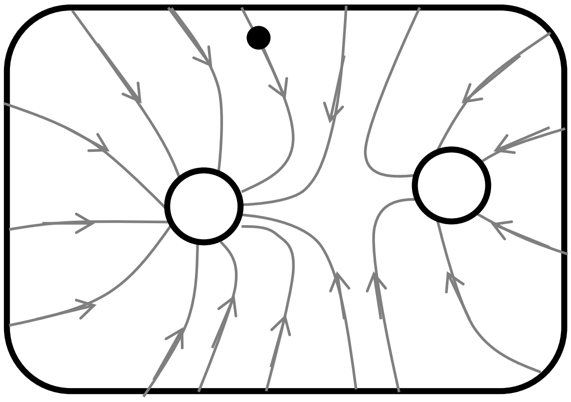
The diagram to the right shows the field between two planets.

(i) Using the field lines, determine whether the mass of one of the planets is bigger than the other, or if they are the same. Explain your reasoning.



Consider the following student dialogue, between two students (S_1 and S_2) concerning a small meteor initially at rest placed at a location shown by the small black circle.

S_1: *The field lines indicated the direction of the force, so the meteor will be forced along the line until it hits the left planet*

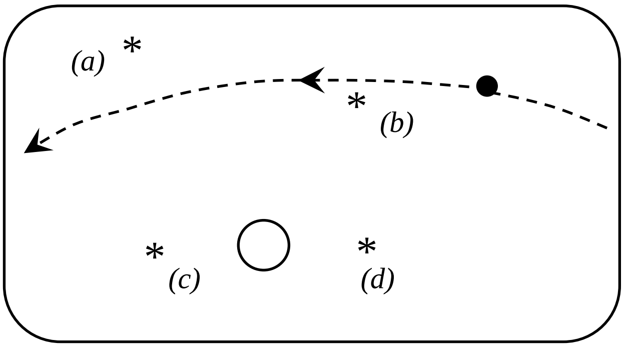


S_2 *As the small meteor begins to accelerate, its gained velocity will make it move away from the field line that it was on originally, so we can be sure it'll*

(iii) Which student, if any, do you agree with. Explain why your reasoning.

(v) Using your answer from (iii), draw in the likely path followed by the meteor on the diagram above.

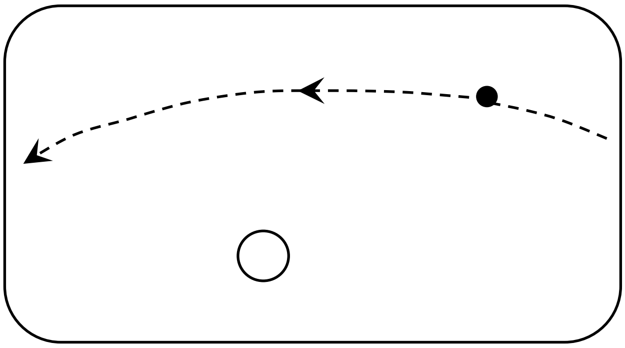
1. The following diagram shows a small planet, denoted by the white circle. A meteor passes by the planet in the path shown. A number of points (a) – (d) are also highlighted. The planet has mass of 3×10^{24} *kg*.



(i) Use field lines to represent the gravitational field caused by the planet.

(ii) Rank the field strength, from lowest to highest, at the points (a) – (d). Justify your ranking, ensuring you reference the field you drew in (i).

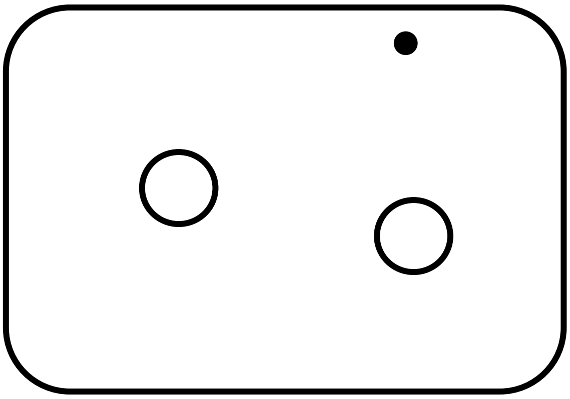
(iii) If the planet had a mass of 6×10^{24} *kg*, draw in the field lines that represent that this planet has an increase in mass on the diagram below.



(iv) Draw in the new path taken by the meteor in this scenario, in which the mass of the planet has increased. Explain why you drew the new path as you did. (If you think it follows the original path, which is shown in the diagram, explain exactly why you think this).

(v) What effect, if any, does increasing the mass of the meteor have on the path taken? Explain. (If you think it does not affect the path taken, explain exactly why you think this).

3. A meteor is placed at rest between two planets of equal mass, that are close to each other.

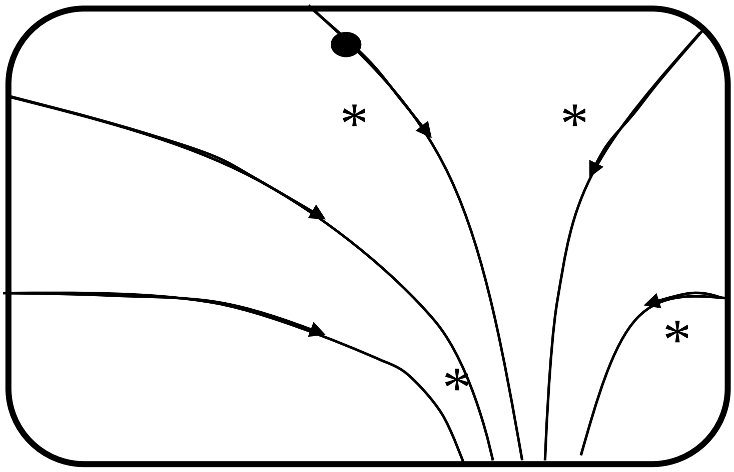


(i) Draw the field lines to represent the gravitation field caused by both planets in the diagram above.

(ii) On the diagram on the top of the page, show the path taken by the meteor as it accelerates under the influence of the gravitational field. Explain why you drew the path as you did.

(iii) Highlight the point between the two planets where the gravitational field is zero using a circle. Explain why this point exists.

1. Here is a small snapshot of a section of field lines.



(i) Trace you finger to the end of any field line, starting where the lines are closest together, so that it travels against the direction of field line. How the field strength varies as you move your finger. Justify your answer.

(ii) Use vector arrows to show the direction of the force the points highlighted.

(iii) A small body is placed at the point marked with a black circle. show the path taken by the body as it accelerates under the influence of the gravitational field. Explain why you drew the path as you did.

Appendix D

Coulomb's law and electric field tutorial materials.

Coulomb's Law explains the attraction or repulsion between two charges. It is given by the formula

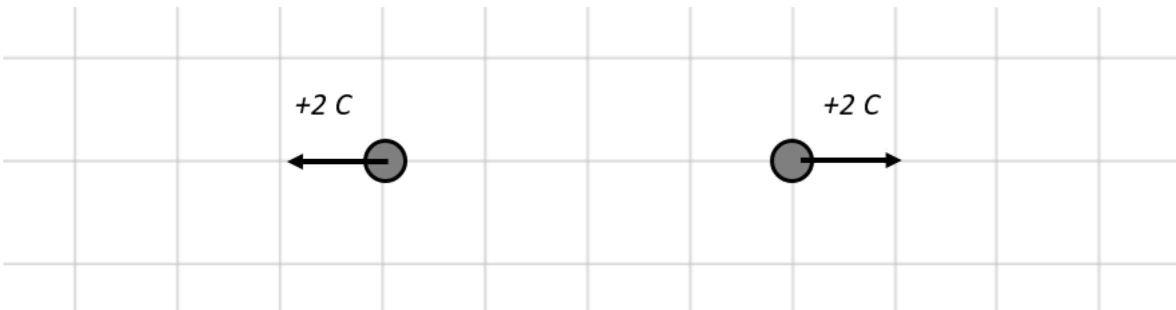
$$F = k\,\frac{q_1 q_2}{d^2}$$

1. Using the formula above, explain the relationship between the force between the two charges and their magnitudes.

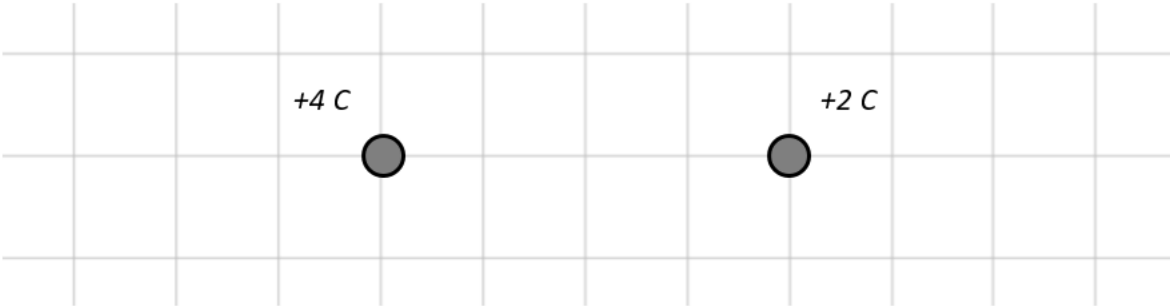
2. Using the formula above, explain the relationship between the force between the two charges and the distance between them.

3. Two *+3 C* charges are held a distance of *10 cm* from each other, and the force acting on both charges is *10 N*. If one of the +3C charges is replaced with a +9C charge, what is the new force acting on the charges? Explain how you know what the change in force is.

4. A *+2 C* and a *+2 C* charge are placed 20 cm from each other. The vectors to show the force are shown in the following diagram.



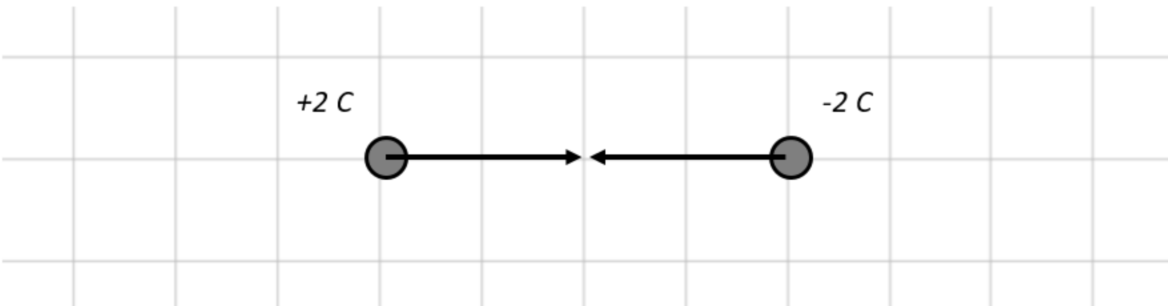
If the +2 C charge on the left is replaced with a +4C charge. Draw the vectors to represent the forces now acting on the charges.



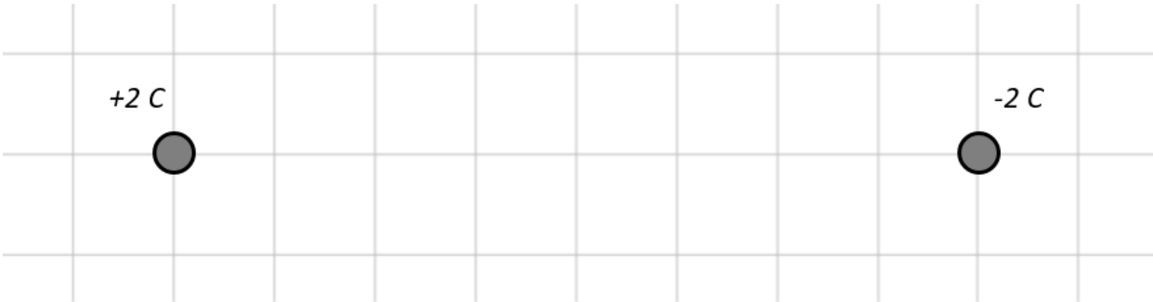
Explanation:

5. Two *+8 C* charges are held a distance of *10 cm* from each other, and the force acting on both charges is *90 N*. The charges are moved so the distance between then is now *30 cm*. What is the new force acting between the charges? Explain how you know what the change in force is.

6. A *+2 C* and a *-2 C* charge are placed 20 cm from each other. The vectors to show the force are shown in the following diagram.



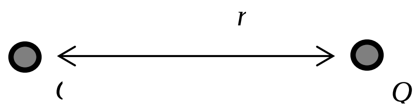
If the distance between the charges is increased to *40 cm*, draw the vector arrows to show the force acting between the charges now.



Explanation:

I. **Looking at the relationship between the charges and the force in Coulombs' Law.**

Two charges are held a distance apart from each – other, a constant distance "r." There is a force exerted between the charges. Each charge, q_1 and q_2, are replaced and with various stronger charges and the forces are recorded as shown. (The product of the charges (q_1*q_2) is shown in the third column)



q_1 (C)	q_2 (C)	$q_1 q_2$ (C^2)	F (N)
1	1	1	2
1	2	2	4
2	1.5	3	6
4	1	4	8
1	5	5	10
2	3	6	12

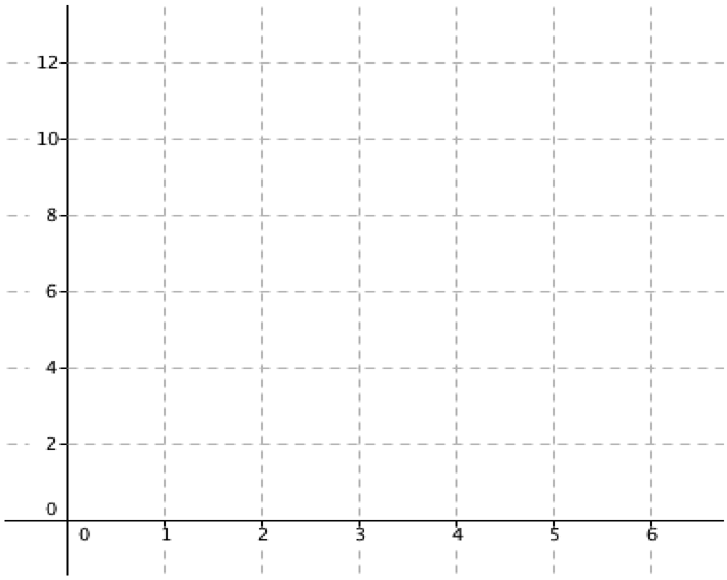
(i) Can you see any pattern between the first column (q_1), the second column (q_2) or the third column ($q_1 q_2$) with the fourth column (F)?

If so, how would you describe this pattern.

(ii) Linear patterns through the origin follow the form y = mx, where m has a constant value. By letting the values for column 4 (F) represent your y values, and which-ever of the other three column you chose to in (i) for the x-values, show that $\frac{y}{x}$ is a constant.

y	F (N)	2	4	6	8	10	12
X							
$\frac{y}{x}$	$\frac{F\ (N)}{\ }$						

If you identifies any patterns in sections (i) – (iii), graph it / them on the graph shown. (Label the axis)



(iii) Explain how this graph shows a directly proportional relationship.

(iv) What force is exerted between the charges when the product of the charges is 2 C^2?

(v) What is the force exerted between the charges when the product of the charges is 6 C^2?

(vi) From part (iv) and (v), explain what affect does tripling the product of the charges have on the force exerted between the charges?

(vii) Is this in agreement to your answer from part (iii) that this is a directly proportional relationship? Explain your answer.

II. Calculations involving a directly proportional relationship.

The force, F_A, exerted by the two charges shown, can be calculated in the manner shown below, if q_1 has a magnitude of $6\ \mu C$ and q_2 has a magnitude of $3\ \mu C$, and the distance between the charges is 1 cm. (Use $F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$, where ε = 8.85 x 10^{-12} $F\ m^{-1}$)

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

A

$$F = \frac{1}{4(3.14)(8.9\ x\ 10^{-12})}\frac{(6\ x\ 10^{-6})(3\ x\ 10^{-6})}{(1\ x\ 10^{-2})^2}$$

B

$$F = \frac{1}{1.12\ x\ 10^{-10}}\frac{(1.8\ x\ 10^{-11})}{(1\ x\ 10^{-4})}$$

C

$$F = (8.9\ x\ 10^{9})(1.8\ x\ 10^{-7})$$

D

$$F = 1{,}602N$$

(i) Explain the mathematical step that occurs in A. (include explanation as to the use of scientific notation for the values)

(ii) Explain the mathematical steps that occur in B (there are 3).

(iii) Explain the mathematical steps that occur in C (there are 2).

(iv) Explain the final steps that occur in D.

(v) From your steps outlines in (i) to (iv), calculate the force, F_B, exerted between the charges if the 3 μC charge is replaced with a 9 μC charge.

(vi) By what factor is the force between the charges increased?

(vii) How does your answers show that the force experienced by the charges is directly proportional to the product of their magnitudes?

(viii) Explain how there would there have been a quick way for you to determine the new force after the replacement?

III. **Looking at the relationship between the distance and the force in Coulombs' Law.**

Coulomb's law states that the force between two charges is directly proportional to the product of the magnitude of the charges, and inversely proportional to the square of the distance between them. This is given by the following formula.

$$F = k\,\frac{q_1 q_2}{d^2} \qquad k = \frac{1}{4\pi\varepsilon}$$

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

In the last 4 pages, you learnt how to show the first relationship by, between the force exerted and the product of the charges by using tables, graphs and calculations. Using what you learnt, your task is to prove the relationship between the force exerted between the charges, and the distance between them, using whichever methods you choose. Attempt all of them. If you get stuck with a method, ask for help in using it.

You can use the following to help you.

Directly proportional general equation: $y = mx, \quad \frac{y}{x} = m, \quad \frac{y}{x} = constant$

Directly proportional to square equation: $y = ax^2, \quad \frac{y}{x^2} = a, \quad \frac{y}{x^2} = constant$

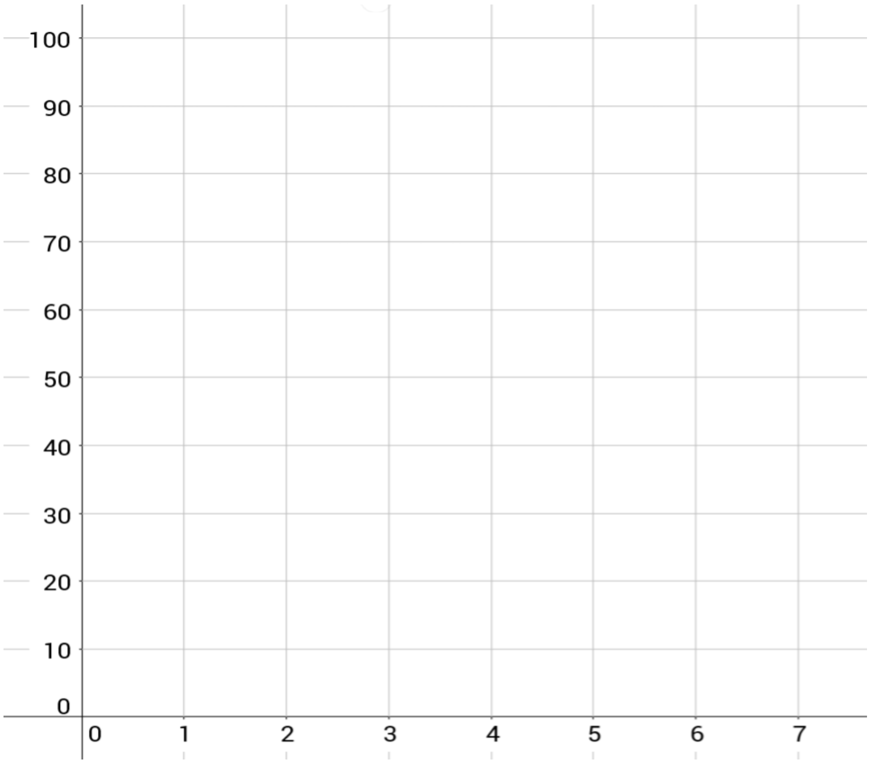
Inverse proportional general equation: $y = \frac{k}{x}, \quad xy = k, \quad xy = constant$

Inverse square proportional equation: $y = \frac{k}{x^2}, \quad x^2 y = k, \quad x^2 y = constant$

Table:

y	F (N)	100	25	11.11	6.25	4	2.78
x	d (m)	1	2	3	4	5	6

Graph:



Equation and values for calculation:

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

$\varepsilon = 8.85 \times 10^{-12}\ F\ m^{-1}$, $q_1 = 6 \times 10^{-6}\ C$, $q_2 = 4 \times 10^{-6}\ C$.

$d_1 = 4\ cm$, $d_2 = 8\ cm$

Which method do you think is the most effective? Why?

Which method is the easiest do use? Why?

Which method would you use, if you have to choose one? Why?

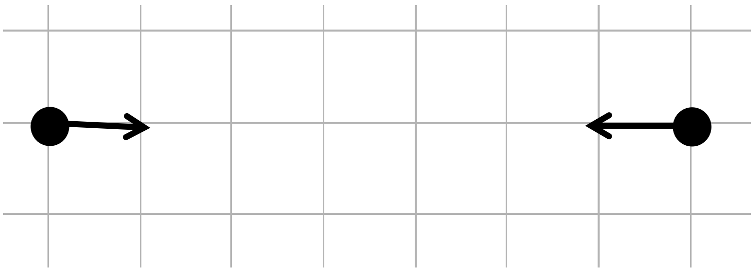
I. **Numerical calculations involving Coulomb's Law. (Formula is in equation tables, as is value for Ɛ)**

A *+3 C* and a *-3 C* charge are placed 10 cm from eachother.

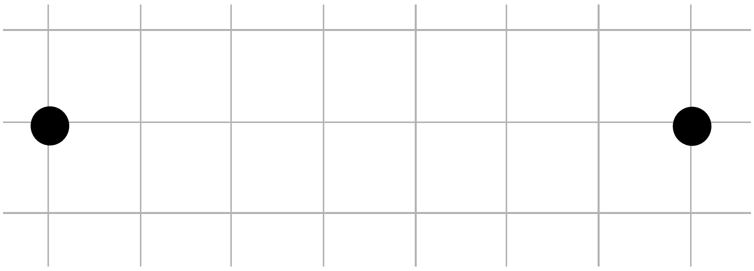
(i) Calculate the force between these two charges.

(ii) If one of these charges is replaced with a *+9 C* charge, what is the new force acting between these two charges.

The force between the two charges from (i) is shown in the following diagram.



(iii) Using your answers from (i) and (ii), draw the vector arrows to how the force acting between the charges from (ii).



(iv) Explanation how your answers from (i) – (iii) show the force between two charges is directly proportional to their magnitude.

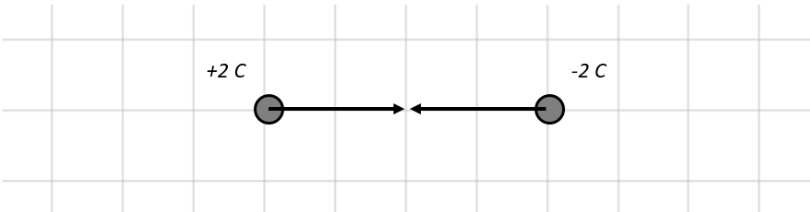
I. **Numerical calculations involving Coulomb's Law. (Formula is in equation tables, as is value for Ɛ)**

A *+2 C* charge and *-2 C* charge are placed 20 cm from eachother.

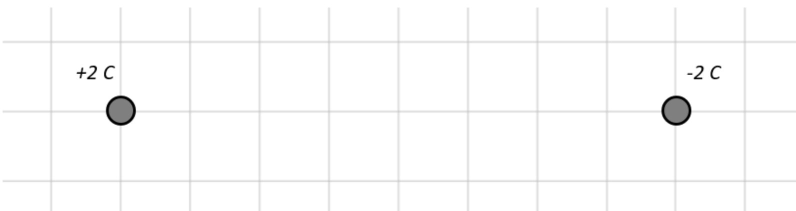
(ii) Calculate the force between these two charges.

(ii) These charges are moved to 40 cm from eachother. Calculate the new force acting between these two charges.

The force between the two charges from (i) is shown in the following diagram.

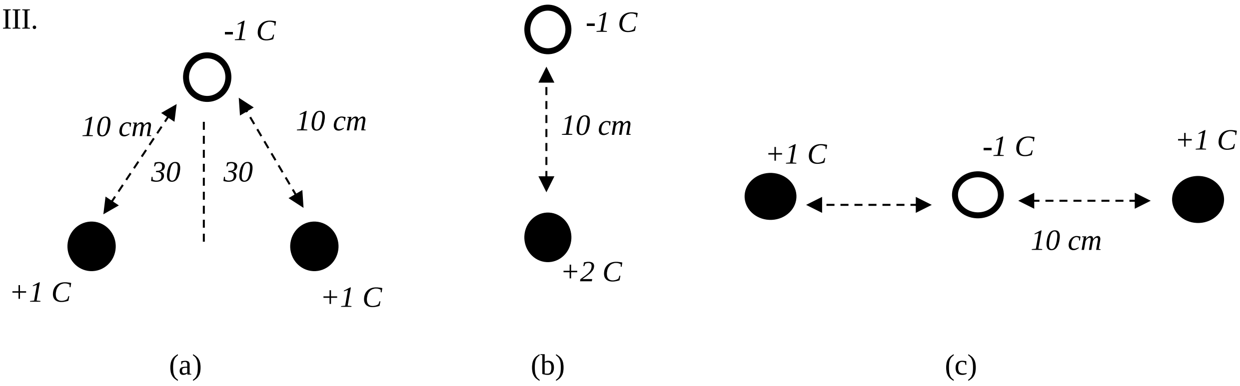


(iii) Using your answers from (i) and (ii), draw the vector arrows to how the force acting between the charges from (ii).



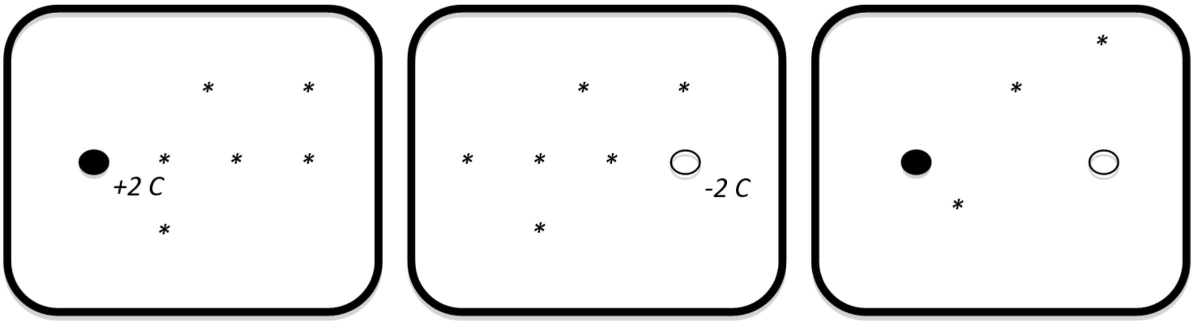
(iii) Explanation how your answers from (i) – (iii) show the force between two charges is inversely proportional to the square of the distance between the charges.

III.

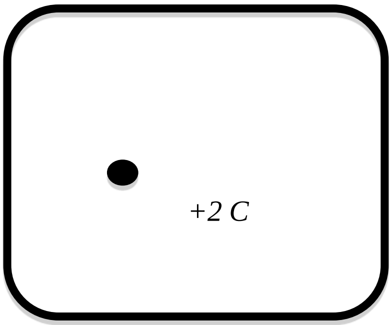


(i) 3 setups of charges are set up as shown in the above diagram. How does the magnitude of the net force acting on the negative charge (white) in setup (a) compare to the magnitude of the net force acting on the negative charge in (b) and (c). Explain your answer, using vectors, calculations or any other manner you see fit.

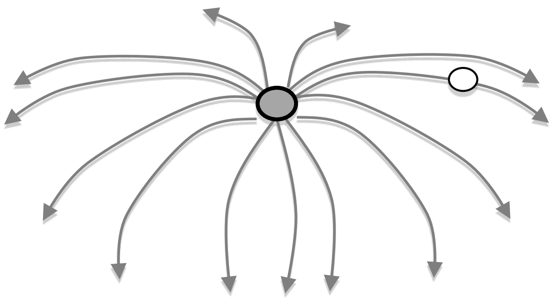
1. Use vector arrows to draw the electric field surrounds charges, at the points highlighted with stars, in the following three diagrams.



2. Represent the first diagram you drew, for the +2 C positive charge, using field lines.



3. A collection of unknown charges are located in the grey circle, which produce an electric field as shown to the right. An electron is placed at the position marked with the while circle.



(i) Draw in the path followed by the electron as a result of it is position in the force field.

(ii) Explain why you drew the direction as you did.

(iii) Rank the magnitude of the electric field strength, from highest to lowest, between a, b and c. Explain your ranking.

(iv) Draw vector arrows to represent the electric field at the points a, b and c on the diagram above.

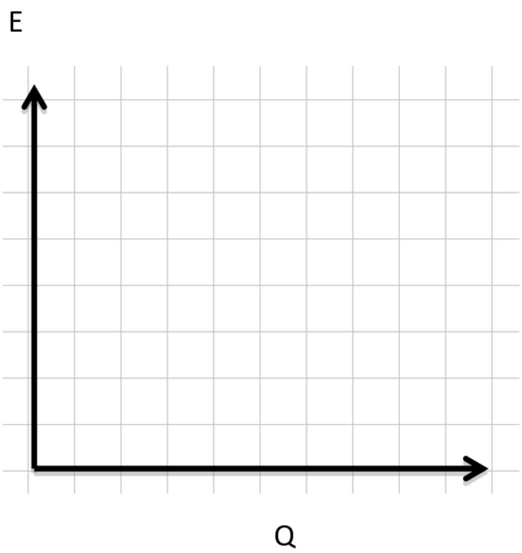
4.. The electric field strength around a charge is given by the formula $E = \frac{1}{4\pi\varepsilon}\frac{Q}{d^2}$.

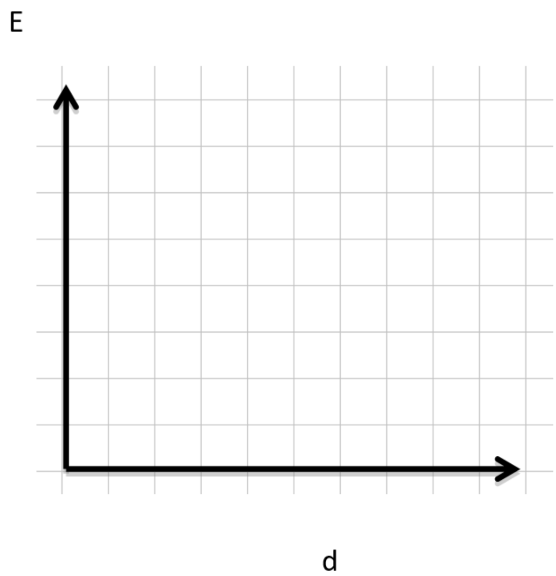
What is the relationship between the electric field strength and (i) the magnitude of the charge causing it, and (ii) the distance from the charge.

(i)

(ii)

Draw these relationships on the graphs below.

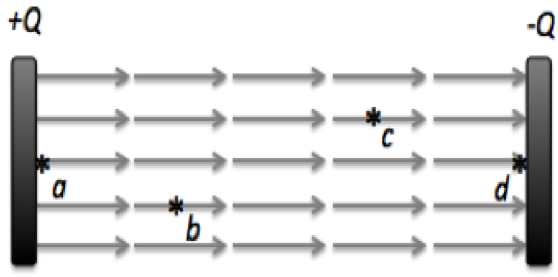




I. The forces experienced by a particle by an electric field – Vector treatment.

An electric field between two charged plates is depicted using vector arrows as shown. 4 points (*a*, *b*, *c* and *d*) are highlighted as shown.

(i) How would you describe variation of direction and strength of the electric field?



The picture above represents a ***uniform electric field***. This means that the strength of the electric field is the same at all points, and is in always in the same direction.

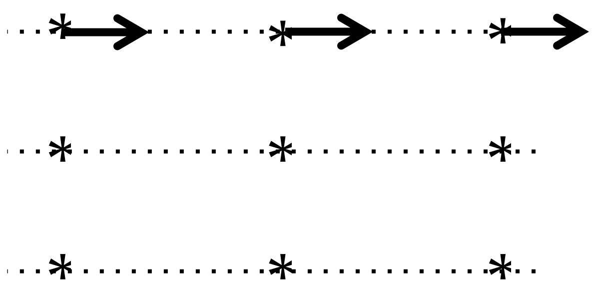
(ii) How does the representation show that the electric field strength is the same at all points?

(iii) Would the force experienced by a *+4 C* charge placed at *b* be *stronger*, *weaker* or *the same* compared to it being placed at *c*. Explain.

(iv) If the electric field has strength, $E = 20{,}000\ N\ C^{-1}$, find the force experienced by a *+4 C* charge when placed in the field.

(v) On the diagram on the right, the first line shows vector arrows, going left to right, for an electric field that is uniform.

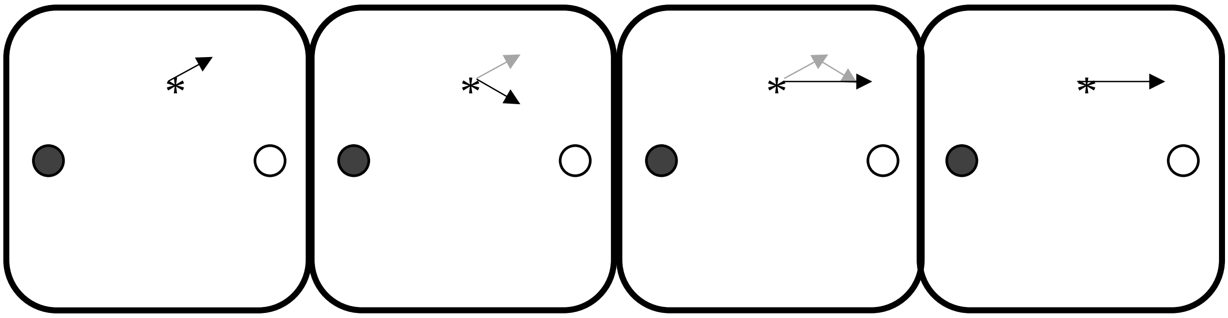
On the second and third line, use vector arrows to show (a) an increasing electric field and (b) a decreasing electric field.



II. Electric field vectors – Principle of Superposition.

One positive and one negative charged particles are placed a small distance from each – other. Both charged particles are have an equal magnitude. The electric field at a point marked with a star (*), are shown using vectors in the following order.

- The first field vector represents the direction of the electric field based on the position of the positive charge only.
- The second diagram represents the direction of the electric field based on the position of the negative charge, but the vector from the first diagram is represented with a grey arrow.
- The third diagram shows the net electric field vector, at this point, based on the two individual electric field vectors shown in the first and second diagram.
- The final diagram shows the net resultant electric field vector without referencing the two vector arrows that were used to construct it.

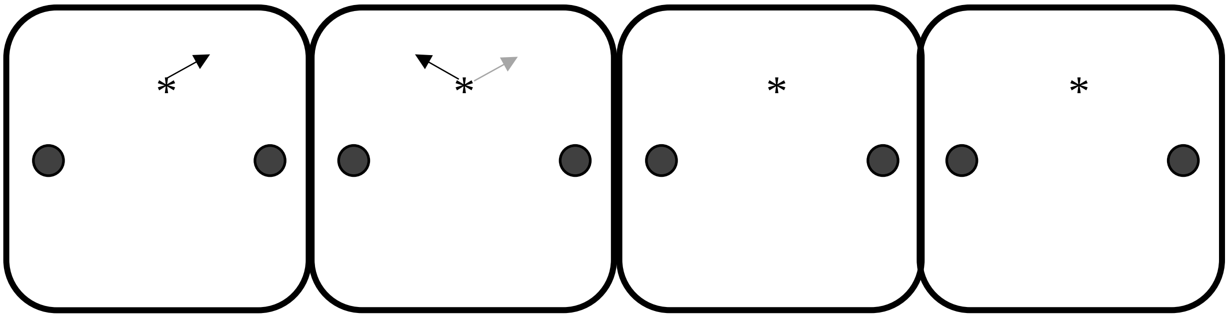


(i) Explain the process in the third diagram that allows us to find the net electric field vector, at this point.

(ii) In the first and second diagram, we can see the electric field vectors point in diagonal directions. However, the net vector in the third and fourth diagram is only horizontal. Explain why this is the case reference the horizontal and vertical vectors of the first two vectors.

(iii) If we measure the magnitude of the vectors in the first, second and third diagram, we will find the net electric field in the third diagram has a weaker magnitude than the sum of the vectors in the first and second diagram. Explain why this is the case, referencing the horizontal and vertical vector components of the first two vectors.

The negatively charged particle is replaced with a positively charged particle of equal magnitude. The first and second diagram represent the electric field of the positive charge on the left and right respectively.

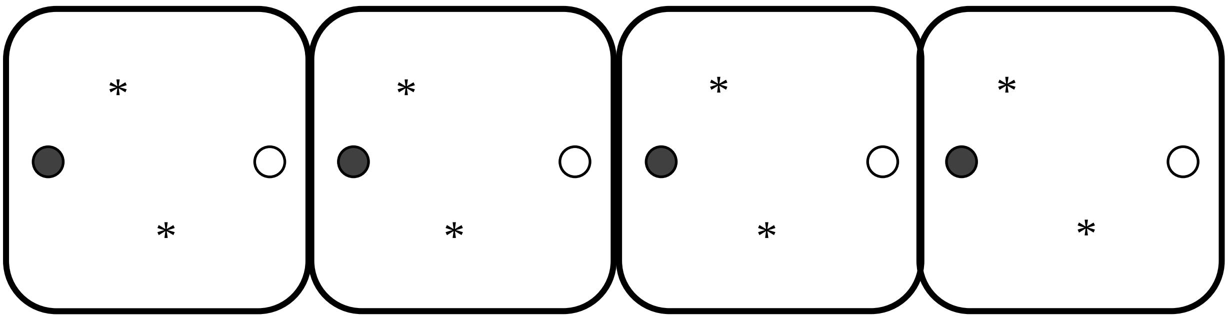


(iv) Draw the net electric field at this point in the third and fourth diagrams using the vector arrows presented in the first two diagrams, in the same manner as shown on the previous page.

(v) Is the net electric field you drew pointing in a horizontal, vertical or diagonal direction? Explain why it points in this direction, referencing the horizontal and vertical components of the first two vectors.

(vi) If we measure the magnitude of the vectors in the first, second and third diagram, we will find the net electric field in the third diagram has a weaker magnitude than the sum of the vectors in the first and second diagram. Explain why this is the case, referencing the horizontal and vertical vector components.

(vii) In the diagram below, use vectors to show the net electric field at the two points points shown, between a positive and negative charge. Show the initial component vectors and how you use them to construct the net electric field in the fourth diagram.



III. The forces experienced by a particle by an electric field – Field treatment.

When representing an electric field, it can be easier to use field lines. We have seen in mechanics that we can use field lines in replacement, or combined with vectors to show the direction of the force at a point, and the relative strength in a field.

The electric field always points in the direction that a small positive charge would feel a force. From this, we determine that an electric field line will always point away from a positively charged object, and towards a negatively charged object.

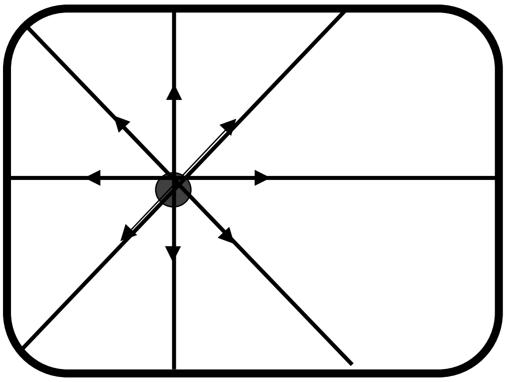
The rules you’ve already covered about field lines is as follows.

- The closer field lines are, the stronger the field.
- When a field line curves, the direction of the force is tangential to the field lines.
- The field line represents the direction of force acting on a body, no the path taken by a body in the field.

Other rules for using field lines are as follows:

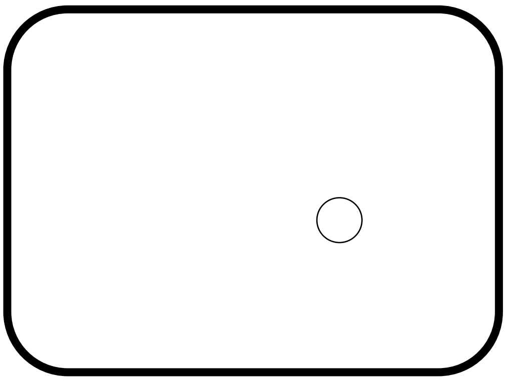
- Field line patterns do not overlap or intersect. Two bodies that cause a field both contribute to one overall field surrounding both objects.
- Field lines do not finish, or terminate. They should extend to infinity / off the page / to the end of the diagram boundary.

(i) Taking this into account, identify the charge on the following particle. Explain how you can tell.



(ii) Does the electric field strength increase, decrease or stay the same as you move away from the charge? How can you tell?

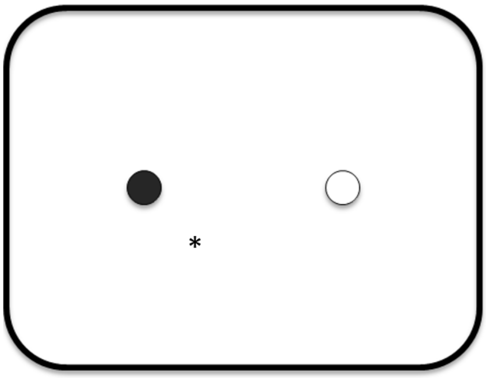
(iii) Draw the electric field for an oppositely charged particle, as shown to the right.



(iv) Explain the differences and similarities for the field for the two charges.

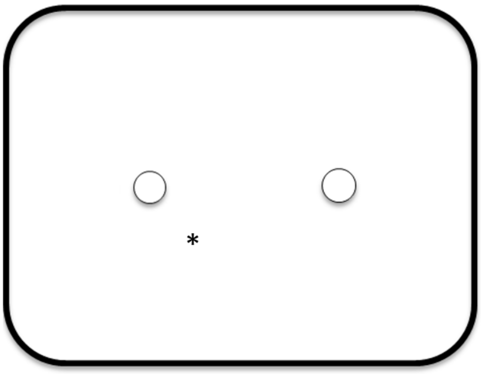
IV Electric field vectors – Principle of Superposition.

(i) Using the principle of superposition (how you determined the net vectors on pages 2 and 3) sketch the electric field between the positive and negative charge of equal magnitude. Ensure that you draw one field line going through the star.



(ii) If a positively charged particle were to be placed at rest at the star, sketch the path it would take. Assume the initial two charges do not move. You can represent the path taken in any manner you see fit (a bold line, a strobe diagram, vectors, etc). Explain why you drew the path as you did.

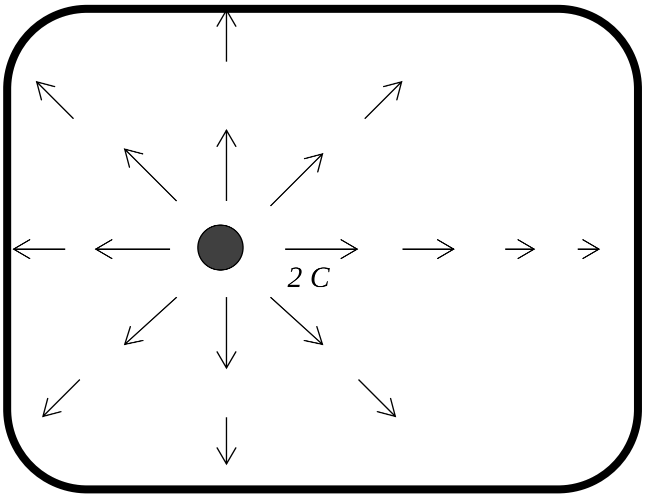
(iii) Using the principle of superposition (how you determined the net vectors on pages 2 and 3) sketch the electric field between two negative charges of equal magnitude. Ensure that you draw one field line going through the star.



(iv) If a negatively charged particle were to be placed at rest at the star, it would take. Assume the initial two charges do not move. You can represent the path taken in any manner you see fit (a bold line, a strobe diagram, vectors, etc). Explain why you drew the path as you did.

I. Electric field of a particle.

The electric field surrounding a *2 C* charge is shown in the diagram to the right, using vectors.

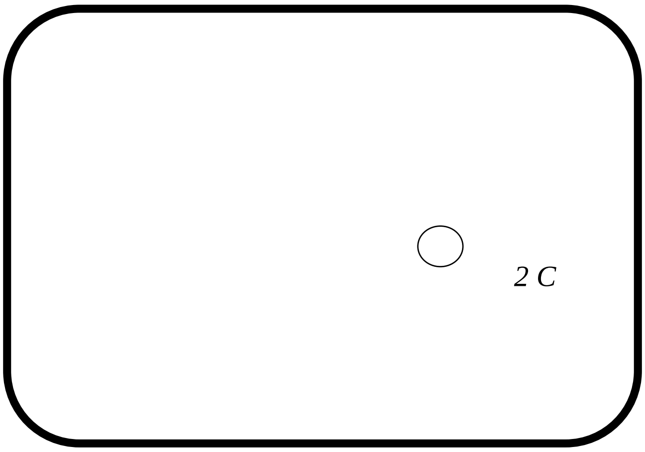


(i) What can you tell about the electric field strength as the distance from the charge increases. How can you tell?

(ii) By using the arrow directions, detemine whether the 2 C charge is positive or negative.

The charged particle is removed and an particle that **is oppositely charged** is placed down, as shown in the following diagram.

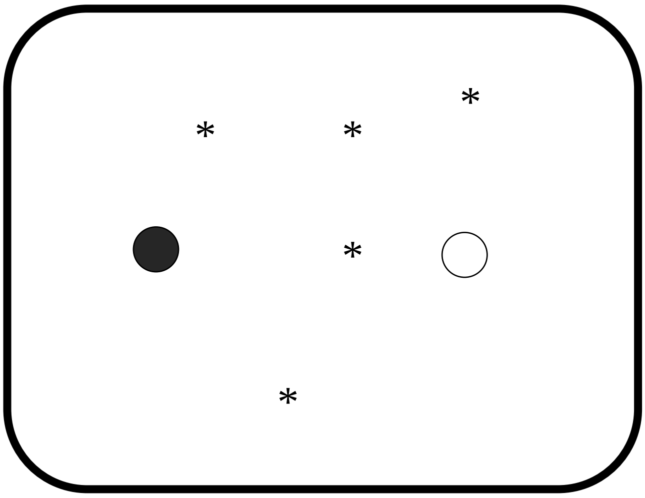
(iii) Sketch the electric field, ***using vector arrows***, the electric field around the new charge.



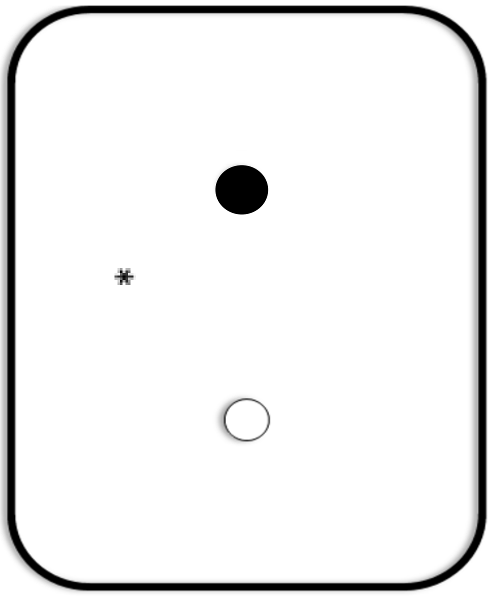
(iv) Explain how your arrows show the change in electric field strength, if any, as you move away from the electric charge.

(v) Explain why you drew your arrows pointing either ***towards or away from*** the negative charge.

(vi) ***Construct vector arrows*** at the points marked with stars to show the electric field around a positive and negative charge. (use the diagrams you did already in ***I(i)*** and ***II(i)*** to help you)



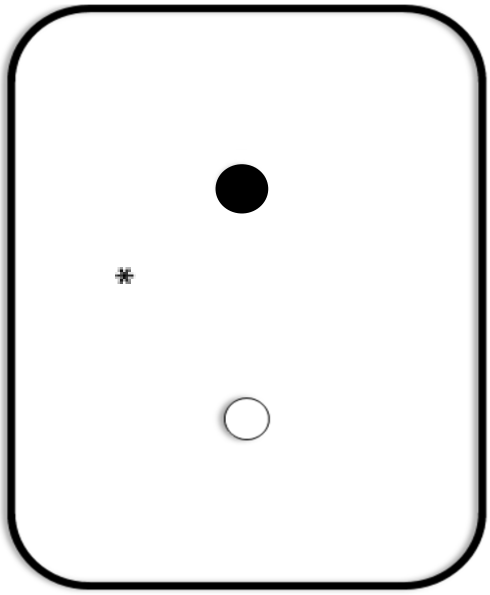
II. Path taken by a charged object in an electric field.



(i) In the above diagram, represent the electric field between the two charged particles, using any manner of your choosing. In this setup, the positively charged particle (black) has twice the magnitude of the negatively charged particle.

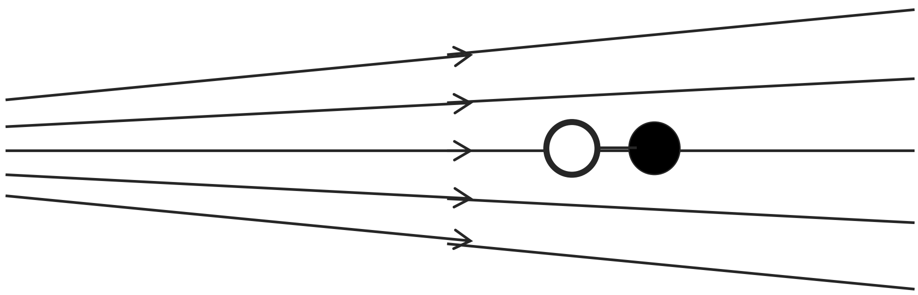
(ii) Draw in the path a positively charged particle would take, if it were placed at the star. Explain why you drew it as you did.

(iii) On the second diagram, draw in the path the particle would take if it has an initial velocity to the right, as shown in the diagram with the arrow. Explain why you drew as you did.



III. Behaviour of charges in an electric field.

An electric field is shown in the following diagram. Within this electric field, a negatively charged particle (white circle) is attached to a positively charged particle (black circle) so they cannot be separated. They are initially at rest.



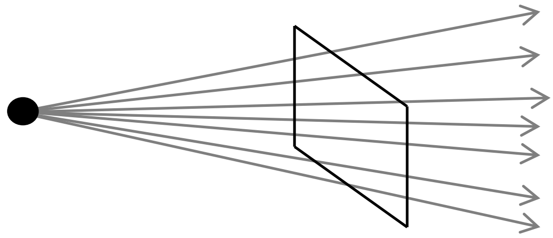
(i) As you go from left to right, does the electric field strength increase, decrease or stay constant? Explain how you determine this?

(ii) What direction will the force on the negatively charged particle act, caused by the electric field? Explain.

(iii) What direction will the force on the negatively charged particle act, caused by the electric field? Explain.

(iv) Using your answers from (i) to (iii), determine whether two charges will move to the left, right or will remain at rest. Justify the outcome you pick.

I. Investigating the variation of electric field strength with charge.



The above diagram uses lines to represent an electric field, of a *+1 C* charge, passing through a *1 m^2* frame. While not accurately sketched, assume there are 100 field lines coming from the charged particle passing through the frame. ***There should be more lines coming symmetrically from the charge in all directions, but for simplicity, we have not drawn on the diagram above.***

(i) If we doubled the magnitude of the charge (*+2 C*), we would double the amount of field lines that are coming from the charge. How many lines pass through the *1 m^2* frame?

(ii) If we had a charge of +4 C, how many lines would pass through the *1 m^2* frame?

(iii) What effect does increasing the charge generating a field have on the electric field strength at a point? Use (i) and (ii) to justify your answer (include the type of relationship observed).

(iv) If we used a *2 m^2*, frame, we would see 200 lines passing through it, as the lines are coming out of the charge symmetrically. Does using a bigger frame change the intensity of how many lines pass through a 1 m^2 frame? Explain your answer (may help to consider how many lines are in a *3 m^2*, *4 m^2*, etc frame)

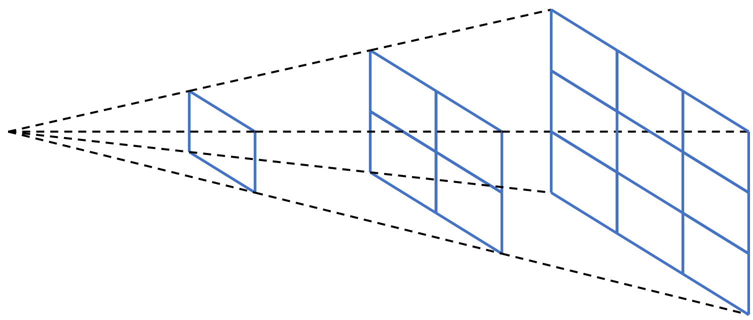
(v) Using your answer from section I (iv) and section I (iv), explain why changing the magnitude of the test charge used to measure electric field strength has no effect on the electric field strength at that point.

(v) If the +1 C charge was replaced with a – 1 C charge, what affect, if any, would you have to make to the field lines?

(vi) How would replacing the positive charge with a negative charge affect the number of field lines passing through *1 m²* frame?

(vii) Using your answer from (v) and (vi), explain why changing the sign of the test charge used to measure the electric field strength at a point has no effect on the electric field strength.

II. Investigating the variation of electric field strength with distance.



A second frame is placed away from the charge, so that all its outer corners are twice the distance from the charge This frame is made up of smaller frames that are identical to the frame in section II.

(viii) Explain why this second frame has an area that is four times bigger than the original frame used on the last page.

(ix) Determine the how many field lines through each of $1 m^2$ frames, for this second overall frame.

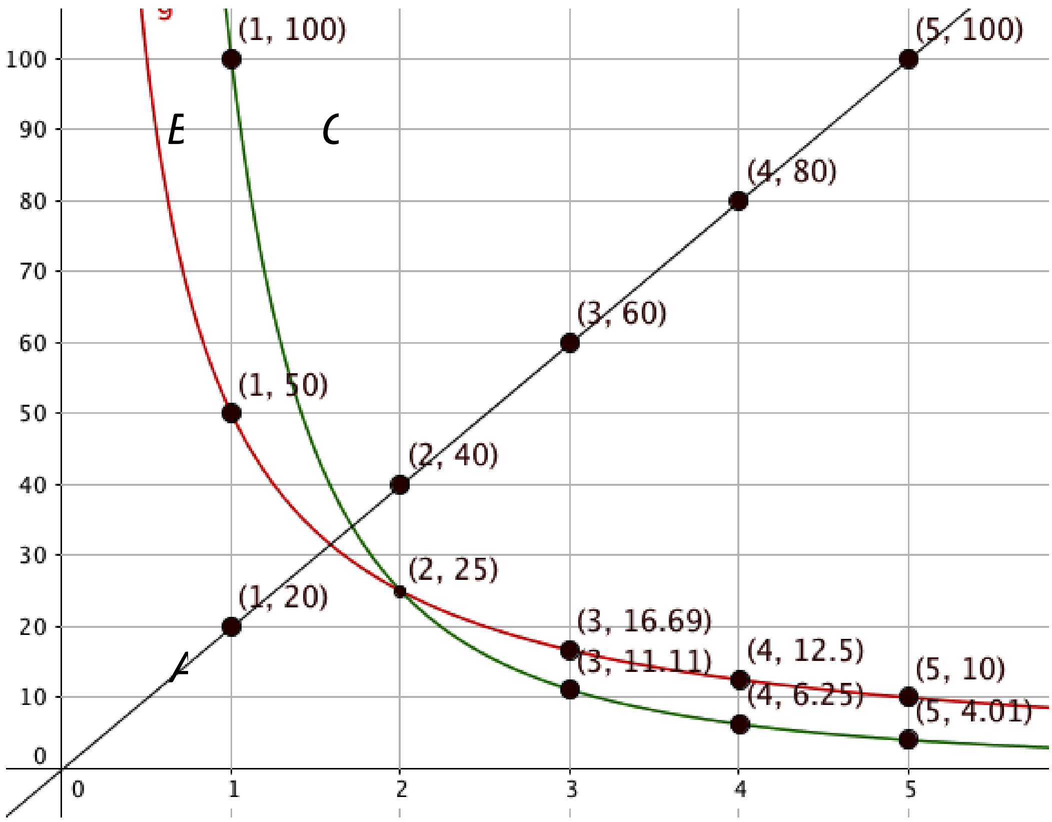
A third frame is placed away from the charge, so that all its outer corners are triple the distance from the charge. This frame is made up of smaller frames that are identical to the frame in section II.

(x) Explain why this third frame has an area that is nine times bigger than the original frame used on the last page.

(xi) Determine the how many field lines through each of $1 m^2$ frames, for this third overall frame.

(xii) As the distance from the charge increases, the number of lines passing through a $1\ m^2$ area decreases. Using your answers from the previous questions, explain why this occurs. (Explain the relationship involved)

1. The following graph shows 3 patterns for different types of functions, *A*, *B* and *C*.



(i) Determine which of the three functions is of the form: $y = mx$. Justify your answer using whatever reasoning you wish.

(ii) Determine which of the three functions is of the form: $y = k\frac{1}{x^2}$. Justify your answer using whatever reasoning you wish.

(iii) Which graph represents the relationship between the force (y-axis) and distance (x-axis) between two charged objects following Coulomb's law?

(iv) Explanation:

(v) Which graph represents the relationship between the force (y-axis) and product of two charged objects (x-axis) following Coulomb's laws?

(vi) Explanation:

2. A $+2\ \mu C$ charge and a $+3\ \mu C$ charge are held 20 cm away from each other. The force of repulsions between the two charges is 1.35 N (rounded to two decimal places). (remember, $1\ \mu C = 1 \times 10^{-6}\ C$)

(i) If you replaced the $+3\ \mu C$ charge with a $+\ 12\ \mu C$ charge, what would the magnitude of the force of repulsion be?

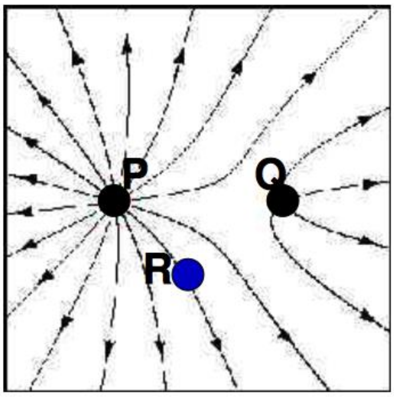
(ii) Use Coulomb's law (equation on last page) to calculate (to 2 decimal places)the force of repulsion between the $+2\ \mu C$ and $+\ 12\ \mu C$ charge at a distance of $20\ cm$ from eachother. Use your answer to verify your answer from (i) .

The original +2 μC charge and a +3 μC charge are held 20cm away from each-other again.

(iii) If the distance between the charges was increased to 40 cm away from each-other, what would the magnitude of the force of repulsion be?

(ii) Use Coulomb's law to calculate (to 2 decimal places)the force of repulsion between the $+2\ \mu C$ and $+\ 3\ \mu C$ charge at a distance of $40\ cm$ from each-other. Use your answer to verify your answer from (iii).

3.



Consider the electric field above, where P and Q are charged particles and R is a point in the electric field.

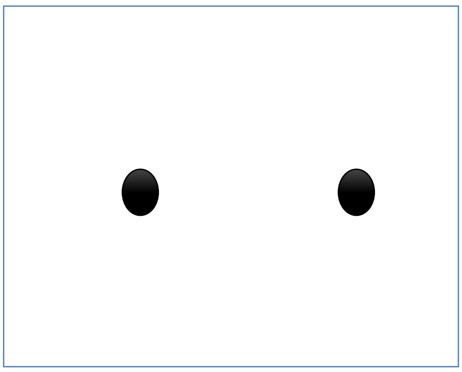
(i) Determine the charge on P and Q. Explain how you can tell.

(ii) Is the magnitude on the charge P greater than, equal to or less than the magnitude on the charge Q. Explain how you can tell.

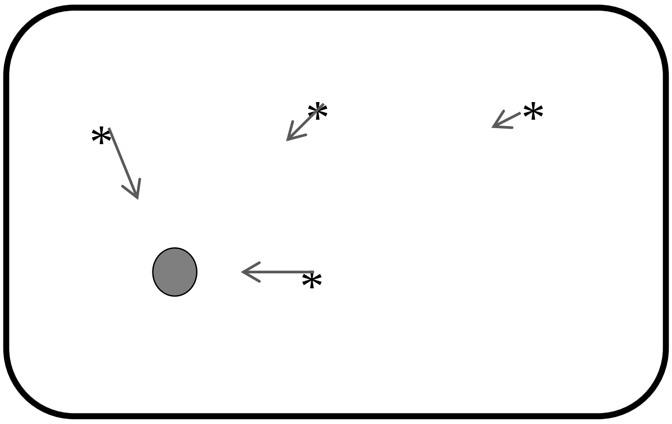
(iii) Place a finger on P and follow one of the field lines coming out of P. As you trace out the path, does the electric field strength increase, decrease of remain unchanged? Explain how you can tell.

(iv) If a negatively charged particle was placed at R, draw on the diagram the path you think the particle would take. Explain why you think it would take this path.

(v) Use vector arrows to above to represent the field shown above, at the following points.



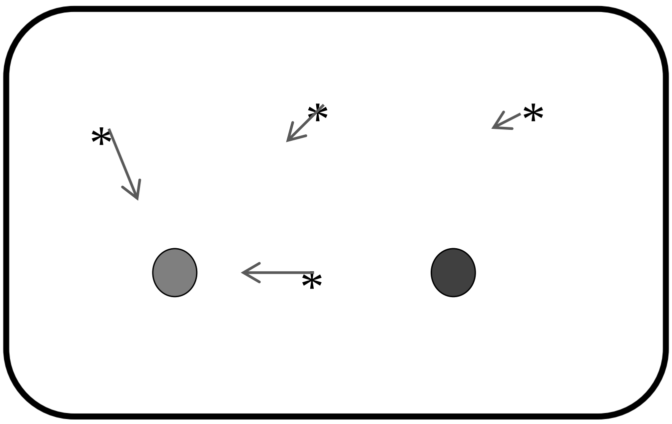
4. An electric charge is nailed down and the electric field at points around it is shown in the diagram, using vector arrows.



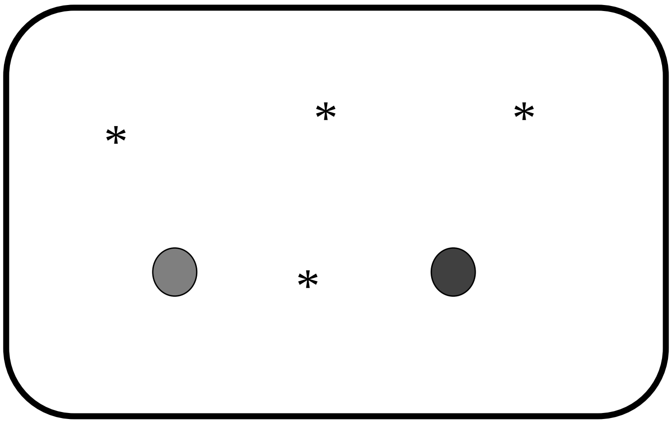
(i) What sign is the charge? Explain how you can tell.

(ii) Explain why the lengths of the vector arrows are not all the same.

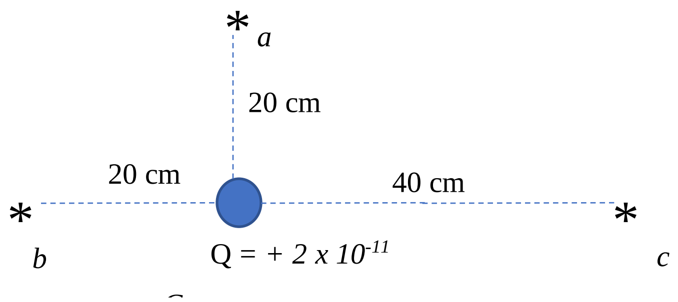
(iii) Another charge, of opposite sign and equal magnitude, is placed at the position shown in the following diagram. ***Construct*** the vectors that show the magnitudes and directions of the electric field of the two charges.



(iv) Use field lines to show the field produced by these two charges.



5. A positively charged object is placed down as shown above. 3 points are marked, a – c, are marked around the charge. The magnitude of the charge, and the distances to the points from the charge are shown on the diagram.

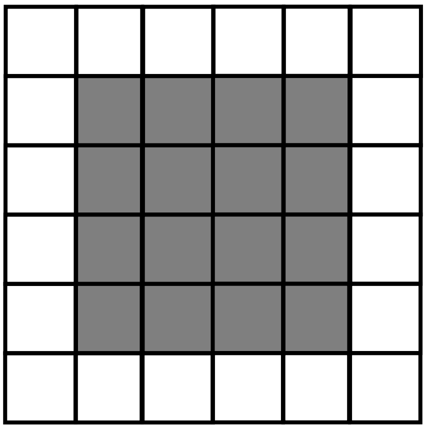


(i) Rank the strength of the electric field, *from lowest to highest*, at the positions *a*, *b* and *c*. Justify your ranking.

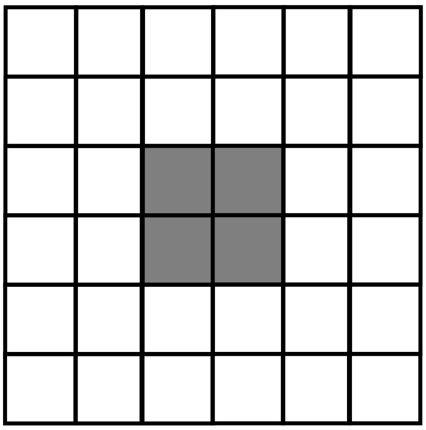
(ii) Show that the electric field at a point around an electric field is given by the formula $E = k\frac{Q}{d^2}$, where $k = \frac{1}{4\pi\varepsilon}$.

(iii) What is the ratio of the magnitude of the electric field at *c* to *a*? Show your workings.

6. A charge is placed down and a square shaped grid is held a distance of 10m from it. 100 electric field lines go through the shaded area of the grid.



(i) At what distance do I need to move the grid from the charge to get the 100 electric field lines to pass through the following shaded area. Explain your answer.



Distance to get 100 lines in this area:

Justification:

Formulae:

You may still want to have your mathematical tables handy, unless I missed something but these are the formulae you should need to complete this test.

$$F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{d^2}$$

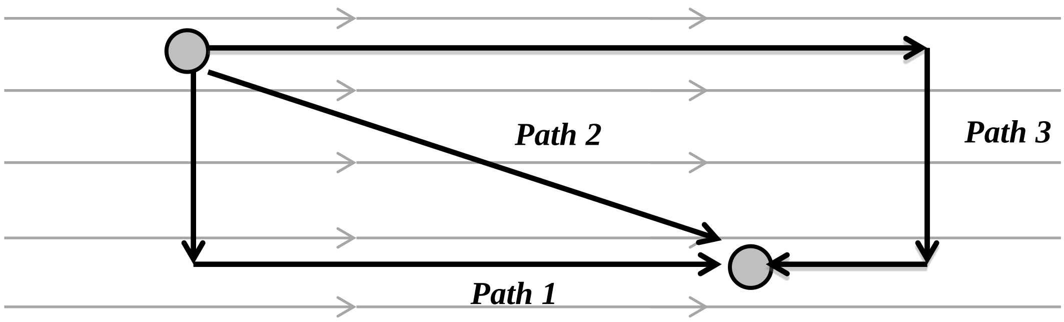
$$E = \frac{F}{q}$$

$$E = \frac{1}{4\pi\varepsilon} \frac{Q}{d^2}$$

Appendix E

Work and potential difference tutorial materials.

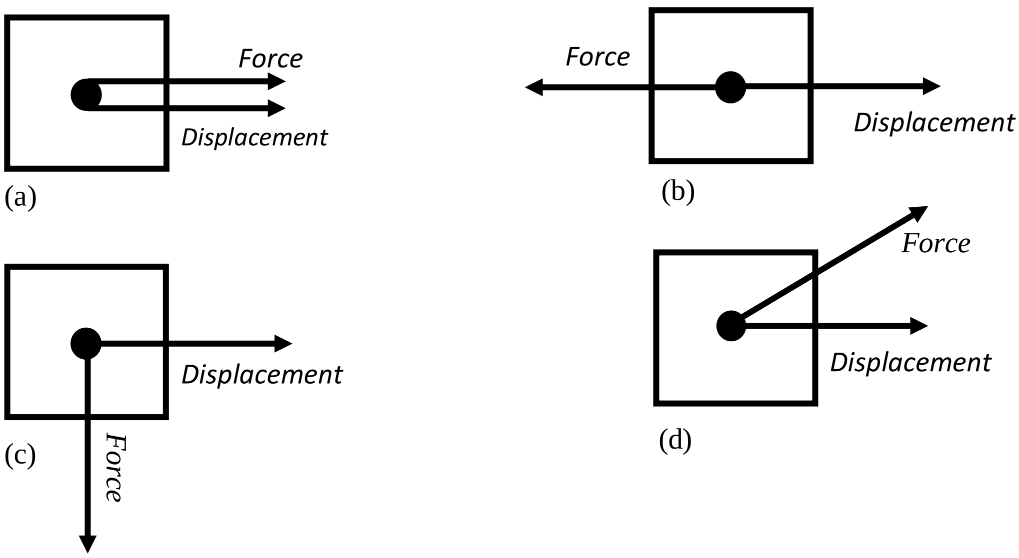
1. The electric field shown is uniform. A positively charged particle can be moved from point *a* to point *b* in one of the three paths as shown. Rank the net work done, by the field, in moving the charge from point *a* to point *b* in the different paths it can take. Justify your ranking.



Ranking:

Justification of ranking:

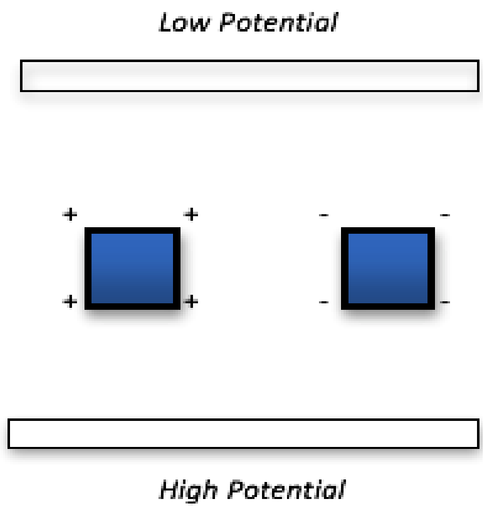
2. Rank the magnitude of the work done for the following pairs, (a) to (d), of Force – Displacement vectors.



Ranking:

Justification for ranking:

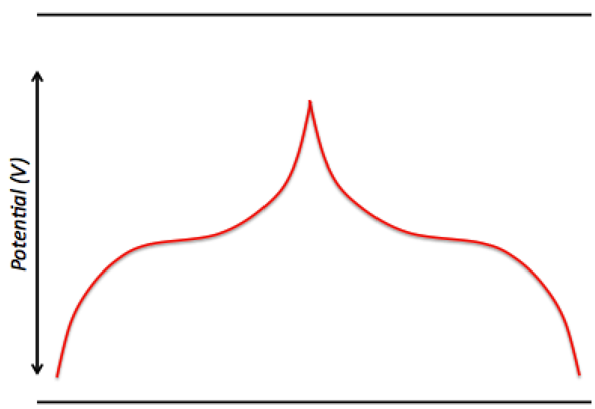
3. A positively charged box and a negatively charged box are suspended between two charged plates, one which has high potential and the other has low potential.



(i) When the positively charged box is released, will it move (a) towards the high potential or (b) towards the low potential. Explain

(ii) When the negatively charged box is released, will it move (a) towards the high potential or (b) towards the low potential. Explain

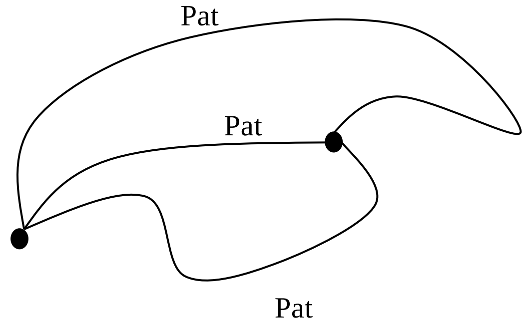
4. On the top line, draw the charges that need to be placed down to show the change in potential as you move from left to right.



Explain why you drew as you did:

I. Distance vs Displacement.

A person can move from point *A* to point *B* using one of the three paths shown.



(i) In which path, if any, is the distance travel greatest?

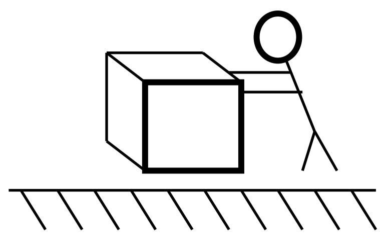
(ii) In which path, if any is the displacement greatest?

(iii) Explain why the distance travelled is not the same as the displacement.

II. Calculating work.

A person pushes a cube shaped block and ground as shown. The block weighs *100 N* and the person is pushes it with a force, F_{Push}, of *50 N*. The block moves a total displacement of *6 m*.

(i) On the diagram, draw vector arrows, to show the direction of the force and the direction of the displacement.



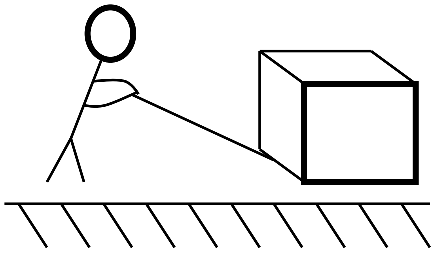
(ii) Calculate the work done by ther person in pushing the block.

(iii) What work is done, if any, on the block by gravity? Explain

III. Resolving a force vector into component vectors.

The person attaches a rope to the block to pull the block, to the left, with a force, F_{Pull}, of 50 N. The rope makes an angle, θ, of 30° with the ground. The block is pulled a displacement of *6 m,* to the left.

(i) On the diagram, draw vector arrows, to show the direction of the force and the direction of the displacement.



(ii) From your answers in (vi), is the work done in pulling the block *positive*, *negative* or *zero*? Explain.

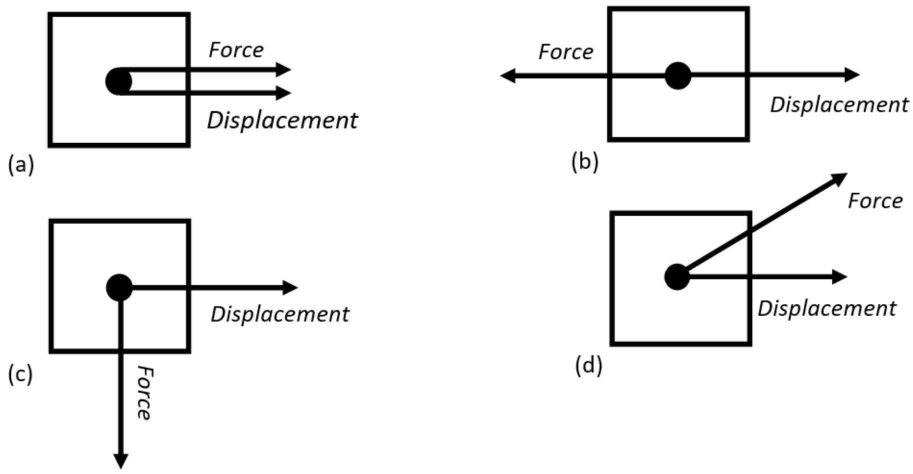
(iii) Copy the force vector into the box to the right. Resolve the vector into it is horizontal, F_x and vectical, F_y, components.

(iv) Which component, if any, contributes *zero* work to moving the block to the left.

(v) Show that the horizontal force moving the block can be given by $F_x = F_{Pull} Cos\theta$

(vii) From part (v), show by multiplying the horizontal force, F_x, component by the displacement, *s*, that the work done on the block is 260 J.

(viii) Rank the magnitude of the work done for the following pairs of Force – Displacement vectors. You may use a ruler to record the relative magnitudes, if necessary. If required, resolve the force vectors in the following into horizontal components.

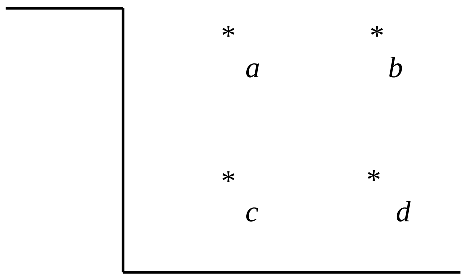


Ranking:

Justification:

III. Work done in a gravitational field.

An object, of mass 1kg, is moved between the points shown in the diagram.



(i) What is the direction of the gravitational force acting on the mass when it is held at *a, b, c* and *d?*

(ii) When the mass is moved from *a* to *c*, is the displacement *in the direction of, against the direction of* or *perpendicular* to the gravitational force?

(ii) Is the work done by the gravitational field, in moving the mass from *a* to *c*, *positive, negative* or *zero*. Explain.

(iii) When the *1 kg* mass is moved from *c* to *a*, does the potential energy of the mass increase, decrease or remain the same.

(iii) When the *1 kg* mass is moved released from *a* to fall to *c*, how does the kinetic energy at *c* compared to it is potential energy at *a*.

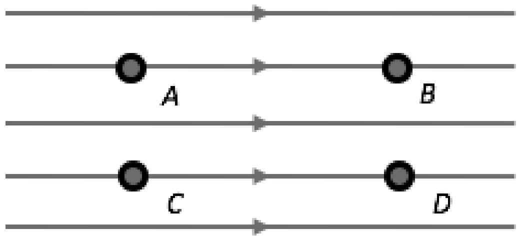
(iv) When the mass is moved from *d* to *b,* is the displacement *in the direction of, against the direction of* or *perpendicular* to the gravitational force?

(v) Is the work done by the gravitational field, in moving the mass from *d* to *b*, *positive, negative* or *zero*. Explain.

(vi) Is the work done by the gravitational field, in moving the mass from *a* to *b*, *positive, negative* or *zero*. Explain.

I. Work done by a field on a charge.

A positive charge (+Q) is placed in a uniform electric field, and is moved in the following paths.



(i) In which paths is the net work done on the charge, by the field, positive, negative or zero. Explain you reasoning.

A to *B*

C to *B*:

D to *C*:

A to *B* to *D* to *C*:

A to *C*:

A to *B* to *C* to *D:*

(ii) How does the <u>*net work done on the charge, by the field*</u>, compare when it moves from *A* to *B* as when it moves from *C* to *D*? Explain.

(Iii) How does the <u>*net work done on the charge, by the field*</u>, compare when it moves from *A* to *D* as when it moves from *A to C to D*? Explain.

Two people are considering what occurs when the charge in moved through the two paths outlined. Their understanding is shown below.

Person 1: *When we move the charge from A to D directly, there is less work done than moving it from A to C to D as we add up the work done moving from A to C directly to the work moving from C to D directly.*

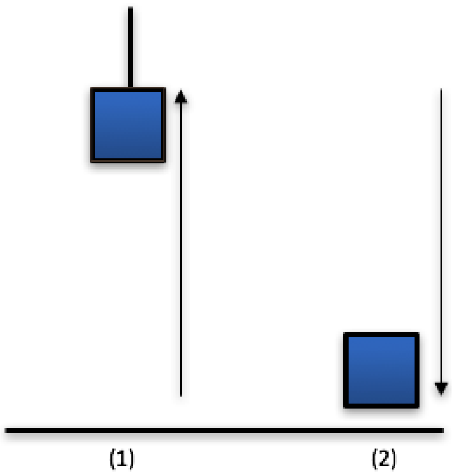
Person 2: *When the charge if brought from A to C and C to D, the displacement has a vertical component which gives zero work. This makes the work done independent of the path taken.*

(vi) Which person do you agree with?

What error in understanding did the other person make?

II. Work done on mass in gravitational field.

A 1 kg box is lifted a distance of 3 m into the air, as seen in (1). The box is then released so it falls to the ground (2). (W = Fs)



(i) Calculate the work done, by gravity, when the 1 kg box falls to to the ground.

(ii) Calculate the work done, by gravity, if the box had a mass of (a) 2 *kg* and (b) 3 *kg*.

(iii) Find the ratio (fraction) of *work done* per *mass of the box* for your answers in (i) and (ii), and write them in their simplest form.

(iv) What do you notice about all of the ratios?

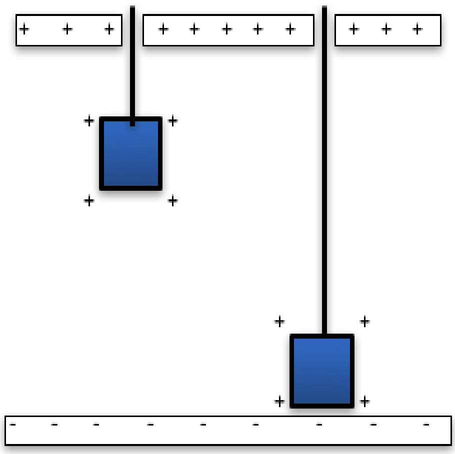
(v) How does the potential energy of the box in (1) compare to the kinetic energy of the ball just before it hits the ground in (2).

(vi) Express your answer from (v) mathematically.

*****Checkpoint****

III. Work done in a charge moving between plates.

A $+1\ C$ box is lifted a distance of $3\ m$ towards a positive plate (3) and then released towards a negative plate (4). The electric field between the two plates is uniform and has a magnitude of 2 $N\ C^{-1}$. ($W = Fs$, $F = qE$)



(i) Calculate the (a) force and then (b) the work done, by the electric field, when the $+1\ C$ box is brought to the negative plate.

(ii) Calculate the work done, by gravity, if the box had a charge of (a) +2 C and (b) +3 C.

(iii) Find the ratio (fraction) of work done per mass for your answers in (i) and (ii).

(iv) What do you notice about all of the ratios?

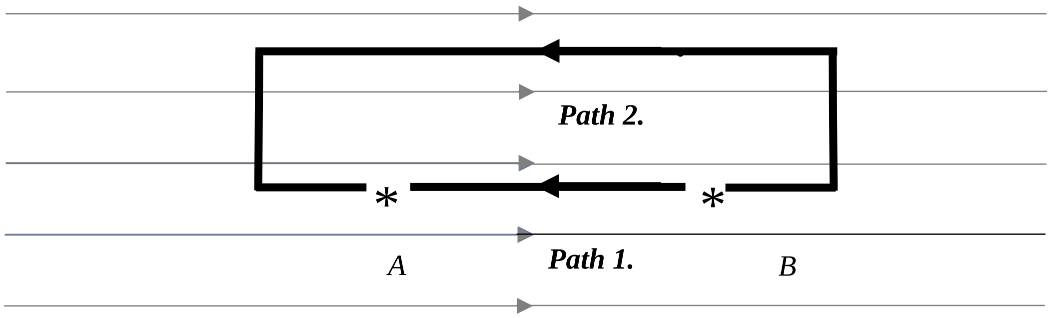
(v) How would you define this ratio, in your own words?

******Checkpoint******

In electricity, there is a special name given to the ratio you just calculated. This is called **potential difference (V),** and is defined as *the work done in moving a unit of charge (+1 C) between two points in an electric field or electric circuit.*

$$V = \frac{W}{q}$$

A uniform electric field is shown below. Two points, A and B, are highlighted. There are two paths between the points, also shown on the diagram.



(i) Explain how you can tell it is a uniform electric field.

(ii) It takes *6 J* of energy to move a -1 C charge from *B* to *A*, along *Path 1*. What is the potential difference between *A* and *B*?

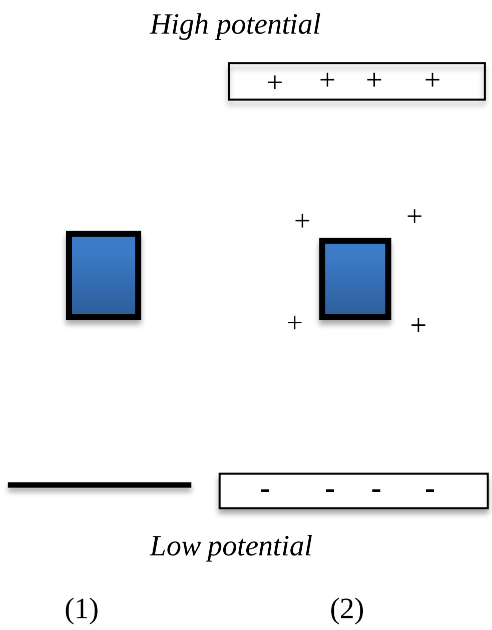
(iii) What is the work done in moving the *-1 C* charge from *B* to *A*, along *Path 2*? Explain your answer.

(iv) How much energy would it take to move a *-4 C* charge?

(v) If I use 36 J moving a charge from *A* to *B*, what is the magnitude of the charge moved?

(vi) If a charged particle of magnitude -*3 C* has a mass of *0.5 g*, determine the magnitude of the velocity it would have at *A* when it travels along *Path 1* from *B to A*. ($W = {}^{1}/_{2}\, mv^2, V = \frac{W}{Q}$)

I. Positive and negative charges in a potential difference.



A box held is held a height above the ground, as seen in (1).

(i) As it is height above ground increases, what change is seen in it is gravitational potential energy?

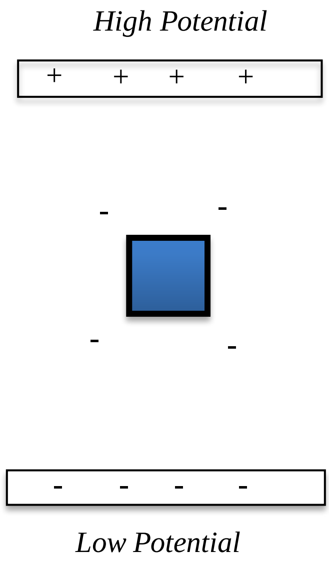
(ii) When it is released, will the box move up, fall down or remain at it is height?

(iiI) From your answer in (i) and (ii), does the box move from (a) low to high potential (b) high to low potential or (c) is not affected. Explain.

A positively charged box is held between a postively charged plate and a negatively charged plate. We associate a high potential with positively charged plate and a low potential with the negatively charge plate.

(iv) When the box is released, will it move towards the positively charged plate, or the negatively charged plate. Explain your answer, ***referring to either the forces involved or the electric field between the plates.***

(v) In terms of potential, is the positively charged box moving from an area of high to low potential or low to high potential? Explain.

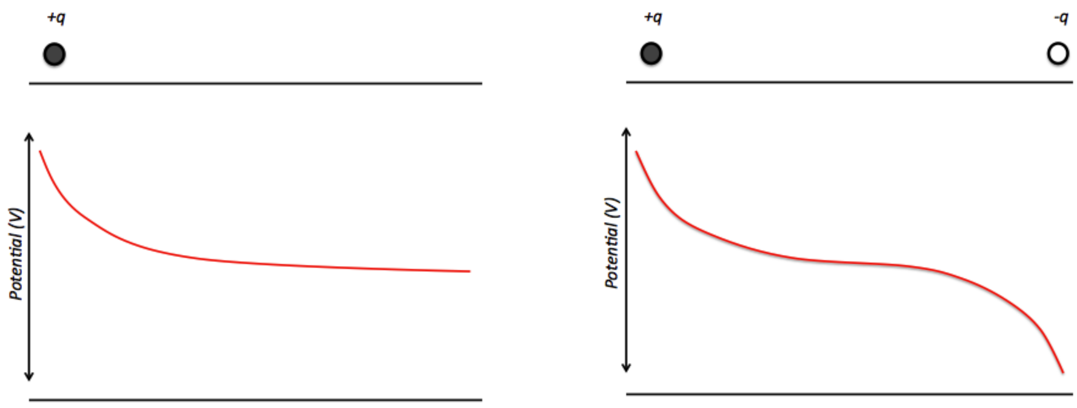


(vi) If the box was replaced with a negatively charged box, which way would it move?

Explanation (refer to potential):

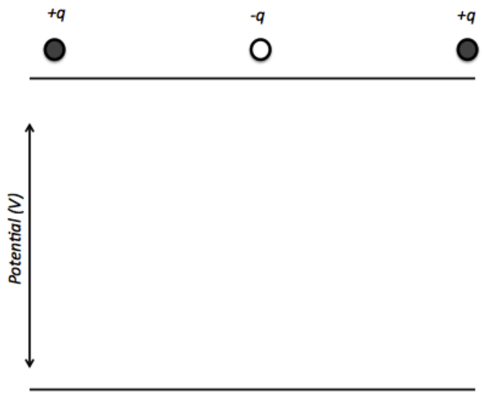
II. Graphing potential.

We have seen that regions of positive charge are considered to have high potential and areas of negative charge are areas of low potential. Consider the following graphs of potential for the setups shown, as you moves from left to right in each.

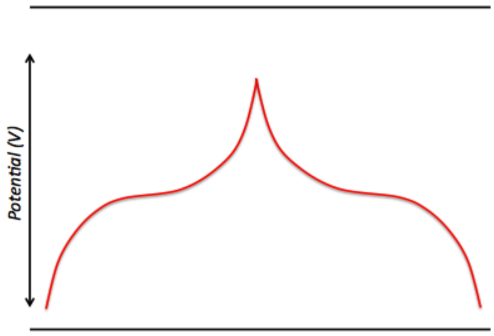


(a) *(b)*

(i) Explain the shape of both graphs as you move from left to right.



(c)



(d)

(ii) Draw on the graph how the potential varies from going from left to right for the setup shown in (c). Explain why you drew it as you did.

(iii) Draw the setup that produces the graph for potential as seen in (d). Explain why you drew the setup as you did.

III. Understanding a battery.

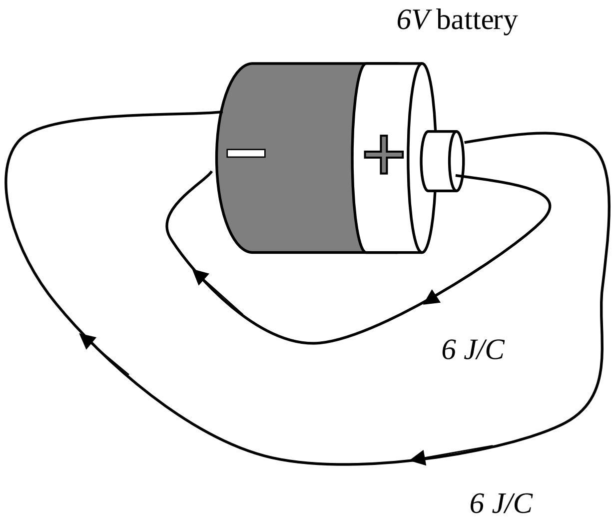
In a battery, there is a positive and a negative terminal. The potential difference that exists between these is usually printed on the battery (typically *1.5 V*, *6 V*, *9 V* and *12 V*). Use your understanding of some or all of the following:

- work,
- potential,
- electric fields,
- electric field lines,
- vectors,
- the behaviour of negative charges in electric fields,

- the behaviour of negative charges between a potential difference

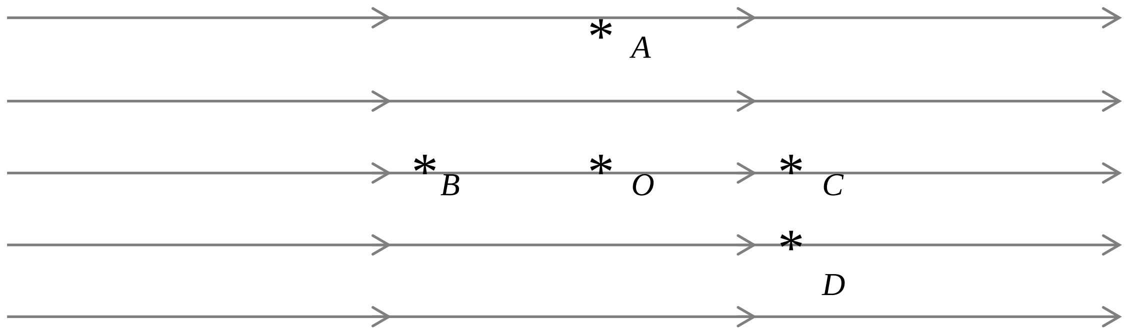
to explain, In the space below, explain why:

3. Current (which is moving negative charge) flows from the negative to the positive terminal.
4. The *work done per charge* in moving current from one terminal to the other is constant, regardless of the length / layout of the wire.



Relevant formulae: $E = \frac{F}{Q}$ $W = F.s$ $V = \frac{W}{Q}$

1. A ***negative charge*** is placed in an electric field as shown, at position *O*. It can be moved to any of the positions marked on the field as shown.



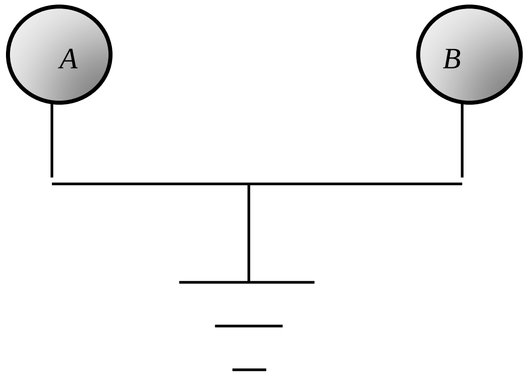
(i) Draw in the direction of the force acting on the negative charge, when placed at *O*.

(ii) State whether the work done in each case is *positive, negative or zero,* when ***the negative charge*** is moved from *O* to *A*, from *O* to *B,* from *O* to *C* and from *O* to *D*. Explain.

(iii) How does the work done, by the field, in moving the charge from "*O* to *C*" compare moving from "*O* to *D*" and "*O* to *C* to *D*?" Explain.

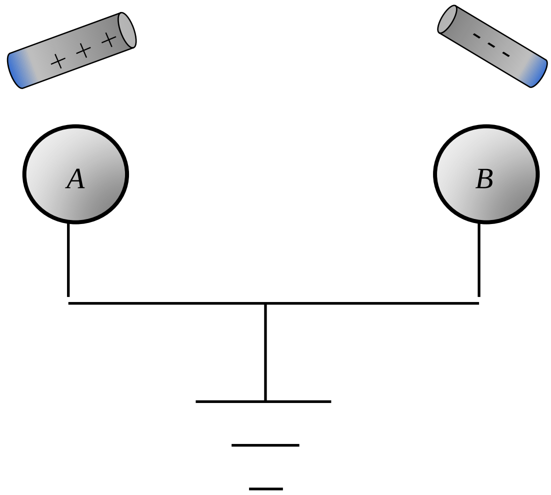
(iv) A student says that the potential difference between the B and O, is the same as the potential difference between O and C. Do you agree with this student? Explain why you think they are correct / incorrect.

2. Two conducting balls are connected to the ground as shown in the diagram.



(i) How does the potential at *A* and *B* compare to each-other and the ground. Explain.

A positively charged rod is held about *A* and a negatively charged rod is held above *B*.



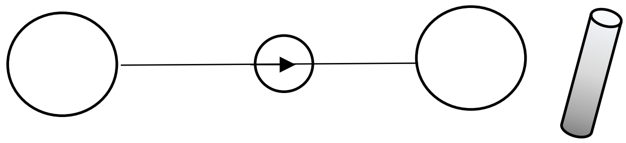
(ii) What effect does this have on the potential of *A*, initially.

(iii) After some time passes, what type of charge will build up on *A*. Explain, **referencing the potential** on *A* you gave in (ii).

(v) What effect does this have on the potential of *B*, initially.

(vi) After some time passes, what type of charge builds up on *B*. Explain, **referencing the potential** on *B* you gave in (iv).

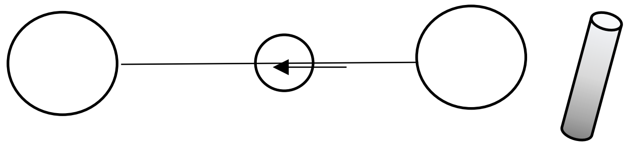
3. Two metal domes are connect with a conducting wire. A device with shows the direction of the movement of charge is placed on this wire.



When a charged rod is placed beside the rightmost dome, the negative charge begins to move, in the initial moments, in the direction as shown on the device.

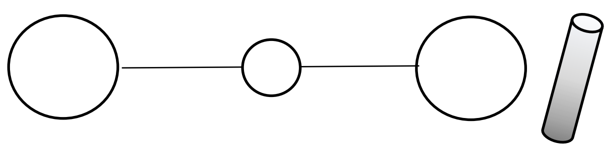
(i) Using this information, compare the potential on the left dome to the potential on the right dome. Explain your comparison.

The rod is taken away replaced with a rod that causes the negative charge to move, in the initial moments, in the direction as shown on the device.



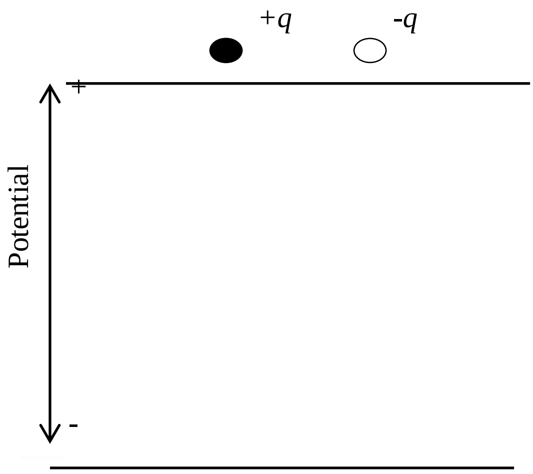
(ii) Using this information, compare the potential on the left dome to the potential on the right dome. Explain your comparison.

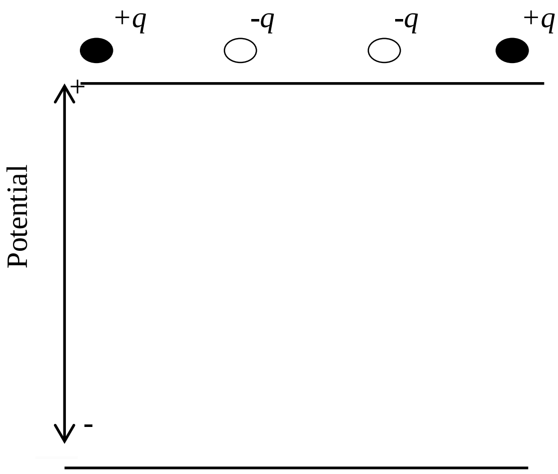
After a long period of time has past, the device registers that there is no electric charges moving, as shown in the diagram.



(iii) What does this tell you about the potential of both domes? Explain.

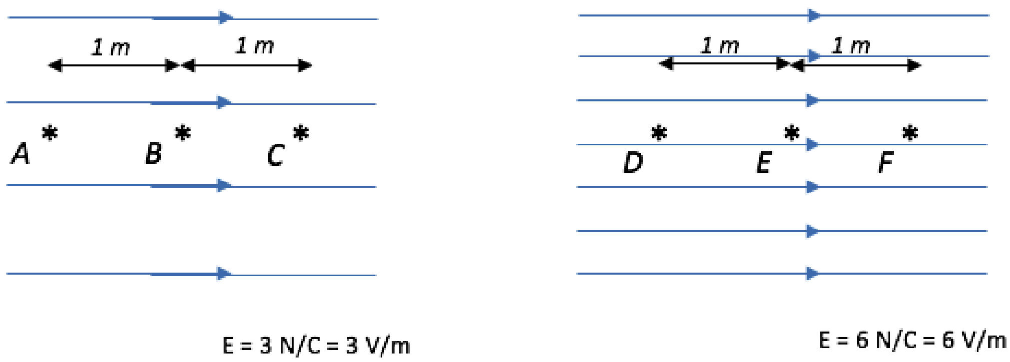
4. Draw on the graph how the potential varies from going from left to right for the setups shown. Explain why you drew it as you did.





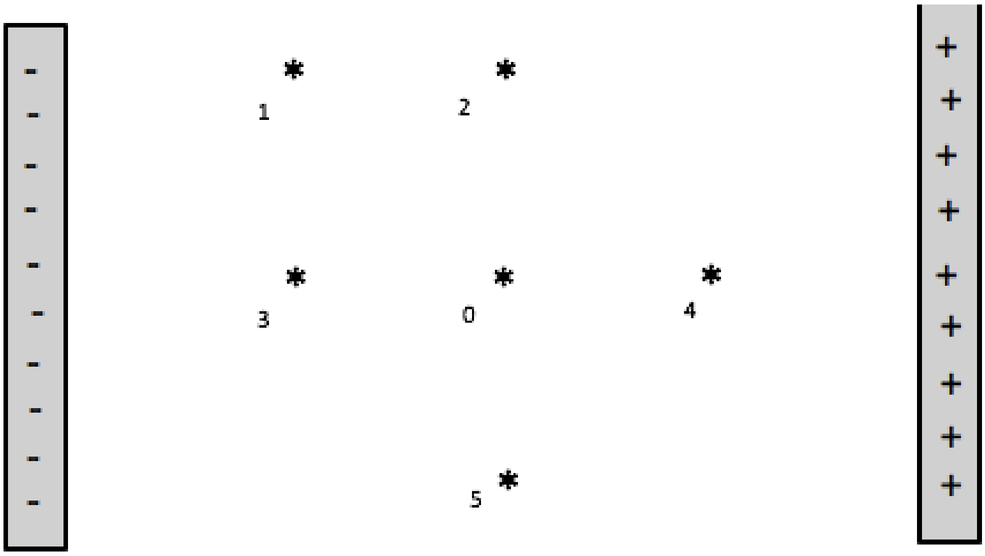
Explanation:

5. Two electric fields are shown in the diagram below. Both fields uniform. The field strength and distance between the points is shown on the diagram.



(i) Rank the Potential difference, from highest to lowest, between AB, AC, DE, and DF. Justify your answer using whatever reasoning you think is necessary (there are multiple ways to justify your answers correctly – you need only use one. If you choose to try use calculations, use a test charge of magnitude +1 C)

6. A positively and negative charged plate at held across from each-other. A uniform electric field exists between these plates. A number of positions are shown between them.



(i) If a positive charge is laid down at position 0, rank the magnitude, from lowest to highest, as it is from along the following paths:

[01], [02], [03], [04] and [05].

[01] means the particle starts at 0 and moves directly to 1. You may draw on the diagram in any manner you see fit.

Ranking:

Justification:

(ii) Rank the potential difference, from lowest to highest, between the points [01], [02], [03], [04] and [05], where [01] means the potential different between the point 0 and the point 1.

Ranking:

Justification:

Appendix F

Legend for codes used in line plots to display extent of conceptual change.

Code.	Learning outcome	Figure.
V 1	Ranking vectors based on magnitude.	4.47
V 2	Use of vector constructions.	4.47
V 3	Consideration of vector components in vector addition	4.47
In 1	Graphically representing inverse square relationship.	4.47
In 2	Reasoning for change in area using scale model.	4.47
In 3	Use of inverse square relationship mathematically.	4.47
Fl 1	Using field line density to determine relative field strength.	4.47
Fl 2	Drawing vectors are points based on field line diagram.	4.47
Fl 3	Reasonable sketches of trajectories of bodies under influence of a field.	4.47
V 4	Variation of electric field strength represented with vectors	5.43
V 5	Use of superposition principal using vectors and electric fields.	5.43
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Developing and assessing student's conceptual understanding of electrostatics in upper secondary Physics

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Declaration

I hereby certify that this material which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work, that I have exercised reasonable care to ensure the work is original and does not to the best of my knowledge breach any law of copyright, and has not been taken from the work of others, save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed:

ID No.: 54312678

Date:

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Publications.

1. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. (2016) *Developing student understanding of attraction between charged and uncharged objects in a lower secondary setting.* School Science Review, No. 363, pg 101-108.

2. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. (2016) *Helping students explore concepts related to the electric field at upper secondary level science education.* Proceedings of GIREP-EPEC 2015, pg 195 – 201.

3. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. (2017) *Developing second level student's understanding of the inverse square law, and electric fields.* Proceedings of GIREP-ICPE-EPEC 2017. (Currently under review).

Conference presentations.

1. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *The application of tutorial based worksheets to enhances student understanding of static electricity and magnetism at lower second level education.* SMEC, 2014.

2. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *Helping students explore concepts related to the electric field at upper secondary level science education.* GIREP-EPEC, 2015.

3. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *Progress and difficulties in student's understanding of vector and field concepts in electrostatics: A qualitative study of a small group of upper secondary students.* SMEC, 2016.

4. Moynihan. R., van Kampen. P., Finalyson. O., McLoughlin. E. *Developing second level student's understanding of the inverse square law, and electric fields.* GIREP-ICPE-EPEC, 2017.

Abstract

Richard Moynihan - 54312678

Developing and assessing student's conceptual understanding of electrostatics in upper secondary Physics

This thesis presents research studies carried out with upper secondary level physics students (n=14) over a timeframe of four months, with the aim of promoting their conceptual understanding of the electrostatic concepts, Coulomb's law, electric fields, and work and potential difference. Teaching and learning materials were developed that adopted Inquiry Based Learning (IBL) with structured inquiry tutorials and Multiple Representations (MR) approaches to examine the extent of conceptual change in the student's understanding of these topics. The research utilizes a case study methodology, with various sources of evidence recorded. These sources include pre-test and post-test comparison, retrieval of student artefacts used during lessons, audio recordings of student's discussions during lessons, teaching and learning interviews and teacher field notes.

In the research design, vector concepts, the inverse square law and field line representations were identified as concepts that students need to be familiar with, to develop their understanding of electrostatics. The students completed structured inquiry tutorials for these topics in the context of mechanics. The findings of these studies show that conceptual change generally occurred in the student's understanding, for these three concepts, with evidence of conceptual extinction and extension occurring. However, the findings of these studies show students encountered difficulties in transferring their understanding of vectors, the inverse square law and field lines to the electrostatic context. By revisiting these concepts during the electrostatic tutorial lessons, evidence of conceptual exchange and extension was observed in the student's understanding of Coulomb's law, the electric field and work and potential difference.

The research presents evidence that the use of structured inquiry tutorials and multiple representations can be an effective approach in promoting conceptual change in student's understanding of electrostatics in upper secondary physics. The overall findings of this research suggest that this approach may have significant benefit for the teaching and learning of other physics / science topics at both upper secondary and lower secondary levels.

Chapter 1. Introduction

As a branch of science, physics is a human endeavour to study the nature of energy, matter and their interactions. It allows for the construction of models, theories and laws that explain observable phenomena on scales as small as the sub-atomic to as large as galaxies and beyond. Physics helps generate fundamental knowledge needed for technological progression that can directly be used to develop new products or influence the progression of other disciplines. In Ireland, physics and physic-trained people underpin a range of sectors from medical technologies to ICT and web-services (Institute of physics, 2012). There are many branches of physics; the ones encountered by upper secondary school students in Ireland are optics, thermodynamics, mechanics, electricity and electromagnetism with introductions to quantum and particle physics. These branches of physics can give students a fundamental foundation that they can use as a platform for further study in more advanced fields of science, technology, engineering and mathematical fields.

Electrostatics concepts are important domains of science teaching that deserve attention. In Ireland, the Chief Examiner's Reports (2013; 2010; 2009; 2008; 2005a; 2005b) for physics at senior level and science at junior level show that students have above-average difficulties with electrostatics. If possible, they tend to avoid questions relating to electricity, as seen in the reports. As the domains of static and current electricity are intrinsically linked, the teaching and learning materials developed in this thesis aim to develop student's understanding of electrostatics concepts. The expected outcome of this approach is that students can transfer their understanding of these concepts to concepts in current electricity, such as the behaviour of current, potential-difference and resistance in circuits. This can aid the development of the student's understanding of these domains, instead of treating electrostatics and current electricity as two separate and exclusive phenomena.

The use of multiple representations of these physics concepts has been recognised as beneficial to learners. Hestenes (1996) explains that a complete understanding of a model requires a student to be able to coordinate information between multiple representations to complete their understanding. Although this research does not employ modelling methodology explicitly, the use of a structured inquiry approach is used to support student's development of models. The utilisation of multiple representations in this work aims to develop similar gains in the student's understanding of electrostatics to those that a modelling methodology would aim to produce. Jackson, *et al*., (2008) note that the use of a modelling method involves students developing conceptual understanding through graphical and diagrammatic representations, before moving onto the algebraic treatment of problem-solving. In the design of the teaching and learning materials in this research, this instruction method is employed in numerous lessons, to aid student's conceptual development.

Using multiple representations in developing education materials can help achieve other goals. Student difficulties that may not be apparent in one representation may easily be assessed in others. McDermott, *et al*., (1986) showed students can have a relatively good grasp of concepts, but numerous issues of interpretation can cause confusion when interpreting graphs. This inability to successfully translate between representations can hinder student's ability to completely understand a concept. Kozma (2003) notes that that experts are fluid in their transitions between different representations when problem solving, while novices typically use one or two. Students focus on surface features of the concept and don't develop understanding at a deep level. This suggests that helping students translate between representations could enable them to develop expert–like understanding, and tackle problems using multiple methods.

1.1. Context and background

This thesis concerns the teaching of electrostatics in the Irish secondary system. Irish secondary education comprises a Junior Certificate and a Leaving Certificate programme. The Junior Certificate programme is a three-year programme, in which students typically aged 12-15 take ten subjects, including Science. In some schools, Science is mandatory, in others it is optional. At the time this research was completed, Junior Certificate students completed the 2003 science syllabus (NCCA, 2003). The Leaving Certificate programme is a two-year programme, in which students typically aged between 16-18 take seven subjects. Leaving Certificate Physics is optional to all students who undertake the programme, subject to availability of facilities and suitably qualified teachers. Leaving Certificate Physics follows the 1999 syllabus (NCCA, 1999), which is briefly discussed in this section. The physics course is set at two levels; ordinary level and higher level. The difference between these two levels is the following:

- Ordinary level consists of a defined set of concepts. Higher level consists of the ordinary level concepts, additional concepts and a particle physics or applied electricity optional section.
- Ordinary level provides an overview of physics and its applications, while at higher level, there is a deeper, more quantitative treatment of physics.
- At ordinary level, equations must be used, while at higher level, some equations must be used and derived. Calculations for higher level are more challenging that those found at ordinary level.

(NCCA, 1999)

As electrostatics concepts are the focus of this research, Table 1.1 provides an extract from the physics syllabus and a discussion of the extract and a commentary of an examination question, shown in Figure 1.1, from the state exams commission (SEC, 2015) is presented.

In recent years, the teaching of Science and Physics in Ireland has undergone changes in both classroom syllabus and examination style. First examined in 2002, the Leaving Certificate Physics Syllabus, reduced the content to be covered in mechanics and electrostatics. This was to accommodate for the introduction of a new section on modern physics. The Junior Certificate Science syllabus (NCCA, 2003), first examined in 2006, also reduced the amount of content to be covered, to allow time for an investigative approach in the delivery of the syllabus. This was justified under the rationale that it allowed for the allocation of class time to allow for an investigative method of learning, but it has been seen that typically, traditional methods of teaching still prevailed in classrooms (Wemyss, 2009).

In developing teaching and learning materials for this research, the learning outcomes for the students, as listed in the Leaving Certificate Physics syllabus (NCCA, 1999) were used to consider what concepts should be addressed. The syllabus lists the topics to be completed in the Leaving Certificate Physics course, the depth of treatment the topics are to be taught to, and suggested activities and examples of science and technology that are appropriate for the topics. There is no prescribed methodology suggested for the teacher to use in completing the syllabus. For instance, there is no indication of suggested explanatory models to understand the concepts underpinning Coulomb's law and the electric field. The Leaving Certificate Physics syllabus is designed to be completed over two academic years, in approximately 180 hours of class contact time. This contact time generally breaks down to just under three hours a week to include laboratory work, teaching and learning of content and concepts and practice solving numerical problems in physics. The syllabus is assessed in a three-hour written examination, which is drafted by the State Examinations Commission and is corrected anonymously. The Physics examination papers give little weight to student's conceptual understanding of the content covered. Typically, students are given mathematical questions which are typically solved using algorithmic problem-solving methods. Any questions that are conceptual tend to revolve around the direction of an electric field in a given setup, with one or two charges, or explaining how Coulomb's law is an example of an inverse square relationship. Table 1.1 presents an extract of the electric field section of the Leaving Certificate Physics syllabus. Items denoted in bold indicate that they are part of the higher-level physics course only.

Topic	Depth of treatment	Suggested activities	Science, Technology and Society
Static Electricity			
1. Force between charges.	Coulomb's law - $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$ - As an example of an inverse square law. **Forces between collinear charges.**	**Appropriate calculations.**	
2. Electric fields.	Idea of lines of force. Vector nature of electric field to be stressed. **Definition of electric field strength.**	Demonstration of field patterns, using oil and semolina, or other method. **Appropriate calculations – collinear charges only.**	Precipitators Xerography Hazards: effect of electric fields on integrated circuits.
3. Potential difference	Definition of potential difference: work done per unit charge to move a charge from one point to another. Definition of the volt. Concept of zero potential.	Appropriate calculations.	

Table 1.1 Extract from Leaving Certificate Physics syllabus (NCCA, 1999) detailing static electric learning outcomes.

When considering student understanding of potential and potential difference, it is defined in the syllabus as work done when moving a unit of charge from one place to another in an electric field. However, it has been shown that students at third level have difficulties in the understanding of concept related to work (Doughty, 2013). This has also been seen to be a topic of difficulty to students exploring potential difference (Hazelton, 2012), so it is not unreasonable to speculate that some of the difficulties could be allayed by helping students to develop a conceptually sound understanding of work and potential at second level. Figure 1.1 presents an extract from the 2015 Leaving Certificate Physics examination paper, which illustrates a number of these points.

Define electric field strength. (6)

Both Van de Graaff generators and gold leaf electroscopes are used to investigate static electricity in the laboratory.

Draw a labelled diagram of a gold leaf electroscope.
Describe how it can be given a negative charge by induction. (20)

A Van de Graaff generator can be used to demonstrate point discharge.

Explain, with the aid of a labelled diagram, how point discharge occurs.
Describe an experiment to demonstrate point discharge. (18)

The polished spherical dome of a Van de Graaff generator has a diameter of 40 cm and a charge of +3.8 μC.

What is the electric field strength at a point 4 cm from the surface of the dome? (12)

Figure 1.1. Electrostatics question from Leaving Certificate Physics examination, 2015. (SEC, 2015)

The initial question requires students to define electric field strength, in which they need to recall that it is the force experienced per unit charge in an electric field. Acceptable answers to this question were in written word, or mathematical notation in which the variables were explained. The following pair of questions required students to sketch a diagram of a gold leaf electroscope and describe how it can be given a negative charge by induction. While the latter question can be answered by understanding the principle involved in this process, both questions can be answered by recalling rote-learned material. This also applies to the following pair of questions which required the students to describe the process of point discharge and describe an experiment to demonstrate the process. The final question required students calculate the electric field strength at a point from the surface of the surface of the Van der Graaff generator. Relevant formulae for this question are provided to the students in the examination. The student must correctly determine the distance from the point to the centre of the Van der Graaff generator, and otherwise, substitute variables into the formulae and evaluate the expression on their calculators. A student who has spent time rote-learning the theory for the initial questions and practised the substitution and calculation process could score highly on this question, without demonstrating the depth of their conceptual understanding to the examiner marking the paper.

1.2. Research approach

In this research, the researcher is also the teacher that develops the teaching and learning materials used to promote the student's understanding of electrostatics. At the time the lessons in this thesis were implemented, I had been employed as a full time second level teacher for 8 years. In this

time, I gained experience in teaching multiple cohorts of students the Junior Certificate Science, Junior Certificate Mathematics and Leaving Certificate Physics syllabi. As a teacher, I developed classroom activities that in which the students would predict the outcome of observable phenomena, share reasons for their predictions, discuss alternative outcomes with their peers. When completing quantitative exercises, I would ask students to consider whether numerical answers appeared reasonable, based on their understanding of underpinning theory they were applying. As the second level syllabi were prescriptive in the material that is to be learned, opportunities to develop lessons in which the students were given a high degree of autonomy in their learning were rare. This research provided an opportunity to develop a sequence of such lessons and gather data on the student's understanding of electrostatics.

The aim of this research is to develop a suite of research-informed and research-validated educational materials to improve student learning of electrostatics concepts, by improving their conceptual understanding of Coulomb's law, the electric field, work and potential difference. The teaching and learning materials employed during the tutorial lessons in this research are, unless referenced otherwise, of my own original design with the primary inspiration for their design coming from *Tutorials in Introductory Physics* (McDermott, *et al.*, 2003) and *Conceptual Physics* (Hewitt, 2009) In this research, the students completed tutorial lessons in groups for three/four, in which they explore different concepts. This allows opportunities for peer tuition between the students, as they discuss and debate different ideas relating to the content in the tutorials and reach conclusions that the group agrees to. The teacher gets real-time insight to student ideas about the concepts covered in the tutorial, as they circulate the classroom, reviewing the student's responses and using probing question. This is discussed in-depth within the specific electrostatics topics covered in chapters four, five and six. Analysis of student's responses and worksheets also allows the opportunity to critically evaluate the efficacy of the tutorials and for redesign of the materials. The theory underpinning this style of research is discussed in chapter 3. There has been no published research completed on the teaching and learning of these electrostatics topics in the Irish second level context, but the use of activity and inquiry learning, has been shown to be useful in the effective teaching of science in Ireland (Wemyss, 2009; Broggy, 2010; Flynn 2011).

The design of the educational materials utilises a multi-representational approach (discussed in section 2.2.3), to promote the student's development of their conceptual understanding. Representations are anything that symbolize an object, a collection of objects, interactions, etc (Rosengrant, *et al*., 2007). Different representations can be used to display different information and can have various levels of difficulty for learners to interpret information from. Ainsworth (1999) states when learners encounter multiple representations related to a topic, the different representations in which the topic is presented allows for different functions in learning. They can show different processes, different information, constrain the learner's interpretation of the material presented and enable them to construct deep understanding. Using multiple – representations,

abstract concepts can become more accessible to learners (Dienes, 1973). Kozma (2003) notes that experts can transition from one representation to another without difficulty, while novice learners struggle with this skill. The use of multiple representations in lessons can help learners develop this skill. For instance, learners may initially struggle to understand a relationship displayed in an equation, but scaffolding using tabular data and graphs can allow students to consider common characteristics between the representations and equations and to establish the relationship in the symbolic form (Project Maths, 2011). To develop student's understanding of Coulomb's law and the electric field, the representations utilised were vectors, field lines and multiple representations of the inverse square law, which were used to develop their understanding of the properties of both topics and relate the concepts to mathematical formulae. The inverse square law, as a mathematical function, allowed for opportunities to employ the use of diagrammatic model, tabular data and graphical representations. The use of field lines, vectors, diagrammatic models and graphs were employed in the lessons for work and potential difference.

1.3. Research Questions

As presented in Sections 1.1. and 1.2, there are difficulties encountered by students developing their understanding of vector concepts, the inverse square law and field lines. To help promote student's understanding of these topics, structured inquiry with multiple representations was used. Therefore, this thesis was designed to focus on the following research questions (RQs).

RQ 1. To what extent does the use of a structured inquiry approach develop student understanding of vector concepts?

RQ 2. To what extent does the use of a structured inquiry approach develop student understanding of the inverse square law relationship, by employing multiple representations?

RQ 3. To what extent does the use of a structured inquiry approach develop student understanding of the field, when utilising field line representations?

These first three research questions address these topics outside of an electrostatic context. The lessons on these topics were completed during the mechanics section of the Leaving Certificate Physics course. The student's understanding of the application and transfer of these topics to an electrostatic context is addressed in the following two research questions.

RQ 4. To what extent does the use of a multi-representational structured inquiry approach develop student understanding of Coulomb's law and electric fields?

RQ 5. To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

Chapters 4, 5 and 6 address these questions, and discuss the considerations that were employed in drafting each of these research questions. The considerations were used to both design the instructional materials and construct a boundary to which these questions could be answered. An overview of the evidence collection methods employed to address these questions is presented in the following section.

1.4. Research Design

In designing this research, it was decided that the students would encounter vector concepts, the inverse square law and field lines outside of the context of the electrostatic topics, and then apply the concepts during the later tutorial lessons. This in line with lesson sequences for how the Leaving Certificate Physics course was delivered to student groups taught in years prior to this research. Student groups first encountered vectors and the inverse square law in mechanics, as an introductory topic and as a concept related to Newton's Universal Law of Gravitation respectively. As field lines can be applied to gravitational contexts, this representation was also explored by the participants of this research in this topic. This allowed the students to develop their understanding of the three representations in relatively familiar contexts. They then later applied the representations to electrostatics to develop their understanding of the Coulomb's law, the electric field, work and potential difference. Figure 1.2 illustrates the design of the lesson sequence employed over the course of this research.

Students commence the Leaving Certificate Physics programme, at age 15-16 years after they have completed a three-year Junior Certificate programme (depicted in blue in Figure 1.2). As part of the Junior Certificate Science syllabus, students explore forces, motion and static electricity (NCCA 2003). In the Junior Certificate Mathematics syllabus, they explored algebra operations, linear and quadratic patterns, trigonometric ratios and graphing functions (NCCA, 2012). These Junior Certificate topics lay a foundation for Leaving Certificate Physics. The prior learning of the students is explained in-depth in Section 3.5.

Figure 1.2 shows how I sequenced core syllabus topics relevant to this research. The five concepts, depicted in green, represent the topics addressed by this research. The sequence of teaching the first three of these concepts were first introduced to the students while teaching of some different

topics. The final two are specific to teaching the electrostatics section of the Leaving Certificate Physics syllabus.

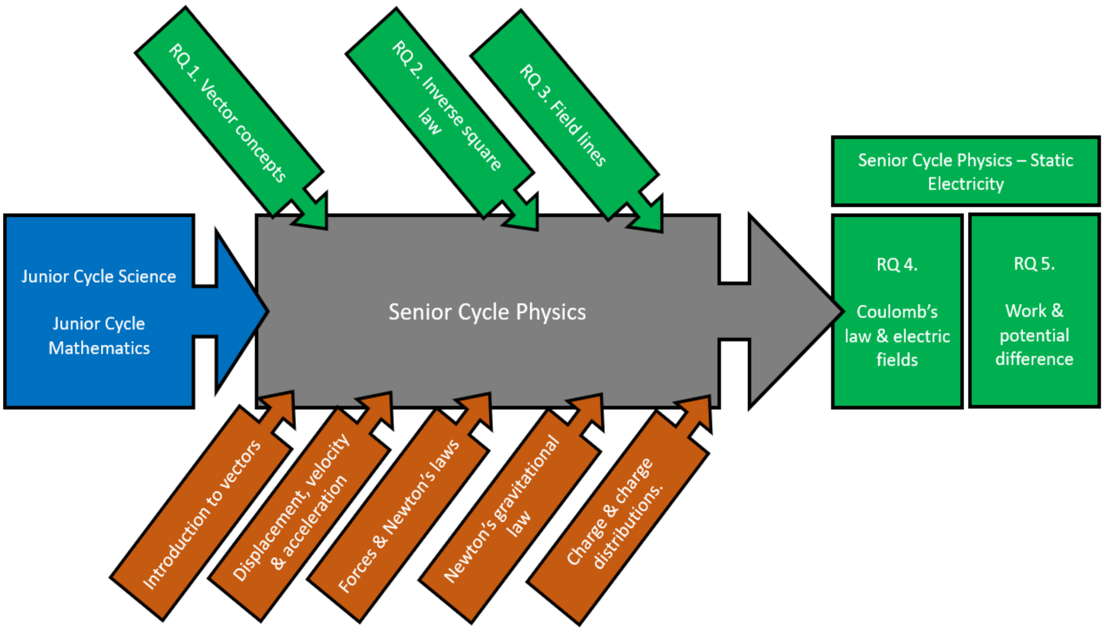


Figure 1.2. Leaving Certificate syllabus topics relevant to this research.

In my classroom, I started by defining vectors and basic mathematical operations with them. The first tutorial lesson was implemented after this initial introduction, in which students were given the opportunity to develop their understanding of vector magnitude, vector addition and vector components. This allowed for evidence collection to address RQ 1. Upon completion of the vector concepts tutorial, the students applied these concepts to the context of motion. The students explored the concepts of displacement, speed, velocity, and acceleration. They practised quantitative problems involving the three equations of linear motion. They used $s = ut + {}^{1}/_{2}\,at^2$ to experimentally calculate the acceleration due to gravity, by using apparatus designed to record the fall-time as a ball bearing is in free-fall. Upon completion of motion, I gave them a lecture-style introduction to momentum, law of conservation of momentum, forces and Newton's three laws of motion. The students were presented with definitions, formulae, videos and scenario's involving collisions between bodies in which they were to predict the outcome based on their understanding of the topics. The students also experimentally demonstrated Newton's second law and the conservation of momentum. Motion, forces and momentum are treated as 2-dimensional quantities but typically only 1-dimensional calculations are completed. The students completed various problem-solving exercises involving these quantities before being introduced to their next topic, gravitational forces, where the second tutorial lesson was implemented, the inverse square law.

This point was chosen as Newton's gravitational law is the first opportunity to apply an equation of the form $y = k\frac{1}{x^2}$ to a physics context. The students would not have used an equation of

this form in their prior formal education, so the tutorial was designed to employ a multi-representational format to allow the students to develop conceptual understanding of the inverse square law. This allowed for evidence collection to address RQ 2. After the tutorial, they applied their understanding to Newton's gravitational law and completed mathematical exercises.

Proceeding from Newton's gravitational law, the students completed a tutorial on field lines, in a gravitational context. While field line representations are not required for the Leaving Certificate Physics course for gravitational contexts, this presented an opportunity for students to use the representation before learning the electrostatics topics in this research. It also allowed for students to differentiate the behaviour of forces, acceleration, velocity and displacement in a gravitational context, and evidence was collected to address RQ 3.

In the interim between the field line tutorial and the initial lessons on charge and charge distributions, lessons on remaining mechanics topics were completed. As these topics were outside the scope of this research, they do not appear on the lesson sequence shown in Figure 1.2. When the students completed the introduction to charge and charge distributions, they completed the tutorials on the electrostatic topics. This allowed for evidence collection to address RQs 4 and 5. These topics were presented in a series of tutorial lessons. The students were required to employ their understanding of vector concepts, the inverse square law and field lines to Coulomb's law and electric fields. They also were required to apply vector concepts and field lines to work and potential difference.

This section overviewed the overall research design as the tutorial lessons were implemented into the student's Leaving Certificate Physics course. The next section details the overall structure of the thesis.

1.5. Structure of the thesis

This section briefly outlines the structure of this thesis. Following this introductory chapter, chapter two presents the theoretical basis for the research and presents the literature on developing student's conceptual understanding in physics, the use of inquiry learning in physics, difficulties encountered by students in vectors, the inverse square law, field lines and their transfer to electrostatics and potential difference, how students learn and a review of various inquiry teaching and learning materials that were used to influence the design of the tutorial lessons employed in this research. Chapter three discusses the research methodology utilised in this research. It discusses case study methodology, justifications and limitations for the use of case studies and the various sources of evidence collected for analysis are identified. This is followed by the implementation

methodology, which illustrates what a tutorial lesson looks like and why it was chosen to be used in the research lessons. The chapter finishes with an overview of the analysis methodology used and a description of the participants in the study.

Chapters 4, 5 and 6 present student's developments and understanding of vectors, the inverse square law, field lines, their application to Coulomb's law, the electric field, work and potential difference. Chapter 4 presents the results of the tutorial lessons in the topics of vectors, the inverse square law and field lines. For each of these topics, a narrative and discussion of the pre-test results, the lesson tutorials, homework assignments and the post-test results are given. These were composed using the sources of evidence presented in Chapter 3. These are followed by a discussion comparing the results, highlighting evidence of student development and persistent student difficulties. Chapter 5 uses a similar structure to present the findings from the tutorial lessons used during Coulomb's law and the electric field. In addition to the student's understanding of these topics, the chapter discusses the student's ability to transfer and apply vectors, the inverse square law and field lines to these two topics. There is also some discussion on the student's ability to transfer between vector and field line representations. Chapter 6 follows the same structure as Chapters 4 and 5 but looks at students understanding of work and potential difference in an electrostatics context.

Chapter 7 presents the final conclusions of this research, in which answers to the five research questions, presented in Section 1.4 are discussed. Implications for teaching and avenues for further research are identified.

Chapter 2. Research Basis

2.1. Introduction

In this chapter, a review of literature detailing how students learn concepts in Physics is presented. The initial section of this chapters discusses how students learn, discussing the information processing model, constructivism and the use of scaffolding in teaching and learning. This is followed by how student develop conceptual understanding and the use of Inquiry as a method to promote conceptual understanding.

The middle section of this chapter presents the theoretical framework that underpins this research. It defines conceptual change, based on the work of Hewson (1992), and describes the conditions necessary for conceptual change to occur in teaching and learning. How inquiry is employed in this research, as a method to promote conceptual change is discussed. As multiple representations are employed in this research, the use of multiple representations in teaching and learning is also discussed.

The final section of this chapter details student difficulties recorded from literature that relate this research. Student difficulties in vectors, the inverse square law, field lines, Coulomb's law, the electric, work and potential difference are detailed. Finally, a pedagogical framework is presented to inform the implementation of the teaching and learning materials over the course of the lessons for this research.

2.1.1. How students learn

Learning can be defined as the process through which relatively permanent change in behaviour or knowledge occurs because of experience (O'Donnell, *et al.*, 2009). Joyce, *et al.*, (2002) state that students learn in "human settings" which are assemblies of teachers and students in environment created for learning purposes. They state that effective schools gather students together to learn, but also engage in specific kinds of inquiry (Rutter, *et al.*, 1979; Mortimore, *et al.*, 1988 and Levine and Lezotte, 1990). As classroom engagement tend to be at the centre of education discussion, the question of how to teach is central to discussions of effective methodologies. Yet traditional "chalk and talk, drill and recite" (CTDR) and didactic teaching still dominate the methods used by teachers, and the minds of critics of education (Race and Powell, 2000; Joyce, *et al.*, 2002). This contrasts with student's perceptions of their own learning: they feel learning occurs when they engage in activities such as class debates, and little learning occurs during intervals when the teacher is talking

(Jensen and Kostarova-Unkovska, 1998). Joyce, *et al*., (2002) suggest that, in the US, the combination of recitation and lectures contributes to approximately one third of learners being unable to complete secondary education, and approximately one fifth being unable to read and write to a standard that allows them to acquire professions that require literacy. They also suggest that evidence supports that this situation is reflected in Great Britain. In these type of classroom environments, Cooper and McIntyre (1996) suggest that students tend not to have strategies for learning, and instead adopt a passive role, in which it is the teacher's responsibility to ensure they learn, and they are "made to work" in the classroom.

Regardless of the way a teacher attempts to engage the student in leaning, the information-processing model (IPM) can explain how students learns (Huffman, 2004). The IPM describes how learners develop internal representations of the external world. It illustrates how stimuli from the environment is transferred from the sensory memory, to the short-term memory (STM) and to the long-term memory (LTM). The model is presented diagrammatically in Figure 2.1.

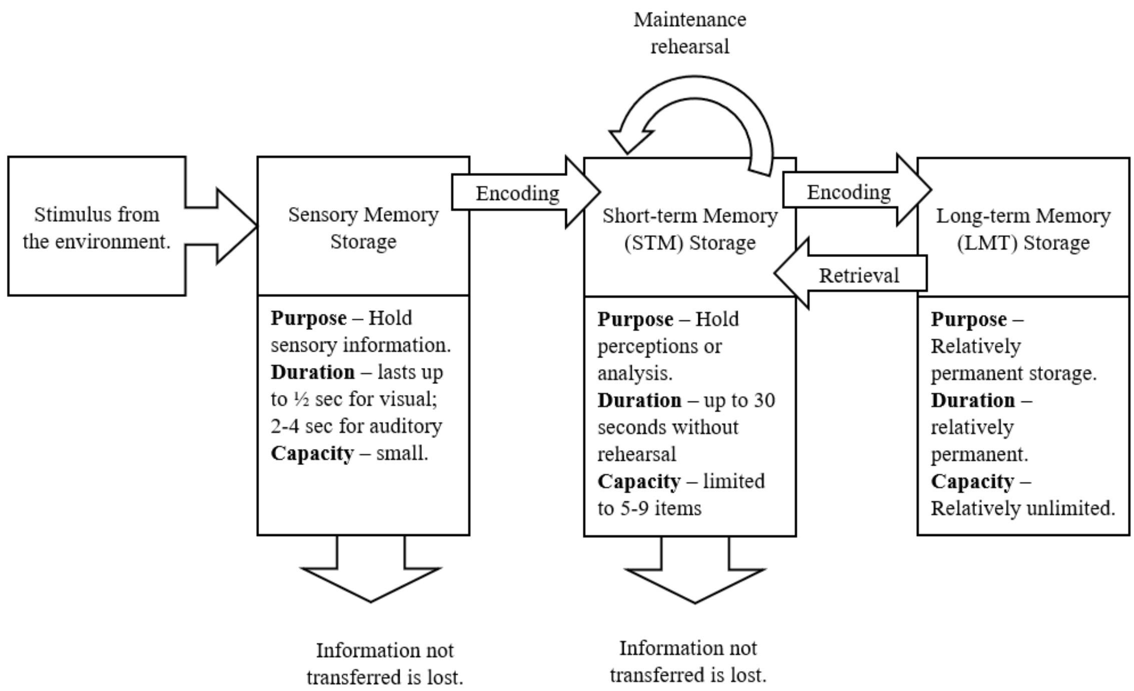


Figure 2.1. Information-processing model (O'Donnell, et al., 2009), reproduced from Huffman (2004).

Sensory memory (SM) is very brief and tends to be applied in two forms in the classroom, visual and auditory. This information is passed to the STM, where if it is not processed, it is lost. However, the information in the memory can be rehearsed to keep it active, in the STM. This information is subject to decay, when information not used is lost, and interference, when something else gets in the way of the recall. Information that is transferred to the LTM potentially has permanent and unlimited storage for the life-time of the learner. O'Donnell, *et al.*, (2009) explain how LTM can take the forms of episodic (events), semantic (verbal information), declarative (details about a structure) and procedural (details about how to do something). This model suggests varied and

complex processes occurring cognitively in the learner, but didactic teaching and CTDR can be limited in how they allow students to engage in the encoding process to transfer from SM to STM to LTM.

Reid (2009) references Yuan, *et al.*, (2006), stating that there is a consensus that STM is a function of working memory (WM). Reid (2009) expands upon this point that the STM not only retains information, but also allows the information to be worked on. Information can be retrieved from the LTM into WM and worked on, pending the capacity of the WM is not overloaded. The capacity of WM can be considered in terms of the number of items that can be processed at any one time. Reid (2009) states that people aged 16 years and older can have 7±2 items of information available to work on in their WM capacity, and processes that involve using more of these can lead to capacity overload, in which the learner can no longer effectively process all the information and will be unable to complete the task they are participating in. Over the course of their education, a learner can "chuck" information together into one item, and there is virtually no limit to how much information a person can "chuck" together to form an item. This frees up WM items for the learner to process new information and complete their task. If the learner is unsuccessful in this, their ability to develop understanding ceases as there is too much information for them to process.

Joyce, *et al*., (2002) list numerous models for learning based on the work of Jean Piaget's theory constructivism. Constructivism is a theory about how students learn, and has many sub-theories related to information-processing, the construction of knowledge from prior knowledge and new experiences and knowledge construction because of social interactions (O'Donnell, *et al*., 2009). In the theory of constructivism, Piaget (1967) discusses learners use of mental processes, called schemas, to organize their knowledge and experiences.

Through observations and experiences, children learn about the world they live in, in which prior knowledge and understanding are used as a lens through which they view new knowledge and experiences. In constructivist learning, no information learned is independent of the experience of the learner and the context in which it is learned. Hein (1991) outlines some principles of constructivist learning as the following:

- Learning is an active process, in which learners use sensory input and construct meaning out of it.
- People learn to learn, as they learn. Learning consists of both constructing meaning and constructing systems of meaning.
- Constructing meaning is a mental action. Hands on activities are not sufficient unless the mind is also engaged.
- Learning is a social activity. We are more likely to be successful in instructional efforts if we recognise the use of conversations, interactions with others and the application of knowledge as an integral part of learning.

- Learning takes time. It is not an instantaneous process and for significant learning, we need to revisit ideas, ponder them, try the out and use them.
- Motivation is a key component in learning. Unless we know "the reasons why," we may not be very involved in using the knowledge learned.
- Learning is contextual. We do not learn facts and theories in an isolated fashion but instead link them and relate them to other facts and theories.
- One needs knowledge to learn. It is not possible to assimilate new knowledge without first having knowledge to build upon.

Transmission models of learning (Mestre, 1991) tend to ignore learners' prior knowledge and experience, while constructivist models provide opportunities for students to perceive an event, and the student's perception informs what learning occurs for them. Generally, students make sense of these perceptions and assimilate them into, and extend, their mental schemata. Green and Gredler (2002) note that from Piagetian and Vygotskyian perspectives, constructivism helps learners develop logical thinking, self-regulated attention, conceptual thinking and logical memory. They note constructivist classrooms may employ student-directed experimentation, or interaction between students and subject matter concepts to develop advanced cognitive capabilities. The learners manipulate objects and ideas, which may lead to cognitive conflict between one's ideas and experimental results, and they can interact with the teacher to develop conscious awareness of, and mastery of, one's thinking and learn to think in subject matter concepts.

The degree of prior knowledge required to process an event and complexity of the event being perceived can lead to difficulty in extending a mental schema in an accurate fashion. Vygotsky (1978) presents a model for learning based on student's development that addresses this. Students have 3 zones in this model; the zone of actual development, the zone of proximal development and the zone of no development. If a student is presented with tasks that they can complete unaided, the skills required to do so are in their zone of actual development. When a student requires assistance or instruction to complete a task or develop a skill, it is said to be in the zone of proximal development. If a task or topic is in the zone of no development, this is currently beyond the capabilities of the student, and no form of instruction will aid the student. This model is useful for educators as it allows them to account of what students can do, what students are capable to doing with aid, and consider what teaching and learning needs to occur for the students to be able to complete more difficult aspects of their courses.

The zone of proximal development is of importance as this is where cognitive development occurs. To support students in this zone, scaffolding can be employed by the teacher. Scaffolding is the guidance, support and tutelage provided by a teacher during social interaction, designed to advance student's current level of skill and understanding (O'Donnell, *et al.,* 2009). Scaffolding provides support for the learner, extends the range of what the learner can do to enable

accomplishment of tasks that would otherwise not be possible. It can be reduced or removed from lessons as learners develop the skill and understanding to learn, or complete a set of tasks, on their own. Lynn, *et al.,* (2013) discusses 4 central tenets of scaffolding in science education, in which it can be used to make science accessible to the learner, make thinking visible, help students learn from others and promote autonomy and life-long learning.

Lynn, *et al.,* (2013) explain that making science, and scientific reasoning, accessible to learners using scaffolding can involve using tangible examples familiar to students, as opposed to abstract models that are employed in science. This allows learners to develop scientifically normative views and explain them using familiar contexts. Learning can be made visible as learners can be encouraged to explain their ideas to others. Multiple representations utilise various models for the students to engage with. Scaffolding can involve helping students listen to their peers, to take advantage of the collective knowledge of the group. Class discussions where students are required to respond to each-other and critique to one another can allow for individual learning, in learning to consider alternative views, expand their own knowledge and engage in effective communication with one another. Autonomy and life-long learning is promoted through scaffolding, as who students who reflect and explain their ideas learn more (Chi, Bassok, *et al.*, 1989), and gain a more robust understanding when revisiting concepts in new contexts.

2.1.2. Developing student's conceptual understanding

As discussed in Section 2.1.1, traditional methods of instruction tend to focus on transfer of facts from teacher to student, while little emphasis on students constructing their own knowledge and understanding. When discussing traditional instruction Mestre (1991) discusses the transmission model of education and notes that it is not a model of learning, but an instructional practice. He highlights that a central assumption of the instructional practice is that the message that the student receives is the same in which the teacher intended for them to learn. In this practice, difficulties in student's understanding are due to the manner the material is presented and the teacher needs to augment their presentation of the material. Roth (1990) notes that primary and secondary level education has struggled to develop student's conceptual understanding when traditional methods of teaching and learning are employed. She notes that while students in these environments are proficient in memorizing facts and procedures, they struggle to build arguments, make predictions and explain observable phenomena, both inside and outside the classroom. When considering conceptual development in a traditional classroom, she states that student's conceptions are largely invisible to both teachers and students, the instruction focused on student's learning explanations and terminology already developed by scientists and test questions revolve around repeating these ideas. Students are rarely asked to apply the concepts to explain everyday situations and the emphasis of

the learning was students developing the "right" answer rather than exploring the nature of the student's conceptions. As detailed in section 2.2.1, Roth (1990) suggests a conceptual change model of instruction that promotes the student's abilities in these areas, allowing them to overcome the difficulties that traditional instruction do not address.

To develop learner's conceptual understanding, Mestre (1991) suggests a constructivist approach, which considers the principles discussed in Section 2.1.1, which sees learning as a process of constructing knowledge and understanding. Meaningful learning occurs when students interpret and apply knowledge in novel contexts, in which the students are mentally engaged. Roth (1990) suggests that the teaching and learning materials should focus on allowing the students to develop deep understanding of a limited number of concepts, that they can apply in novel contexts, which does typically not occur in traditional classrooms. A constructivist approach allows the students to engage with new concepts and can provide opportunity for students to resolve their prior understandings/misunderstandings with new concepts. Johnston (2010) suggests strategies to successfully engage students in developing their conceptual understanding in the classroom, such as actively engaging students and providing regular feedback, focusing on the observable phenomena, explicitly exploring misconceptions, using various problem-solving skills and strategies and providing homework tasks that involve qualitative and conceptual analysis of phenomena. This sort of constructivist environment promotes students to use active learning to support their construction of knowledge and understanding. The students are not being viewed as passive recipients of knowledge but of active participants in its creation (McDermott, 1991).

Regardless of the teaching style of the teacher, students will have constructed their own models of understanding from both their formal education and their interactions and observations of the world. This prior learning can result in their models of the world not being scientifically accurate, and the students cannot be considered "blank slates" when they partake in science lessons (Knight, 2004). Students have relied on these models to explain the world for some time before entering classrooms. These models, which may contain misconceptions, can be resistant to change, so constructing a conflict between the student's model and the scientific model may be required multiple times before successful change takes effect.

The approach chosen to facilitate the student's conceptual development in this manner is structured inquiry. This approach is discussed in the Section 2.1.3.

2.1.3. Inquiry learning in Physics

To promote development of conceptual understanding as discussed in Section 2.1.2, inquiry-based learning (IBL) can be utilised (Tabak, *et al.*, 1995; Blanchard, *et al.*, 2010). There are many

definitions for IBL in science education. The National Research Council present a general definition for inquiry that does not necessarily preclude the use of any one specific teaching / learning method.

> *Scientific inquiry refers to the diverse ways in which scientists study the natural world and propose explanations based on the evidence derived from their work. Inquiry also refers to the activities of students in which they develop knowledge and understanding of scientific ideas, as well as an understanding of how scientists study the natural world.*
>
> (National Science Education Standards, 2006, p23)

An aspect of this definition is that student's experiences in a classroom setting should reflect scientists' experiences when seeking to expand the human scientific knowledge. In using inquiry in the classroom, learning involves the students thinking scientifically. While this is not an exhaustive list, inquiry can take the form of designing and critiquing experiments, performing research, engaging in scientific debate, discourse with peers, construct models, search for information, and/or some combination of these (Lynn, *et al.,* 2013).

As there as many classroom activities that can be used to engage students in inquiry, there is a wide interpretation of what inquiry is. Banchi and Bell (2008) present inquiry on a four-point scale, "limited, structured, guided and open." Limited inquiry involves following a set of predetermined instructions to arrive at a pre-determined conclusion and is typically referred to as "cook book". The aim is for students to confirm something they have already learned. Structured inquiry occurs when there is no pre-determined conclusion to the task. It is purely based on the student's construction of knowledge through whatever activity was completed, such as an investigation. Guided inquiry has no predetermined method presented to the students, and they must determine how to complete the task, as well as draw conclusions from it. In a manner, guided inquiry involves presenting students with a learning objective, such as a phenomenon to be investigated, but allowing them to complete the objective in a manner in which they see fit. Open inquiry involves giving the students no predetermined questions to answer, and instead they generate their own questions which they answer themselves.

The choice of inquiry to use in a set of lessons can depend on the outcomes the teachers wishes for their students to achieve. Tabak, *et al.,* (1995) and Blanchard, *et al.,* (2010) showed structured and guided inquiry practices are beneficial for students to develop conceptual knowledge and engaging with nature of science skills. Research into open inquiry showed that learners gained independence and autonomy over their learning, relying less on their instructors (Krystyniak and Heikkinen, 2007). Students may develop meta-cognitive reflection skills, engage in higher order thinking and improved motivation to complete investigations (Berg, *et al*., 2003). The several types of inquiry allow students to engage with several types of autonomy (Stefanou, *et al*., 2004). In guided and open inquiry, students are given an opportunity to display an elevated level of cognitive

autonomy. They can, amongst other things, discuss multiple ways to think about an issue, debate thoughts freely, re-evaluate errors and ask questions. Open inquiry, additionally, allow students to display an elevated level of procedural autonomy, such as design investigations freely, choose how to demonstrate competency and design their learning outcomes.

Inquiry learning is not accepted as best practice by all academics in the educational sphere. Clark *et al.,* (2012) argue when learners are presented with information, they should be fully instructed on what to do and how to do it, basing their argument on a review by Mayer (2004) and overloading short term memory (STM). Mayer's review identifies numerous studies since the mid twentieth century which compared unguided to guided instruction. In all cases, the results indicated that guided instruction resulted in better students gains, over discovery learning, in the tasks specified in the studies. However, the studies referenced by Mayer do not consider the various inquiry types (Banchi and Bell, 2008), and when considering these, the studies can be interpreted as utilising structured and guided inquiry. The argument that Clark, *et al.,* (2012) make regarding STM relates to the limited number of elements it can process at any one time. They state that instruction should aim to impart as much knowledge and skills into a learner's long-term memory (LTM) as they have relatively limitless access to this, as opposed to their STM. They suggest a "worked-example" approach, in which the learner uses their STM resources to develop comprehension instead of both comprehension and discovery which directs attention to storing essential information and understanding. However, they state that this approach is only fruitful for introducing new knowledge, and it can have a detrimental effect for material the learner is familiar with. Their article does not appear to address limited or structured, in which case the design of the teaching and learning materials can present learners with the introductory concepts and knowledge, to which the learners initially only need to comprehend. After this introduction, the learners can then apply the initial concepts and develop a deeper understanding in a limited or structured environment.

Rocard, *et al.,* (2007) identifies IBL as a method of best practice for implementation in school classrooms. Bevins and Price (2016) state that there is a wide range of empirical evidence that reports positive outcomes for learners in terms of achievement, enthusiasm, ownership and scientific skills development. They reject that inquiry must be entirely student driven and unsupported by the teacher. They propose that inquiry can develop a new sophisticated model, to reap further benefits. They propose a three-dimensional model to inquiry practices, as shown in Figure 2.2.

The first dimension focuses on the body of knowledge in science. This informs how scientists think about the natural world and generate questions for inquiry. The second dimension focuses on procedural knowledge. These allow for the reliable generation of data and evidence, ensure they are reliably interpreted, and the data and evidence is communicated appropriately. The third dimension focuses on learner's psychological energy, in which they engage with the inquiry process, which generates energy to create and manage authentic inquiry process. Bevins and Price propose that these

three dimensions are not intrinsically interlinked but instead the model of inquiry in made up of these dimensions as the sum of their individual parts.

An early implementation of structured inquiry as defined by Banchi and Bell, which the authors however term guided inquiry, is the Physics by Inquiry worksheets developed by McDermott, *et al.,* (1995). It comprises a series of modules across various physics topics. Each module is structured so students develop their understanding based on their own observations and encourages them to develop explanatory models. There are gaps between each narrative that each student must bridge as they complete the experiments and exercises in the module. The primary emphasis is on teaching by questioning rather than the transmission of knowledge. This helps students to see physics as a process of discovery. The student is central to the learning process, developing sound qualitative understanding, which compliments and improves student’s ability to tackle quantitative problems, and overall, sets a higher standard of learning (McDermott, 2001). However, the time required to complete a module in any one topic can be quite demanding, making it inappropriate for direct implementation in a secondary level setting where multiple topics are to be taught in a relatively short amount of time.

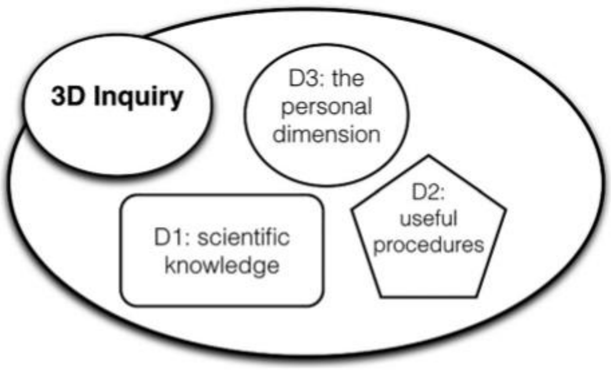


Figure 2.2. Three-dimensional model for inquiry (Bevins and Price, 2016).

Within the field of Physics Education research, there have been numerous studies in recent years (Shaffer and McDermott, 2005; Close and Heron, 2010; Heron, *et al*., 2004; Hazelton *et al.*, 2012) show that student who learn through inquiry methods develop better conceptual understanding of physics than students who learn using traditional approaches. These approaches focused on development of mental models, student reasoning, conceptual reasoning and conceptual change.

To illustrate the efficacy, the first of these four cited studies is reviewed which employed a structured inquiry methodology for teaching and learning. When looking at student’s understanding of vectors, Shaffer and McDermott (2005) showed students can have trouble with one dimensional vector subtraction when shown in a real-life context, such as collisions, and these student difficulties in vectors appeared to increase when transferred to a two-dimensional problem. Their lesson was designed to engage students with using vectors to identify changes in velocity and determine the

direction of acceleration as a result. In a separate lesson, students apply the same skills in a two-dimensional context to provide commentary about velocity and acceleration for a body in motion at points on a closed loop path. They found upon completion of the lessons, they helped not only undergraduate students transfer vector analysis from one-dimension to two-dimensions, but also the postgraduate students who acted as teaching assistants during the tutorials. This example illustrates how structured inquiry lessons can help inform instructors of student difficulties and provide an environment for students to overcome them.

2.2. Theoretical framework

This section discusses the theoretical framework that underpins this research. As a mechanism to promote conceptual change, this research employs the use of structured inquiry and multiple representations in the design and implementation of the teaching and learning material used in this work.

2.2.1. Conceptual change

Konicek-Moran and Keeley (2015) describe traditional forms of instruction as promoting "literal understanding". Literal understanding allows students to memorise and reproduce knowledge without understanding the meaning behind it, or the power to use it to argue, predict or delve deeper into the ideas involved. They argue that to do these things, a learner must develop a conceptual understanding of what they learn. Although they do not give a formal definition, Konicek-Moran and Keeley (2015, pg. 6) state that when a learner develops, or is developing their conceptual understanding, they can partake in or demonstrate any of the following attributes with their conceptual knowledge:

1. Think with a concept.
2. Use the concept in areas other to that in which they learnt it.
3. State it in their own words.
4. Find an analogy or metaphor for it.
5. Build a mental or physical model of it / to explain it.

To develop conceptual understanding, the learner must be exposed to a concept in some manner. Concepts are packages of meaning; they capture regularities, patterns, or relationships among objects, events, and other concepts (Novak and Canas, 2006). They are formed when a student engages in an intellectual function in which memory, attention, inference and language all

participate. From experience, students naturally construct their own conceptions, which may or may not be scientifically accurate, to make sense of the interactions they see in the world around them. This can also result from methods of instruction that do not aim to reduce the occurrences of students developing misconceptions (McDermott and Shaffer, 1992). Assessments and evaluation based on recall of content and solving quantitative problems can hinder development of conceptual understanding, as educators tend to teach to the test instead of focusing on their students developing coherent understanding of the material they learn in the classroom setting. For this reason, misconceptions do not tend to be addressed by traditional instruction (Dykstra, *et al*., 1992). As students spend considerable time and energy constructing these concepts, they can develop an emotional and intellectual attachment to them, which is not easily overcome. To overcome such an attachment, a conceptual change model of instruction is proposed.

Hewson (1992) discussed conceptual change as three mechanisms, conceptual extinction, conceptual exchange and conceptual extension.

- **Conceptual extinction** is when an alternate idea is challenged to the point where it is no more, and all that remains is the new learned concept.
- **Conceptual exchange** occurs when a student is presented with a conception that challenges their current understanding. Through the learning process, the status they associated with their current understanding is lowered and the status of the new conception is raised. The original idea is still present, but the student becomes aware of its inaccuracies or limitations and disregards it in favour of the more useful robust concepts that were learned.
- **Conceptual extension** is when a person goes from not knowing an idea to knowing the idea. A person makes connections between the things they already know and extends their understanding to account for new material achieves this.

To promote conceptual change in the context of conceptual exchange and extension, the following proposed four conditions generally tend to be present:

- *There must be dissatisfaction with existing conceptions.*
- *The new conception must be intelligible*
- *The new conception must be initially plausible*
- *A new conception should suggest the possibility of a fruitful research programme.*

(Posner, *et al.*, 1982, pg. 214)

Under these conditions, student's models are effectively challenged and allowed to fail. This allows the students to adjust their models of understanding of the topic being studied. Vosniadou and Brewer (1994) consider conceptual change as it applies to a student's overall understanding of a topic and see it *"as the product of the gradual lifting of constraints, as presuppositions, beliefs, and mental*

models are added, eliminated, suspended or revised during the knowledge acquisition process". By encouraging students to evaluate their initial ideas over time and adjust it by adding the elements of scientific explanation, students are facilitated to allow their models to undergo conceptual change to create models that extend to more fields of study and possess greater explanatory power.

In some cases, cognitive conflict arises between the new knowledge and the existing schema (Piaget, 1975, Posner, *et al.*, 1982, Chan, *et al.,* 1997). This results in the student altering their schema to accommodate the new knowledge. This reorganization of mental schema is required to make sense of the world, as they learn more about the world around them. Schemata can be altered or replaced with other schemata that better explain the environment the learner is in. This results in the construction of new knowledge that furthers learner's understanding of a particular experience / concept / etc. In this view, constructivism can be understood "in terms of a shift in the location of the meaning of what is found in our environment" (Taber, 2011, pg. 40). However, the understanding constructed by the students is not always what the teacher wishes, which can lead to the construction of misconceptions, which can be difficult to deconstruct.

The following sections discuss structured inquiry as a method to promote conceptual change in the classroom, the use of various inquiry resources employed in the development of the research instruments and the use of representations as an aid to promote conceptual change.

2.2.2. Approach adopted in this research

Structured inquiry is adopted as the approach used in the research to promote conceptual change for the student's understanding of Coulomb's law, the electric field, work and potential difference. Structured inquiry generally allows students to develop conceptual understanding (Tabak, *et al.*, 1995; Blanchard, *et al.*, 2010). As seen in the pre-test discussions in chapters 4 and 5, many students produced answers consistent with difficulties recorded in literature. Therefore, structured inquiry was chosen to be utilised in this research. Section 3.3 discusses how structured inquiry was implemented in this research, through the development of tutorial lessons. To avoid confusion, the term inquiry is used as an umbrella of the inquiry types used in the materials developed, instead of identifying each type of inquiry as they arise.

In developing the materials, numerous approaches to inquiry tasks that could have been employed in the lessons were reviewed. After consideration of the research questions, learning objectives of the Leaving Certificate Physics curriculum, the Leaving Certificate Physics examination format and ability of the students, the following approaches were considered for use in the development of the teaching and learning materials designed in this research to be employed in the second level classroom: Tutorials in Introductory Physics and Conceptual Physics.

Tutorials in Introductory Physics (McDermott, *et al.,* 2003) is a set of supplementary activities to accompany lectures or a standard textbook in a standard university physics course. The emphasis is on student understanding of concepts and scientific reasoning skills, as opposed to rote learning theory or solving quantitative problems. A tutorial consists of a pre-test, worksheets, homework assignments and a post-test. The pre-test is typically given after the lecture to determine what concepts the students do understand, and what they are expected to understand at the end of the materials. This indicates student's initial conceptions that can be targeted for conceptual change. The worksheet questions are designed to guide students to construct concepts and apply them to real world situation, when contextually appropriate. These are completed in groups to allow for peer tuition when constructing answers. When students run into difficulty, the teacher uses prompt questioning to guide student thinking instead of volunteering answers themselves. While the teacher can explicitly state when a student's reasoning is diverging from what is intended, they ultimately encourage the students to find their own correct answers. Through completing the worksheet and engaging in discussions, students encounter the conditions for conceptual change to occur (Posner, *et al.,* 1982). The homework exercises are designed to reinforce what is covered in the worksheet, applying the concepts in both familiar and unfamiliar contexts, and in some cases, extend student knowledge. Questions used in the pre-test may or may not appear in the tutorial lesson, or the homework assignment, depending on the design of the materials. This can be used for comparative purposes, and to allow the students to apply their developed understanding to a previous question they may have struggled with or completed in error. The post-test can then be used to gauge any development in student understanding. To be effective, the post-test is written to emphasize the concepts and reasoning skills used in the tutorial lesson and can be used as a comparative tool to determine how a student's understanding has developed since completing the pre-test. Student responses can be individually, or as a group, compared with the pre-test responses. This allows for the identification for the extent of conceptual change that occurred, and determine if the conceptual change was extinction, exchange or extension (Hewson, 1992). Any difficulties persistent in both the pre-test and the post-test can then be redesigned in a future edition of the tutorial, with the research informed of specific difficulties encountered by students. This cycle of results-based redesign has been used to develop more robust materials that increase the number of students able to access difficult concepts (McDermott and Shaffer, 1992; Wosilait, *et al.,* 1998).

The use of materials based on Tutorials in Introductory Physics as the intervention allows for a strong targeting of specific concepts and topics (Ambrose, 2004). For this reason, the tutorial lesson format of Tutorials in Introductory Physics Tutorials in Introductory Physics (McDermott and Shaffer, 2003) address multiple topics in Physics and they could also be used as a guide to draft and develop lessons that adopt the tutorial approach in the second level context, as they were for this research is adopted in this research. However, Tutorials in Introductory Physics is aimed at the introductory undergraduate level, instead of upper secondary level. The material that is covered is

too advanced for a second level classroom; and the worksheets developed in this research were of original design, utilising the structured inquiry approach utilised in the Tutorials in Introductory Physics materials.

Another resource that was considered was Conceptual Physics (Hewitt, 2009), as the material is more accessible to a second level student, and utilises familiar everyday contexts, representations and analogies to illustrate physics concepts.

In 1964, Paul Hewitt began teaching at City College, San Francisco, in which he taught Physics to non-scientists. His approach focused on teaching concepts and relationships in physics using English words, and using little or no mathematics (Hewitt, 2011a). The approach uses analogies and imagery from real-world situations to promote student conceptual understanding of physics principles. When students explore equations, they learn to reveal information about the relationships involved and then develop the ability to manipulate formulae to substitute values in during the last step. This allows students to observe relationships that are otherwise not typically seen when the equations are represented in their standard form.

> "When problems are couched in symbols, and the numbers held for later, a student's task calls for thinking that calculators cannot supply. They think concepts."
>
> (Hewitt, 2011b, p. 264)

By giving students a solid foundation in the concepts involved in Physics, they are equipped to understand various formulae, and to make connections between the concepts of physics and their everyday world.

Exercises from the Conceptual Physics practice book, 10th edition (Hewitt, 2009), were used to develop the contexts used in exploring concepts related to the electric field and potential difference. The material presented in the practice book was further developed, to align it with the current Leaving Certificate Physics curriculum, so the students would not be at a disadvantage when they complete their terminal examinations at the end of their second level education.

2.2.3. Using representations to develop conceptual understanding

A representation describes something that symbolizes or stands for an object, a collection of objects, interactions and/or a process (Rosengrant and Etkina, 2007). Representations can come in many forms, such as words, diagrams, mathematical equations, tables, graphs, animations, simulations, etc. In this research, the use of multiple external representations (MERs) is employed as an aid in the tutorials, to promote student's conceptual development of the different electrostatic

topics. Presenting the same concepts in separate ways provides the learner with the opportunity to build abstractions about mathematical concepts (Dienes, 1973). This is a fundamental step in successful learning. Numerous studies show the benefits of using MERs (Cox and Brna, 1995; Mayer and Sims, 1994; Tabachneck, *et al.,* 1994). The use of MERs is not a "silver bullet," as others have failed to find benefits (Chandler and Sweller, 1992; Van Someren, *et al.*, 1998). This section briefly discusses design parameters to consider when using MERs, cognitive tasks related to the use of MERs and the functions of using MERs in a learning experience for learners.

Ainsworth (2006) states in designing educational materials that employ the use of MERs, the designer should consider number, information, form, sequence and translation. The number of representations must be at least two, but more can be employed. They can be presented simultaneously to the learner, or in sequence. The designer must consider the information they wish to convey to the learner. MERs allow for flexibility in how information is distributed across representations. Information can be redundant, where each representation presents specific information that is isolated from others, partially redundant, where some information is common to all representations and some in unique to each representation, or all the representations express the same information and the only difference is how the information is represented. This must influence the design of teaching and learning materials, as redundant information in representations can make it difficult to transfer between representations, relate representations together and overload their cognitive load capacity. The form of MERs considers what type of representations are chosen and how the learner will interpret how the interactions interact with each other upon completing their lesson. When considering the learner's translation when using MERs, the designer needs to consider how the learner will interpret the information and consider if the student will increase their understanding of the representational format chosen, or the domain knowledge it represents.

When leaners are presented with, or use their own MERs to complete a task, they must face and master several cognitive tasks (Ainsworth, 2006). These cognitive tasks are presented in order, but they are not necessarily the order a learner should approach them.

1. Learners must understand the form of a representation. They must know how a representation presents information, and how to use the operators of the operation. Learners can have difficulties in both learning the operators of a representation and how the operators are applied in the contexts the representation models. An inability for students to use representation operators can hinder the learners' ability to complete a task, understand the concepts and/or context they are studying or achieve the targets learning from an activity that employed MERs.
2. Learners should understand the relation between the representation and domain. Interpretation of representations is a contextualized activity. For instance, students need to be aware that the slope of a line on a distance – time graph represents the velocity, as opposed to the height of the graph (Leinhardt, *et al.,* 1990). This can be a challenging task during a learning process, as opposed to problem solving, as the learner is applying MERs upon incomplete domain

knowledge. Deciding how the learner will use the representation to model the context of the domain knowledge needs consideration. Otherwise, difficulties could be encountered by the students, and the envisioned learning using MERs may be hindered.

3. Learners need to understand how to select an appropriate representation. Learner may have the opportunity to select which representation is most appropriate to complete a task, and they consider factors such as the representation, task characteristics and outcomes and individual preferences. While students are often competent in this (diSessa, 2004), selecting appropriate representations tends to be more difficult for novices as opposed to experts (Chi, Feltovich and Glaser, 1981). Understanding, and accounting for, learners limited ability to select appropriate representations can be used to inform the design of the tasks that employ MERs.
4. Learners need to understand how to construct appropriate representations. In certain tasks, learners may need to construct their own representations. DiSessa (2004) argues that learners are good at doing this, and Grossen and Carnine (1990) showed that children solved problems more effectively when they constructed their own diagrams, rather than select from a series of pre-drawn diagrams. This can be incorporated in material design by providing learner the opportunity to model a process using a representation and have them explain how it represents the process.

When MERs are employed in a teaching and learning sequence, it is important the designer considers why they wish to employ MERs. When carefully planned and applied, such as being cognisant of the cognitive tasks involved in using MERs and their limitations, Ainsworth (1999) suggest the use of MERs allows for the employment of multiple parallel functions in learning. However, without considering the 4 cognitive tasks that Ainsworth (2006) outlines, the result of using MERs could range from fruitful to detrimental to the desired learning outcomes. To consider the functions of MERs, she developed a functional taxonomy of multiple representations that illustrates these functions, shown in Figure 2.3.

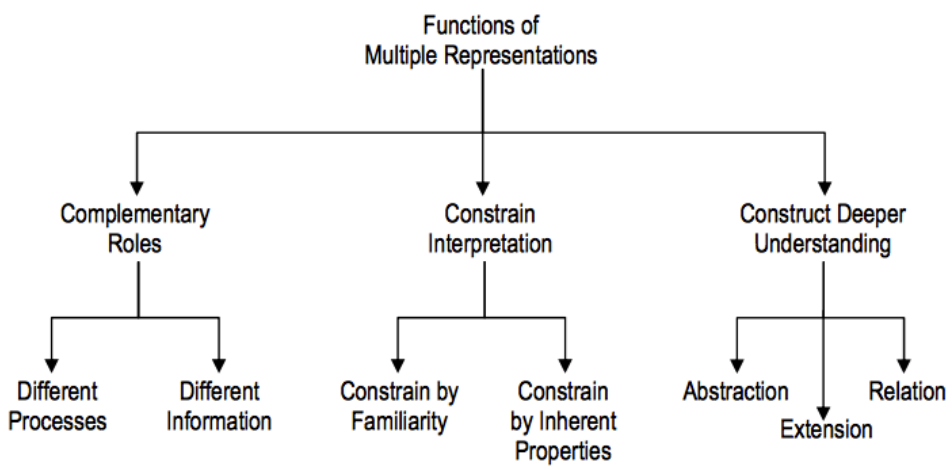


Figure 2.3. Functions of Multiple representations (Ainsworth, 1999)

To understand the complementary roles of multiple representations, consider how different information can be represented in diverse ways, as is appropriate to that information. Representing the velocity and acceleration against time is appropriate in a table or a graph, while inappropriate for representing the mass of the body. Different processes used in multiple representations allow the student to take advantage of their computational properties. Diagrams allow students to group together relevant information, graphs and tables allow students to determine values from reading off, patterns can be determined by analysis of the table and graph, while algebraic equations summarize the relationships between various variables in a given context.

Constraining information involves using the properties of one representation to focus on information taken from another. This allows the familiar representation to be used as a scaffold to show the same or similar concept in a more difficult representation. For instance, using a matching game with several types function in graph form, tables of graphs typical of those functions and general equations for those functions can give students the opportunity to identify key coefficients, variables and parameters in the equations that are characteristic of each type of function. When designed at the appropriate level, there are only a limited number of combinations and reasoning the student can arrive at, and can further be facilitated by instructional guidance.

Multiple representations can also support deeper understanding of the material covered by reinforcing concepts and information that is common to multiple representations of the same material, but also by highlighting features that is most notably prominent in a representation. Velocity – time graphs for non-uniform displaying increasing and decreasing motion can be relatively easy for students of all levels to understand and interpret, but when given various algebraic equations, with corresponding domains for time can be quite difficult to interpret for students. The use of multiple representations in this research primarily aims to employ this construction of deep understanding. Many sections of the tutorials require the students to represent information as tables, graphs, apply algebraic reasoning, and draw and interpret vector and field line diagrams.

Kozma (2003) discussed one difference between understanding concepts by novices and experts, being that experts are fluid in their transitions between representations while novices typically use one or two. Students focus on surface features of the concept and do not develop understanding at a deep level. To advance their understanding, Hestenes (1996) suggests that a complete understanding of a model of a physical system requires a student to be able to transfer between multiple representations. This ability to engage in transfer between different representations leads to increased conceptual understanding.

2.3. Overview of the research study

2.3.1. Teaching and learning electrostatics

As discussed in chapter 1, this research focuses on upper secondary level student's understanding of vectors, the inverse square law, field line representations, Coulomb's law, electric fields and work and potential difference. This section presents a review of literature related to these domains, detailing difficulties and misconceptions typically encountered by students in electrostatic forces, fields and potential difference. Section 2.1.3.1 discusses issues of vector concepts vector addition and vector components, and then difficulties of using vectors in Coulomb's law and electric fields. Section 2.1.3.2 details issues around the inverse square law, focusing on learner difficulties surrounding scaling. Section 2.1.3.3 discusses difficulties related to field lines, and how they are applied to represent electric fields. Finally, section 2.1.3.4 discusses issues related to work and potential difference.

2.3.1.1. Difficulties in understanding of vector concepts and their application to the electrostatic context

Student difficulties are encountered in the understanding of vector concepts. Flores, *et al*., (2008) showed that highlighting the vector nature of forces, and acceleration, in kinematics can increase student's ability to use vectors to solve problems that otherwise prove difficult, but that the overall improvement of vector understanding is quite a challenge. Nugyen and Meltzer (2003) showed that students have difficulties with vector addition, in cases of collinear vectors and vectors in two dimensions. Illustrations of these difficulties are shown in Figure 2.4. Difficulties seen with collinear vectors included adding vectors to form two-headed arrows, connecting vectors "tail to tail" or "tip to tip," incorrectly attempting to find the resultant between two vectors. They also showed another difficulty in student's understanding, such that when finding the superposition of two vectors, students re-orientate vectors arrows if they were not perpendicular to each-other to apply Pythagoras' theorem to determine the resultant vector. This shows an inability for students to correctly combine vectors, both diagrammatically and mathematically. Conceptual difficulties with vector addition in two dimensions include adding magnitudes as in scalar addition instead of vector addition, not taking into account direction of a vectors, or the angles between which the vectors act, not conserving vertical / horizontal components or not acknowledging their contribution to the resultant vector and using a "split the difference" algorithm, in which the resultant is always along the bisector of two vectors, regardless of their magnitude.

Students must be aware of how vectors sum to form resultant vectors, by the principle of superposition. Flores, *et al*., (2008) showed students have difficulties applying vector concepts to forces in which they treat them as scalar quantities, in the domain of mechanics. It is reasonable to postulate these difficulties would transition to electrostatics.

As an overall aim of the research was to improve student's overall development of the electrostatics, an isolated tutorial covering vector concepts was developed. It was expected that the students could transfer their understanding to electrostatics and reduce the cognitive load on their working memory. By initially developing understanding of vector concepts, this would free up items in the student's working memory capacity, to be devoted to Coulomb's law and the electric field. Nguyen and Meltzer (2003) highlighted the common errors that were likely to occur, and these were considered in both the design of the materials, and as conceptual difficulties to identify in student responses as they progressed through the vectors teaching and learning materials.

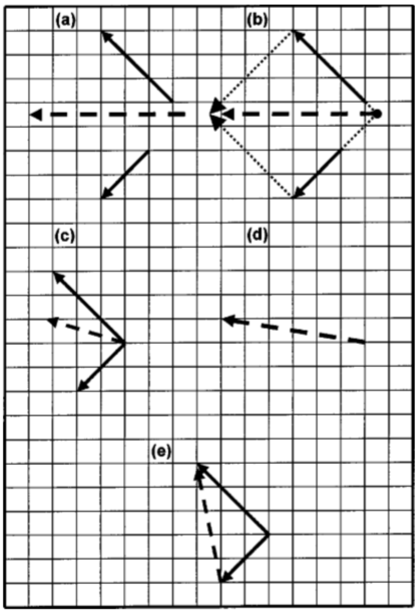


Figure 2.4. depictions of student errors during vector addition (a) zero vertical vector components (b) "split the difference" (c) incorrect parallelogram addition (d) incorrect horizontal component and (e) top – to – toe error (Nguyen and Meltzer, 2003).

Maloney, *et al.*, (2001) showed that undergraduate students can struggle with vector concepts such as superposition in an electro-static context. In question 6 of the Concept Survey of Electricity and Magnetism (CSEM), as shown in Figure 2.5 (i), students were asked to find the direction of the net force acting on the charged particle labelled B. In question 8, shown in Figure 2.5 (ii), the students were required to determine the outcome that adding a charge +Q at (b, 0) would have on the charged particle, q_1.

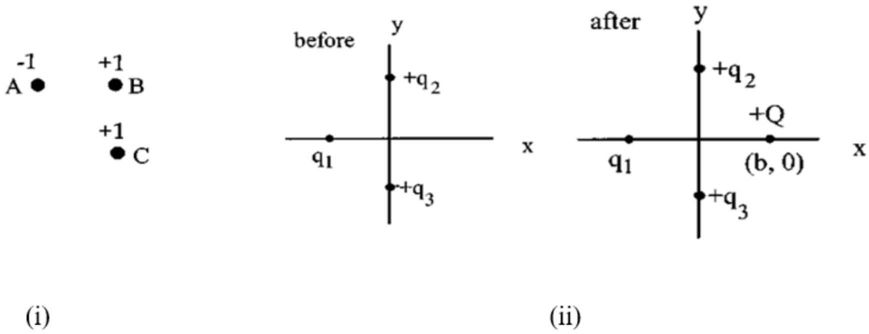


Figure 2.5. Question 6 (i) and question 8 (ii) from the CSEM (Maloney, et al., 2001).

These questions rely on the understanding of vector magnitude, vector direction and the superposition of vector quantities, and their application to electric forces and fields. They noted that in question 6, during the post-test, 33% of students in an algebra-based course and 27% in a calculus-based course could not determine the superposition of the force acting on the charged particle, B. They also reported that noted, in question 8, 47% and 34% of students respectively could not determine how the addition of a new charged particle would affect the direction of the force felt by q_1. The majority of incorrect answers suggested there would be a change in the magnitude and direction of the force, suggesting student struggled to differentiate between the quantities, and not consider the interaction of vertical and horizontal components.

2.3.1.2. Difficulties in understanding of the inverse square law and its application to the electrostatic context

Coulomb's law is considered the fundamental law of electrostatics. It is the first mathematical treatment that students in Leaving Certificate Physics engage with, after completing charge transfer and demonstrate some charge phenomena qualitatively. The Leaving Certificate Physics syllabus (NCCA, 1999) outlines that students should appreciate it is an example of an inverse square law, without explicitly defining what constitutes as an appreciation. Marzec (2012) and Arons (1997) notes that students find it difficult to understand unless exposed to it repeatedly. Maloney, *et al*., (2000) found that students have difficulty understanding the mathematical implications of Coulomb's law pre-instruction. Post-instruction, they found that student gains in understanding were not as one would expect, for those which have completed exercises on Coulomb's law.

The application of the inverse square law to electrostatic forces was disputed for 40 years (Heering, 1992), it has since been proven and can now be demonstrated in a laboratory using charged domes and an electronic balance of precision to 4 decimal places (Cortel, 1999) or measuring the distance between charged and uncharged pith balls (Wiley and Stutzman, 1978). A simple approach for students to investigate the inverse square law is to use an electric field sensor held at various

distances from a charged body and plot the electric field against distance and analyse the data. This approach was taken for light intensity (Bohacek and Gobel, 2011). Other methods include presenting students with data and getting students to determine the relationship based on graphing activities (Hestenes and Wells, 2006). However, data alone does not enable students to develop a conceptual understanding of the inverse square law, nor provide a tangible context for them to relate the relationship to. In Conceptual Physics – 10^{th} Edition (Hewitt – 2009), the context of a spray paint can spraying drops of paint over different areas is used to illustrate the inverse square law. This model is easily understandable by students and adopted in the approach taken in this research.

Research literature also looks specifically at students understanding of the inverse square law, as applied to electric fields. Cao and Brizuela (2016) demonstrated that learners can qualitatively develop an appreciation that the closer charged particles are together, the stronger the force exerted between the charged particles. Conversely, Maloney, *et al.,* (2001) delivered a question involving Coulomb's law where the distance between a pair of charges was increased by a factor of three. It was seen after instruction, 54% and 32% of students struggled to apply the inverse square law to the increase in distance, with the most prevalent error submitted by the students was that the force would reduce by a factor of three. Marzec (2012) suggests that students misunderstanding of spherical scaling could lead to student to difficulties in this area. While there are many methods to experimentally demonstrate the inverse square law, research looking at students conceptual understanding of the law are scarce. Previous findings in this research, as part of an initial pilot study (Moynihan, *et al*., 2015) noted the that students have an over-reliance on the use of formulae and tend to unintentionally ignore the index value on the distance variable in the equation for Coulomb's law. These difficulties appear to be in line with Arons (1999), in which he notes that learners struggle understanding proportional scaling for area when individual dimensions of a geometric shape increase. For example, in the absence of formulae, learners struggle to explain how increasing the sides of a square by a factor of 3 results in an increase in the area by a factor of 9. When this reasoning is applied to a gaussian sphere to model an electric field, these struggles can indicate that scaling can cause difficulty in understanding the inverse square law.

2.3.1.3. Difficulties in understanding of field line concepts and their application to the electrostatic context

Törnkvist, *et al.,* (1993) paraphrased the work of Newton, Faraday and Maxwell in explaining that field lines presented an explanation of "action at a distance" for non-contact forces. For some, field lines presented a physical medium in which the force could act, where others saw them as representations. For a course that introduces the use of field lines, understanding the relationship between force, field and trajectory provides solid foundation for field theory. Greca and Moreira

(1999) showed that in a small group of undergraduate students taking an introductory course in electromagnetism, the students that could form working models of electromagnetic fields were comfortable exploring the concepts of a field mathematically or used models for field lines and solve problems using constructed images. Conversely, students who struggled to form working models of electric and magnetic fields appeared to overly rely on definitions without understanding their implications. They were unable visualise a field, using field lines or vectors, and in some cases, represented the fields incorrectly. Student's inability to apply the concept of field hinders their understanding and ability to progress to more advanced applications of electromagnetism in physics, electrical engineering and other relevant avenues of study.

As a diagrammatic representation of a vector field, a lot of information can be gleaned from a field pattern, that otherwise, would require complicated calculations to determine. Such examples are determining relative field strength at various positions, identifying relative magnitudes of charges / masses of bodies, identification of charge types, construction of reasonable paths taken by various charged and uncharged bodies in the fields and identification of charge on a particle moving in a field. While such a simple representation can be used to determine a vast amount of information, this also presents opportunities for students to develop confusion about the representation and errors in understanding (Maloney, *et al*., 2001).

Furio and Guisasola (1998) presented their student's difficulties in differentiating between field intensity, and the force acting on a particle in a field. One possible source of this difficulty is suggested by them that interpreting the definition of the magnitude of the electric field strength, $E = \frac{F}{q}$, the students interpret the proportionality between the electric field strength and force for an equivalence. Similar findings were presented by Cao and Brizuela (2016) who demonstrated that students can accurately produce canonical electric field lines, but struggle to attribute meaning such as force directions, velocity directions and trajectories to them. In some cases, students can attribute combinations of these meanings simultaneously to the field lines. These difficulties contrast the

convention that field intensity is based on the charge / mass generating the field and is independent of the test-charge / small-mass placed in the field. This is typically represented by the density of the field lines. Törnkvist, *et al.*, (1993) note that students struggle to transfer between field line representations and vector representations, in which some students produce curved vector force arrows to represent a curved field line, instead of producing a tangential arrow. They also showed that student errors include field lines overlapping, instead of use of the superposition principle to show a resultant field. Galili (1993) found that some students perceive field lines to be real tangible structures, which mediate the force acting on a particle in a field. Both Galili (1993) and Törnkvist, *et al.*, (1993) showed student represent the path taken by a body in a field to follow the curvature of a field line.

While the difficulties presented be used to inform development of instructional materials, Cao and Brizuela (2016) discuss how students do not always express in-depth reasoning initially but can develop sophisticated understandings whilst explaining and reinterpreting their work. They illustrate an example of a student making additions to their work during an interview to expand on their initial explanations and form an accurate model of force, motion trajectories using electric field line representations, but point out that they do not imply that students should be expected to develop sound reasoning without the aid of some manner of instruction.

2.3.1.4. Difficulties in understanding of work and potential difference

This section discusses literature related to student's understanding of work, potential and potential difference. Literature on these topics tends to focus on the work-energy theorem and how it applies to system interactions. Potential and potential difference are looked at from a calculus-based context, or their applications to electrical circuits. As this research focused on second level students, the literature reviewed focuses on students conceptual understanding of the mathematical implications of work, applying it to potential difference, and applying potential difference to systems, using an electrostatics context and reasoning, as opposed to current electricity.

Doughty (2013) conducted research with undergraduate students, in which she presented student difficulties in determining whether work done was positive, negative or zero when a test charge moves between various points in the electric field. Difficulties were observed for student determining the direction of the electric field, and force, relative to the displacement and assuming the direction of the force is collinear and in the same direction to the displacement. During student interviews, she noted students use energy conversion as a mechanism to explain the concept of positive work (increase in translational KE), negative work (decrease in translational KE) and zero work (no change to gravitational potential energy). This reasoning can be applied to systems, in which work can increase or decrease the energy in a system. Lindsey, *et al*., (2009) conducted a study with over 4000 undergraduate students over numbers of years, where they developed tutorials focusing on systems involving mechanical work and springs. They identified numerous difficulties in student's understanding of work, such as belief that energy in any system is constant, thinking in terms of kinetic and potential energy and ignoring the cause and effect of the work-energy theorem, and associating work with change in kinetic or potential energy, as opposed to a system. They also identified specific difficulties related to work, such as treating the sign of work as dependant on a coordinate system and failing to consider the displacement of the point at which the force is applied. The approach taken in this research does not discuss systems, or internal energy, so it is expected to see students associate work with the change in kinetic and potential energy of charges in electric field, in the models presented to the students. Other approaches that were taken include a multi-

representation approach by van Heuvelen and Zou (2001), which focused on the use of verbal, pictorial, graphical and mathematical representations, and showed graphs and charts can be useful in helping student develop understanding of these processes.

Hazelton (2013) showed that students struggle to associate higher and lower potential, relative to the ground, to metal spheres with charged rods adjacent. While students could associate high and low potential to positive and negative charge, they incorrectly applied them to the metal spheres, both as the charged rod was initially placed over the sphere, and as time went on so charge was no longer moving between the ground and the rod. Student also demonstrated that they associate the potential with the charge on the body, but do not consider the overall system when multiple bodies of various charge are present. Maloney, *et al.*, (2003) showed that students struggle to understanding how a negative charge will move based being at a position of high potential, but not being aware of the potential in other regions, indicating they do not consider the movement of charged to be based on potential difference, as opposed to potential. Some of these difficulties can be explained by work completed by Guisasola, *et al*., (2002), in which they found that, when phenomena are related to the process of charge, students feel more comfortable when they talk in terms of charge, rather in terms of potential. This can lead to difficulties when students must consider the interactions with objects outside the system they are dealing with, and resort to imagining some form of contact or interaction with external bodies.

2.4. Implementation of lessons

The last section of this chapter discusses the overall teaching and learning approach adopted in this research. Section 2.3.1 highlighted that there are many issues and difficulties in the teaching and learning of Coulomb's law, electric field and work and potential difference. Difficulties and misconceptions were identified from literature and assessed using pre-tests to identify student's initial conceptions that can be targeted for conceptual change. Inquiry methods of learning were employed to direct the learning to promote conceptual change in the lessons (Tabak, *et al.*, 1995; Blanchard, *et al.*, 2010). The use of inquiry is employed in the research using tutorial lessons. The tutorial lesson structure is based on the lesson format as described in Tutorials in Introductory Physics (McDermott and Shaffer, 2003) and the design of some of the questions was influenced by Conceptual Physics (Hewitt, 2009).

Multiple external representations are utilised in these tutorial lessons, using various representational tools to enable the students to encounter information and display their understanding of the concepts underpinning Coulomb's law, the electric field and work and potential difference. It was aimed that the students would have been enabled to iso-morphically transfer the understanding

concepts in these topics between all the representations employed in the tutorial lessons, i.e., students can transfer proficiently between all representations without difficulty in any case. The tutorial lessons utilise representations to allow the students to (a) explore concepts and relationships in different manners, (b) glean different information from similar scenarios and/or (c) predict the behaviour of charged particles in different scenarios using the different representations. Homework and post-tests gauge the extent to which conceptual change occurred, and gather evidence for indications of conceptual extinction, exchange and/or extension. The details of how the tutorials lessons are implemented are discussed in detail in chapter 3.

Vygotsky's (1978) zones of development were considered to determine the order of the lessons. Teaching Coulomb's law, electric field, work and potential difference, without having covered any other underpinning topics would not be feasible, as without some foundational domain knowledge, these topics are in the zone of no development. In this research, vectors, the inverse square law and field lines are identified as element topics, as in they are elementary to the understanding the electrostatic topics. Once their understanding of these element topics is developed, the student's zones of proximal development extends to encompass the electrostatic topics. The student's development of the element topics is discussed in chapter 4, and the development of their understanding of the electrostatic topics is discussed in chapters 5 and 6.

Table 2.1 summarises this section in a pedagogical framework for the implementation of these lessons. It presents the several topics covered by the research and links them to the research questions that underpin this research. The target concepts within each topic are identified and the representations are presented. The later topics of Coulomb's law, the electric field and work and potential difference display which element topics are employed, and the necessity for the students to have completed them prior to the electrostatic topics. The various teaching and learning (TandL) materials that were developed and utilised for this research are listed for each topic, with a reference to a copy of each set of the TandL materials listed in the Appendix column.

	Contextual / conceptual domain of physics	Research questions	Target concepts	Representations & elements used	Teaching & Learning (T&L) Materials	Appendix
Elements	Vectors	RQ 1	• Vector magnitude • Vector addition constructions • Horizontal / vertical component superposition.	Mathematical / algebraic. Vector diagrams.	Pre-test. Tutorial lesson. Homework. Post-test.	Appendix A
	Inverse square law	RQ 2	• Increase in x-variable reduces the y-variable • Change in y variable is invers square proportional to x-variable • Area and paint model to explain concepts.	Diagrammatic model. Tabular data. Graphs. Mathematical / algebraic.	Pre-test. Tutorial lesson. Post-test.	Appendix B
	Field lines	RQ 3	• Difference between force and field. • Field line density for strength. • Path taken not represented by field lines.	Field lines.	Pre-test. Tutorial lesson. Homework. Post-test.	Appendix C
Application of elements	Coulomb's law & Electric field	RQ 4	• $F \propto q_1 q_2$ • $F \propto \frac{1}{d^2}$ • Vector addition of electrostatic forces. • Difference between force and field. • Field line density for strength. • Path taken not represented by field lines. • Negative charge moves against field lines. • Field patterns for attraction and repulsion. • Drawing vectors from field lines. • Drawing field lines from vectors.	Tabular data. Graphs. Mathematical / algebraic. Diagrams. Vectors. Field lines.	Pre-test. Tutorial lesson. Homework. Post-test. Pre-test. Tutorial lesson I. Homework I. Tutorial lesson II. Homework II. Homework III. Post-test.	Appendix D
	Work & potential difference	RQ 5	• Use of vectors to differentiative between displacement and distance. • Use of vectors and field lines to identify positive, negative and zero work. • Potential difference as a mathematical ratio.	Field lines. Vectors. Mathematical / algebraic.	Pre-test. Tutorial lesson I. Homework I. Tutorial lesson II. Homework II. Post-test.	Appendix E

Table 2.1, Pedagogical framework for the research studies.

Chapter 3. Research Design

3.1. Introduction

This chapter presents four sections: the research methodology, the implementation methodology, analysis methodology and description of participants. The research methodology overview focuses on the case study approach and outlines how and why it was adopted in this research. The propositions used to focus the research are outlined and the types of evidence collected for analysis to explore the propositions are detailed. The implementation methodology introduces the background to the tutorial lessons used in this project, outlines what occurs in a tutorial lesson, and justifies the use of structured-inquiry tutorials as an educational method. The ethical considerations for the students participating in the case for research is also discussed. The analysis methodology discusses the use of qualitative analysis in this project, whilst the description of participants illustrates the background of the fourteen students who took part in this project.

3.2. Research methodology

This section of this chapter discusses the research methodology employed in this research. It discusses published research on the use of case studies, primarily looking at the work of Yin (2009). The chapter then discusses how the case study methodology is adopted in this research, identifying the case central to the research and several propositions of the research. The final part of this section overviews the several types of evidence collected during this research and provides a descriptive commentary of each.

3.2.1. The use of case study in qualitative research

A case study is a study of something that occurs over time, with the subject case being a person, a group of people, an organization, other possible groups and even specific events. As empirical research into a chosen phenomenon within a context, a case study can gather both qualitative and quantitative data that can be analysed to construct conclusions (Given, 2008). Factors that affect this research include the sample size of the case being studied, and whether a single-case study or multiple case studies take place. As case studies look at individual contexts, it is not always possible to identify and control all variables that could affect the outcomes in the study, especially when research

involves multiple cases. While this can be considered a weakness of using case studies, they afford a unique opportunity to gather in-depth data that other research methods may not afford; moreover, it is often simply not possible to control all variables.

Yin (2009, p13) suggests a two-fold technical definition for a case study:

> "*A case study is an empirical inquiry that investigates a contemporary phenomenon in-depth and within its real-life context, especially when the boundaries between the phenomenon and context are not clearly evident*"

and

> "*The case study inquiry copes with the technically distinctive situation in which there will be many more variables of interest than data points and as one result, it relies on multiple sources of evidence, with the data needing to converge in a triangulating fashion, and as another result, benefits from the prior development of theoretical propositions to guide data collection and analysis*."

A case study is an appropriate methodology when the following considerations are applicable to the research:

1. A "how" or "why" question is being asked.
2. The events being researched are contemporary, i.e., they are events occurring in the present.
3. They are events in which the investigator is unable to control all the relevant variables that may affect the outcome. (Yin, 2009)

Other methods of qualitative research were considered for this research, such as social experiments, surveys, archival analysis and narrative historical accounts. However, due to the small sample size of the student cohort, the evidence collection employed in the research and the propositions use to guide the research questions discussed in section 1.3, a case study methodology was deemed appropriate.

When considering the rigor and reliability of using case study, the following advantages were identified: they

1. cope with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result,
2. rely on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result;
3. benefit from the prior development of theoretical propositions to guide data collection and analysis. (Yin, 2009)

3.2.2. Case study design

A case study is a research study of a person, group or situation over a period of time. Nisbet and Watt (1984) note that there are many strengths and benefits to the case study methodology. Case studies catch unique features that may otherwise be lost in large data-sets, and these features may hold the key to understanding the situation. As case studies tend to use relatively small sample sizes, numerous sources of data are recorded and analysed which can give a wide set of results that show the interaction of many factors being studied. They are strong on reality, in that they provide insight into an event being studied and seek to analyse the event. They can also provide insights into other, similar situations, thereby assisting interpretation of similar cases. Unlike alternative forms of research, case studies can be undertaken by a single researcher, instead of requiring a team of research for completion. Finally, they can embrace and build in unanticipated events and uncontrolled variables. When designing a case study, Yin (2009) suggests the following aspects of the research need to be considered, to implement the method efficiently:

- The research questions.
- The propositions of the research.
- The units of analysis.
- Linking of data and propositions.
- The criteria for interpreting the findings.

The research should be guided by an overall research question or set of research questions. This can guide the researcher to perform an effective literature review to inform and aid their research design. As case studies can produce a wide amount of varied data, when the researcher develops a research question, they should also identify propositions that guide the data analysis and provide a structured purpose to their research. As the sample size in this research is small, any hypothesis developed to guide the research would not be verifiable / rejectable when considering the general population. For this reason, the research is framed around propositions. Baxter and Jack (2008) explain that in a case study, propositions can be equated with hypotheses. This is justified as both have a predictable power to determine possible outcomes of the experiment/research study. Propositions are useful in that they allow the research to place limits on the scope of the study and increase the feasibility of completing the project, with specific propositions allowing for the construction of boundaries in a case study. Propositions may come from the literature, personal/professional experience, theories, and/or generalizations based on empirical data (Baxter and Jack, 2008). A case study may contain several propositions to guide the study. They are distinct from each-other and allow for a specific purpose when determine what data to collect and how to use that data to inform discussion. Each proposition serves to focus the data collection, determine

direction and scope of the study and together the propositions form the foundation for a conceptual structure/framework (Miles and Huberman, 1994).

The unit of analysis involves identifying the case that is to be researched and how it will be analysed. Over the course of research, the unit of analysis can change, due to improved designed research questions, or new opportunities for inquiry presenting themselves. When data is collected, the results can be linked back to the initial proposition stated in the research design. This allows for purposeful data analysis that links to the overall research question(s). The last consideration is criteria for interpretation, which allows the researcher to develop a manner for interpreting the analysed data to ensure it reflects what is being asked by the research. In the implementation of a case study, many sources of evidence can be collected, such as documentation, archival records, interviews, direct observation, participant-observation and physical artefacts (Yin, 2014).

Documentation involves the analysis of any documented material related to the case study. It can take the form of letters, memoranda, agendas, administrative documents such as reports, formal studies and/or evaluations. Archival records involve the analysis of archival data such as "public use files" (example, census data), service records and organisational records. Interviews allow for direct inquiry with the research participants and allow for a fluid generation of qualitative data from the interview participants. Direct observations involve directly observing the case participants, using data collection activities that range from casual to formal. In participant-observer data collection, the researcher assumes a role within the fieldwork situation and may participate in the actions being studied and record their own field notes as the research progresses. Finally, physical artefacts are the retrieval of physical evidence from the case study, with can be analysed upon completion of the evidence collection. The different strengths and weaknesses of each of these evidence collection types are presented in Table 3.1.

Using multiple sources of evidence allows for triangulation of data. The requirement for multiple evidence sources in case studies far exceeds that of other research methods (Yin, 2014), due to the method's limitations. Multiple evidence corroborating the same finding gives more weight to their validity. By triangulating data, the researcher constructs validity in their case study, and increases confidence that the research accurately renders the event being studied. Figure 3.1 illustrates the convergence of triangulated data to the same findings, and the non-convergence of un-triangulated data from separate sub-studies.

As every classroom is different and student's experiences and prior knowledge may be different, it is difficult to control all the variables that could affect student performance. To determine the progression of student's conceptual development, multiple sources of evidence are employed, discussed in-depth in section 3.4, such as pre-test – post-test comparisons, analysis of student's artefacts, teacher observations, teacher feedback, student feedback and teacher-student interviews.

This allows for the identification of patterns in the student's progression in their conceptual development and attribute this to the students completing the materials.

SOURCE OF EVIDENCE	Strengths	Weaknesses
Documentation	• Stable – can be reviewed repeatedly. • Unobtrusive – not created as a result of the case study. • Specific – can contain exact names, references, and details of event. • Broad – can cover a long span of time, many events and many settings.	• Retrievability – can be difficult to find. • Biased selectivity, if collect is incomplete. • Reporting bias – reflects (unknown) bias of any given document's author. • Access – may be deliberately withheld.
Archival records	• *[same as documentation]* • Precise and usually qualitative.	• *[same as documentation]* • Accessibility due to privacy reasons.
Interviews	• Targeted – focuses directly on case study topics. • Insightful – provides explanations as well Insightful into interactions and personal views (e.g., perceptions, attitudes and meanings)	• Bias due to poorly articulated questions. • Response bias. • Inaccuracies due to poor recall • Reflexivity – interviewee gives what interviewer wants to hear.
Direct observations	• Immediacy – covers actions in real time. • Contextual – can cover the case's context.	• Time – consuming. • Selectivity – broad coverage difficult without a team of observers. • Reflexivity – actions may proceed differently because they are being observed. • Cost – hours needed by human observers.
Participant – observation	• *[same as direct observations]* • Insightful into interpersonal behaviour and motives.	• *[same as direct observations]* • Bias due to participant-observer's manipulation of events.
Physical artefacts	• Insightful into cultural features. • Insightful into technical operations.	• Selectivity. • Availability.

Table 3.1. Six sources of evidence: Strengths and Weaknesses (Yin, 2014)

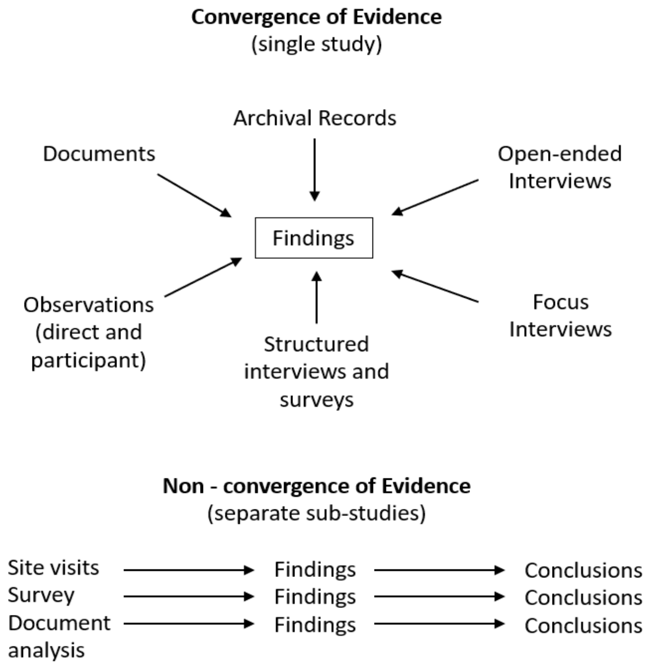


Figure 3.1. Convergence and non-convergence of evidence (Yin, 2014)

3.2.3. Case study limitations

Case studies produce unique research that can allow for understanding complex phenomena in particular contexts (Simons, 1996). The contexts tend to be unique and dynamic, allowing for investigation into complex and unfolding interactions of events, human relationships and influences and other unique factors (Cohen, *et al.*, 2000). However, the case study methodology is not without its limitations. Some researchers consider it to be a less than desirable form of research methodology, to which Yin (2009) suggests the following reasons:

- The lack of rigor of case study research, possibly due to a lack of literature on the subject when compared to other research methodologies.
- Single-case studies do not provide enough evidence for generalization.
- The time required to complete case studies, which can produce massive unreadable documents.
- Case studies do not provide enough evidence to determine “causal” relationships.

Nisbet and Watt (1984) note that results from this research (i) may not be generalizable to the wider population but may be useful to studies in which the contexts are similar, (ii) are not easily open to cross checking, and may be selective, biased, personal or subjective and (iii) prone to problems of observer bias. Simons (1996) notes that the first weakness is often cited as a case study

weakness, but this view assumes a polarity in research. However, an alternative perspective of this is that case studies allow for the development of descriptions and explanations to the case being studied, whilst acknowledging the numerous variables in the context that can affect the study. While it is outside the scope of this research to attempt to resolve these limitations of case study research, limitations of the case study methodology are acknowledged.

3.2.4. Applying a case study to the research

In this research, the case to be studied is the conceptual development and understanding of one class of upper secondary level physics students in the academic year 2016/17, in the topics of vectors, inverse square law and field lines, and how they transfer their understanding of these topics when learning about Coulomb's law, electric fields and potential difference. As described in-depth in section 3.5., the group consists of mixed gendered, mixed ability students which varies from gifted to weak students.

A case study was chosen for this group for the following three reasons. The sample size of the case being studied consists of fourteen students. Any findings generated from data from a sample of this size are not reliable to make any generalisations that apply to the wider population of upper secondary physics students. Outside of instructional design, student abilities, student attitudes and previous learning experience from different teacher influences are factors that could influence the outcomes of this case study. A case study is an appropriate method to use when extra variables that influence the findings cannot be controlled. As the influence of these variables changes from school to school, the research can also define the case as a contemporary grouping of students, which is difficult to study in other forms of qualitative analysis.

As suggested by Yin (2014), numerous sources of evidence are collected for triangulation to determine conceptual development and link how the tutorials promoted the development. Using all the evidence collected, would produce a report of unreasonable length. Collating the data and providing a varied sample of student responses results in a concise overview of student's progressions and difficulties encountered when completing the tutorial lessons. However, it is acknowledged that in some cases, the omission of some students work in favour of other's, may not provide a complete picture of what occurred during the lessons, but it is endeavoured to keep this to a minimum. A final consideration is that while the tutorials are likely the only intervention the students use to explore concepts related to the research, it is acknowledged the evidence can only provide indications that attribute the developed materials to the student's conceptual gains but cannot provide a causal link between the two.

In drafting the propositions that underpin this research, relevant sections of the thesis that cover the background used to inform them are referred to. The propositions used in the case study to frame the research and determine how to collect data are:

1. Tutorial lessons are a good teaching methodology to enable students to develop conceptual understanding in physics topics. (Section 3.1)
2. Students that learn a concept through, and develop the ability to transfer between, multiple representations to develop their understanding of that concept. (Section 2.2., section 2.3., section 3.3
3. Vectors, the inverse square law relationship and field line representations are tools, that when mastered by students, will enable them to describe electric fields, and the interactions of charges with electric fields. (Chapter 4)
4. Vectors and field line representations are tools, that when mastered by students, will enable them to describe the work done in moving a charge in an electric field, and conceptually explain the potential difference between two points in an electric field. (Chapter 5, chapter 6)

3.2.5. Evidence collection in this study

In this study, five types of evidence were collected. These evidence types were categorized as primary and secondary evidence. The primary evidence is used in all aspects of the data analysis through all concepts covered by the students and is used to glean the conceptual understanding developed by the students. The secondary evidence is used to support of the findings from analysing the primary data, but in and of itself, it not rigorous enough to construct meaningful findings on their own. The primary evidence types are used to pre-tests / post-tests, student artefacts and teacher – student interviews, while the secondary evidence are recordings of student dialogues and teacher reflections.

3.2.5.1. Pre-tests and post-tests

Pre-test / post-test comparisons are used in this research to gauge conceptual development of student's models over the course of the inquiry lessons. Each question on the pre-test is designed to elicit student's understanding of one aspect of a concept. An ideal pre-test question is structured in a manner that only correct understanding of the concept being asked about can bring about a correct answer. However, when asked to predict outcomes or produce rankings, it is possible for students to guess a correct answer, and so students are also required to display the reasoning they used to inform

their answer. The contexts of the questions are typically simple ones, so students are not influenced by the context itself. Additionally, by designing the pre-test questions to be as simple as possible, the students would be less likely to be put off answering the question by the perceived difficulty of the pre-test.

The post-test questions were designed in the same format as the pre-test questions. In some questions concepts were also asked in a manner that only one aspect of a topic was tested in any one question and correct understanding of the concept was required answer to the question correctly. The questions are not designed to guide students through the question, as they are in the tutorial. Additionally, there were questions asked that required the use of multiple concepts to answer correctly. By comparing student's answers between their pre-tests and post-tests, indications to the level of conceptual change that occurred could be observed. This provides evidence to judge the effectiveness of the instructional materials and the determination of what revisions need to be considered for redesigning future materials.

Students completed pre-tests in class, before completing the tutorial and after the teacher presentation of the topic. These tests were scanned and stored on an external hard-drive. Each pre-test was analysed, and student responses were categorized based on concepts / misconceptions used in answering the questions. These categories were used in the post-test questions, allowing for direct comparison between student answers in both tests. Further analysis of student's understanding was then gleaned from specifically looking at the student's articulation of their answers and their direct application of the concepts when answering questions. In this case, student's understanding must be interpreted from their written responses, especially in cases where the student used common colloquial language, instead of physics specific terminology and it is acknowledged that while this is a strong indication of student thinking, it cannot provide a complete representation of it.

3.2.5.2. Student artefacts

Samples of the student tutorial materials and the homework assignments were collected and scanned. The homework assignments were categorized in the same manner as the pre-tests and post-tests, while the tutorials were reviewed to identify where particular students encountered difficulties. When this occurred, students were requested to not destructively erase their earlier writing, and instead use extra space, to allow for the comparison of incorrect and correct reasoning produced by the student. This gives insight into student thinking, that is not afforded in the pre-test/post-test comparison. The use of the pre-test/post-test model can struggle to identify the source of difficulties in student reasoning. In analysing student artefacts, it is possible in some instances to pinpoint where student thinking diverged from what was the intended reasoning. In class, this was used by the teacher

to consider what line of questioning and prompts were required to enable the student to revise their thinking and develop news lines of thought to arrive at the correct conceptual understanding.

The homework assignments allowed students to review and extend their understanding of concepts covered in the tutorial. In some cases, questions from the pre-test appeared in the homework assignments. This gave the opportunity to observe how students answer the same question to determine if there is any change in their answers. When the homework assignment was designed to allow students to review the lesson material, it was possible to gauge whether there was divergence in student thinking when in a group setting in the tutorial or reaffirm correct reasoning between both the tutorial and the homework assignment.

3.2.5.3. Teacher-student interviews

The teacher-student interviews were conducted after the students completed the tutorial lessons. Students who completed the interviews were generally identified as having difficulties with concepts covered in the tutorials and post-tests. This allows for the use interviews as an extra mechanism for the student to help students alleviate their difficulties and provide insight to the source of their difficulties.

A small 20-minute tutorial worksheet is developed for these interviews, which explore the same concepts as those seen in the tutorials, but in an unseen context. As the interviewer, I provided prompts and scaffolds to the students, as they complete the worksheet. As students answered questions, I could ask the students to explain the reasoning used by the students to develop their answers. This gave insight into what the students were thinking and allowed them to articulate their reasoning more in depth than the reasoning some students submitted in the tutorials. I could also observe the students discussing the questions with each-other and take note of how the students developed and supported each-other's reasoning over the course of the teaching and learning interview. The interviews also allowed me to produce a record of what interventions and prompts I used that were effective in facilitating student conceptual development.

Engelhardt, *et al.*, (2004) highlight this as an advantage of using teaching-learning interviews. The teaching-learning interview also provides an opportunity to continually probe student's understanding in a manner not afforded when using post-tests. When students provide assertions, the interviewer can question the nature of these assertions. If the student's explanation is not complete, the interview can take time to explore their explanations, so the student clearly articulates their thinking, in a manner that they do not consider in the tutorial lesson. However, as this focus on student's specific explanations of their reasoning is atypical of the depth feedback generally provided in the tutorial lessons, the interview does not accurately reflect the learning experience the student

encounters in a classroom. Chini, *et al.*, (2009) note that this is a limitation of the interview, in that, what interventions work in this setting may not work in a classroom setting.

3.2.5.4. Recordings of student dialogues

Student conversations that took place during the lessons were recorded. As the students worked together to complete the materials, developments in student's conceptual understanding that were verbalised may not be recorded in the artefacts. Having recorded student's conversations, it is possible to analyse the reasoning used by students in their discussions to observe how student difficulties were overcome. Due to the massive amount of data generated, and the time required to analyse the data, the students were asked to record the time on the dicta-phones when they felt that other students helped them overcome difficulties they encountered during the lessons. The teacher also recorded the time in which they used questions and prompts to help students develop and consolidate their thinking.

3.2.5.5. Teacher reflections

Upon completing of the tutorial lessons, the teacher drafted reflections, based on his observations and feelings on what felt worked and did not work in the lesson. In a teacher reflection, the teacher engages in a cognitive process, which involves providing a commentary on difficulties encountered during the sequencing of the lesson. Reflective thinking generally addresses practical problems, allowing for doubt to be used as a mechanism to identify problems and shortcomings before solutions are reached (Hatton and Smith, 1995). By reviewing the teacher reflections, a comparison of the teacher's feedback and recordings of student dialogues can allow for the identification of student difficulties to take place. Specific difficulties that the teachers encountered during implementation can also be identified and open a conversation space to discuss ways to alleviate encountered difficulties.

3.3. Implementation

This section of the research discusses the implementation methodology for this research. It describes different influences used to develop the tutorial lessons implemented in this research. The structure and format of a tutorial lesson and details their implementation are then presented. The justifications of using tutorial lessons with the second level students is discussed. Additionally, as

the participants in this research were under 18 years of age, the ethical considerations that were made, and the approval by DCU's ethics committee is outlined in this section.

The approach adopted in this research is the use of structured inquiry. Instead of students being presented with the content and required to learn it off, learning occurs through students answering questions, solving problems and working through challenges (Lynn, *et al*., 2013). As seen in section 1.3, improving student's conceptual understanding is central to the research questions of this research, and inquiry has been shown to be effective in this pursuit (Tabak, *et al.*,1995; Blanchard, *et al.,* 2010). The tutorial lessons developed in this research as the teaching and learning materials, are based on the approach from Tutorials in Introductory Physics (McDermott and Shaffer, 2003). The use of tutorial lessons has been shown to promote gains in student's conceptual understanding of physics concepts (McDermott and Shaffer, 1992; Wosilait, *et al.,* 1998). As stated in section 2.2.2, exercises from the Conceptual Physics practice book, 10^{th} edition (Hewitt, 2009), were used to influence the design of the tutorials in this work, as the way Hewitt approaches contexts would be more accessible to the second level students. Sections 3.3.1 and 3.3.2 discuss how inquiry was employed in this research, by detailing what a tutorial lesson looks like, and justifying the use of inquiry in this research.

3.3.1. What does a tutorial lesson look like?

The following description of a tutorial has been patterned after Tutorials in Introductory Physics – Teachers Guide (McDermott and Shaffer, 2003). It summarizes what occurs when students complete a tutorial lesson, from start to finish. The logistics of implementing tutorial lessons into a second level classroom are also described.

A tutorial lesson is designed to supplement lectures and textbook materials in Physics courses. Students do not focus on reciting definitions, listing how to complete demonstration experiments, or completing quantitative problems. Instead, the focus is shifted on students developing their understanding of physical concepts and the use of their scientific reasoning skills. Tutorials provide a structured format for students to determine what they do and do not understand when learning physics. In small groups, students complete a series of questions in which they are guided through the reasoning necessary to construct scientifically valid models of a chosen topic in physics. They provide opportunity for students to interpret and represent concepts using a variety of representations, such as formulae, graphs, diagrams and verbal descriptions. Students take part in the tutorial lesson after they have been initially introduced to a concept through a lecture, presentation and/or laboratory, but can be used to introduce a concept or extend it.

As discussed in-depth in section 2.2.2., this research developed tutorial lessons to promote the student's conceptual understanding. The tutorial lessons comprise of a pre-test, worksheet assignment, homework assignment and post-test. The pre-test examines student's prior knowledge and understanding of a concept. This allows for the identification of difficulties to be targeted for conceptual change. The worksheet assignment is completed by the students in small groups. They work through the exercises together, engaging in dialogues between themselves or with the teacher when difficulties are encountered. The conditions of conceptual change (Posner, *et al.*, 1982) are generally encountered and met at this stage. The homework assignments are completed by the students, which gives them the opportunity to revisit the concepts from the worksheet or in some cases extend them. The context of the homework may or may not be altered from contexts in the worksheet exercises. The post-test is given at the end of the tutorial lesson. It is written to focus on the concepts and skills used in the tutorial lesson. Comparisons between the pre-test and post-test allow for the generation of evidence that can be used to identify and indicate the extent to which conceptual change occurred.

With the participants of this research, they completed a lesson in which concepts were introduced through power-point presentations, demonstrations and practicing qualitative problems and classroom discussion. These lessons occurred in either a single lesson period, (40 minutes in length) or a double lesson period (80 minutes in length). The students then completed tutorial lessons in a double lesson period. The materials themselves were written with the materials designed to take up to 50 minutes of class time, which affords time at the start of the lesson to be used to take the pre-test, and time at the end for a quick round up with the class in their entirety.

When covering inverse square law, vectors and field lines, the students completed one tutorial a week, and the pre-tests, homework assignments and post-tests are completed around the tutorial. During the section on electric field, a traditional lecture style class introduced concepts related to charge and Coulomb's law, and the pre-test and tutorial on Coulomb's law followed. Students were then given another lecture style lesson introducing the electric field and completed quantitative problems. Students then completed all the electric field tutorials in succession and a post-test based on Coulomb's law and the electric field was completed. When the students completed the tutorial lessons on work and potential difference, they completed the pre-test before completing the work tutorial, engaged in two tutorial lessons, one on work and one on potential difference, before completing the final post-test for these two topics together. An in-depth timeline for the implementation of the tutorials is presented in Chapters 4, 5 and 6.

3.3.2. Justifications for using inquiry tutorials

In section 2.1, the role of inquiry in science and physics education was discussed, and in the previous section, the application of inquiry in this research, using tutorial lessons, was outlined. This section provides justification for the use of structured inquiry in this research, in terms of the efficacy in promoting targeting and promoting conceptual understanding, how it aligns to some of the aims of the Leaving Certificate Physics syllabus (1999) and how structured inquiry allows for balance between content requirements and conceptual depth of treatment.

Structured inquiry was utilised in this research to promote conceptual change in the students understanding. Structured inquiry was chosen as it has been shown to be a favourable method to develop conceptual knowledge in students (Tabak, *et al.*, 1995; Blanchard, *et al.*, 2010). Ambrose (2004) notes that it is effective for targeting specific concepts for conceptual change. Section 2.1.3 illustrated the effectiveness of structured inquiry tutorials by detailing one example from research (Shaffer and McDermott, 2005) and referencing others (Close and Heron, 2010; Heron, *et al*., 2004; Hazelton, *et al.*, 2012). This research initially targets students conceptual understanding of vector concepts, the inverse square law and field line and then targets how they employ these topics to develop their understanding of Coulomb's law, the electric field, work and potential difference. Structured inquiry is employed in this research as evidence from the research literature suggests it is an effective method to achieve the teaching and learning outcomes that unpin this research.

Tutorial lessons involve learners working in small groups to discuss their thoughts, ideas and understanding of concepts that underpin the topics they are studying. This provides them the opportunity to articulate their thinking and communicate to each other to collate their understanding. This is in line with the aims of the Leaving Certificate Physics syllabus (NCCA, 1999, pg2) which states students should develop "an understanding of the fundamental principles of physics" and "develop the ability to observe, to think logically, and to communicate effectively." The tutorial lesson format provides students the opportunity to attain these aims.

Finally, as tutorial lessons allow for the targeting of specific concepts, the lessons were developed in a manner that the core learning outcomes for the Leaving Certificate Physics syllabus (1999) were met. The depth of understanding targeted by the research was balanced with the content the students were required to learn for completion of the course. This way, the students have completed the required content to complete questions on their terminal examination. Other forms of inquiry, such as guided and open, can result in students learning content not targeted by the teacher, and may miss specific content designated by the learning outcomes of the course they follow, but develop their competencies in their cognitive and procedural autonomy (Stefanou, *et al*., 2004). Guided and open inquiry was not employed in this research, as developing these competencies were not targets in this research.

However, there are notable difficulties in applying general inquiry methods to a classroom. These difficulties include, but are not exclusively, the workload involved, lack of resources and the effect the increased workload has on the teaching time required to finish a syllabus (Higgins, 2009). To address these difficulties, the materials are designed and prepared during school breaks, to not interfere with the time allocation to the teacher in preparing lessons. The topics in which the lessons are used, electric field and potential difference, take up a relatively small part of the overall physics course, and thus, the time allocated to completing the classes is minimal. Ambrose (2004) notes that structured tutorials allow for an inquiry implementation in a relative short duration of time. As the tutorials are mainly paper and pen focused, no special laboratory equipment needed to be procured to complete the material and any practical equipment the can be of use is generally readily accessible in the standard fit-out of a second level physics laboratory.

3.3.3. Ethical considerations for research involving second level students

To conduct this research, ethical approval was sought from the Ethics Committee of Dublin City University. As this project involved the analysis of data taken from students, it was important for the university to ensure that every effort was taken to ensure the educational experience provided to the students was well planned and would not hinder their academic progress in their physics course. The allotted time that could be spent on the research was also considered, to ensure they experienced minimal disruption to the lessons required to complete their physics course.

Other issues considered when seeking ethical approval for this project was honesty in reporting results, both positive research and negative results, identification of any potential bias, as the funding body are involved in the promotion of inquiry based learning, and human subjects protection, provided as an opt out of the research and communicating with the school care team regarding student welfare during the project, in the event that students were to find the teaching approach adding undue stress to their educational environment. Approval for this project was granted December 2013 (DCUREC/2013/197) for the pilot trail of the materials and amended September 2014, to include the additional schools for garnering data for external validation purposes.

All students were given two copies of an informed consent form, two plain language statements and a cover letter briefly outlining the aim and rationale of the project. As the students were under 18 years of age, their parents / legal guardians were required to sign the consent forms and return one copy to the teachers involved. Parents were informed of the aims of the project and given the opportunity over a two-month time frame to contact me with regards to any concerns regarding the project, prior to its commencement. Parents were also informed that their children could opt-out of the research at any time, in which any data recorded would be destroyed electronically, and paper copies of materials would be shredded.

Each student's name was known only to the teachers involved in the classrooms. This allowed for the comparison of pre-test data, post-test data, class worksheets and homework worksheets of individual students. Each student was given a unique reference code, which is used to identify him or her in this document. The reference code is made up of a number and a letter. The number referred to the year of the research and the letter referenced the student. In the case of the 2014/15 academic year, the second group to complete the material were given two letters. For example, student 2C took part in the 2014/15 editions of the materials, is from the second group of students to complete the materials and is the third person on the class roll as denoted by C being the third letter of the alphabet.

3.4. Analysis Methodology - Qualitative explanatory and qualitative descriptive

There are numerous types of case studies, as outlined by Baxter and Jack (2008), such as explanatory, exploratory, descriptive, intrinsic, instrumental and collective. The type of case study devised in this research contain elements of both an explanatory and a descriptive case study. Explanatory is when a case study would be used if you were seeking to answer a question that sought to explain the presumed causal links in real-life interventions that are too complex for the survey or experimental strategies. In evaluation language, the explanations link program implementation with program effects (Yin 2003). Descriptive is a type of case study is used to describe an intervention or phenomenon and the real-life context in which it occurred (Yin, 2003).

As the sample size of the students is small, quantitative data and quantitative data analysis tools cannot be used reliably. Instead descriptive qualitative data analysis is used, as it gives insight into understanding the context, participants and interventions encountered when the research was conducted. In this manner, a collection of data that allows for understanding the learning process that occurs during the tutorials lessons applied during the tutorial lessons could be generated. The data analysis allows for the drawing of patterns based on the progression of student conceptual development and provides illustrative explanations of their conceptions, based on their individual responses.

In the research, the educational instructional intervention used in the research is the developed inquiry materials used in a tutorial lesson. A case description (Yin 2009) is developed on the theoretical propositions discussed in section 3.2.2. Over the course of chapter 4, 5 and 6, the implementation of the tutorial lessons is described as an observational narrative. An analysis of the student's conceptual developments in vectors, inverse square law and field line representation is detailed in chapter 4 and provide a commentary on how these developments enable students to explore Coulomb's law, the electric field and potential difference.

Through this narrative, content analysis (Carley, 1993), is used to describe how the tutorial lessons promoted conceptual gains in the student models, using multiple forms of evidence gathered, as described in-depth in section 3.2.3. In the content analysis, the evidence was primarily used to populate matrices of student responses and concepts/misconceptions apparent in the data, to determine whether students are using key concepts in their correct contexts or overlapping their understanding (Yin, 2003). These matrices were analysed to produce the results tables shown in Chapter 4, 5 and 6. As a consideration to remove potential bias, initial blind marking was used when populating these matrices. This was done by removing the codes from the scanned student artefacts and initially recorded the frequency of the concepts used and difficulties observed in the student's responses. The artefacts were then analysed again, with the student codes intact, and were used to populate the student responses, as shown in the tables of results throughout chapters four, five and six.

An assumption of the research that students developing a functional conceptual understanding of vectors, inverse square law and field line representation will result in positive conceptual gains in electric field and potential difference, as explained in various sections of chapter 2. This is tested for using a type of pattern matching, determining whether the concepts used in the later topics are transferred from the former topics, and provide a narrative on how the students use these initial concepts to build their understanding of the electric field and potential difference. In the discussions of each section, instances of conceptual change will be discussed, collating evidence from as many of the various the evidence types as possible to identify conceptual extinction, exchange and extension where possible. If conceptual change did not occur, difficulties are identified and suggestions for redesign are given.

Four descriptors are selected in this research to indicate the extent of conceptual change that occurs over the course of this research. As shown in Figure 3.2, these descriptors are based on the total number of 14 students that were studied to measure their conceptual change for a given concept. Minimal conceptual change refers to instances in which between one and three students demonstrated that conceptual change occurred. Partial conceptual exchange refers to instance where between four and seven students were observed to have engaged in conceptual change. Moderate conceptual change is referred to when between eight and eleven students engage in conceptual change and ideal conceptual change is referred to when between twelve and fourteen students demonstrate conceptual change.

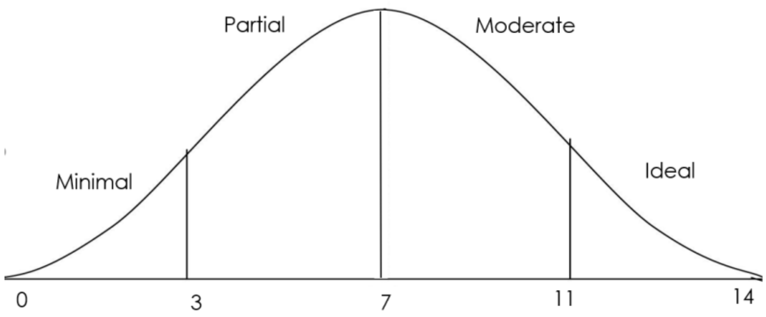


Figure 3.2. Illustration of descriptors used to describe extent of conceptual change within a group of 14 students.

3.5. Description of participants

The group of students that undertook this research was a group of 14 mixed ability students, aged 16 – 17 years old. The group is mixed gendered, but predominantly male (female = 4 students, male = 10). Three of the students speak English as a second language but are fluent in their use of the language. One of the students took part in the educational system of New South Wales, Australia for all the years they spent in formal education, before enrolling in the upper secondary Irish educational system in the academic year 2016/17. To illustrate the general ability of the students in science and mathematics, Table 3.2 shows the student's results from their lower secondary level final examinations in mathematics and science, which were produced by the State Examinations Commission. A copy of the results was obtained from the school where the research took place. These exams are unseen by students and teachers in Ireland, and all students partake in these so-called Junior Certificate examinations at the exact same time. The grades are presented as categorical ordinal data, with A being the highest grade and NG being the lowest grade. The table also includes their results from an in-house Christmas physics examination. This is an indication of the general ability of the students, having studied Leaving Certificate Physics for 4 months. This examination took place after the vector, inverse square law and field line tutorial lessons, but before the electrostatics tutorials. The grades for the in-house test are presented as categorical ordinal data, with H1 being the highest grade and H8 being the lowest grade, mirroring the grading system of the Leaving Certificate exam they undertake at the end of 6th Year.

Figure 3.3 presents the Junior Certificate exams results in bar charts, and the in-house Physics examinations results are shown in Figure 3.4. This allows for identification of the clusters of the results, to create a profile of the student's academic achievements in these exams. As both charts show the cluster of data centralized towards the higher grade, the student's results suggest that the students are a mix of high achieving and average achieving students.

	Junior Results		Senior Results
Student Code	**JC Science**	**JC Maths**	**5th Year Christmas**
4A	C	C	N/a
4B	B	B	H1
4C	B	B	H2
4D	B	D	H5
4E	A	B	H3
4F	N/a	N/a	N/a
4G	A	A	H1
4H	B	C	H4
4I	B	B	H1
4J	C	D	H2
4K	A	B	H3
4L	C	D	H6
4M	B	D	H6
4N	A	A	H2

Table 3.2. Summary of student's results pre-research.

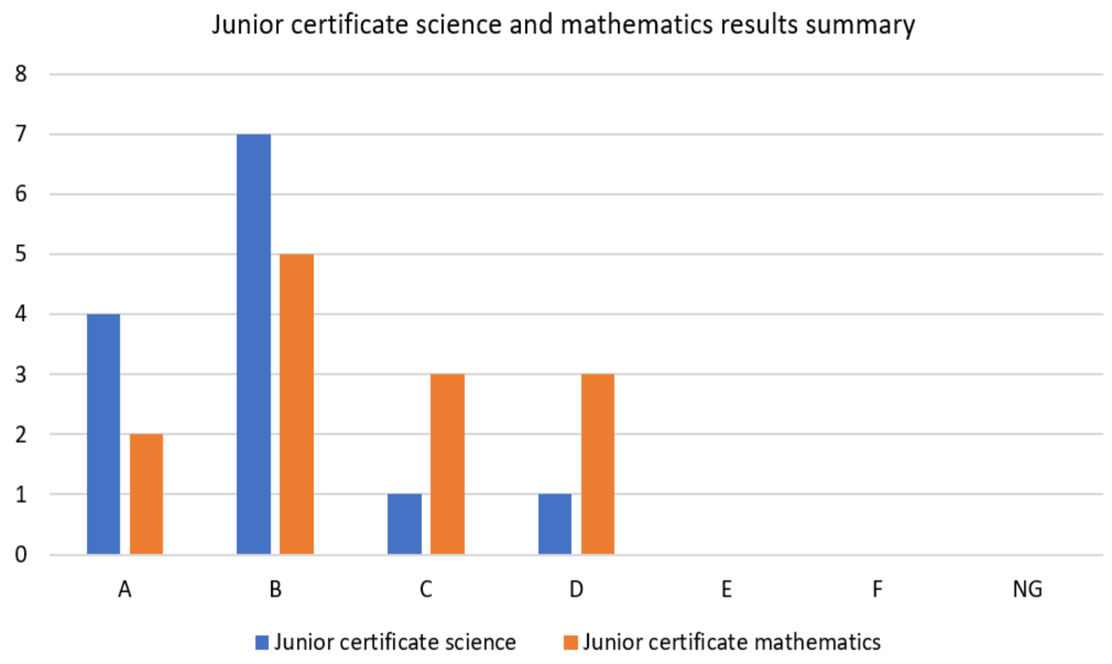


Figure 3.3. Student's results for Junior Certificate Science and mathematics examinations.

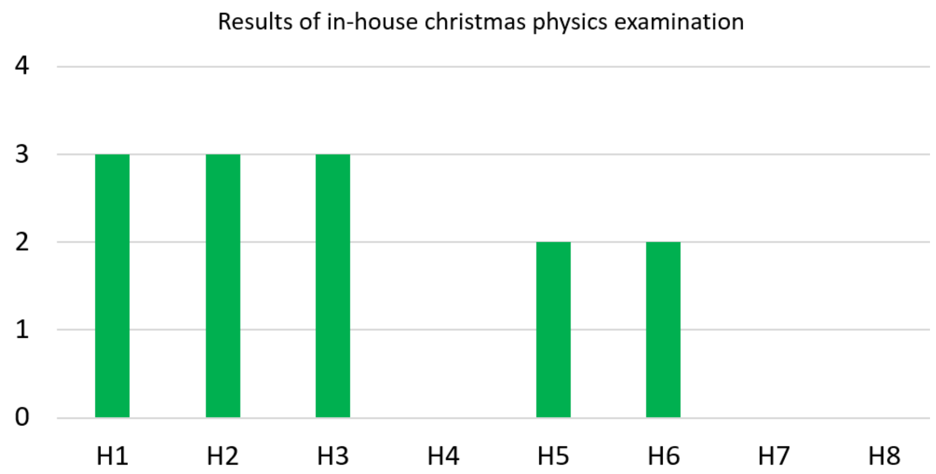


Figure 3.4. Student's results of in-house physics Christmas examination.

3.5.1. Participants' prior learning

This section discusses aspects of the student's prior learning in formal education, for lower secondary level science and mathematics. As the lower secondary level science and mathematics course are delivered for a duration of three years, this section will only reference the content covered in these subjects that are relevant for this research. As the students had different teachers for lower secondary science and mathematics, the aims and learning objectives from the syllabi for Junior

Certificate Science (NCCA, 2003) and Junior Certificate mathematics (NCCA 2012) will be referenced to illustrate the student's prior learning in these subjects.

According to the Junior Certificate Science syllabus (NCCA, 2003), the study of science contributes to a broad and balanced educational experience for students, extending their experiences at primary level. The course aims to promote student's development of scientific literacy, science process skills, and an appreciation of the contribution that science had on the humanity and the planet. The junior science course was designed to be investigative and activity based, and the courses' aims (NCCA, 2003) seek to encourage student development of manipulative, procedural, cognitive, affective and communication skills. Student are to be provided with opportunities for observing and evaluating phenomena and processes and for drawing valid deductions and conclusions, which aids to enable students to acquire a body of scientific knowledge appropriate to their age, and an understanding of the relevance and applications of science in their personal and social lives. The nature of the activities and investigations also aim to help students develop a sense of enjoyment in the learning of science. However, the extent to which teachers use lessons that allow student to engage in activity or investigative based exercises is not always in line with the extent intended by the syllabus (Weymms, 2008).

No matter the manner of instruction chosen by a lower secondary science teacher, the students must have completed the same learning objectives, as part of their lower secondary science course. The design of the tutorial materials identified the following objectives are relevant to the research, and considered the following to be required prior learning completed by the students:

- appreciate the concept of force; recall that the newton is the unit of force; describe forces and their effects.
- investigate examples of friction and the effect of lubrication.
- investigate the relationship between the extension of a spring and the applied force.
- recall that weight is the force due to gravity and that weight can vary with location; recall that mass in kilograms multiplied by 10 is approximately equal in magnitude to weight in newtons on the surface of the earth.
- carry out simple experiments to show attraction and repulsion between magnets and test a variety of materials for magnetism.
- plot the magnetic field of a bar magnet.
- demonstrate that the Earth has a magnetic field and locate north and south.
- use simple materials to generate static electricity; demonstrate the force between charged objects and the effect of earthing.

In the Junior Certificate mathematics syllabus, (NCCA 2012), it states that mathematics is the study of quantity, structure, space and change. In Mathematics, students develop skills in numeracy,

statistics, basic algebra, shape and space, and technology that have many uses in society. These skills can be utilized to allow students to make calculations and informed decisions based on information they come across in their everyday lives. Students also develop the skills to become good mathematicians, which allows them to compute and evaluate a calculation, follow logical arguments, generalize and justify conclusions, and apply mathematical concepts learned to real-life situations.

In the design of the tutorial materials, I took note of the learning objectives (NCCA, 2012) in the mathematics syllabus as relevant to the research. The mathematical aspects of the tutorial lessons were designed so they would align with the student's prior knowledge. This include students understanding co-ordinate geometry, the application of the rules of indices, the use of scientific notation. The algebraic understanding included the use of tables to represent a repeating-pattern situation, generalizing and explaining patterns and relationships in words and numbers. Writing arithmetic expressions for terms in a sequence and using tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations. When using tables, graphs and equations, the students would have learned to distinguish those features that are appear in the different representations, use the representations to reason about situation from which relationships can be derived and communicate their thinking to others and recognize problems involving direct proportion and identify the necessary information to solve them without formulae. Finally, the students have completed the using of graphs to represent various phenomena in different contexts including motion graphs, interpreted quantitative graphs, make connections between the shape of a graph and the story of a phenomenon and describe both quantity and change of quantity on a graph. In addition to considering the learning objectives, all the mathematical understanding and operations related to algebra (NCCA, 2012, pg. 28) is considered prior learning for this research.

This section of the description of participants illustrated the student cohort who agreed to undertake in this research. A description of the students was presented, results from both state examinations and an in-house exam were presented to help gauge the progress of the students in formal education in science, mathematics and physics thus far. An in-depth look at the specifics of the Junior Certificate Science and mathematics courses was presented, highlight the aims of both courses, and the objective completed by the students which are relevant to this research.

Chapter 4. Vectors, inverse square law and field lines

4.1. Introduction

This chapter addresses the development of student's understanding of vector concepts, the inverse square law, and field line representations as they completed the tutorial lessons. This chapter aims to address the first three of the research questions:

1. To what extent does the use of a structured inquiry approach develop student understanding of vector concepts?
2. To what extent does the use of a structured inquiry approach develop student understanding of the inverse square law relationship, by employing multiple representations
3. To what extent does the use of a structured inquiry approach develop student understanding of the field, when utilising field line representations?

These questions are important to the overall research as vectors, field lines and the inverse square law are the foundational topics for promoting conceptual understanding of Coulomb's law, the electric field and potential difference. We use structured inquiry tutorials to develop the student's understanding of three representations, as this allows us to (i) target specific concepts for the students to develop, (ii) design lessons that can easily be designed to be completed in a 60-minute timeframe and (iii) put the emphasis on the students to explore and develop their own understanding of vectors, field lines and the inverse square law. Having developed an understanding of these topics, the cognitive demand on the students should be lessened when they apply and utilise them in a electrostatics context, and instead they can focus on their understanding of the electrostatics concepts. In Vygotskian terms, when the students come to develop understanding of electrostatics, the required concepts for vector representation will have been moved from the student's zones of proximal development to actual development (Vygotsky, 1979). Multiple representations are employed, as Ainsworth (1999) shows that they can have many positive roles in the learning process, such as displaying difference processes and information, constraining the content and concepts learned by the nature what the representation can communicate, and constructing deeper understanding.

The tutorials, in the style of Tutorials in Introductory Physics (McDermott and Shaffer, 2003), utilise scaffolding questions to help students construct and develop understanding of a particular topic or concept. Breaking a topic down into small questions prevents the students from being overloaded by trying to assimilate too many new concepts at any one time. This approach also allows

for the observations of student difficulties as they progress through the tutorial, allowing for identification of the cause of the conceptual difficulty.

Figure 4.1 depicts where the topics fit into the student's broader understanding of Coulomb's law, the electric field and potential difference. Items presented in the left box are completed at Junior Certificate, items at the bottom are completed during Leaving Certificate Physics course, items on top are addressed specifically by this body of research and items on the right are the electrostatic topics covered by this research, discussed later in chapter 5.

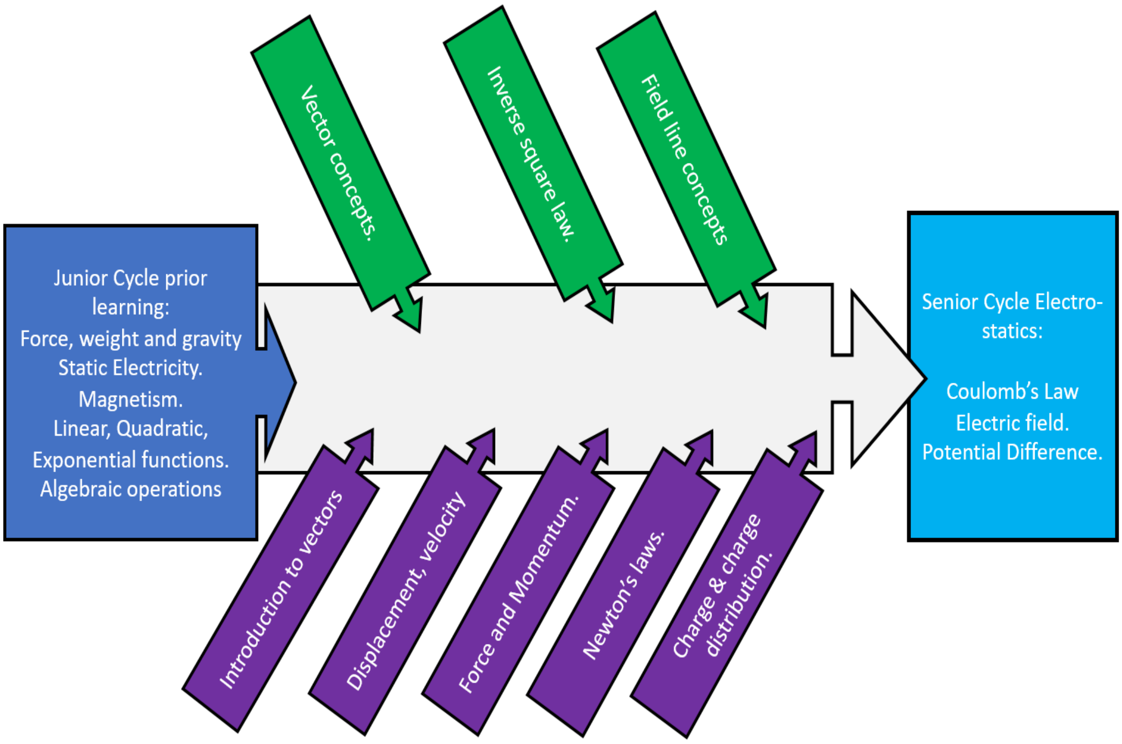


Figure 4.1. Flowchart depicting how the topics in this chapter contribute to electrostatics.

The timeline of this section of the project is shown in Table 4.1. Sections in bold refer to materials covered as they related to the research. Sections that are not presented in bold are required to be covered for completion of the required syllabus for Leaving Certificate Physics, but did not contribute to the collection of data used in this research. Week 1 denotes the beginning of the students studying mechanics, in their physics course. This coincided with the commencement of the first half of the research. These classes ran from the second last week in October to the second week in December 2016.

The order of the tutorials, (vectors, inverse square law and field lines) followed the order in which I normally teach the Leaving Certificate Physics course. This allowed for minimal disruption to the annual planning of topics. The vectors tutorial was implemented first, as it is one of the initial mechanics topics presenting in the Leaving Certificate Physics syllabus (NCCA, 1999). The inverse

square law and field lines tutorials followed later as Newton's gravitational law was contextually the first topic these representations could be applied to.

Week 1.	**Vectors Pre-test, vectors worksheet, vectors homework and vector constructions.**
Week 2.	Topics unrelated to project: Inclined planes, Momentum and Forces
Week 3.	**Vector Post-test.** Topic unrelated to project: Forces and Motion.
Week 4.	**Inverse square law pre-test**. Newton's law of gravitation. **Inverse square law tutorial.**
Week 5.	**Field lines pre-test. Field lines tutorial. Field lines homework. Further Newton's law of gravitation.**
Week 6.	**Inverse square law and field lines pre-test.**

Table 4.1. Timeline of the implementation of the first section of the research.

Section 4.2 covers the development of the student's understanding of vectors. Section 4.3 reports on the intervention used to promote the student's understanding of the inverse square law. Section 4.4 presents the development of the student's understanding of the use of field line presentation. In each of these three sections, a brief introduction presents common difficulties seen in literature by students used to form the learning objectives of the tutorial lessons. This is followed by a discussion of the student's initial understanding is elicited from the pre-test results, their development of the concepts is presented with a narrative of the tutorial lessons and homework assignments and their final understanding is delivered through an analysis of the post-test results. Each of the sections close with a discussion that compares the pre-test and post-test results. Difficulties targeted for conceptual change are identified and instances of Posner's conditions for conceptual change (1982) are referenced. Discussions of the post-test illustrate instances of conceptual change, when apparent. Finally, Section 4.5 presents the conclusions of this chapter, which discusses the progression of the student's understanding of the three different representations and implications for the use of the concepts in Coulomb's law, electric fields and potential difference.

4.2. Vector concepts

This section presents a narrative and analysis of the development of student's understanding of vectors. Section 2.1.3.1 detailed difficulties encountered by learners in understanding vector concepts

identified in the literature. The design of the vector tutorials focuses on student understanding of three introductory vector properties. Upon completion of the teaching and learning material, the students should be able to:

1. Differentiate between the magnitude and direction of a vector (Nguyen and Meltzer, 2003; Ivanov, 2011).
2. Demonstrate vector addition using the parallelogram and/or "tip to tail" constructions (Nguyen and Meltzer, 2003; Hewitt, 2009).
3. Consider vector components when adding vectors that are non-colinear nor perpendicular (Flores, *et al.*, 2008).

The inquiry approach consisted of a pre-test, a tutorial lesson, a homework and a post-test. This intervention ran over three weeks. A timeline for the implementation of this part of the study, including the target concepts for the intervention, is shown in Table 4.2. The vectors pre-test completed by the students was the first pre-test for the research, and the first experienced by the students. It was explained to them that the function of the pre-test was to allow for an indication of what they did and did not know about a given topic, that could later be used with the end of topic test (post-test) for comparative purposes.

Section 4.2.1 presents the pre-test results, looking at the difficulties the students have with representing vector magnitude, use of the parallelogram and "tip to tail" constructions and their prior understanding of horizontal and vertical components. Section 4.2.2 presents a narrative of the development of the student's understanding of vector concepts during the tutorial lesson. Section 4.2.3 presents an analysis of the homework assignment, which was developed to allow the students to practice the skills and apply the understanding they developed in the tutorial. Section 4.2.4 presents an analysis of the post-test results which, like the pre-test, focused on student's understanding of representing vector magnitude, the use of the parallelogram and "tip to tail" constructions and their understanding of vector components. Section 4.2.5 presents a comparison of the pre-test and post-test results, and a commentary of the student's progress during the tutorials. Examples of student progression and difficulties that persisted throughout the tutorial and post-test are discussed, and instances in which the conditions for conceptual change (Posner, *et al.*, 1982) were apparent during the tutorial lessons are discussed.

Time		Research Implementation	Target Concepts
Week 1	Class 1.	Pre-test	Relating magnitude to length of vector.
	Class 2.	Tutorial Lesson Homework	Parallelogram / Tip to tail construction of vector addition. Horizontal and Vertical components.
	Class 3		Lesson reserved for addressing difficulties observed during tutorial and homework, and extra time to practice using vector constructions.
Week 2		*N/a*	*Topics unrelated to project: Inclined planes, Momentum and Forces*
Week 3	Class 1	Post-test	Relating magnitude to length of vector. Parallelogram / Tip to tail construction of vector addition. Horizontal and Vertical components.

Table 4.2. Timeline of the implementation of the vector concepts study.

4.2.1. Pre-test: Vector Concepts

The pre-test on vector concepts was completed by the students before any instruction was delivered for vector concepts. As 12 of the 14 students study Applied Mathematics, they were familiar with some concepts related to vectors. Therefore, the pre-test could be used to identify what concepts the students understood before the tutorial, and what conceptual gains observed could be attributed to them completing the tutorial lesson. The students were given 15 minutes to complete the pre-test. The target concepts were magnitude of a vector and vector addition.

The first question, shown in Figure 4.2, looked at student's understanding of vector magnitude. The students were asked to rank the magnitude of five vectors represented by horizontal arrows from weakest to largest. Student's responses are presented in Table 4.3. Results highlighted in bold indicate a correct response or reasoning.

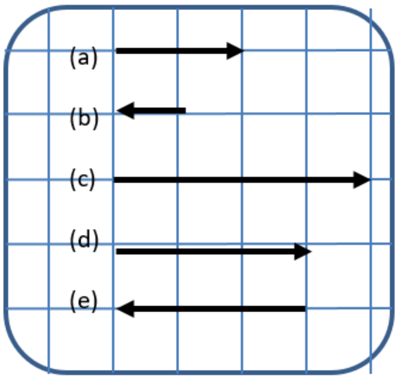


Figure 4.2. Pre-test vector magnitude ranking question.

A common difficulty (11/14) was that students thought the vectors that pointed to the left had a smaller magnitude than vectors pointing to the right. Students explain their reasoning as the vectors pointing to the left having a negative magnitude and the vectors pointing to the right having a positive magnitude. This was not addressed in their physics course, but introductory vectors using $\vec{\imath}$ and $\vec{\jmath}$ notation is covered in the Leaving Certificate Applied Mathematics course (NCCA, 2006), which all but students 4A and 4J completed. However, as Table 4.3 shows, student 4A produced the correct outcome but gave no reason to suggest they related the vector length to magnitude based on their understanding, and could have been the result of a reasonable guess. For the 11 students that did not produce the correct outcome, this indicates that students do not separate out the mathematical signs from the magnitude of the vectors, in this case, assigning negative to left and positive to right, and reasoning that negative integers have lesser value than positive integers. Nguyen and Meltzer (2003) also showed that when students were given scaled diagrams of various vector arrows, and asked to identify vectors of equal magnitude, their responses were influenced by not just the length of the arrows, but also the direction in which they pointed.

Responses	Students.
b < a < d = e < c	**4A**
c > e = d > a > b **(ranking from highest to lowest)**	**4C, 4M**
e < b < a < d < c	4B, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4N

Table 4.3. Summary of responses to the Vector magnitude ranking pre-test question.

In the second pre-test question students were asked to draw the resultant vector when two vectors at an acute angle are added (see Figure 4.3). Finding resultant vectors using the superposition principle is outlined as a mandatory skill in the Leaving Certificate Physics syllabus (NCCA, 1999). This pre-test question was designed only to determine if the students could combine vectors

diagrammatically, and if so, determine what vector construction they would use. The responses to this question are summarized in Table 4.4.

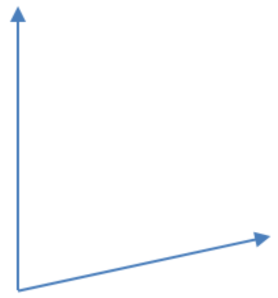


Figure 4.3. Pre-test vector construction of resultant question.

<u>Construction Method Used</u>	***<u>Students</u>***
"Tip to tail" construction (Nguyen and Meltzer, 2003)	**No responses**
"Parallelogram" construction (Hewitt, 2009)	**4B, 4H, 4I**
"Split the angle" construction	4C, 4D, 4G, 4M
Connects tails	4J
No attempt	4A, 4E, 4F, 4K, 4L, 4N

Table 4.4. Student's construction methods to find resultant of two vectors.

The results show us that only three of the students used a parallelogram construction, one of whom (4H) made a minor error in positioning the arrow head of the vector. Four of the students attempted to "split the difference" in which they bisected the angle but did not use any construction to determine the relevant magnitude (Nguygen and Meltzer, 2003). While these vectors may initially appear to be correct, there is no indication by the student's responses that they could determine any information about the resultant vector, other than its approximate direction. One student (4J) joined the resultant vectors with a curve but provided no reasoning, which indicates they made a guess to attempt to answer the question, without an understanding of what was being asked. The remainder of the students did not attempt the question. A sample of the student's constructions is presented in Figure 4.4.

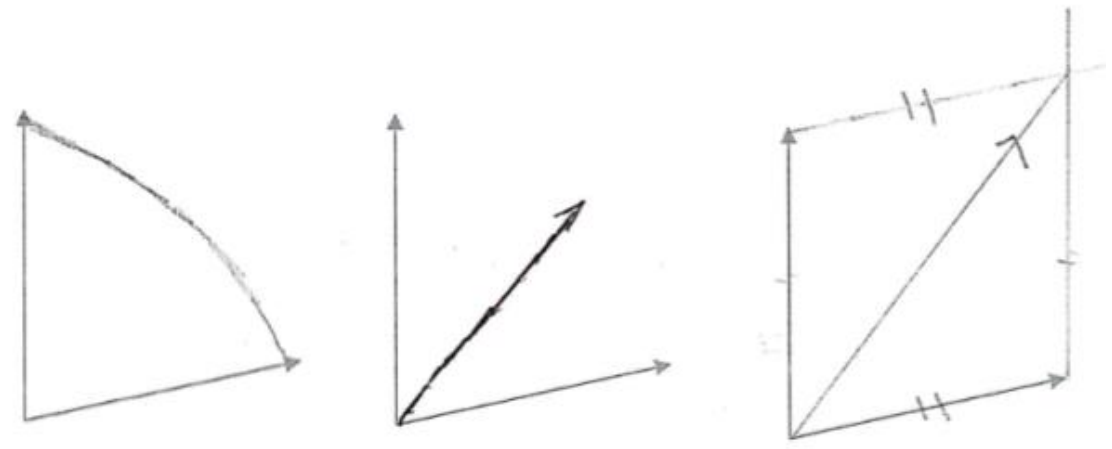


Figure 4.4. Examples of student responses, from student 4J, 4M and 4H respectively.

The last set of questions of the pre-test were used to determine student's understanding of vector addition in a conceptual vector question. Nguyen and Meltzer (2003) discussed common problems in vector addition, such as forcing results to align with the horizontal / vertical axis, or combining the magnitude of the horizontal components inaccurately, suggesting some attempt at the triangle / parallelogram rule. The students completed two questions looking to elicit their understanding of vectors. Both questions were published in Tutorials in Introductory Physics (McDermott and Shaffer, 2003). The first question, shown in Figure 4.5, asked students to determine which body will experience the highest net force, based on the vectors shown.

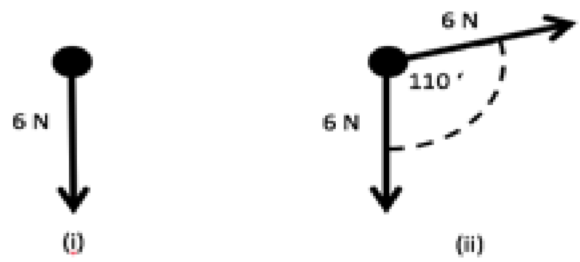


Figure 4.5. Vector pre-test question to elicit student understanding about vector addition.

The student's reasoning could be used to identity difficulties with vector addition. Table 4.5 shows the results from this question, with the students who used the correct concepts highlighted in bold. Where a student used the correct concept but made an error in applying it, their code is in bold and italics.

Concepts Used	Student Responses
Vector Addition	***4H,***
Parallelogram rule (Hewitt, 2009)	***4H***
Split the angle	4E
More forces lower the magnitude	4I
More forces increase magnitude	4D, 4G, 4K, 4M, 4N, 4L
Scalar addition	4C, 4B, 4F
No Reasoning Submitted	4A, 4E, 4J

Table 4.5. Student reasoning used in vector addition pre-test question.

While all the students identified that the net force for the vector diagram in Figure 4.5 (i) would have the highest resultant magnitude, only one gave an answer that explicitly referenced the use of vector addition. As the students were asked to submit their reasoning, the question allows for the identification of the reasoning used by the students. This enables and allows for differentiation between students that guessed the correct outcome, and those that determined the correct outcome. Table 4.5 shows a range of reasons submitted by the students; with the most common being that the more forces that act on the charge, the higher the magnitude of the resultant vector. In some cases, the reasoning given suggested vector addition, in other cases, it was suggestive of scalar addition, and in some cases the reasoning submitted was not explicit enough to include responses under either category, as seen in the following examples.

Student 4D: (ii) experiences the most force, because it experiences 2 forces in the diagram. (Not explicit to put into either category)

Student 4G: (ii) experiences the most force, because it experiences 2 forces in different directions, that when combined, are stronger than the force in (i). (Suggestive of vector addition)

In the case of the three students (4B, 4C and 4F) who used scalar addition, they explicitly stated the final vector would sum to 12 N, in effect disregarding the direction of the vectors. 4I stated that the angle between the vectors would reduce the resulting magnitude but gave no indication as to why. 4E gave a similar response, suggesting the resultant was "split between the angle" between the vector. When asked to explain what this reason meant, 4E volunteered that the combined force would spread across the 110°, so it'd be weaker as it went over a large angle, such as how intensity quantities get weaker as they are spread over larger areas. These responses show that the students not only used incorrect reasoning to attempt the question, but their reasoning was inconsistent with their choice of which of the two vector diagrams would result in the highest net force and did not recognise the inconsistency in their responses.

A fuller picture of student's understanding of 2D vector addition emerged from the second pre-test question This question specifically looks at their conceptual understanding of the addition of horizontal and vertical components. This question was adapted from Tutorials in Introductory

Physics (McDermott and Shaffer, 2003), in which the students were told the vectors represented forces acting on a body; they were asked to state which body experiences the most force, and to justify their choice. The vector diagrams are shown in Figure 4.6, and the student responses are summarised in Table 4.6.

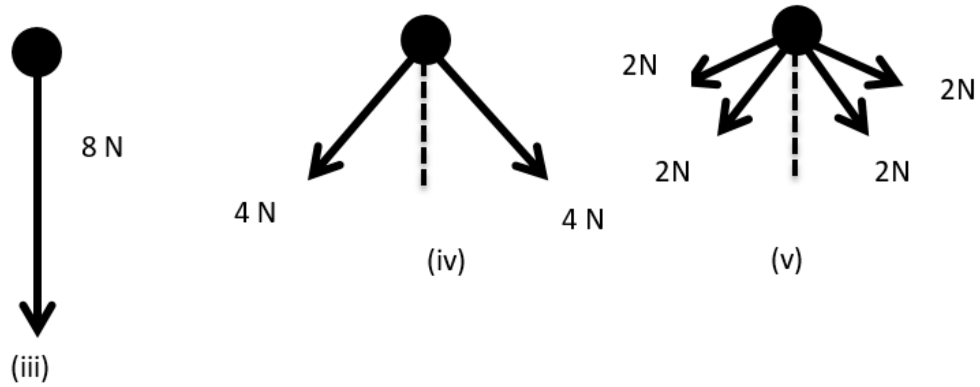


Figure 4.6. Vector pre-test question, to elicit student understanding of vector components

This question gave more clear results about student thinking. As can be seen in Table 4.6, eight of the 14 students added the vectors as if they were scalars, not accounting for the direction of vectors, nor the horizontal or vertical vector components. Only one student, 4H, attempted to use vector addition to complete this question, but errors in their application were apparent, as seen in Figure 4.7.

Concepts Used	Student Responses
Vector Addition	**4H**
Scalar Addition	4A, 4B, 4C, 4F, 4G, 4I, 4K, 4M
More angles mean more force	4D
No Reasoning Submitted / Reasoning unclear	4E, 4J, 4L, 4N
Correct Outcome for Setup	**4E**

Table 4.6. Student reasoning used in vector addition pre-test question, related to vector components.

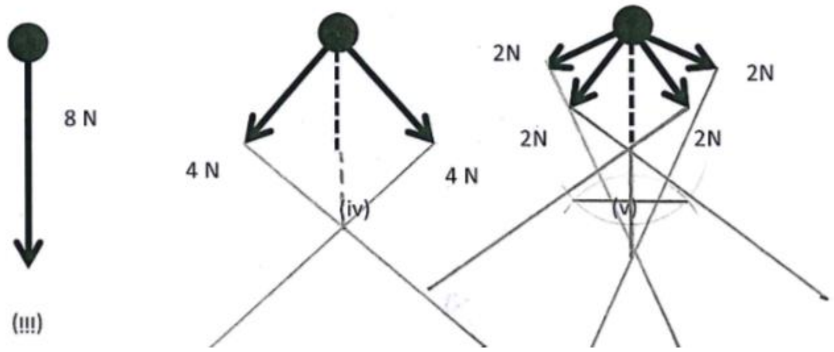


Figure 4.7. Student 4H's response to the vector component pre-test questions (iii)–(v).

Student 4H attempted to use the parallelogram rule to determine the resultant vectors, and in the case for (iv), their sketch is accurate. However, in (v), their attempt to resolve four vectors proved to be difficult and instead of using the parallelogram rule for the two most horizontal vectors, they translated them long an axis through the tips of the other two vectors, when attempting to find the resultant. This is a creative attempt for a student who never attempted a question with 4 vectors, but ultimately highlights an incomplete understanding of the contribution the horizontal and vertical components have in determining the net force.

Two students who used scalar addition were consistent in their responses, in which they found the sum of the magnitudes, as shown Table 4.7.

Student 4B:	Student 4C:
I think all the forces are the same because: *8 = 8* *4 + 4 = 8* *2 + 2 + 2 + 2 = 8.*	*They're all the same* *8N = 4N + 4N = 2N + 2N + 2N + 2N.*

Table 4.7. Student reasoning used in vector addition pre-test question, related to vector components.

Doughty (2013) showed students who completed introductory physics course in a third level setting struggled with vector addition, with vector components being a notable difficulty. It is not surprising to see the second level students in this study to have difficulties with vector addition questions, and due to their lack of experience with vector operations, it is not unreasonable to observe students disregarding the direction of vectors.

The pre-test results indicate that the student's understanding of vector concepts is undeveloped, even though 12 of the students were introduced to vector concepts in their Applied Maths course (NCCA, 2006). This is consistent with the work of Flores, *et al.,* (2008) and Nguyen and Meltzer (2003). The students did not consider direction and magnitude of a vector separately, had difficulty

in determining the superposition of two vectors, and appeared to lack conceptual understanding of the difference between adding vectors and scalars, and in their use and understanding of horizontal and vertical components to combine vectors.

4.2.2. Tutorial lesson: Vector Concepts

Vector concepts were introduced to the students during a lesson which consisted of a 15-minute teacher led class discussion and a 65-minute tutorial. The fourteen students were placed into two groups of four and two groups of three. During the class discussion, students were introduced to vectors by showing them a short clip of a plane flying in crosswinds, on a digital projector. A discussion between the students and teacher on the behaviour of the plane in the crosswinds led to informally discussing the difference between vector and scalar quantities. Formal definitions for each were then presented and examples of each were discussed on a PowerPoint presentation. On the presentation, students were presented with a grid with horizontal and vertical vectors. Students had to attribute positive and negative signs to represent the direction of a series of vector arrows (with reference to the right being positive, and up being positive). They then had to identify the longest vectors. This led to a discussion between students and teacher in which the magnitude of the vector was separated from its direction. An analogy of a strongman pushing a crate with 1000 N of force in various directions was used to aid the discussion. Students were also shown the use of the tip to tail method for vector addition and the parallelogram rule. Due to time constraints, the students were not afforded the opportunity to practice these constructions multiple times during the tutorial, and it was decided to complete them in the following class period.

The tutorial lesson was developed to allow the students to build up an understanding of how two-dimensional vector addition operates differently to scalar addition, explicitly exploring the use of vector components. Initially, students were guided through the labelling and drawing two vectors, $\vec{a}$ and $\vec{b}$, on a graph and sketching the vector components for each vector on the graph. Students were asked to match and draw vectors on graphs from coordinates. Upon sketching the components and producing two right angled triangles, the students had to apply Pythagoras' theorem using the components to determine the magnitude of the two vectors, $\vec{a}$ and $\vec{b}$. The students were then guided through the addition of vectors, using the tip to tail method, using the vectors from Figure 4.8, (i) and (ii).

The students showed an ability to apply the "tip to tail" method on the page to determine the resultant vector, $\vec{c}+\vec{d}$. However, all the students encountered difficulties when attempting to produce the horizontal and vertical components and use them to produce the resultant vector, $\vec{e}+\vec{f}$. Instead of adding the horizontal and vertical components, two groups of students attempted to add

the magnitudes of $\vec{e}$ and $\vec{f}$ directly to produce the resultant, while the other groups directly asked for assistance in completing this section. In all cases, the teacher requested the students sketch the horizontal and vertical components for $\vec{e}$, and then $\vec{f}$. Students were asked what the resulting horizontal magnitude would be if they were to combine the horizontal components only, and then apply the same reasoning to the vertical components. At this point, the student groups were comfortable to combine the resultant horizontal and vertical components to construct the vector $\vec{e} + \vec{f}$.

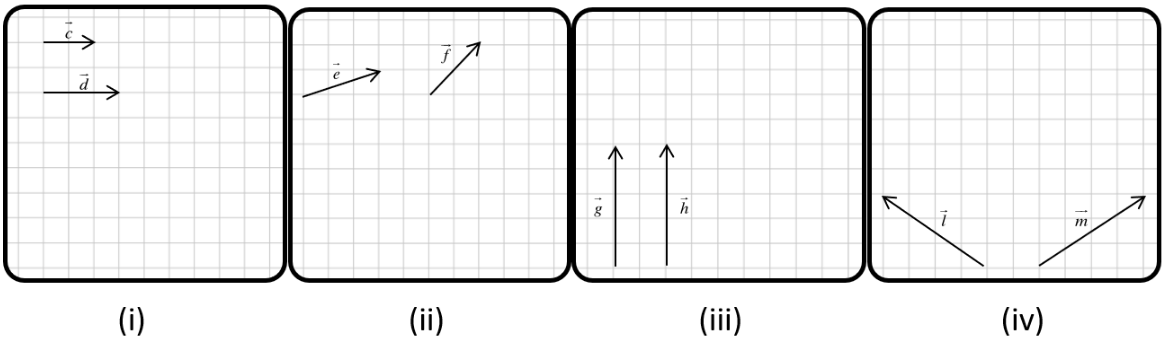


Figure 4.8. Vector addition diagrams from the vectors tutorial.

The students were then required to add the pairs of vectors shown in Figure 4.8 (iii) and (iv), using both the tip to tail method and by adding the horizontal and vertical coordinates. Upon completing this part of the exercise, students were asked to explain, in their own words, why the magnitudes in setup (iii) could be added directly and why they could not be added directly in (iv). In some cases, this proved to be the most challenging task on the whole worksheet, but ultimately, the only guidance needed for the students was to highlight the keywords required to give a full answer (horizontal and vertical components, magnitude, resultant), and they were able to construct their own valid meaning to this question. The following transcript, which was recorded on a dicta-phone during the tutorial lesson, illustrates this type of intervention.

Teacher (reading student's work) "Because they are not going in the same direction so the"… ok, so the direction is important?... Ok I'm going to show you some keywords for your answer. (Teacher highlights the words horizontal and vertical components on the tutorial page).

Teacher allowed students to work on this section for 5 minutes before returning.

Student 4K: Is it cause the horizontal components are going in different directions?

Student 4L: *Yeah*

Student 4J: Well, I said, that way one goes up and then it returns, so then it is equal to zero. Like four plus six would be equal to ten, but we're taking them away, so I said it is zero here. This is why there is like, no change there.

Teacher: I didn't hear you, what were you saying?

Student 4J: Nothing (brief laughter)

Teacher: No, go with it. It sounded like you were right, I just came in at the end.

Student 4K: [4J] was saying that these parts are going in a different direction.

Student 4J: And it is returning, so there is no change…

Teacher There's no change in what?

Student 4L: In the horizontal components.

Student 4J: Yeah… (brief laughter) I came up with that!

This transcript of the dialogue between the teacher and the students showed that initially the students were not necessarily dissatisfied with their initial answer, but they were unable to reason through the task. Upon being prompted to consider the necessary keywords and being given the appropriate time, the students developed reasoning that was plausible and intelligible to them (Posner, *et al.,* 1982) and allowed them to obtain the correct answer of the task.

Upon reviewing the tutorial solutions, there is evidence that the basic skills required in understanding vectors, such as drawing vectors and combining them using the tip to tail method, were relatively straightforward for the students. The difficulties they encountered when using horizontal and vertical components to combine vectors indicates that this was a more challenging task for the students. The study showed that, unless guided, the students did not consider using the components to combine vectors and needed prompting questions to lead them through the process. At the end of the class the students demonstrated the ability to apply this process to vector pairs, as shown in Figure 4.8 (iii) and (iv) and explain how vectors of equal magnitude can produce resultants of various magnitude in terms of the combination of the vector components.

4.2.3. Homework: Vector Concepts

The homework assignment for vectors followed the format of the tutorial lesson. It allowed the students the opportunity to practice the skills developed in the tutorial and class discussion. The questions required students to (i) draw horizontal vectors of various magnitude (ii) combine non-collinear vectors arrows (iii) combine vectors in terms of their components mathematically and (iv) rank the net force acting on bodies with 2 forces acting on them, at different angles.

The first question on the pre-test, as shown in Figure 4.9, presented students with a vector, $\vec{b}$, and asked them to sketch the vectors defined as follows: $[2\vec{b}, 4\vec{b}, -\vec{b}, -3\vec{b}, 2\vec{b} - 3\vec{b}\ ]$. The students showed good ability to correctly represent the vectors, with only some minor errors shown by 4 students. Both student 4F and 4N included multiple arrow heads in their vectors, but otherwise produced valid answers to represent the required vectors. Students 4C and 4K did not use the scale

of $\vec{b}$ having a magnitude that corresponded to two boxes, as seen in Figure 4.9. Both student's vectors were relatively correct, for example, their representations of $4\vec{b}$ were longer than $2\vec{b}$, which were longer than $\vec{b}$, and $-\vec{b}$ pointed to the left with roughly the same magnitude as $\vec{b}$. These difficulties were briefly discussed with the whole class, where they commented on the errors produced by the four students and suggestions to how they may have represented the vectors more accurately were noted by the four students.

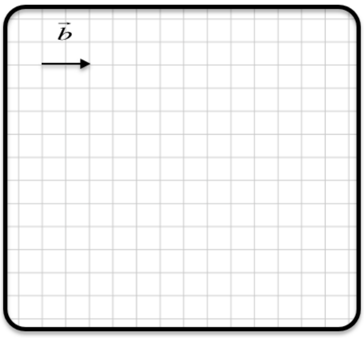


Figure 4.9. Homework question in which students sketch vectors.

In the second homework question, students were given 3 sets of vector pairs to add, as seen in Figure 4.10. In general, it was seen that students answered this section adequately; with all attempting to use the methods they learned in the tutorial, or class exercises, to find a resultant vector. Their results are summarized in Table 4.8.

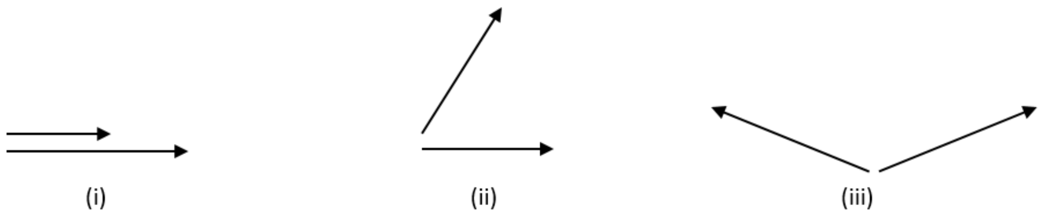


Figure 4.10. Homework question seeking to determine what construction students employ.

Concepts Used	Student Responses
Tip to Tail construction.	**4C, 4D, 4E, 4F, 4I, 4L, 4M, 4N**
Parallelogram construction.	**4A, 4B, 4G, 4H, 4K**
Errors in application of either construction	4E, 4F, 4I, 4K, 4L, 4N

Table 4.8. Constructions used by students to find the result of 2 vectors in homework exercise.

While all students attempted to use a vector addition construction to find the resultants, some did not draw their vectors to scale, and some did not complete arrow heads. In a same number of cases, the tip to tail / parallelogram was sketched but the resultant vector itself was not drawn, leading to an incomplete diagram. As shown in Table 4.2, the students were provided with an extra lesson to practise the vector constructions further. In this lesson, I set the students to work in pairs, to complete a series of exercises from their textbook over the course of 15 minutes, allowing them to attempt to overcome the issues observed in the homework assignment. Examples of student errors are presented in Figure 4.11.

(a) (b)

Figure 4.11. Examples of errors and incomplete diagrams from students 4L (a) and 4E (b).

The third homework question showed students the same vectors as seen in the second question but gave the students the vectors in terms of their horizontal and vertical components, as shown in Figure 4.12.

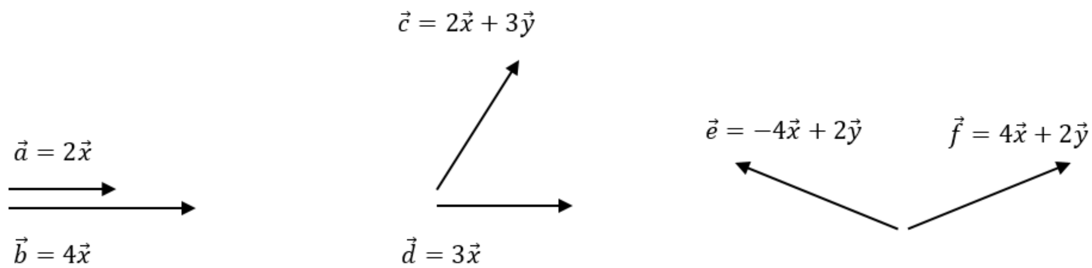


Figure 4.12. Homework question for students to add vectors using components.

The students were required to find the resultant vectors for $\vec{a} + \vec{b}$, $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$. All students demonstrated they could add the vector components in each combination, as this addition is similar to scalar addition in one dimension. Difficulties were observed in the addition of $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$, which were designed to resemble the vector addition pre-test questions, where the student had to consider the vector components in more depth. 8/14 of the students correctly used Pythagoras' theorem to calculate the magnitude of $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$. One student (4C) attempted to use the theorem but made errors in their work. For instance, instead of applying Pythagoras' theorem to $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$, they applied the theorem to $\vec{c}$, $\vec{d}$, $\vec{e}$ and $\vec{f}$ individually. The remaining students (4A, 4E, 4F, 4L and 4N) appeared unable to apply the theorem correctly. Difficulties seen in these students work showed error in picking appropriate values for the *x* and *y* component to use in Pythagoras' theorem (4A, 4F and 4L) or the student did not attempt the question (4E and 4N).

In the next section of the question, the students were asked the following question:

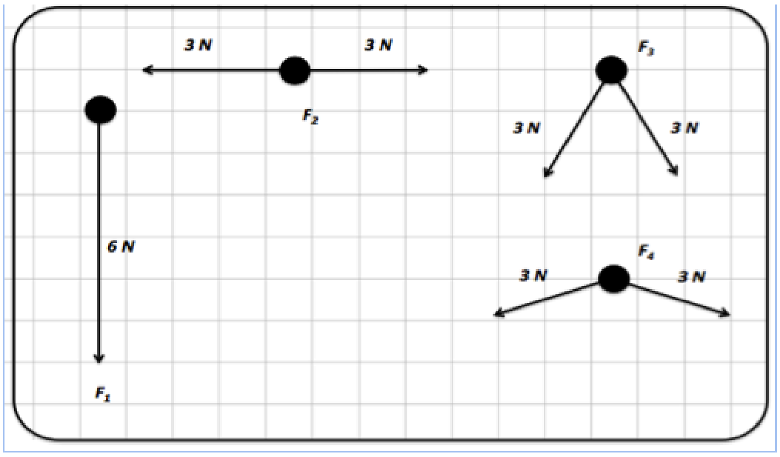
Explain, referring to the addition of horizontal and vertical components, explain why the magnitude of $\vec{c} + \vec{d}$ *is greater than* $\vec{c}$ *and* $\vec{d}$*, individually, but the magnitude of* $\vec{e} + \vec{f}$ *is less than* $\vec{e}$ *and* $\vec{f}$ *individually*

6/14 of the students referenced how the components of the vectors would affect the resultant, where horizontally cancelling components was observed in the $\vec{e} + \vec{f}$ combination. 3/14 of the students referenced the general direction of the vectors being opposite, and this would lead to a reduction in the overall magnitude but did not explicitly refer to the components. The remaining students did not attempt, and were likely unable to give any explanation. This highlights student difficulties to reason qualitatively, even when they are presented with information to help form their reasoning. This suggests the students would require more practise to develop the ability to reason scientifically, and as described by Hewitt (2011b), think in terms of concepts. As the tutorials progress, as shown in chapters 5 and 6, the students are given multiple opportunities to develop their qualitative reasoning skills and apply concepts qualitatively, as well as quantitatively.

In the last homework question, students were given a setup like the final pre-test question, with only a maximum of two vectors acting on a given body at any one time. The questions are shown in Figure 4.13, and the student's results are summarized in Table 4.9. 7/14 students gave the correct outcomes for the ranking. Two of them (4G and 4M) referenced the horizontal components cancelling out, while also using the parallelogram rule. A further three students relied solely on the parallelogram construction, without referring to the addition of either the vertical or horizontal components.

Concepts Used	Student Responses
Correct Outcome for Setup	**4B, 4E, 4G, 4H, 4J, 4K, 4M**
Horizontal / Vertical vectors referenced.	**4G, 4M**
Parallelogram / tip to tail	**4B, 4E, 4G, 4H, 4M**
Scalar Addition	4A, 4C, 4F, 4N
Incorrect ranking with no reasoning	4D
Not attempted	4I, 4L.

Table 4.9. Summary of student responses for homework vector addition conceptual question.



A small ball has a force or numerous forces acting on it as shown in the diagram to the right. The net force (resultant force) is labelled as F_1, F_2 and F_3.

(i) Rank the net forces acting on the ball, from highest to lowest. Explain your ranking.

Figure 4.13. Extract from vector homework.

Figure 4.14 presents a homework response from student 4E, which illustrates the use of the "tip to tail" construction to complete this task. The student applied the parallelogram rule to construct the resultant and presented 4 bullet points that commented on the resultant of the vectors. However, in the absence of a vector construction, there is no indication that the students could reason conceptually the correct ranking for this question. As discussed with the previous question, this illustrates the student's lack of practise in using qualitative reasoning in physics questions, or thinking in terms of concepts when approach qualitative exercises.

Students 4A, 4C, 4F and 4N submitted that all forces would be the same in all cases, contrary to the reasoning submitted in their tutorials. For all vector diagrams, they added the magnitudes of the vectors, ignoring the directions of the vectors. In a class review of the material, students 4F and 4N stated they did not think it would be the same as what they did in the tutorial, as in the tutorial they were dealing with magnitudes of vectors, while this was dealing with force. This can be explained by one any of the following three difficulties:

(i) The students could not connect the meaning of the term magnitude and its application to vector quantities.

(ii) The students could not connect the mathematical understanding to a physics concept.

(iii) The students did not consider force to be a vector quantity (4F and 4N explicitly stated this difficulty).

As part of the class discussion to review the homework, other students voiced their understanding of the terms magnitude, direction and vectors, and explained how the magnitude was represented by the length and how the length is a diagrammatic representation of the numerical measurement of the vector quantity. During this discussion section, students 4D, 4I and 4L were encouraged to communicate with the people beside them. Horizontal and vertical vectors were sketched on the board and these students were requested to use the diagrams to determine a ranking for the outcomes. Student 4D quickly realized that the horizontal vectors would sum to zero, whilst the other students came to this reasoning with the aid of their partners.

The homework assignment showed that the students were competent in representing vectors of various magnitudes and using the parallelogram and tip-to-tail constructions to combine vectors. Difficulties persisted in the conceptual understanding of combining vectors when considering the components of the vectors, and instead, students preferred to rely on the constructions to solve problems related to vector addition, for non-collinear vectors.

Student 4E: *F_1 – Longest vertical components*

F_3 – Second longest vertical components

F_2 – Third longest vertical components.

F_4 – That's just nil: $+3N+(-)3N = 0N$

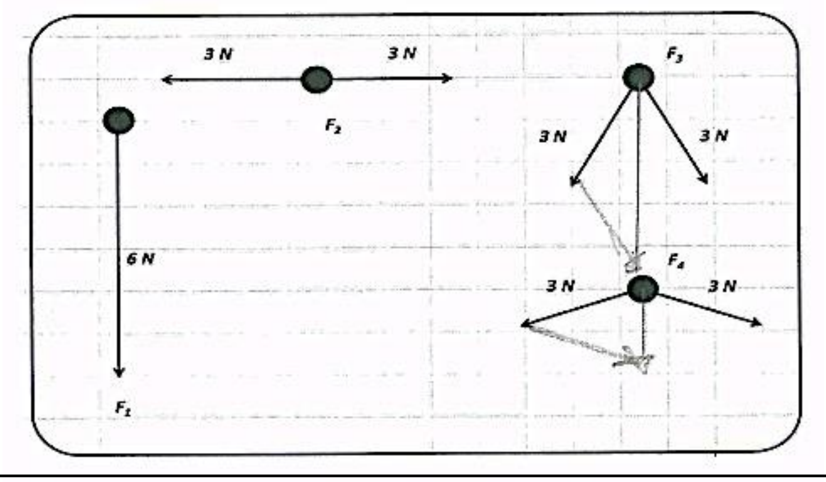


Figure 4.14. Student 4E's homework response, showing their work using the tip to tail to construct their ranking.

4.2.4. Post-test: Vector Concepts

The vectors post-test was written in a manner like both the tutorial lesson and the homework assignment. It tested the student's understanding of vector magnitude, by means of a ranking question, their ability to combine vectors using the "tip to tail," or parallelogram construction, and their understanding of vector components. The post-test took place approximately two weeks after the tutorial lesson. The large time duration was not due to research purpose, but due to a break in the tuition term.

In the first question on the post-test, the students were given a set of vector arrows and asked to rank them, from lowest to highest, based on their magnitude. This question was similar in nature to the first question they saw on their pre-test, to allow for a direct comparison, while also using result from their homework assignment to provide commentary on other difficulties that student may have, that were not seen in the post-test. The vectors are presented in Figure 4.15, and the results are shown Table 4.10.

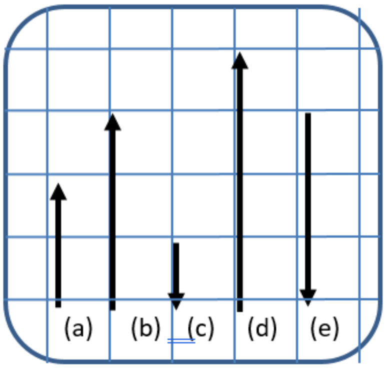


Figure 4.15. Post-test vector magnitude ranking question.

Vector Ranking	Student Responses.
C, A, B=E, D	**4A, 4B, 4E, 4F, 4G, 4H, 4J, 4K, 4L, 4M, 4N**
D, E=B, A, C	4C, 4D
E, C, A, B, D	4I

Table 4.10. Student responses from the post-test vector magnitude ranking question.

From Table 4.10, it is clear to see that all but one of the students ranked the vectors based on the lengths of the arrows, whilst acknowledging the direction of the arrow was not relevant to the magnitude. Students 4C and 4D may have submitted the incorrect ranking, but this is likely based on

misreading the question, as their submissions are correctly ranked from highest to lowest. Student 4I was the only student who based their ranking both the direction and the length of the vectors, as opposed to the length of the arrows alone when determining their magnitude. This shows evidence that most students overcame the difficulty that was apparent in the student's pre-test results.

In the second post-test question, students were required to construct resultant vectors by combining two vectors using the tip to tail, or parallelogram construction. The question is shown in Figure 4.16 and the student results are shown in Table 4.11.

All the students appropriately used one of the vector constructions to find the resultant vector. Some minor issues were observed in so far as some of the constructions were free hand sketches and slightly inaccurate in terms of scale (students 4E, 4F, 4I and 4J). Student 4A used the tip to tail method but extended the resultant beyond where it should have been. This was not expected as in the homework exercises, student 4A used the parallelogram construction and correctly represented the vectors using that representation.

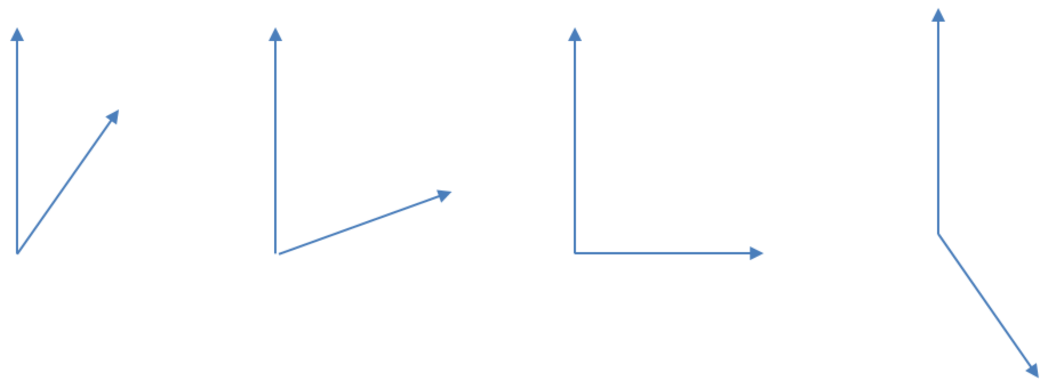


Figure 4.16. Post-test vector construction question

Construction Used	Student Responses.
Parallelogram construction	**4B 4I 4J 4K 4L**
Tip to tail construction.	**4A 4C 4D 4E 4F 4H 4M 4N**

Table 4.11. student's construction methods used in post-test to find resultant of two vectors.

Overall the responses indicate that students are aware of when to apply the vector constructions, but some students did not appear to consider the importance of using rulers to ensure they conserved the magnitude of their vectors, or the slopes of the vectors, when translating them to a new position, when completing the constructions. This was consistently seen in both the homework and post-test by students 4F and 4J, and in the post-test (but not the homework) by student 4E. All other students

consistently took care to draw the vectors to scale, using a ruler, in both the homework and post-test, or in the post-test only.

The final question on the post-test was developed to determine what conceptual understanding students had of adding vectors at angles. Students were given a scenario where an object was being pulled by two ropes, all with the same magnitude. Students were required to determine which of the three scenarios showed the strongest net force acting on the object and the weakest net force acting on the object. All the student's responses, except for 4N, showed they could determine the strongest and weakest force from the three diagrams. Student 4N reversed their answers, in which they incorrectly stated that the setup in which the force vectors were parallel would produce the weakest force, and the force vectors which were diverging with the largest angle would produce the strongest force. The questions are depicted in Figure 4.17, and the reasoning used by the students is presented in Table 4.12.

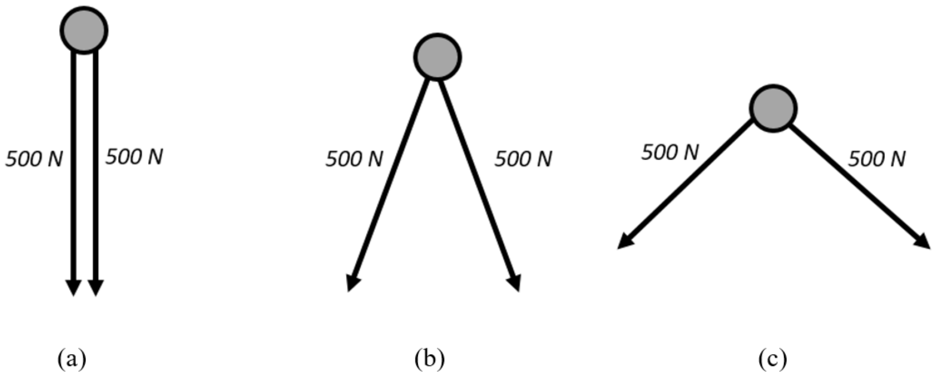


Figure 4.17. Vector post-test question to elicit student understanding of vector components.

Concepts Used	Student Responses
Parallelogram / tip to tail	**4E, 4F, 4G,**
Horizontal and Vertical components explicitly referenced.	**4B, 4C, 4D, 4E, 4G, 4I, 4J, 4K, 4M, 4N**
Horizontal and vertical components suggested / answer incomplete.	4A, 4F, 4L
Angle affect magnitude	4N

Table 4.12. Student reasoning used in vector addition post-test question, related to vector components.

The post-test results show that all but one of the students correctly determined the correct outcome. Of all the correct answers, the use of horizontal and vertical vectors was the primary reason chosen by the students, while a small number of students also included the use of the parallelogram / tip to tail constructions as evidence to support their answers, as shown in Figures 4.18 and 4.19.

Two of the students gave answers that did not fully articulate complete reasoning but were suggestive of the student's understanding about the horizontal components of the vectors cancelling out, while one student incorrectly related the magnitude to the angle and gave an incorrect ranking as a result.

In the cases where the students provided both justifications for their answers, it was seen that students tended to refer to both their diagrams and the components in their reasoning. This would suggest to us that students are not completing the constructions as a matter of rote – learned procedure but indicate that the students can determine the utility in both answering the question using vector constructions and in terms of the combination of vector components.

Student 4H:

The resultant for (i) is the strongest. Plus, it has no horizontal components. The resultant for (iii) is the weakest magnitude. The horizontal components cancel out.

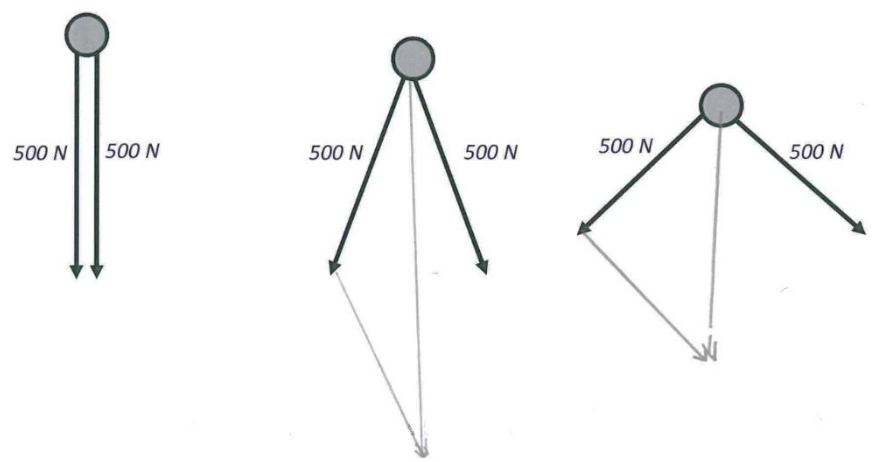


Figure 4.18. Student 4H's response for conceptual vector post-test question.

Other students chose to directly reference the horizontal vectors alone and did not attempt to generate evidence for their reasoning using the parallelogram / tip to tail method.

Student 4B: [(a)]. Both forces are acting the same direction, meaning there are no opposite forces cancelling out.

Diagram [(c)] shows the weakest force as there are large horizontal component forces cancelling out meaning there is less vertical component forces.

Student 4K: The third one, [(c)]. Even though the second one has a vertical and horizontal component, the third one has a wider horizontal component, so the force has to act on two difference horizontal.

Other students submitted the correct answer but gave incomplete reasoning that suggested the use of horizontal and vertical components, but lacked clarity and the use of keywords in the explanation:

Student 4E: A would allow a pulling power of 1000 N, as the combination of the two forces pulling in the same direction would be greater than that of two pulling in different directions, and cancelling each other out to a degree. [Diagram] C [is the strongest]. The forces are pulling in moderately different directions, which causes them to weaken the pulling power, by cancelling most of what the other is doing.

Student 4G:

1 only has vertical components and so is stronger than the others because the x components are 0, therefore all the force are acting the one direction making it the strongest. 3 has larger x values than 1 and 2, there the force acting on the x components are larger and cancel out. When they cancel out, the leave the vertical vector (the resultant vectors) to be weaker than 1 and 2.

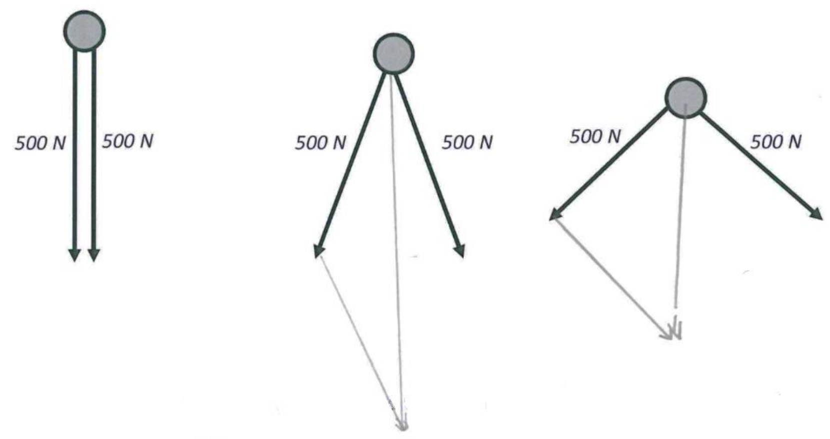


Figure 4.19. Student 4G's response for conceptual vector post-test question.

Student 4N, who did not give correct reasoning, associated the angle between the vectors with the magnitude of the resultant vector. Note that this reasoning was used by students 4D and 4J in the last two pre-test questions, as seen in section 4.2.2. Upon completion of the tutorial lesson, these students no longer produced work that indicated this type of reasoning. However, student 4N (who did not give a response to the pre-test question) was under the impression that a larger magnitude would result from a larger angle between the vectors. From discussion with the student, it was apparent that the student did not grasp the meaning of the term magnitude after completing both the tutorial and general class exercises. While they understood that numerically the magnitude was the number in any given quantity, they did not associate it with the length of the arrows explicitly and instead, associated it with any variable that could be quantified, such as in this case, the angle between

the vectors. Therefore, this reasoning produced a line of thought that a bigger angle had a bigger numerical measurement which gave the largest magnitude in the three setups.

The post-test results indicate that the students developed an understanding of vector magnitude and overcame the difficulty of not separating out the direction and magnitude of a vector (Nguyen and Meltzer, 2003). All the students also showed competency in combining vectors using a vector construction. The students were also able to apply horizontal and vertical component reasoning to the final conceptual question.

4.2.5. Discussion

By comparing the pre-test and post-test results, there is evidence to suggest that the students responded favourably to developing their understanding magnitude by using vector representations in a classroom discussion. Figure 4.20 presents a comparison of the pre-test and post-test results.

During the pre-test, it was observed the students had difficulties ranking vectors based on magnitude, a difficulty based on associating an influence of the direction of a vector on its magnitude (Nguyen and Meltzer, 2003). The occurrence of this difficulty in the class identifies the need for conceptual change during the lessons and tutorial (Hewson, 1992). During the class discussion, the students were presented with a similar ranking question as seen in the pre-test. The students submitted their rankings and reconsidered their rankings in terms of the definitions of vectors and scalar quantities. Through discussion with the teacher, it was seen that most of the students were easily able to identify that vectors associated with a direction that is denoted with a negative sign were equal in magnitude to vectors associated with a direction denoted with a positive sign, when the vectors are of equal length. The prompt to students to review their understanding of the term magnitude allowed become dissatisfied with ranking. They were then able to develop a correct ranking, that was in line with the correct understanding of the term magnitude. Students discussed their rankings in terms of their understanding of the word magnitude, and engaged in conceptual exchange, from which they ranked vector magnitude in terms of length and direction to just length in this task (Hewson, 1992). Based on the increase of correct reasoning used by the students in Figure 4.20, moderate conceptual change was recorded for this concept.

The gains in student understanding were apparent in both the homework and the post-test questions, in which it was seen that most of students submitted correct sketches and the correct rankings with valid reasoning. This classroom discussion appears to have been sufficient for students to engage with the visual representation and overcome the misconception that a negative sign on a vector is associated with a lower magnitude. The post-test results indicate that for most students there

are no persistent misconceptions for the student's understanding of how magnitude is represented by the length of a vector.

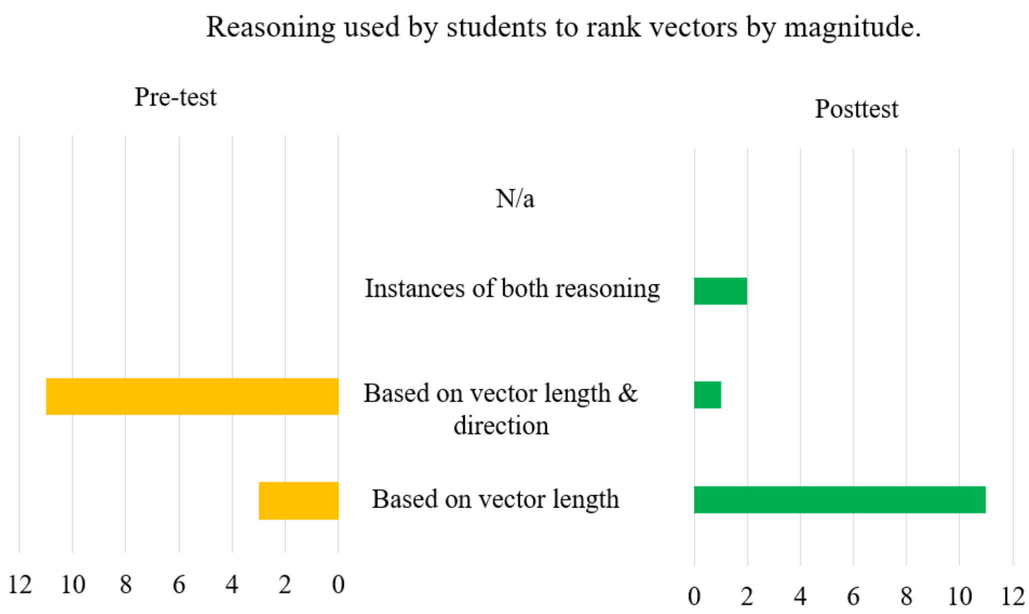


Figure 4.20. Comparison of reasoning used by students to rank vectors.

The next section of the discussion focuses on the students use of vector constructions to combine vectors graphically. Figure 4.21 presents a comparison of the constructions used by the students in both the pre-test and the post-test. During the pre-test, students had difficulties in combining vectors graphically. The most common error seen was students attempting to "split the angle" followed by connecting the tails of the vectors. These difficulties were encountered in literature (Nugyen and Meltzer, 2003) and were identified for conceptual change. In the homework assignment, discussed in section 4.2.3, that followed a tutorial lesson, all the students attempted to use either a tip-to-tail or a parallelogram construction to find the resultant between two vectors, as opposed to only3 students attempting the construction in the pre-test. However, numerous errors were observed in the student's application of the constructions in the homework. These were mainly, but not exclusively due to the lack of use of a scale or correct use of a ruler in completing the constructions. Some of the incorrect vectors also appeared to show some of the errors as seen in Nugyen and Meltzer (2003) such as "split the difference." As a response to the persistent difficulties that were observed in the homework assignment, extra time was additionally given to the students in a separate lesson. The students worked in pairs on completing more vector constructions and allowed for instances of peer tuition to occur. This allowed for an extended response to the student's initial difficulties to correctly implement the vector constructions.

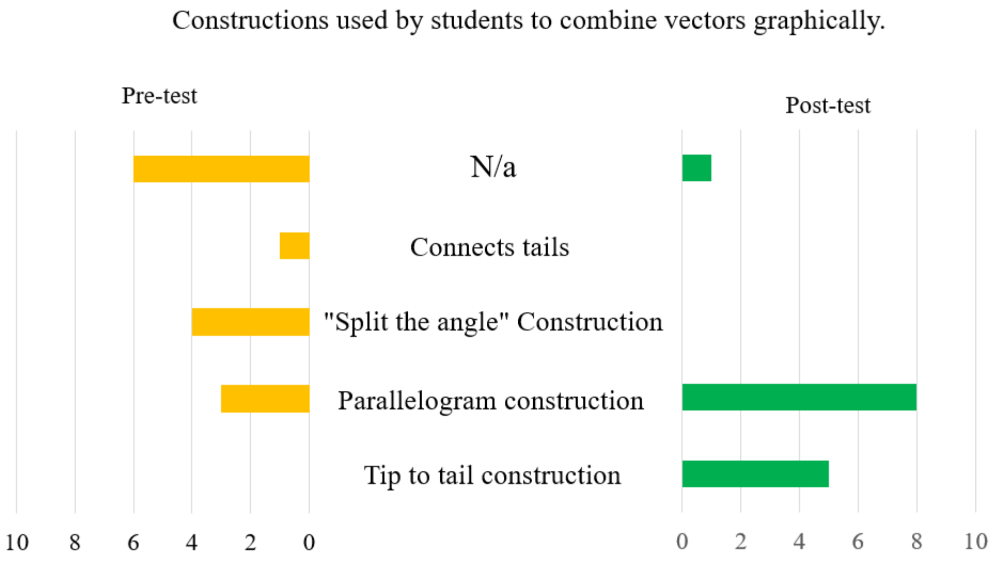


Figure 4.21. Comparison of vector constructions used by students

Upon completion of both the tutorial and additional set of exercises, the post-test showed all students using the constructions in their submissions, with most students having overcome the errors made in the homework. The post-test results showed that all, but one student effectively used one of the two constructions. This suggests that the students practise of the constructions during the tutorial and the extra 20 minutes after the homework activity allowed the students to see the constructions as intelligible and appropriate for adding vectors diagrammatically. The shift in results observed in Figure 4.21, indicates that conceptual exchange occurred, as the students were no longer producing the errors of connecting tail, or “splitting the angle” during the post-test. The shift in results also indicates that moderate conceptual change occurred over the course of this tutorial.

The last section of the discussion focuses on the student’s reasoning and understanding of horizontal and vertical components. Figure 4.22 presents a comparison of the reasoning used by the students in vector addition questions, in both the pre-test and post-test. The most prominent gains were seen in the student’s responses to ranking the net force acting on bodies when the forces acting are at obtuse angles to each other. Most students used scalar addition when completing the pre-test conceptual question, and this was targeted for conceptual exchange from their over-reliance on scalar addition (Hewson, 1992) to being able to identify when vector addition and scalar addition are appropriate.

In the tutorial lesson, it was observed that the students developed an understanding of components of the vectors, and they considered how these components affect the resultant of a vector. The discussion quoted in section 4.2.3 shows how the students required time to consider and discuss how all the components affected the resultant, and how the horizontal components summed to zero. In highlighting the reasoning used by the students wasn’t complete, the teacher provided a source of dissatisfaction in the initial reasoning (Posner, *et al.*, 1982), as students would realise that the

reasoning they provided would not provide an accurate outcome to a tutorial task. They then discussed alternatives amongst themselves. In a previous question, they had mathematically shown that the horizontal component vectors can sum to zero, leaving a vertical vector as the resultant, but the students did not initially consider this as evidence to support their reasoning in the final question of the tutorial. When students were prompted to review all their previous answers, and to consider what their mathematical answers could tell them about the resultant vectors, they developed reasoning that was plausible and intelligible (Posner, *et al.*, 1982) to provide an accurate and well thought out reason to the task.

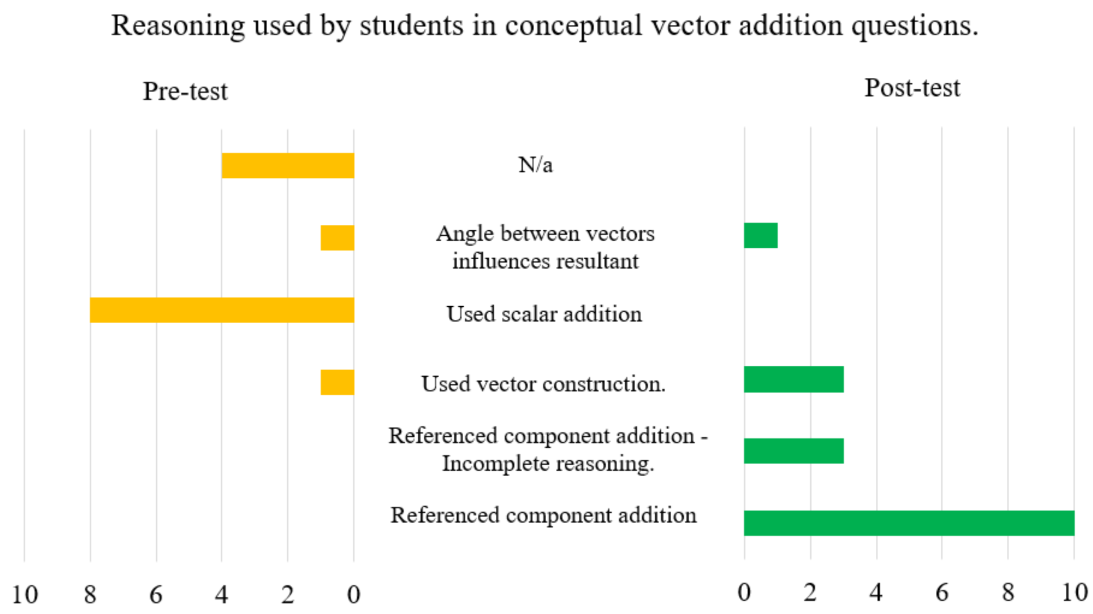


Figure 4.22. Comparison of reasoning used by student in conceptual vector questions.

In section 4.2.4, during a homework assignment it was seen that 7/14 of the students correctly ranked the net forcesacting on a body, at angles ranging from 0° to 180°. Students who submitted correct reasoning either referenced the vector components or used a vector construction. The remaining students reasoned incorrectly about vector components, or used scalar addition without applying the reasoning developed during the tutorial (see also Doughty, 2013). An open class discussion was held, in which the teacher sketched the horizontal and vertical component vectors, gave them the correct ranking and encouraged students to determine why the ranking was correct, referencing the vector components. As the students had the correct outcome presented, this allowed the student to focus on why the outcome was correct and review the reasoning they developed in groups in the previous tutorial lesson. The students worked in pairs in this discussion. As the groups were prompted to consider the components of the vectors, the groups were able to construct a comprehensive, concise and useful explanation that justified the correct ranking (Posner, *et al.*, 1982). The post-test results indicate this teaching approach was effective, as essentially all students gave the correct answers to the final conceptual vector question, with most of the students referencing the horizontal and vertical component summation in their answers. The increase in students

explaining vector addition in terms of vector components and reduction of students applying scalar addition, as seen in Figure 4.22, suggests the students engaged in conceptual exchange (Hewson, 1992) over the course of this tutorial. The increase in correct student responses and reasoning is indicative of moderate conceptual change having occurred. Sections 4.2.1 and 4.2.4 detailed the reasoning produced by the students and the jump from zero to ten students, in the pre-test and post-test, producing reasoning based on vector addition is evidence to support this.

In the homework assignment, the students who submitted a correct ranking tended to favour using the parallelogram or tip-to-tail method to produce their ranking, while only two students considered the vertical and horizontal components. After a discussion of the solution in the next lesson, which primarily used horizontal and vertical components, it was observed that most of the students acknowledged that considering the components provided an efficient method to solving the conceptual vector problem. This discussion could have heavily influenced the student’s choice of reasoning and considering the students may have given both styles of reasoning in the post-test had the discussion being balanced to explore both methods to justify the correct ranking. Additionally, even though the students completed mathematical calculations relating to the summation of the horizontal and vertical vectors, in both the lesson and the homework, no students considered it’s use in the conceptual questions at the end of either the homework or the post-test. This was unexpected, considering they would be familiar with the tools required to complete these calculations, and they draw a parallel to the reasoning the students developed and explored in their first and second year of second level mathematics, in which they covered the topic of coordinate geometry.

This chapter of the research presents evidence to suggest that conceptual change occurred to various extents between the three target concepts. On completion of the teaching sequence, most students showed they associated vector magnitude with the length of an arrow, regardless of direction. This can transfer to Coulomb’s law, to represent the relative strengths of force acting on charged particles, and electric field to represent the field around a charge, and the superposition of two charges, at various points. The use of the constructions can also be applied to electric fields, in which students construct electric field of two, or more, charges and explain how the introducing more charges can increase / decrease the magnitude of the electric field at various points. The student’s gains in understanding of components can be utilised by them to give a deeper explanation of the variation of the electric field strength at points between multiple charges, and develop an understanding of positive, negative and zero work and how it applied to potential difference in uniform electric fields.

4.3. Inverse square law

This section presents a narrative and analysis of the development of student's understanding of the inverse square law. The inverse square law applies to many contexts in physics, such as light intensity, sound intensity, Newton's gravitational law, Coulomb's law , electric fields and radiation. The participants in this study are only required to study three of these examples: Newton's gravitation, Coulomb's law and electric field and sound intensity (NCCA, 1999). Bardini, *et al.*, (2004) showed students learning to graph linear equations using contextual problems can help develop their understanding of the properties of an equation. There are numerous analogous practical activities that can be employed in teaching the inverse square law, subject to the time and equipment constraints of a secondary school environment. Hestenes and Wells (2006) showed that presenting the students with data can be sufficient. For the student to explore the inverse square law using a relatively non-abstract context, the tutorials employed a context of spray paint droplets spraying over various areas, which was adapted from Hewitt (2009).

Section 2.1.3.2 detailed difficulties encountered by learners in their understanding of the inverse square law. These difficulties informed the design of the tutorials, so students could recognise, explain and apply the inverse square law using multiple external representations. These are presented in the following learning objectives, as upon completion of the teaching and learning material, the students would be able to:

1. Accurately sketch and switch between graphical and algebraic representations of the inverse square law (Bardini, *et al.*, 2004; Hestenes and Wells, 2006; Bohacek and Gobel, 2011).
2. Apply a diagrammatic model utilising intensity to explain the behaviour of the inverse square law, and make predictions based on the model (Hewitt, 2009).
3. Demonstrate proportional reasoning using the inverse square law (Arons, 1999, Maloney, *et al.*, 2000; Marzec, 2012).

The inquiry approach developed for promoting student understanding of inverse square law consisted of a pre-test, a tutorial lesson and a post-test. This intervention ran over three weeks. The materials focused on student's ability to graph an inverse square law relationship, and modelling the uniform spreading of paint droplets over an increasing area to explain the behaviour of quantities that obey an inverse square law. A timeline for the implementation of the inverse square study, including the target concepts for the intervention, is shown in Table 4.13. As the field lines tutorial was completed in the same three weeks as the inverse square law tutorial, the field lines classes are also presented but bold font is applied to the classes which only applied to the inverse square law.

Section 4.3.1 presents the pre-test results, looking at the difficulties the students showed in representing an inverse square function on a graph, explaining the increase in the area covered by a

bulb illuminating a wall when the bulb is moved away from the wall, and a calculation based on the inverse square law. Section 4.2.2 presents a narrative of the development of the student's understanding of the inverse square law, by guiding them to develop an understanding of intensity applied to a context of using spray paint, model how the spray paint intensity varies as the distance from the source to the surface changes and present this model on a graph. Section 4.3.3 presents an analysis of the post-test results which, like the pre-test, focused on students representing an inverse square function on a graph, explaining the variation of area covered when a source is moved from a surface, and a calculation based on the inverse square law. Section 4.3.4 presents a comparison of the pre-test and post-test results, and a commentary of the student's progress during the tutorials. Examples of student progression and difficulties that persisted throughout the tutorial and post-test are discussed.

Time		Research Implementation	Target Concepts
Week 4	Class 1.	**Pre-test.**	**Representing an inverse square function on a graph.**
	Class 2.	**Tutorial Lesson**	**Understanding of the increase / decrease in area model that follows an inverse square law.** **Inverse square law ratios calculation.**
	Class 3.		*Topics unrelated to project: Newton's gravitational law.*
Week 5	Class 1.	Pre-test.	*Topics unrelated to project: Newton's gravitational law.* Field lines pre-test.
	Class 2.	Tutorial Lesson Homework.	Field lines tutorial Field lines homework.
	Class 3.		Topic summary.
Week 6.	Class 1.	**Post-test**	**Representing an inverse square function on a graph.** **Understanding of the increase / decrease in area model that follows an inverse square law.** **Inverse square law ratios calculation.**

Table 4.13. Timeline of the implementation of the vector concepts study.

4.3.1. Pre-test: Inverse Square Law

The inverse square law pre-test was designed to elicit student's understanding in various representations. The students were required to show their ability to (1) graph an equation of the form $y = k\frac{1}{x^2}$, (2) explain how the area on a wall, illuminated by a torch, varies as the torch is moved closer to or further from the wall, and (3) answer a mathematical question based on proportional reasoning involving the inverse square law. The use of different representations provides the opportunity to use many ways for students to show and for me to gauge their understanding.

The inverse square law pre-test took place after an introduction to Newton's universal law of gravitation. In a class discussion, which lasted 15 minutes, the students were guided through a series of slides outlining the Cavendish experiment, in which a diagram of the setup was shown, and a summary of the observations were presented. The slides also presented data they could use to qualitatively discuss, as a class group, the effect of varying the product of two masses, and the distance between the masses. The students determined the direct proportional relationship between the gravitational force and the product of the masses, but were unable to determine the relationship between the gravitational force and the distance. The students were informed that this was a relationship not covered in their mathematics courses and they would explore this relationship in the next lesson. The class discussion concluded with the students completing the pre-test, with a time limit of 15 minutes.

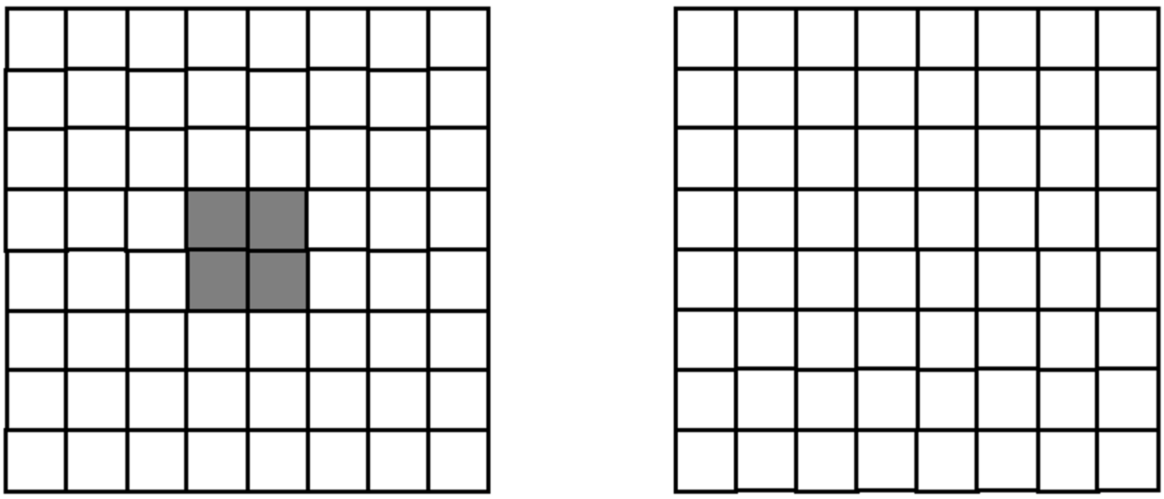
In the first question, the students were presented with blank *x*-*y* axes. They were asked to draw a pattern to match an equation of the general form $y = k\frac{1}{x^2}$. This allowed the students to demonstrate if they were aware how to transfer from algebraic to graphical representation, using the correct general characteristic curve for the function. A summary of the results is presented in Table 4.14.

Only three of the students correctly represented the relationship on a graph. The most common responses were quadratic curves. The responses from 4E, 4G and 4H were U-shaped curves, with an arbitrarily chosen intercept, while the responses from 4B, 4I and 4J were increasing quadratic curves with an intercept of zero. These students referenced the exponent on the *x*-variable in the equation to determine the function is quadratic. Student 4D presented a linear pattern with a positive slope, while student 4A sketched two linear patterns, one positive and one negative, overlapping. Both student's reasoning suggested they guessed the desired pattern. Of the correct graphs submitted, students 4C and 4M presented mathematics-based reasoning. Both students referenced that the function took the form of a fraction, and as the *x*-variable increased, the *y* value would approach zero, without reaching it, effectively defining an asymptotic function.

Response	***Students***
Decreasing asymptotic curve	**4C, 4F, 4M.**
Increasing curve	4B, 4I, 4J.
Quadratic curve	4E, 4G, 4H.
Decreasing straight line	4A.
Increasing straight line	4A, 4D.
N/a	4K, 4N.

Table 4.14. Student responses from the pre-test inverse square law graphing question.

The second pre-test question used a diagrammatic model for light intensity to probe student's understanding of the inverse square law. The students were presented with an 8 x 8 grid, with a torch illuminating 4 squares on the grid when it is 1 m away, as shown in Figure 4.23 (i). The students were asked to sketch, on the grid shown in Figure 4.23 (ii), the pattern observed if the torch was moved to 3 m away. This test was designed to elicit the student's understanding about scaling, as mentioned by Marzec (2012). The student's responses are summarized in Table 4.15.



(i) (ii)

Figure 4.23. Pre-test inverse area question involving scaling.

Response	Students
9 times bigger	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4N**
3 times bigger	N/a
4 times bigger	4M

Table 4.15. Responses for pre-test question seeking to elicit student's understanding of area scaling.

It was observed that all but one of the students were able to correctly represent the area covered when light source distance from the grid was tripled. However, not all student's reasoning directly

reflected this, and in some cases, student's reasoning contradicted their sketches. It was found that there were four lines of reasoning presented by the students:

- Students 4A and 4E submitted that they observed an increase in the "diameter" of the area by 3 times.
- Students 4B and 4H submitted that the area would increase 3 times, which is indicative of an inverse relationship. However, this is not reflected in their diagram, which shows an area that is 9 times bigger. Students 4D, 4F, 4G, 4I, 4J, 4K and 4N did not justify the quantity of their increase in area, but just qualitatively explained why the area of illumination increased, referring to how the light spreads across a larger area.
- Student 4C determined that for every metre the torch was moved back, the area given in Figure 4.23 (i) would increase by the power of the value of the distance from the wall: i.e., if the initial area covered is 4 units and the torch is moved to 3 meters from the wall, the final area would have a value of 4^3 squares on the grid. However, this student shaded 42 boxes, which does not correlate to the reasoning the submitted.
- Student 4M reasoned that in increasing from 1 m to 3 m, for every meter the torch was moved back, the area doubled. In going from 1 m to 2 m, the area increased from 4 boxes to 8 boxes, and in going from 2 m to 3 m, the area increased from 8 boxes to 16 boxes. This reasoning indicates a use of exponential proportional reasoning.

The last pre-test question looked at student's proportional reasoning, involving the inverse square law. Students were given a scenario in which a light sensor was placed 2 m from a bulb, and gave a reading 100 W m^{-2}, and were asked to determine what reading would be given on the sensor, if it was placed 4 m from the light bulb.

A summary of the results is provided in Table 4.16. 11/14 of the students determined that the light intensity would be half the original value, i.e., 50 W. The students did not submit reasoning, but instead produced a numerical value. These answers suggest that students did not consider the inverse square relationship, nor did they consider the patterns they drew in the second question. However, given that most of student's reasoning was incorrect for the second question, this is unsurprising. This indicates that the students had a mental model to help explain inverse relationships but had not developed an extension to the inverse square law.

The pre-test provides evidence to suggest that the students were generally unfamiliar with the inverse square law. 11/14 of the students did not relate the characteristic asymptotic curve associated with an equation of the form $y = k\frac{1}{x^2}$, instead producing a variety of linear and quadratic curves. 13/14 of the students were generally able to determine how the area covered by a source that follows an inverse square law changes, but showed reasoning inconsistent with their diagrams or no reasoning. This suggests some students may have guessed the correct outcomes, based on an

understanding that the area would increase in some manner. None of the students applied the necessary proportional reasoning to a mathematical exercise that required the understanding of the inverse square law. These difficulties in the pre-test agree with the findings of Marzec (2012).

Responses	Students
4 times smaller	**N/a**
2 times smaller	4A, 4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4M, 4N
Other reduction	4F, 4K
N/A	4L

Table 4.16. Responses for pre-test question probing student's proportional reasoning of intensity.

4.3.2. Tutorial lesson: Inverse square law

The tutorial lesson opened with a brief class discussion for 10 minutes, in which the previous presentation of the Cavendish experiment was reviewed. A formal definition for Newton's Gravitational Law was introduced and the discussion highlighted the similarities when comparing the equation $F = G\frac{m_1 m_2}{d^2}$ and functions of the form $y = k\frac{1}{x^2}$. Various physical phenomena that follow inverse square laws were listed by the teacher, such as light intensity, sound intensity, gravitational, static electrical force and the emission of particles from radioactive sources. This provided context for the importance of the mathematical relationship, which the students would explore in the tutorial lesson, which took up the remaining 70 minutes of the lesson.

The tutorial on the inverse square law was designed using an analogy of paint being sprayed over an increasing area, in which the students determined the number of particles of paint spraying over individual segments. In this way, spray paint "intensity" is used to conceptually model the inverse square law. This educational model was developed, and expanded upon, from Conceptual Physics, Practice Book (Hewitt, 2009).

Initially the students were presented with a scenario of a spray can emitting 100 drops of paint per second over a given area and were required to calculate the amount of paint droplets landing each second on a uniform area of 1 m^2. The students were then required to expand their model to the increase of area covered when distance from the can to the area is increased, as depicted in the Figure 4.24. This section of the tutorial was developed to promote the student's conceptual understanding of scaling (Arons, 1999), in which they were guided to reason why the area of the frames increases quadratically, instead of linearly, as the spray moves from left to right in the diagram.

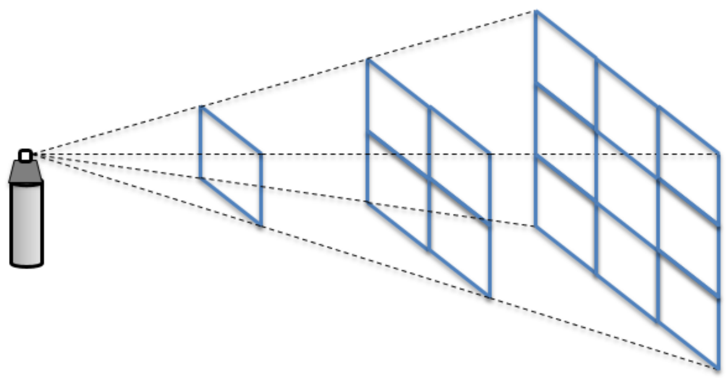


Figure 4.24. Diagram representing spray paint droplets passing through frames.

The students were required to explain, in the case of the second frame which was twice the distance from the can as the first, why the area of the second frame was four times the size of the area of the first frame. From this, they were required to determine how many droplets of paint pass through the individual smaller frames of the second setup, if 100 droplets passed through the first in 1 second. By determining that there were 25 drops passing per second in an individual frame in the second setup, the students were required to determine how this showed the spray paint "intensity" was following an inverse square law. Many discussions took place within groups to develop this reasoning, generally taking somewhere in the region of 15 minutes for the student groups to develop the reasoning to explain how the intensity drops. In the cases where students struggled to progress, the teacher asked the student groups to consider the increase in the length of the overall frame, the width of the overall frame, and discuss how both these increases affected the area of the overall frame.

Upon completion of this question, students had to consider the frame that was 3 times as far from the paint can as the first frame. Again, they were asked to determine why the area was 9 times the area of the first frame and use this to determine the spray paint "intensity" for one frame in one second, on the third setup. The students determined that there were 11 droplets per second, rounded to nearest whole number, and again used this to demonstrate the inverse square relationship. In completing this, the students demonstrated how the growing distance from the can decreases the number of droplets passing through an individual frame. To illustrate these points, the following quotes from student 4A, 4D and 4I were obtained by scanning the student artefacts.

Student 4A: Doubling the distance and the height, fits 4 plates in.
Distance triples and height triples, fits 9 plates in.
The drops are being divided (through the frames) as it grows

Student 4D: The farther away from the can, the bigger the area is because the lines are expanding, meaning more boxes (frames) can be filled in from each side.
As the distance from the pain can increases the drops per second decreases because the same number of drops pass through each part but distributed equally into each frame.

Student 4I: Because you can fit 3 more square in horizontally and vertically, as it gets further away.
The droplets have to spread between the area. The more area there is, the less droplets of paint passing through 1 m^2.

This section of the tutorial gave the students a conceptual grounding in the inverse square law, using a tangible model which they could easily picture. The initial difficulty encountered by students suggested students recognized the limits of their understanding and became dissatisfied with it. Whilst some students required prompting to consider the length and width of the frames individually, the groups managed to discuss and construct sound reasoning that allowed them to develop explanations of what was described in the presented model.

The tutorial then turned to a graphical treatment, in which the students graphed the data for the paint "intensity" at various distances between the can and the frame, to show the inverse pattern. From this, they chose data points on the graph and use the data points to show the reduction ratio, when the distance from the can is increased by a factor of 2, and then again by a factor of 5. This enabled the students to confirm, using their graph, that an inverse square law is observed. To illustrate, work from student 4I is reproduced in Figure 4.25.

Student 4I: It is a quadratic graph, that is decreasing, a slope that gets smaller.
1 m, it is 100. At 2 m, it is 25.
If you move 2 m away, it will cause a decrease of 4 times the intensity.
5 m, it is 4.
If you move 5 m away, it will cause a decrease of 25 times the intensity.

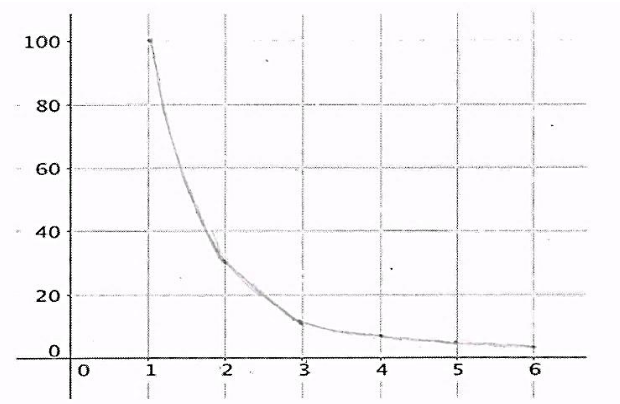


Figure 4.25. Student 4I's Graphical representation of the inverse square law.

There was not enough time for the tutorial to address student's complete quantitative problems the involved the inverse square law. However, in completing quantitative problems in Newton's Gravitational law in subsequent lessons unrelated to this project, the students were afforded the opportunity to practice the mathematical operations involving the inverse square law.

4.3.3. Post-test: Inverse square law

The inverse square law post-test was completed by the students two weeks after completing the tutorial lesson on the inverse square law. It consisted of questions that probed student's understanding of the inverse square law using graphical, diagrammatic and mathematical means. All questions revolved around the context of a spray-paint can spraying on a wall with a grid, which was used to model intensity.

In the first question, the students were presented with a formula for spray paint intensity and asked to produce the shape of an intensity vs distance graph and suggest reasoning as to why they chose the shape that they did. This question was similar in nature to the first pre-test question and would allow for direct comparison. It would allow us to determine if students learned to recognize and transfer the algebraic characteristics of an inverse square function to a graphical one and articulate their justifications. The question is shown in Figure 4.26, and the student's responses are presented in Table 4.17.

While 9/14 of the students answered correctly, only two (4H and 4I) of these students referenced the mathematical relationship between the variables referenced in the questions. These students directly referenced the exponent of the distance variable given in the provided equation, justifying not only the shape of their graph, but the type of inverse relationship in the equation.

Student 4H:	As the distance from the nozzle increases, the intensity decreases by a square of the distance.
Student 4I:	The intensity decreases by a square factor.

One of the students (4B) referenced the area model used in the tutorial to justify the shape of their graph but stopped short of relating this to the equation given. Another two students (4C and 4E) briefly explained the shape of the graph and how it relates the intensity to the distance from the can. This suggests memorization of the graph shape, but an inability to interpret it correctly in the real-world context.

Student 4B:	As the distance from the nozzle increases, the paint is spread over a wider area, meaning the intensity decreases proportionally.
Student 4C:	It shows that intensity is affect by distance.
Student 4E:	As the distance from the nozzle increases, the intensity decreases.

Furthermore, one student (4K) produced a linearly increasing graph but referenced the inverse square law for the intensity of the spray paint. This indicates a lack of understanding and an inability to link the graphical representation with the relationship they encountered in the inverse square law.

This suggests the student resorted to the use of rote memorization of the law but employed a familiar graph shape that they do not realise does not represent the law they stated. The remaining students explained that a linear pattern produces a line on a graph or submitted no reasoning at all.

A can of spray paint emits 200 droplets of paint per second from the nozzle. The amount of droplet from a can of spray paint that fall on a given area (intensity – I) is given by the formula: $I = \frac{200}{0.125\pi r^2}$

Draw a sketch of the graph to show the relationship between the spray paint intensity (I) and the distance from the nozzle (r) and explain how it shows the relationship.

How it shows the relationship:

Figure 4.26. Post-test question asking students to represent inverse square equation on a graph.

Responses	Students
Decreasing asymptotic graph	**4B, 4C, 4E, 4G, 4H, 4I, 4J, 4M, 4N.**
Decreasing linear graph	4L.
Increasing linear graph	4A, 4D, 4F, 4K.

Table 4.17. Student responses from the post-test inverse square law graphing question.

The second question referred to viewing the wall itself, in which they were presented with the grid in which the paint was only landing a small section of it, when placed two meters from the grid. The question layout was identical in nature to the second pre-test question, as discussed in section 4.3.2, but with a different number of boxes shaded in, and different distances referenced in the question. The students told the spray paint was 2 m from the wall, and the paint spray covered 4 boxes. They were asked to sketch the shape of the painted sections on the grid if the distance were to be increased to four meters. This gave us an opportunity to determine if they could apply the

inverse square law to the area covered by the paint. The diagrams from this question are presented in Figure 4.27 and the student results are shown in Table 4.18.

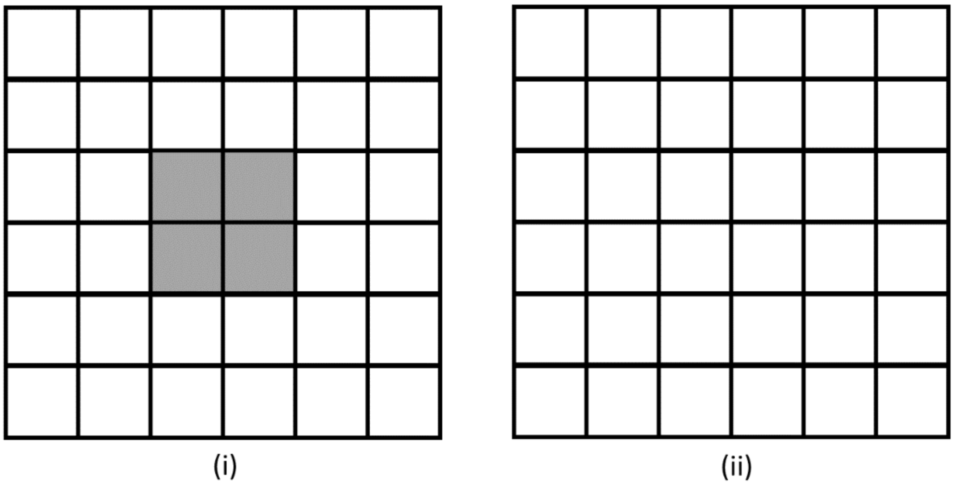


Figure 4.27. Area covered by the spray paint when (i) held 2 m from the wall and (ii) the blank grid.

Responses	Students
Doubling the distance, quadruples the area.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**

Table 4.18. Responses for post-test question seeking to elicit student's understanding of area scaling.

As can be seen from the responses, all students showed that the increasing the distance from the wall increases the area of the spray of paint quadruples. While it its noted that doubling the distance and squaring the distance can result in the same result, if students were to apply this to the grid, it would give an incorrect response (doubling 4 square results in 8 squares being shaded, as opposed to squaring 4 to result in 16 squares being shaded). The students were also asked to determine what the distance the paint spray would be if 36 squares were covered. The diagram presented in the post-test is shown in Figure 4.28, and the student's results are summarized as shown in Table 4.19.

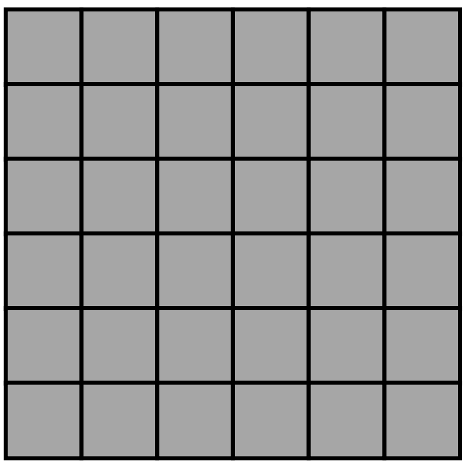


Figure 4.28. Post-test question where students apply proportional reasoning to scaling.

Responses	Students
36 boxes are produced by a radius of 6 m.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4M, 4N**
N/a	4L

Table 4.19. Responses for post-test question in which students determine the distance from the spray paint can to the wall.

Only one student was unable determine the distance required to produce 36 boxes covered in paint. The remaining students produced four lines of reasoning to determine the distance from the can to the wall.

- Mathematical: Students calculated the square root of the area of the shaded boxes to find the length / width of the square. They reasoned, and generalised, in this question that the length / width of the shaded area was equal to the distance from the paint can to the wall. This was valid for this particular question, and is a limitation of it is design. The question was intentionally designed as such, so the ratios would not be difficult for the students to work through when exploring this concept initially. A similar question was also completed by the students during the electric field post-test, in which the numbers are more difficult, and this manner does not directly produce the correct answer. The comparison of these two questions is discussed later in section 6.3.3. As an example, student 4B's reasoning is shown in Figure. 4.29.

Student 4B

$$x^2 = 36$$

$$\sqrt{x^2} = \sqrt{36}$$

$$x = 6$$

Figure 4.29. Sample of reasoning presented by Student 4B.

- Graphical: Students overlaid the paint patterns from the previous two questions. By superimposing these diagrams for 2 m and 4 m, it is easy for the students to extrapolate a pattern across the diagram to determine the distance from the can to the wall. Figure 4.30 shows this solution used by student 4D

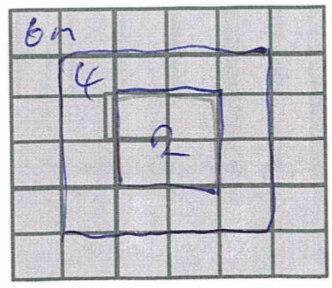


Figure 4.30. Student 4D's graphical reduction used to determine distance.

- Tabular: One of the students, 4E, used a table to determine how many boxes would be shaded at different distances. This acts as a combination of the previous two methods. The student could probably determine how many boxes would be covered for any value of *r*. Note that this student did not employ this method when looking at the electric field, as discussed in section 6.3.3.

Student 4E:

r	1	2	3	4	5	6
r^2	1	4	9	16	25	36

Table 4.20. Data produced by student 4E to demonstrate quadratic change.

- Changing length and width: Only one student (4H) determined the area of the wall when the can was 6 m from the wall using reasoning related to the change in the lengths / widths of the boxes. As the area of 36 boxes has a length of 6 boxes, and the initial diagram showed an area of 4 boxes with a length of 2 boxes, the student divided the lengths to determine that each side of the area grew by a factor of 3. The student then used this factor to multiply the original distance for the area of 4 boxes (2 m) by 3 to determine the correct radius. In their submitted post-test, it was evident that the student struggled to clearly articulate this in their answer.

Student 4H: *If you divide 6* [width and length of shade in question] *by 2* [the original length and width of the shade], *you get 3. So 2 m x 3 = 6 m.* [2m is the original distance]

In the final question presented on the post-test, the students were asked to use the formula presented on the first question to determine the spray paint intensity at both five meters and ten meters from the can. This would afford them the opportunity to use the data to verify that spray paint intensity followed an inverse square law, as developed from their comments in the previous questions in the post-test and use a method that would confirm their reasoning mathematically. The results of this section are summarized in Table 4.21 and Table 4.22.

Responses	Students
Correctly determined both intensity values.	**4B, 4C, 4E, 4F, 4G, 4H, 4J, 4K, 4M**
Correctly determined one value	4A, 4I, 4D
Inverted the distance only.	4N
N/a	4L

Table 4.21. responses for post-test question probing student's mathematical proportional reasoning of intensity.

Values used to verify inverse square law	**4C, 4E, 4G, 4H, 4K**
Incomplete reasoning	4B, 4J, 4M, 4N
Misconception not relating intensity to area	4A, 4D, 4F,
No reasoning given	4I,
N/a	4L

Table 4.22. Students that calculated values to verify the intensity as an inverse square law.

It was seen that 9/14 of the students completed the substitution and evaluation required to determine the spray paint intensity, but only 5 of these used the values to show an inverse square relationship was observed. They used the formula, developed a ratio and commented on how the change in distance from the can to the wall affects the intensity. They could all calculate the intensities and demonstrate that the doubling the distance produces one quarter the intensity.

Student 4C:	Student 4E:
$\frac{200}{0.125\pi(5)^2} = 20.37$ $\frac{200}{0.125\pi(10)^2} = 5.09$ *Distance x 2 =* 2^2 *= 4.* *20.37 ÷ 4 = 5.09* *The intensity is dependent on the distance. If the distance is doubled, it is 4 times less intense.*	$I = \frac{200}{0.125\pi(5)^2} = 20.37\frac{W}{m^2}$ $I = \frac{200}{0.125\pi(10)^2} = 5.092958$ $\frac{20.37\ W/m^2}{4} = 5.092958$

Table 4.23. Post-test calculations presented by students 4C and 4E.

Four students could qualitatively explain the effect of increasing distance on intensity but struggled to completely justify their understanding quantitatively, as seen in the following submissions:

Student 4B:	As the distance increases, the paint intensity decreases proportionally (exponentially).
Student 4J:	It is an example of an inverse square law because 5.0929 is more than half the intensity for another 5 m away.
Student 4N:	The intensity decreases the further out you go.

A misunderstanding that arose in the reasoning in the post-test was seen in the submissions from students 4A, 4D and 4F, in which they appeared to indicate their understanding of intensity was the amount of paint drops being emitted from the can, as opposed to the paint droplets passing through a defined area

Student 4A:	The intensity is the same, only dispersed over a larger area.
Student 4D:	It is the same, just displaced over a larger area.
Student 4F:	The intensity is spread over a larger area.

However, even accounting for this misunderstanding, these students did not demonstrate an inverse square law mathematically. The remaining students who completed the post-test did not submit any reasoning for their answers or did not attempt this question.

The post-test results indicate that the tutorial lesson had a positive effect on aspects of the student's understanding of the inverse. 9/12 of the students correctly represented a pattern that follows the inverse square law on a graph with an asymptotic curve. Some students demonstrated they could correctly determine the increase in area covered by the spray paint droplets and used reasoning indicative of the increase of the lengths and widths, either graphically or in written form, or analysed the pattern to extrapolate the correct answer. In some cases, the students showed they were applying inverse square proportional reasoning in mathematical questions, although these were not addressed in the tutorial lesson. This can be attributed a combination of the students applying the reasoning developed in the tutorial to mathematical questions and applying the skills they developed in solving problems involving Newton's gravitational law.

4.3.4. Discussion

This section discusses the student's understanding of the inverse square law, by comparing the pre-test and post-test results and referencing development shown by the students during the tutorial lessons. It will then discuss the student's conceptual understanding of the inverse square law using graphical representations, diagrammatic representations and algebraic representations.

Initially, the pre-test results indicated that the students were unaware of the shape of the graph and numerical patterns of an inverse square law. This immediately highlighted an issue for conceptual change to be addressed. During the tutorial lesson, the students were guided in mapping an inverse square law graphically. The post-test results show an increase in the number of students who could transfer the mathematical formula to a graph, represent the function using the correct shape, and provide justifications for the graph choice. This is presented in the Figure 4.31, showing the frequency of different responses in the pre-test and post-test.

The gains seen in the student's responses are in line with the findings of Bardini, *et al.,* (2004), in which guiding students through a function in context can help them develop an understanding of the equation and its transfer to a graph. Several difficulties were seen in the student's pre-test submissions, there were no difficulties that trumped any others, and thus, all difficulties were considered for conceptual change. A lack of clear concise reasoning for the graphs drawn in the pre-test would indicate that the students were not satisfied with the reasoning they were using to construct their graphs (Posner, *et al.,* 1982). In the tutorial, it was observed that the students could be guided to represent data that follows an inverse square law on a graph. They then used the data from the graph to develop ratios to determine that it shows an inverse square law.

They also clearly demonstrated that a decreasing asymptotic graph can be used to represent how the intensity decreases as the distance from the can to the wall increased. This evidence indicates that conceptual exchange and extension occurred, as the students demonstrated they could transfer the inverse square law from one representation to another, and then extended their understanding to develop an intelligible method to analyse the data on the graph (Posner, *et al.,* 1982; Hewson, 1992). Comparing the pre-test and post-test results directly from Figure 4.31, the results indicate that partial conceptual change occurred over the course of this tutorial.

Upon completing the lesson centred on the inverse square law, it was observed that students began to associate the general shape of an inverse square graph to any inverse pattern. Some difficulties persisted as five of the students demonstrated that they were still unaware how to either recognise or transfer an inverse square relationship from a mathematical symbolic representation to a graphical representation, in which four of the students sketched a linearly decreasing pattern, and one student sketched a linearly increasing pattern. This difficulty was later re-addressed in the Coulomb's law tutorial lesson, in which the inverse square is highlighted to the students in the

mathematical notation and the student then completing a graphing exercise with force and distance data, to display and analyse the graphical pattern. This is discussed in section 5.5.2.

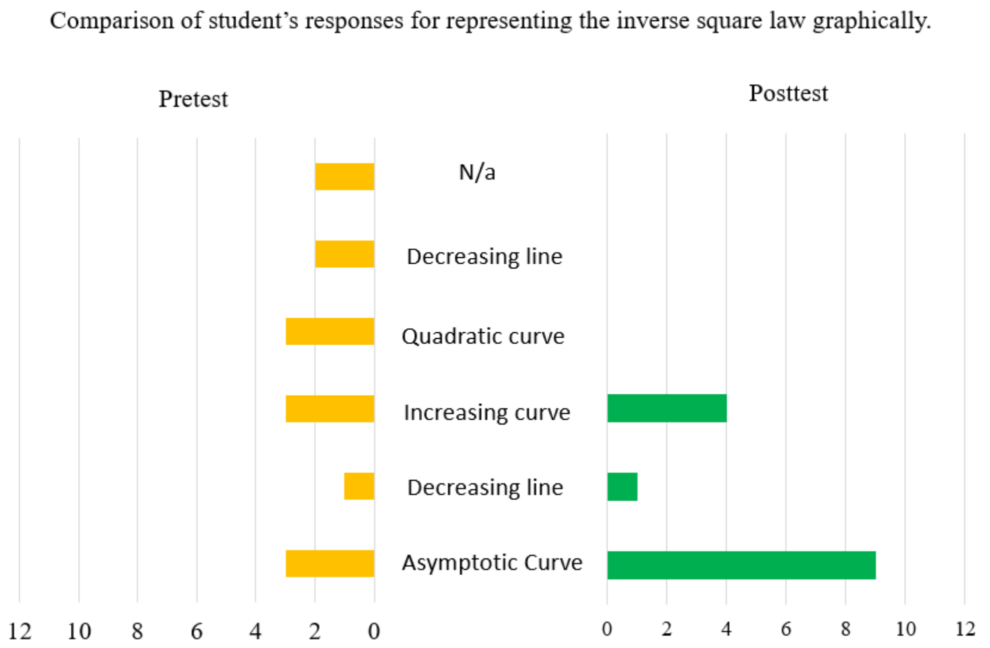


Figure 4.31. Comparison showing for student's graphs of inverse square law.

The development of student's understanding of the area change due to scaling is presented in Figure 4.32. In both pre-test and post-test, it was observed that students could correctly determine the increase in the area illuminated when a light is moved back from a wall. While it was a positive outcome where it was observed that the students could predict the change of the area in the given exercise, it was later observed that this increase did not correlate to student's understanding of a concept like intensity, in which a quantity is spread out evenly over this area. This would indicate that the difficulties to be targeted for conceptual change was not just dimensional scaling, but also applying the scaling area to other quantities and explaining how it applying to concepts like intensity, in which paint / energy is "spread evenly" over the increasing / decreasing area.

During the tutorial lesson, it was found that the model adopted (Hewitt, 2009) of using spray paint passing through square frames helps students visualise the inverse square law, in a relatively simple tangible context. When developing the student's understanding of scaling in the inverse square law they struggled to articulate why the area of the frames grew quadratically with the increase in distance between the nozzle of the paint can and the frame. The students were asked to consider both the increase in the width and height of the frames, and to consider how both these increases could explain the quadratic growth observed in the diagram. Difficulties were also encountered when the students needed to determine the amount of paint droplets passing through the frames when they were presented with 4 and 9 frames. The students tended to consider the total area of all the frames, instead of looking at them individually. The teacher was required to provide initial dissatisfaction to

reasons that focused on the overall area of the frames only. The students were required to extend their thinking to the amount of paint passing through all the frames, and then using their answer to determine how much paint was passing through the individual frame and provide reasoning to scale the “paint intensity” from the larger frames to the smaller individual ones. In the post-test, a small number of the students still considered the overall area, while the remaining students to focused on the change in dimensions of the shape, indicating that conceptual exchange occurred the understanding of these students (Hewson, 1992). This would indicate that minimal conceptual exchange occurred, but an increase in the students focusing on the dimensional scaling is an optimistic finding.

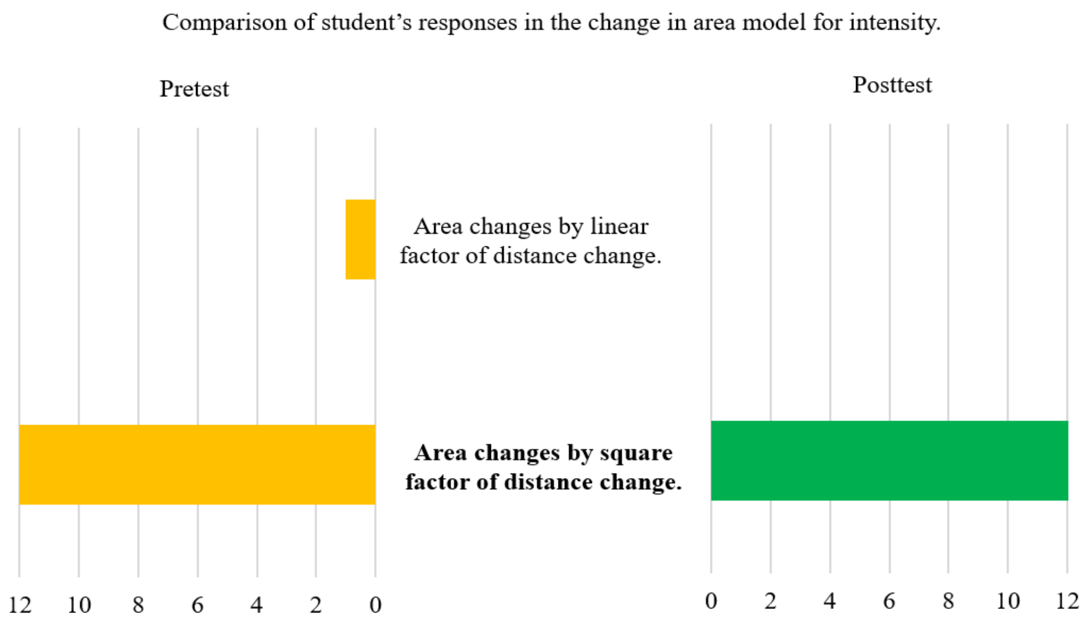


Figure 4.32. Comparison for student’s responses using area model.

The last section of this discussion looks at the student’s mathematical understanding of the inverse square law. A comparison of the pre-test and post-test results is presented in Figure 4.33. The mathematical pre-test question, in which students needed to apply the appropriate proportional reasoning consistent with the inverse square law, they used either proportional reasoning consistent with a linearly inverse law or qualitatively stated that a reduction would occur. The overwhelming occurrence of students applying linear proportionality indicated a difficulty to be targeted for conceptual extension (Hewson, 1992), as the students need to be aware when to use linear proportionality, and non-linear proportionality. In the post-test, the students were presented with a formula for intensity and were asked to prove that intensity followed an inverse square law, using the same skills developed in using ratios as completed in the tutorial when completing the graphing exercise.

It was observed that 12/14 of the students used the formula to produce at least one correct set of results, but only 5/14 students calculated a ratio, as they were directly instructed to do. These five

students demonstrated transfer of understanding between representations, and their consideration of the overall inverse square law in a task unseen from the tutorial. This suggests conceptual exchange (Hewson, 1992) occurred as the students demonstrated conceptual understanding in an unknown context (Konicek – Moran and Keeley, 2015), and that the students have develop intelligible reasoning that is applicable to further contexts (Posner, *et al.*, 1982). Based on these results, the extent of conceptual change observed was partial. The results also suggests that the difficulty for the remaining students was not the mechanics of using the mathematical operations, but how to apply their calculations to demonstrate an inverse square law as it was observed that nine students could produce values using an equation that involved an inverse square law, but only five could use their calculations to demonstrate the relationship. This is indicative of a gap between their mathematical ability and their ability to understand and apply it in a physics context. The Coulomb's law tutorial was augmented to account for this, as discussed in section 5.4.2.

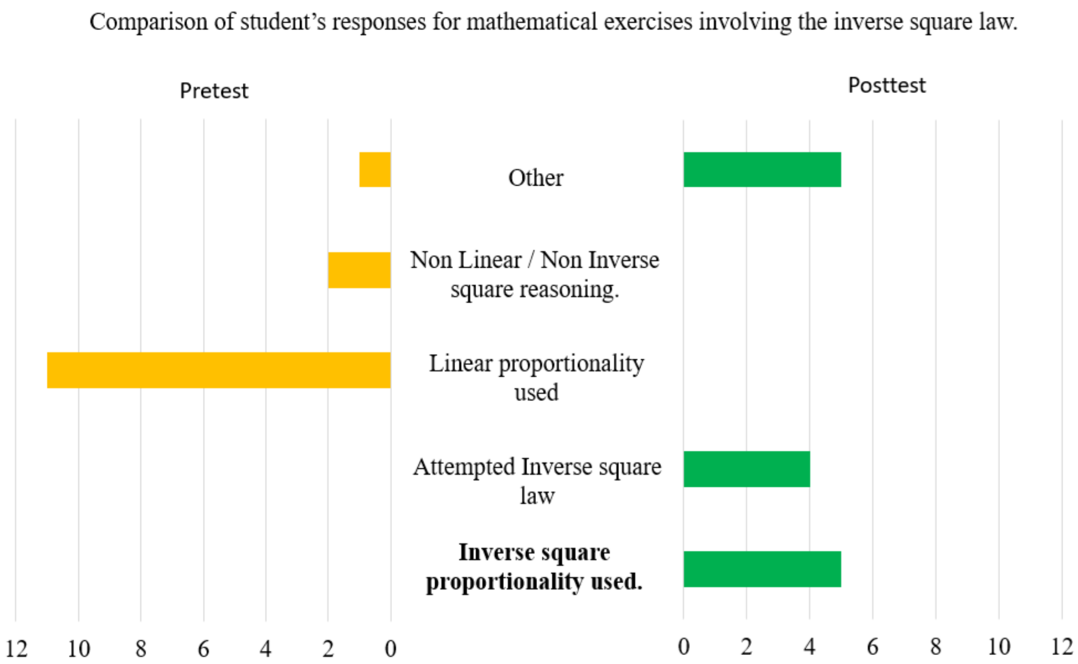


Figure 4.33. Comparison of student's responses for mathematical exercises using the inverse square law.

As discussed in section 4.3.2, the students plotted a graph of data using the inverse square law and completed mathematical calculations based on their graph to demonstrate an inverse square relationship, which they demonstrated. The tutorial itself did not address exploring the inverse square law mathematically. However, the students practised qualitative problems involving calculations involving Newton's gravitational law between the tutorial lesson and post-test. Therefore, we can attribute the increase in understanding of the inverse square law demonstrated mathematically to be a combination of representational transfer utilised in parallel to solving qualitative problems.

This section has shown evidence of gains in the student's understanding of the inverse square law. The discussions show indications that conceptual change occurred, but some difficulties persist with the students. This was to be expected as Arons (1997) and Marzec (2012) showed difficulties can persist beyond initial instruction. The approach adopted allowed for students to progress their understanding of the inverse square law. The reasoning developed by the student can be transferred to Coulomb's law, to explain the variation of the force felt between two charges as the distance between them is varied. This reasoning can also be utilised with students developing their understanding of electric fields, in which the students can explain the variation in the electric field strength at varying distances from a single charge. As discussed in section 5.5.3, one of the electric field homework assignments is written in a format similar in nature to the inverse square law tutorial, where field lines are substituted in lieu of paint droplets, so the students could use the frame model to explain the behaviour of electric field lines, and thus model the variation of field strength with increasing distance.

4.4. Field line concepts

This section presents a narrative of the teaching sequence the students experienced to develop their understanding of field line representations. Field is a key concept in physics, providing a model to explain "action at a distance" for non-contact forces, such as gravitational, electric and magnetism. Greca and Moreira (2006) noted that field theory is rarely covered in high school physics and students are mainly introduced to the theory in the study of electromagnetism third level. When students apply field theory to electromagnetism, the emphasis is placed on mathematical representations. This can lead to student difficulties in their understanding, application and interpretation of field lines as discussed in section 2.1.3.3.

The difficulties identified were used to construct the following learning objectives. Upon completion of the field line tutorial lessons, the students would be able to:

1. Distinguish between force and field (Furio and Guisasola, 1998).
2. Sketch field lines diagrams from vector fields, and vice-versa. (Törnkvist, *et al.*, 1993).
3. Recognise field lines are representational tools and not tangible objects (Galili, 1993).
4. Reasonably determine the trajectory of a body under the influence of a field. (Galili, 1993; Törnkvist, *et al.*, 1993).

The inquiry approach developed for promoting student understanding of field lines consisted of a pre-test, a tutorial lesson, homework, and a post-test. This intervention ran over three weeks. The intervention aimed to promote student's understanding of (1) field strength using field line density,

(2) the direction of force acting on a body in a field and (3) the path taken by a body moving through a field. A timeline for the implementation of the inverse square study, including the target concepts for the intervention, are shown in Table 4.24. As mentioned in section 4.3, this intervention took place in the same period as the inverse square law materials. The sections relevant to field lines are presented in bold.

Section 4.4.1 presents the pre-test results on how students represent field strength, sketch vectors to show the direction of force at different points in a field, and show the path taken by a body in a field. Section 4.4.2 presents a narrative of the development of the student's understanding of the field line conventions, initially looking at uniform fields, then varying fields and finally looking at bodies interacting with a field. Section 4.4.3 presents an analysis of the homework assignment, which was developed to allow the students to practice the skills and apply the understanding they developed in the tutorial. Section 4.4.4 presents an analysis of the post-test results which, like the pre-test, focused on student's understanding of field strength using field line representations, sketching the direction of force at different points in a field and showing the path taken by a body in a field. Section 4.4.6 presents a comparison of the pre-test and post-test results, and a commentary of the student's progress during the tutorials. Examples of student progression and difficulties that persisted throughout the tutorial and post-test are discussed.

Time		Research Implementation	Target Concepts.
Week 4	Class 1.	Pre-test.	Inverse square law pre-test.
	Class 2.	Tutorial Lesson	Inverse square law tutorial lesson.
	Class 3.		*Topics unrelated to project: Newton's gravitational law.*
Week 5	Class 1.	**Pre-test.**	**Field strength.** **Direction of force related to field line.**
	Class 2.	**Tutorial Lesson** **Homework.**	**Path taken by body in field.** **Topic summary.**
	Class 3.		
Week 6.	Class 1.	**Post-test**	**Field strength.** **Direction of force related to field line.** **Path taken by body in field.** **Inverse square law post-test.**

Table 4.24. Timeline of the implementation of the field lines study.

4.4.1. Pre-test: Field line Concepts

The pre-test for field lines was based on gravitational fields, as this is the first topic in Leaving Certificate Physics that students experience in which field line representation can be used. At Junior Certificate level, the would have used field lines to represent the magnetic field of a bar magnet. During the initial presentation of Newton's universal law of gravitation, the students were introduced to field lines as an alternative manner to visually represent the gravitational field of a large mass, as opposed to using vectors.

The students then completed the pre-test, in which the results could be used to determine how much understanding the students develop at Junior Certificate level about field line convention. The results were also used for comparison with the post-test results, to determine any conceptual gains that can be attributed to the students completing the tutorial lessons. The students were given 15 minutes to complete the pre-test and it was administered before the class discussion to introduce the topic. The tutorial mainly revolves around scenarios of bodies moving through space and feeling the gravitational force between themselves and planets. The pre-test probes student's understanding of the following concepts:

- The field strength is represented by the field line density. (Furio and Guisasola, 1998)
- The direction of the field is tangential to the field lines. (Törnkvist, *et al.,* 1993)
- The field lines represent the direction of the force acting on a body, not the path taken by a body. (Galili, 1993; Törnkvist, *et al.,* 1993)

The field line patterns used in probing the student's understanding of both field strength and the direction of the field at various points are shown in Figure 4.34.

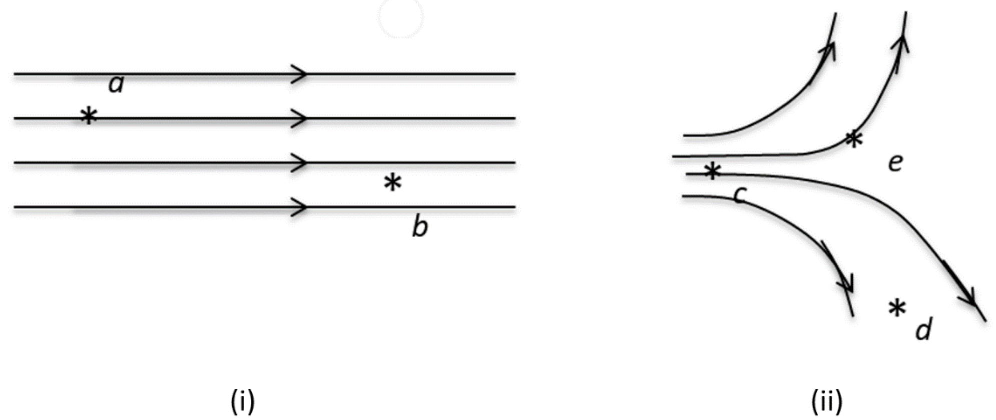


Figure 4.34. Pre-test field line question.

The student's rankings of field strength, from highest to lowest, is shown in Table 4.25. The correct ranking is denoted in bold. The pre-test results indicate that no student had a full understanding that the field line density represented the relative field strengths of the field. The most prominent rankings submitted by students were "d > e> c > a = b" and the converse ranking, "a = b > c > e > d." These rankings could suggest some indication that the field line density represents strength, but some of the reasoning provided by these students indicate otherwise. Reasoning provided included that the straighter the lines, the stronger the force (4B); the more the lines turn, the stronger the force (4H); the more the direction faced downwards aligned with gravity, the stronger the force (4J). The remaining students who chose these ranking did not submit reasoning.

Ranking	Students
c > a = b > e > d	n/a
d > e> c > a = b	4A, 4E, 4F, 4H, 4J.
a = b > c > e > d	4B, 4I, 4N.
e > d > a = b = c	4C
d > e > a > b > c	4G, 4L
d > a > b > c > e	4K
a > c > e > d > b	4M
N/a	4D

Table 4.25. Student's pre-test rankings of field strength, from highest to lowest

The other students who completed the pre-test and gave alternative rankings submitted reasoning such as the angle of the lines determines strength (4K), the apparent (or lack of) movement of the lines as you travel from left to right (4C) and the distance of the points to the field lines themselves (4M). Overviewing these results, it shows that the students were familiar with field line representation, and that when faced with a question on this, approximately half of the students were unable to correctly interpret how field lines representations represent field strength.

The second pre-test question asked the students to determine the direction of the field at the points marked with an asterisk (*) in the gravitational field, and to use vector arrows to represent them. The results are summarized in Table 4.26, with the correct response denoted in bold writing.

Response	Students
Force is tangential	**4M, 4N**
Force follows field line	4B
Force from (a) to (e)	4C, 4G
N/A	4A, 4D, 4E, 4F, 4H, 4I, 4J, 4K, 4L

Table 4.26. Student pre-test responses to representing the field using vector arrows.

Only two students correctly determined the direction of the force at the points labelled, 4M and 4N. Their results showed the force acting in the direction of the uniform field, tangential to the curved field lines. One student, 4B, showed the force following the field lines, not only showing a misunderstanding about the force direction, but also that the vector arrows curve, as opposed to show directions at a single point. Two students appeared to misread the question and drew a vector arrow from the point (a) directly to the point (e), 4C and 4G. The remaining students did not answer the question, suggesting they likely did not possess the understanding required to complete the question. Overall, this suggests that students were unaware of how field lines represent the direction of the force

The final question on the pre-test determined whether students can predict a reasonable path taken by a stationary body when it moves under the influence of a gravitational field of two nearby planets. The field is shown in Figure 4.35, and a summary of the student's responses is shown in Table 4.27.

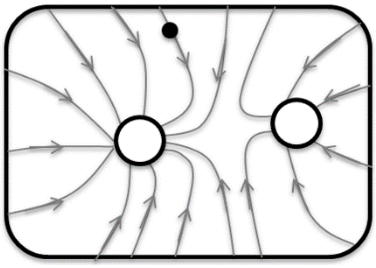


Figure 4.35. Pre-test question in which students were required to draw the path taken by a stationary body under the influence of the gravitational field of two nearby planets

Responses	Students
Path does not follow field line	**N/A**
Path follows field line	4B, 4E, 4G, 4M, 4N.
Directly to planet	4C
N/A	4A, 4D, 4F, 4H, 4I, 4J, 4K, 4L.

Table 4.27. Student's pre-test paths taken by small body in a gravitational field.

Students typically did not understand what information can be gleaned from the field line pattern. No students determined how the field lines represent the direction of the force at the point. Five students appeared to think that the field line represents the path. One student, 4C, indicated that the body would fall directly towards the leftmost planet, ignoring the effect of the gravitational field generated by the rightmost planet, and any acceleration it may have experienced due to it. The remaining students were unable to formulate any reasoning to allow them to attempt this question, and left it blank on the pre-test.

The pre-test results present evidence that students were unaware of the conventions of using field line representations. When presented with a field line pattern, the students were unable to accurately determine the field line strength, using the field line density as a guide (Furio and Guisasola, 1998). 12/14 students showed difficulties in using vectors to represent the direction of the force at various points in the field (Törnkvist, *et al.,* 1993). When required to draw the path taken by a body in a gravitational field, 6/14 students thought the field lines present the path taken by the body, or ignored the patterns of the field lines and directed the body directly to the nearest mass (Galili, 1993; Törnkvist, *et al.,* 1993)

4.4.2. Tutorial lesson: Field line Concepts

The students were briefly introduced to field lines in a class discussion, looking at the use of vectors to demonstrate the inverse square law. They were presented with a planet and vectors shown at points around the planet, getting shorter as the points were further from the planet. The students were asked to explain how the vectors represent that the field was getting weaker as the points were getting further from the planet. In pairs, they discussed this, and all pairs volunteered that the shorter arrows demonstrated a weaker field. They were then informed that drawing vectors in the manner shown can be cumbersome and they were presented with the same planet with eight field lines converging towards the planet. I explained, using diagrams to demonstrate, how field lines and vector arrows could be used to demonstrate the same thing. The conventions of relative field strength being represented by field lines being close together was discussed, and highlighted that they represented the direction of force, as opposed to the trajectory of bodies under the influence of the field lines. I presented three diagrams of field line patterns and the students in pairs practised interpreting the field line patterns, in light of the conventions shown, and verbally gave feedback. Eight of the students initially confused the field strength convention, in which they reasoned field lines further apart were stronger but over the three, they appeared to rectify this error. A depiction of a satellite in orbit was used to illustrate how field lines represent the direction of force experience by a mass at different points, but not the path taken by the satellite, as they clearly were different in the diagram. The students were then informed in the tutorial lesson, the use of field lines would be explored, which they then commenced.

In the first section of the tutorial, the students were presented with a body falling off a cliff, in the path taken as shown in Figure 4.36. The students were guided through a series of questions asking about the force acting on the body as it fell, to guide them to understand that the ball was under a constant force due to gravitational attraction. It was expected that the students would identify that the acceleration due to gravity on the ball was constant and could represent it as such using vector arrows.

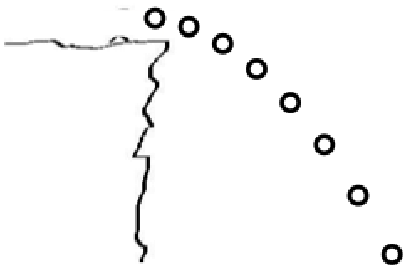


Figure 4.36. Motion diagram of a body falling from a cliff, from the field lines tutorial.

Some of the students highlighted the negligible change in acceleration due to gravity due to the ball being closer to the centre of gravity of the earth as it falls. This was discussed, and they concluded that the change would be extremely small, and practically immeasurable.

|Student 4B: It (gravitational force) changes very slightly.
No (acceleration is not constant), as it changes as it gets closer to the centre of gravity.

Student 4D: Yes (gravitational force is constant), gravity doesn't change.
Yes (acceleration due to gravity is constant), as the force of gravity doesn't change.

Student 4K: It (gravitational force) remains constant.
Yes (acceleration remains constant) because the acceleration due to gravity will be 9.8 m/s^2.

All students presented identical vector arrows to show the acceleration to due to gravity was constant, including students who mentioned the negligible change due to the decreased distance as the body fell. They were then introduced to field lines, and explicitly told that the field line density showed relative strength and the direction of field was in a direction tangential to the field lines. They were asked to apply these conventions to the field line patterns they were shown in the pre-test, as previously shown in Figure 4.34.

As the convention was explicitly presented to the students, there was little difficulty encountered by them to apply the convention correctly to determine the correct ranking and the direction of force acting on a body placed at the points shown. The students were then invited to represent the gravitational field on the body falling from the cliff. It was observed that when representing the uniform field, the following two difficulties were shown by the students, as seen in the sample of their work presented in Figure 4.37.

- These were that the field lines began at the ball when it was falling.
- The field lines terminated before running down beyond the cliff.

These errors were seen in approximately half of the student's responses. This error was mentioned to students, and is explicitly addressed in a later tutorial involving electric fields.

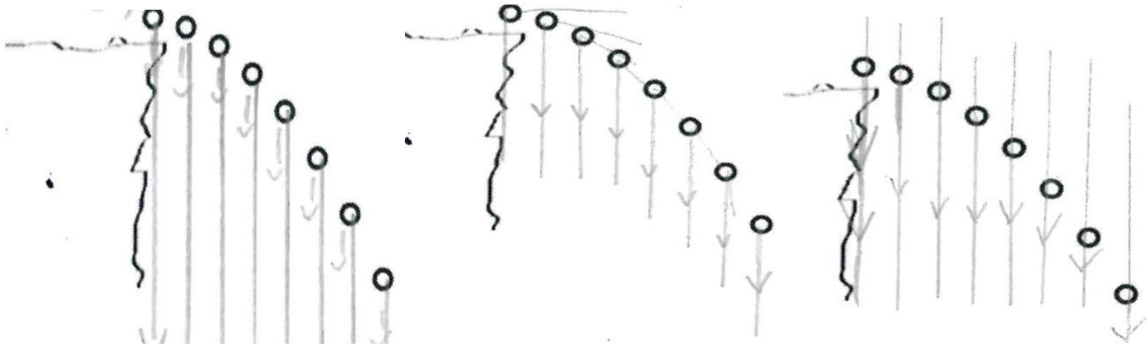


Figure 4.37. Examples of responses, in which field lines begin in body, field lines begin in body and terminate, and an accurate depiction of field lines.

The students were then presented with the diagram shown in Figure 4.38, showing a small meteor moving with an initial velocity and the earth. They were required to draw in the path taken by the meteor and explain how the field shows the variation in field strength, with a suggestion to compare it to the uniform field they previously encountered.

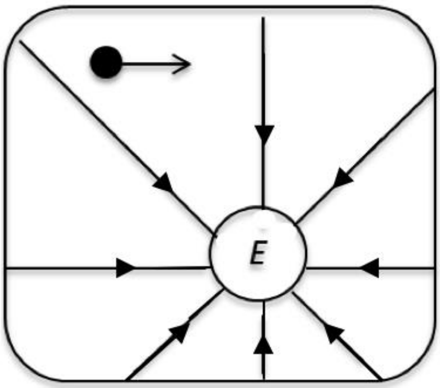


Figure 4.38. Tutorial diagram for difference between the direction of a field line and the path taken by a body.

All the students identified that the field gets stronger at points closer to Earth. Six students referenced that the strength decreases with the square of the distance from the planet referencing the inverse square relationship they encountered in the previous tutorial lesson. However, they could not explain how the field line pattern itself could be used to attributed this relationship, and were instead considering Newton's universal law of gravitation in justifying their reasoning. The remaining students used the field line representation for their justification, as shown in the following quotes taken from scans of student's tutorial worksheets.

Student 4A: No. The field lines are not all equal… the further away they are, the weaker the field.

Student 4E: The lines are far apart at a and b, but c and d are close, so the force is stronger. (Points drawn on diagram by student)

Student 4M: No. The strength decreases. The further from earth you go because the distance between the lines increases, weakening the strength.

The students then had to determine the path taken by the meteor under the influence of the gravitational field. It was expected that students draw their paths so that they would follow one of the field lines, as expected from errors seen in literature where leaners consider the field lines to represent the path taken by a body (Galili, 1993; Törnkvist, *et al.,* 1993). There errors were also observed in earlier versions of this tutorial trailed with pilot groups. However, during the tutorial lesson, the students drew individual paths, and then in their groups, all the students engaged in discussions to determine which of their paths they considered to be valid. While no group said that the meteor would follow a line, there were differences of opinion as to whether the path would be a linear path, a circular path or a curved path due to the initial velocity. When students suggested a deviated linear path for the meteor, the teacher suggested that their chosen path may not be accurate. This allowed other members of their group to explain why a circular or curved path would be an appropriate choice. Although the most accurate paths to represent the paths would have been hyperbolic or elliptical, these types of paths would require a level of depth of understanding the students would not have developed at this point in their education of Leaving Certificate Physics or Leaving Certificate Applied Mathematics. Therefore, both a circular and curved path were considered valid, given the nature of the question, the concept being taught, the lack of numerical details for the necessary calculations to determine the exact path and the prior learning of the students. Each group was asked to explain this in detail to the teacher, to which all the explanations are summarized in the following bullet points:

- The meteor will try to move in the direction it is going with the initial velocity, but the gravitational attraction between the meteor and the earth will cause it to turn.
- This force will cause the meteor to deviate from its original path.
- The field lines will show the direction the meteor will attempt to turn instead of the path.

In the last section of the tutorial, the students were once again presented with the scenario depicted in Figure 4.35. In this case, the meteor has no initial velocity. First, to deepen their understanding of the field line representation, the students were asked to identify which of the two planets had a larger mass, based on the field line density around the planets presented. The students were then required to determine what path would be taken by the meteor. To help them with this process, the tutorial provided a dialogue between two hypothetical students discussing the path taken:

S$_1$: The field lines indicated the direction of the force, so the meteor will be forced along the line until it hits the left planet

S$_2$: As the small meteor begins to accelerate, its gained velocity will make it move away from the field line that it was on originally, so we can be sure it'll hit either planet.

Two groups of students inquired into the meaning of the term "gained velocity," as seen in the reasoning seen by S$_2$. It was explained that this was shorthand for "the meteor would experience a force and accelerate". The students were asked to determine if the velocity would be zero or non-zero, due to the acceleration, after a small amount of time passed.

Initially, students 4F, 4H and 4M agreed with S$_1$, explaining that the meteor has no initial velocity in this case, leading to the force felt by the particle at all points to turn in tandem with the field line. The teacher then suggested they consider the velocity a small instant after the particle starts to accelerate. The students acknowledged that there would be an non-zero velocity after a small instant in time. The students were then directed to review their reasoning from the previous section covered and discuss with their groups.

Upon review, the students agreed with S$_2$, and sketched a path consistent with this reasoning. All other students initially considered that the force would cause an acceleration that would take the path off the field line, into a curved path towards the negative charge. Two examples of student work are presented in Figure 4.39.

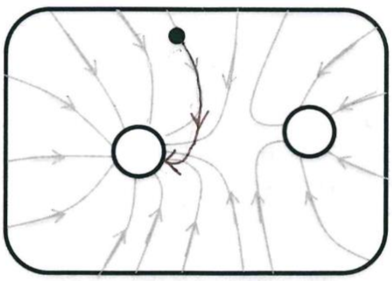
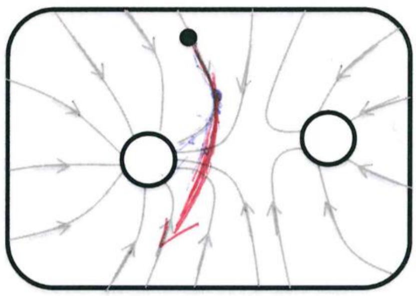
Student 4C:	Student 4H:
S$_2$, because it has gained velocity, it will move off the field line, to hit the planet. 	I agree with S$_2$, because as the meteor begins to accelerate, the gained velocity will make it move away from the field line that it was on originally. 

Figure 4.39. Paths depicted by students 4C and 4H.

In summary, the tutorial discussion illustrates how the students developed their understanding of field line conventions. The students were guided to represent a uniform field using vectors and then

transferred the vectors into field line representation. The students were presented with both uniform and non – uniform fields and guided through the reasoning to accurately determine the relative field strength using field line density as an indicator. With assistance from the teacher, the students developed reasoning to explain why a mass does not follow a field line, referencing force, acceleration, velocity and time, to construct a path taken by a body in a field, under the influence of that field.

4.4.3. Homework: Field line Concepts

The homework assignment was developed to reinforce the target concepts developed in the lesson. The homework assignment questioned student's ability to draw field lines, determine field strength based on field line density, and depict the path taken by bodies in various gravitational field.

In the first question, the students again used the context of meteor travelling past planets, in slightly different scenarios to those shown in the tutorial. The first scenario is presented in Figure 4.40.

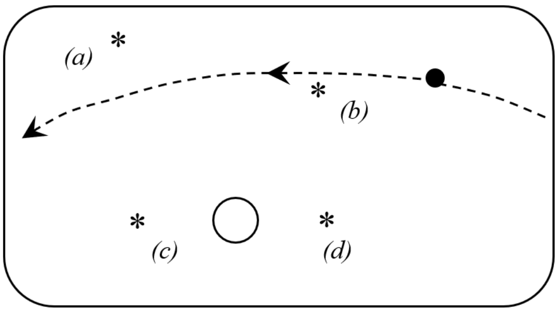


Figure 4.40. Homework question of comet passing planet.

The students asked two questions, in which they were required to draw the gravitational field lines caused by the planet and the second question required them to rank the field strength, from lowest to highest, at the points labelled (a) to (d). The latter question explicitly required the students to reference the field line pattern they sketched for the former question. Their results are summarized in Table 4.28.

Eight of the 14 students sketched a correct field line pattern for the planet's gravitational field, in which straight lines extended from the surface of the planet to the boundary of the diagram, as depicted in Figure 4.38 in the tutorial lesson. Student 4D sketched field lines that terminated in space, before the boundary of the diagram, unlikely realising this depiction suggests the gravitational field terminates at a distance away from the planet. Students 4E and 4F made a similar error, but also sketched their field lines with minor curves, likely not realising that this suggests other masses would be required to be nearby to cause the minor curves they sketched. However, outside of these error in

understanding by the students, they provided an accurate depiction of the field. One student, 4N drew a magnetic field of the earth, but did not provide any reasoning as to why they did this. However, magnetic field patterns are the first field line pattern the students are exposed to (at Junior Certificate level) and it is possible that the student was recalling a pattern they came across two years previously.

Responses	Students
Correct Field line pattern	**4A, 4B, 4G, 4H, 4I, 4J, 4K, 4M**
Used vectors instead of field lines	4C
Field lines terminate	4D
Field lines curve and terminate	4E, 4F
Sketched Earth's B-Field.	4N

Table 4.28. Student's representations of the gravitational field of the planet.

The students were also required to rank the field strength, from highest to lowest, at points marked (a) – (d). Their rankings are summarized in Table 4.29. Seven students provided an accurate ranking. Six students did not explicitly define the ranking between (c) and (d). It is reasonable suspect that they considered the field strength at (c) is greater than (d) in these cases. This would be consistent with reasoning submitted by these students.

Student 4F: C is closest, so it gets affect by the gravitational field the most.

Student 4H: Because as you increase the distance from a planet, you decrease the gravity.

Responses	Students
c = d > b > a	**4A, 4C, 4D, 4E, 4J, 4K, 4M**
c, d, b, a	4B, 4F, 4G, 4H, 4I, 4N
N/A	4L

Table 4.29. Student's rankings of the gravitational field of the planet.

This reasoning was also seen in some students who provided the correct ranking, whilst one student used the field line density to justify their choice.

In the final question, students were asked to sketch the field line pattern of two planets of equal mass and from this, determine a reasonable path the meteor would take, when starting from rest, as depicted in Figure 4.41. There were some difficulties observed in the student's representation of the field lines, such as terminating field lines (4C and 4D), the use of vectors instead of lines (4E) and failing to show the superposition of the two fields (4A, 4B, 4G, 4J and 4K), where lines overlapped instead and two magnetic fields with no superposition (4N). Despite these errors, most of the students

produced a reasonable path to be taken by the meteor, where the path did not follow either the field lines or vectors when used by the students. However, this may have been due to recall of the paths taken in the tutorial questions and applied to this question.

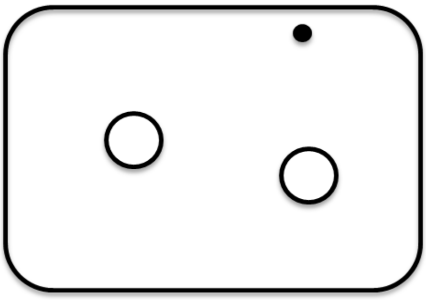


Figure 4.41. Homework question of comet with no initial velocity near two planets.

4D, 4J and 4N submitted paths that were considered unreasonable due to the meteor moving in a direction which does not align with the direction of the net force acting on it (4D), associating a higher gravitational pull to one of the planets at a point equidistant between them (4J) and a path that ignores one of the planets and collides into the other planet (4N).

The homework assignment indicated that the tutorial produced positive gains in some student's understanding. 8/14 students could correctly represent a gravitational field of a planet, with persistent difficulties present such as using vectors instead of field lines to represent the field, field lines terminating inconsistently, field lines being curved when unnecessary or representing the field as the pattern for a bar magnet. 7/14 students could correctly rank the field strength at various points, but in this case, typically used the distance of the points from the planet to justify their ranking. A further 6/14 students submitted answers indicating the correct ranking, also typically referencing the distance from the planet to the points. When representing the field of two planets, student difficulties were more commonly observe, such as representing terminating field lines, drawing vector instead of field lines, not applying the principle of superposition, and drawing bar magnet patterns. Despite these errors, only three students drew paths considered to be unreasonable between the two planets. This indicates that the they were considering the influence of the force of gravity from both planets, the acceleration of the mass, the changing velocity and how these affect the trajectory of the mass.

4.4.4. Post-test: Field line Concepts

The post-test for field lines was undertaken by the students approximately one week after the they completed the field lines tutorial, along with the post-test for the inverse square law. The post-test was designed to elicit student understanding the following three conventions:

- The field strength is represented by the field line density. (Furio and Guisasola, 1998)
- The direction of the field is tangential to the field lines. (Törnkvist, *et al.,* 1993)
- The field lines represent the direction of the force acting on a body, not the path taken by a body. (Galili, 1993; Törnkvist, *et al.,* 1993)

In the first section of the first question, the students were presented with a section of a gravitational field, as shown in Figure 4.42. The students were asked to trace their finger along the lines, starting from where the lines are closest together, so that it travels against the direction of field line, and explain how the field strength varied as they did so.

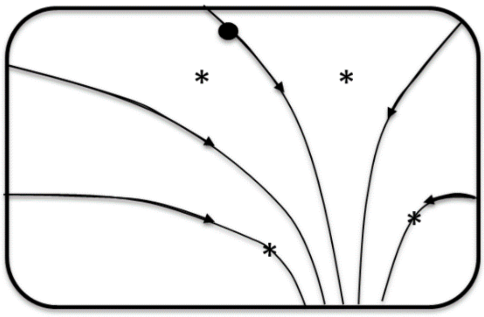


Figure 4.42. Post-test question field lines question.

All the students, except 4I and 4L, explained that the field strength decreases, and this was represented as the field line spreading further apart. Student 4I did articulate that the lines spread out as the field lines were traced but did not reference how this affects the strength of the field itself. Student 4L was not present on the day to complete the post-test.

In the second section of the first question, the students were required to determine the direction of force at the points. The results are summarized in Table 4.30.

Most of the students appeared to have grasped how the direction of the force is represented by field lines. Students 4I and 4N represented the force as acting vertically downwards, which is symptomatic of students who do not consider the field line as a representation of force, and instead assume beyond the bottom of the diagram is a planet in which the gravity uniformly is directed down towards. Student 4G's diagram also suggests a similar reasoning, albeit with the planet just below where the lines appear to converge and the gravitational field acting towards it in a radial pattern.

In the last section of the first post-test question, the students were asked to determine what path a small body would take when it was released from the position marked with a black dot on the diagram. The student's responses are presented in Table 4.31.

Most of the students explained that the meteor's trajectory would not follow the field line, as the bodies inertia would prevent it from directly following the line. They explained that the body would move in the direction of the force acting upon on it and produce a path that is not represented by the

pattern of the field lines in Figure 4.43. The students articulated this by using simpler terms reasoning. The students themselves articulated this as velocity gained by the meteor would cause that path that would carry it from the sketch of the field line. The response summary presented in Table 4.31 indicates that eleven of the students did not think of field lines as a path, and ten of these students could interpret the diagram to draw a reasoning path taken in which the trajectory was influence by, but not identical to, any of the field lines shown.

Responses	Students
Force is tangential to the field lines.	**4A, 4B, 4C, 4D, 4E, 4F, 4H, 4J, 4K, 4M**
All forces point downwards.	4I, 4N
All vectors point to where the line appear to converge	4G
N/A	4L

Table 4.30. Student post-test responses to representing a field using vector arrows.

Responses	Students
Path trajectory sketch diverges from field line pattern in a reasonable path	**4A, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4K, 4M.**
Path trajectory sketch diverges from field line pattern but is an unreasonable path.	4B
Path taken follows the field line.	4N
No path was determined.	4J

Table 4.31. Student's post-test paths drawn taken by a body under the influence of a gravitational field.

However, some students gave additional information about the interaction between the body and the field lines that showed misconceptions. Student 4F correctly reasoned the path and referenced the acceleration, but then also volunteered reasoning that the mass of the body must compete with, and overcome, the force generated by the field lines. Another student, 4H, also gave the field lines a tangible property, in which case the path was a result of the acceleration off the line, the gravitational pull towards the bottom of the diagram and the other field lines pushing the body away to stop the path from intersecting with other field line. This tangible property of field lines has been observed in other research (Galili, 1993)

Student 4F: As the body accelerates, it moves off the field line towards the gravity centre, because its mass is greater that the force of the gravity field.

Student 4H: The body will go off the field line as it accelerates and gains its own velocity. However, the other field lines will push it downwards.

Student 4B reasoned that the area where the gravitational field was strongest would have the most pull, therefore it would pull the body off the field line as shown in Figure 4.43. This suggests confusion to attributing a gravitational force to the field line themselves, as opposed to a representation of which way a body would experience a force. Student 4N acknowledged the body would accelerate, and as it got closer to the bottom, this acceleration would increase. They did not however consider that this acceleration would generate a velocity that would carry it from the field line.

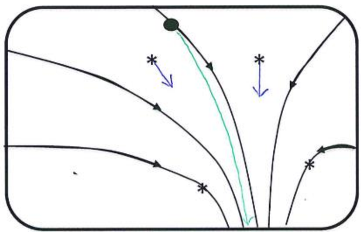
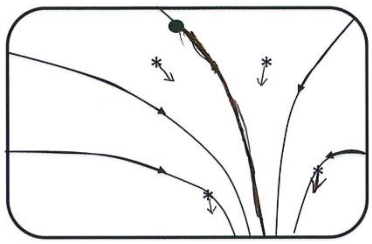
Student 4B:	Student 4N:
The small body is pulled towards the field lines that are closer together, because the strength of the gravity is stronger there.	*The body accelerates faster when the lines are closer together and the body is being pulled down by the force of gravity.*
	

Figure 4.43. Path taken by the body from rest from student 4B and 4N.

The post-test showed that student gains in understanding of how the electric field lines represent the relative strength of the field. 12/14 of the students correctly explained the variation of the field line patterns, with another student providing reasoning alluding to the convention. 10/14 students accurately represented the force vectors as tangential to the field lines, which persistent difficulties in the remaining students observed such as directing the vectors to the point where the field line converge or ignoring the field pattern entirely. The most notable student difficulties were students representing the path taken by a stationary body in a gravitational field. 10/14 of the students sketch the path taken as a pattern that did not follow the field lines, but there were examples of students associating a tangible nature to the field lines themselves, in which they described the field lines are having a gravitational pull of their own. This reasoning appears to treat gravity as a contact force, instead of a non-contact force, and shows the approach did not address the difficulty for these students.

4.4.5. Discussion

From comparing the pre-tests, lessons, homework and post-tests, it can be determined that most of the students made progress in their understanding of field line conventions. The first concept addressed in this discussion is the student understanding of field line density and its representation of relative field strength. Figure 4.44 compares the pre-test and post-test results for this concept.

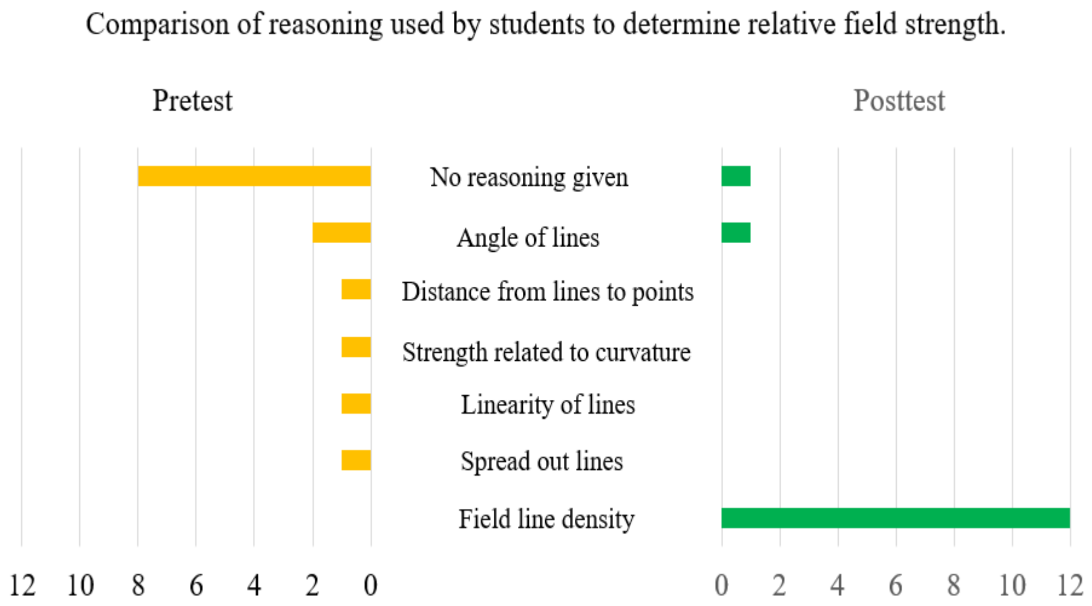


Figure 4.44. Comparison of reasoning used by students to determine relative field strength.

As seen in Figure 4.44, a shift in student reasoning occurred from the pre-test to post-test. In the pre-test, six of the students used incorrect reasoning such for stronger field strength, such as (i) the further spread the field lines, the stronger the field strength, (ii) straight lines being stronger, (iii) the more pronounced the curvature of the line, the stronger the field (iv) the further a point is from a line, the weaker it is and (v) the higher the angle the lines make with the horizontal / vertical, the stronger the field. Eight of them submitted no reasoning for their rankings. The eight submissions with no responses suggest the students had no explanations they were satisfied with to justify their rankings (Posner, *et al.*, 1982), while difficulties the remaining students had were identified. The students were introduced to the convention for field line density representing strength during the tutorial lesson, and section 4.4.3 outlines multiple instances that were sufficient for the students to explore and adopt the convention of field line density to represent relative field strength.

Figure 4.44 indicates that the tutorial lesson was effective in promoting the student's conceptual development, as a clear shift in reasoning submitted by the students was observed in the post-test results. All but two of the students correctly used the field line density to produce the correct rankings, and except for one student, there were no references to the misconceptions observed in the

pre-test. This indicates that conceptual extinction occurred (Hewson, 1992), with the shift in the pre-test and post-test results demonstrating that ideal conceptual change occurred.

The next section of this discussion presents a comparison of the student's transfer from field line representation to vector arrow representation. Figure 4.45 presents a comparison of the pre-test and post-test results for this concept.

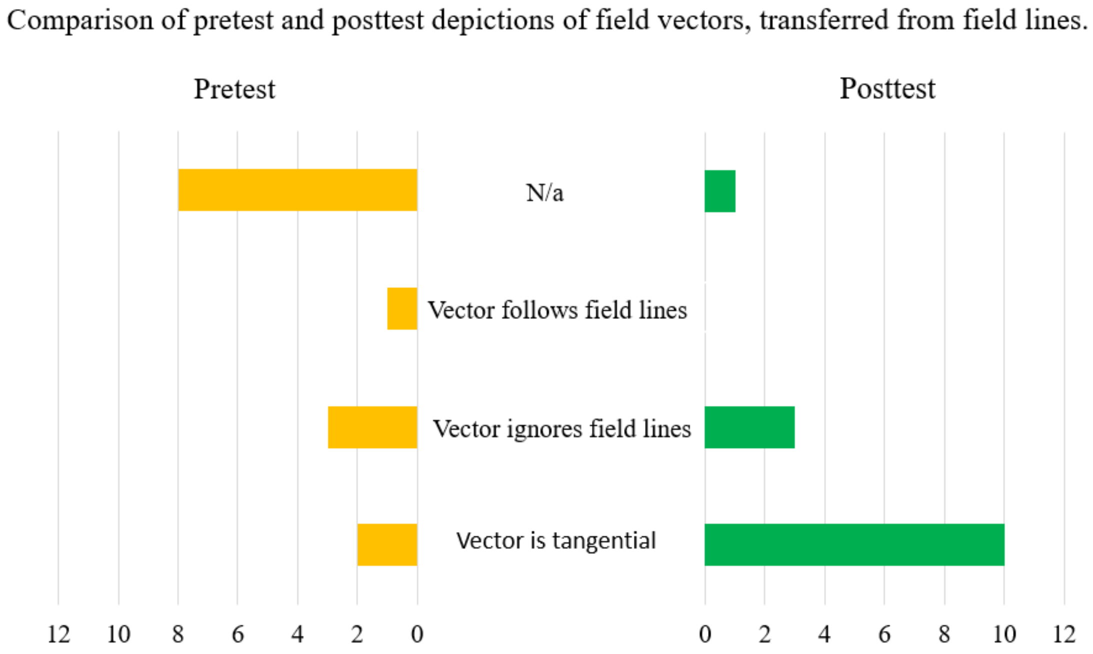


Figure 4.45. Comparison of depictions of field vectors, transferred from field lines.

Figure 4.45 shows that the tutorial lesson also produced gains in student's understanding that the electric field vector at a point is tangential to the field lines. Initially, most of the students were unable to attempt this question in the pre-test and did not have enough understanding to approach the question. The student's inability to reason this question suggests dissatisfaction with their prior understanding (Posner, *et al.*, 1982), while the difficulties observed by the three students who ignored or followed the field lines were identified for conceptual extinction. The students were guided to transfer from field lines to vectors during the tutorial lesson, provided with opportunities to practise the transfer between the representation in the tutorial and homework assignment. During the tutorial lesson, the students used rulers to practise drawing the arrows and repeated this for the homework activity. Figure 4.45 indicates that there was a shift in the number of students that could accurately apply vector diagrams to an electric field context, from the pre-test to the post-test. As the participants demonstrated proficiency of using vectors in section 4.2, this shift indicates conceptual extension occurred (Hewson, 1992), with the shift from two to ten students producing tangential vectors indicating that moderate conceptual change occurred. However, no students attempted an accurate scale, in which the vectors were longer where the field strength was greater, ignoring the magnitude

of the vectors, and instead, focusing on the direction only. This indicates conceptual change occurred, but issues of transfer between the vector representation and the field line representation persisted.

Additionally, in the tutorial lesson, the students built up a model of the gravitational field of the earth and transitioned it to a field line model, but half of the students demonstrated errors such as field lines beginning at the object in the field and terminating before reaching the planet generating the gravitational field. These errors are further discussed in sections 5.6 and 5.7 where the students were given the opportunity to address them.

The last section of this discussion focuses on the student's predictions of the path taken by a body under the influence of a field. Figure 4.46 presents a comparison of the pre-test and post-test responses from the students for this concept. As discussed in section 4.4.1 and 4.4.4, these results are for scenario's in which the body under the influence of the field has an initial velocity of zero.

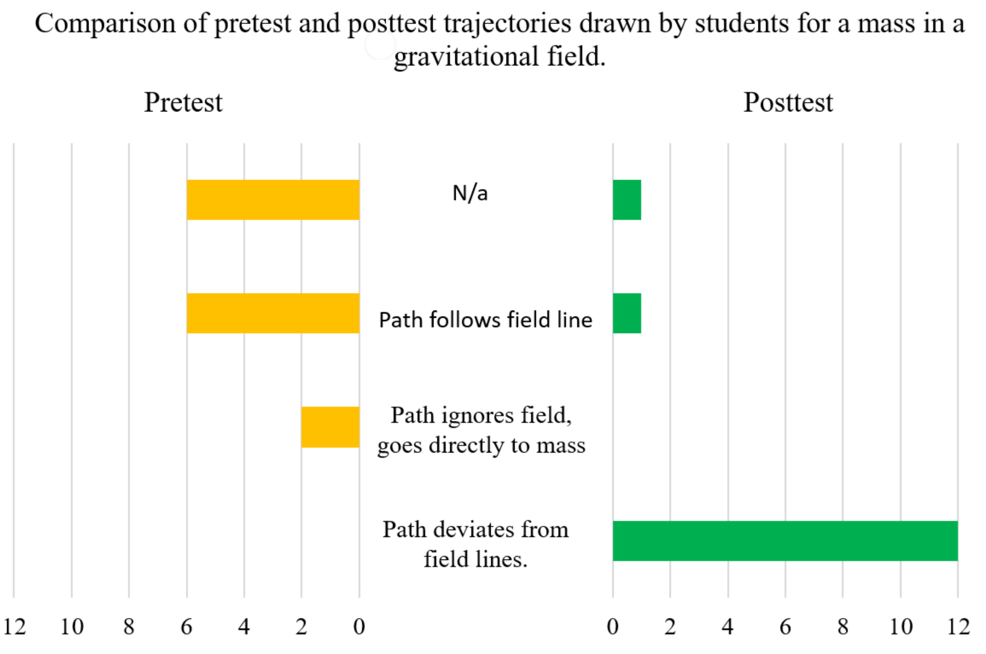


Figure 4.46. Comparison of trajectories drawn by students for a mass in a gravitational field.

The most prominent error by students in the pre-test was that the object would follow the field lines or ignore the field lines and go directly to the mass generating the gravitational field. Both difficulties were predicted from literature (Galili, 1993; Törnkvist, *et al.*, 1993). As the reasoning that produces these difficulties is erroneous, these difficulties were targeted for conceptual exchange (Hewson, 1982). Section 4.4.2 illustrated many examples of the students developing their reasoning for this concept. The tutorial was written with three examples of a body moving with a trajectory influenced by, but not identical to, a gravitational field. The students explicitly looked at this concept on two questions during the tutorial and referenced the first question during discussions with the teacher. As the students were familiar with in the initial scenarios presented in the tutorial, they became dissatisfied with erroneous reasoning that did not explain the observations accurately (Posner

et al., 1982). The students were given ample opportunity to develop and apply the field line representation to explain the observations in the tutorial, and they used it to develop intelligible reasoning to predict the behaviour of objects under the influence of a field, both in contexts they were familiar with and in contexts unseen to them (Posner *et al.*, 1982). In the post-test, all but two of the students depicted a path that diverges from field lines. This shift in student responses from the pre-test to post-test indicates that the tutorial lesson was effective in promoting conceptual exchange in the student's understanding (Hewson, 1982), with the results indicating that ideal conceptual change occurred.

In the absence of presenting students with a field line pattern, such as those seen in the homework activity, it was observed that the students used different reasoning to justify their claims. In the first homework activity given to the students, they were not explicitly presented with a field line pattern, as previously seen in Figure 4.42, and were asked to rank the field strengths. In this case, the students justified the ranking in various manners. 8/14 students based their ranking on using the distance to indicate the gravitational field strength, and all but one of the students ignored their sketches of the field lines and used these to justify their ranking. The students could have also referenced the inverse square law for gravitation to justify their rankings, but this was not seen in any student's responses. Only one student (4B) explicitly based their ranking on the field line density. However, when shown a field line pattern and asked to explain how the field strength varies, as seen in the post-test, 12/14 of the students used the convention for field strength with the representation. This indicates that while the students were comfortable with the representation, they did not always consider it a valuable tool to use, unless directly required to do so.

In the final homework question, the students attempted to use the reasoning they developed in the tutorial lesson, but many errors were observed in the field lines patterns they produced. One tool not observed in the homework responses by the students was the additional use of vectors or motion diagrams to justify the path chosen. When students ran into difficulties in the tutorial lesson, the teacher would ask the students in which direction the body would move and represent this with a vector and sketch the body again at the end of this vector, indicating that this was the body's new position after a short interval of time. The teacher would repeat this process two more times, using the student's answers to define the direction of the movement of the body and its new position after a short interval of time, thus generating a motion diagram with the aid of the teacher. This line of reasoning was not considered by the students in their homework activity. An exercise designed to look which representations students prefer to use is discussed in chapter 5 in which students plot the path taken by a negative charge when placed between two positive charges. In this question, the students are not asked to represent the field in any way, so it will give us an insight as to which representation, or combination they choose to use.

This section has shown evidence of student's gains in understanding of field line conventions. While there are still some difficulties that persist with the students, they appeared in a small number

of the student's responses. Instances of Posner, *et al*., (1982) conditions for conceptual change were indicated and when possible, the manner of conceptual change that occurred was identified (Hewson, 1992). The reasoning developed by the student can be transferred to electrostatics, to represent the fields of one or two charges. They can also use the representation to help explain the behaviour charged particles in an electric field. Field lines are also utilised in an electric field tutorial to aid students in associating the inverse square law to electric field, using a model of field line passing through a unit frame, similar in nature to the tutorial discussed in section 4.3.3. Students will also employ the use of electric field lines to develop an understanding of positive, negative and zero work in an electric field, to develop their understanding of potential difference.

4.5. Conclusions

The results of the students presented in this chapter show that the student's understanding of vectors, inverse square law and field lines improved by the employing tutorial lesson. Evidence provided supports that conceptual change occurred in some of the student's understanding of these topics. The tutorials both introduced students to the topics, and specifically addressed difficulties typically encountered by students, as seen in literature. This approach has been shown to be effective to address student difficulties, over using traditional instructional methods. (Dykstra, *et al*., 1992; McDermott and Shaffer, 1992).

The results indicate that the approach adopted promotes conceptual understanding of vector magnitude, vector addition and the implications of adding horizontal and vertical vector components. While some of the students preferred to use vector constructions over reasoning based on vector components, it was observed that students engaged with, and overcame, difficulties in the vector concepts such as linking the magnitude of a vector to its direction (Nguyen and Meltzer, 2003), incorrectly combining vector arrows (Nguyen and Meltzer, 2003) treating vector addition as scalar addition, with no consideration for either the directions of the vectors or the summation of the vector components (Doughty, 2013). Regarding these difficulties, the evidence presented in this chapter indicates conceptual exchange (Hewson, 1992) occurred in the student's models, although some difficulties persisted. As the students became more proficient in these vector concepts, the representation could then be utilised by the students to explain the direction and variation of electric field strength between two charged sources, and the forces acting on the charges.

When the students completed the inverse square law tutorial, it was observed that the students could recognize the general shape of a graphical pattern that follows an inverse square function. However, it was noted that students upon completion of this topic, the students could not differentiate between data that followed an inverse square pattern, and an inverse pattern when they complete

investigations into Boyles' law and the focal length of a concave lens. This indicates that students had an over-reliance on the general shape of the pattern and did not consider to mathematically analyse the data presented on a graph. Mathematically, it was seen that the students could evaluate an equation for intensity, but also struggled to analyse their produced values to show an inverse square law. This lack of consideration to find meaning in the numbers produced echoes student's experience of solving quantitative problems that is typically found in traditional methods of instruction in physics education. The students were given more opportunity to consider the inverse square law and practice their ability to analyse the data, both on a graph and using algebraic evaluation, when they completed the Coulomb's law tutorial lesson, as these skills are employed in that lesson, as discussed in section 5.5.2.

Conceptually, it was observed that students could explain the variation of area covered in intensity contexts, when the distance between an object and the frames were varied. Section 4.3.3 narrates how students could reason that area change of frame was a quadratic factor of the change in the distance between the objects due to a change in bother the length and width of the frame, demonstrating their understanding of quadratic pattern observed when scaling up the area of a surface. Difficulties were seen when applying this to intensity when considering how much of a given phenomenon, in this case droplets of paint, passing through the frame. This highlights confusion between their understanding of intensity of a phenomenon and the phenomenon itself. This can also lead to confusion about which factors follow a quadratic increase, in the case of the tutorial; area, and which follow an inverse square law, in the case of the tutorial; paint intensity.

From the student's exploration of field lines, it was shown that the students are reasonably proficient in the use of the representation and can interpret information from a field lines pattern. It was demonstrated, that upon completion of the materials, they could determine the field strength based on field line density, determine the direction of force acting on a body at a point in a field and reasonably plot a path taken by a body in a field, based on this information. However, in the absence of a field line pattern, when asked to construct the field for two masses, the students produced errors in their field lines patterns. The most persistent difficulty seen was the students not applying the principle of superposition of the two fields, field lines overlapping and using vector arrows in lieu of field lines. These persistent difficulties are addressed in a later tutorial, discussed in sections 5.6 and section 5.7.

In the cases of all three of the topics, there is evidence that conceptual change occurred. The pre-test, tutorials and post-test, student - student discussions and teacher-student discussions presented in this chapter provided many instances of the 4 conditions presented by Posner, *et al*., (1982) required for conceptual change and when the evidence was presented, the type of conceptual change; extinction, exchange and/or extension (Hewson, 1992) was identified. The extent of conceptual change that occurred varied from minimal to ideal, and the instance of each descriptor is presented a

line plot, shown in Figure 4.47. A legend of the codes used in Figure 4.47 can be found in Appendix F.

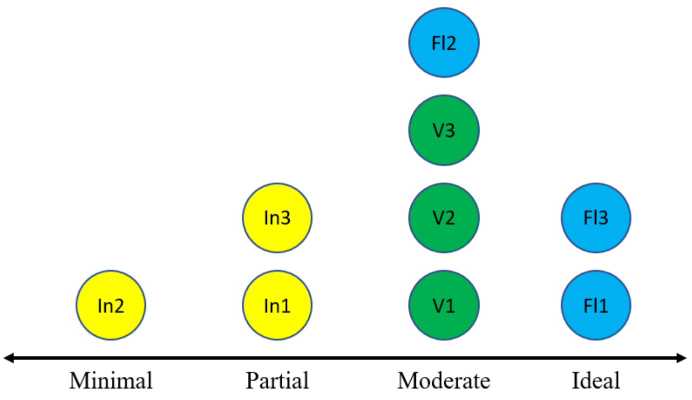


Figure 4.47. Line plot of extent of conceptual change for vectors, inverse square law and field lines.

The extent to which the student's engaged in conceptual change over the course of the tutorials varies depending on which topic the approach addressed. The tutorials that were most effective in promoting conceptual change for the concepts related to the field lines, as moderate and ideal instances of conceptual change were recorded. There were moderate instances of conceptual change for the vectors concepts and minimal and partial instances of conceptual change for the student's conceptual understanding of the inverse square law. The vectors and field line tutorials generally omitted the requirement for students to use mathematical reasoning, while the inverse square law required both mathematical and scientific reasoning to be employed by the students. This requirement to employ dual reasoning could have overloaded the number of items being processed in the students working memory (Reid, 2009) and halted their ability to fully develop their understanding.

By developing the student's understanding of vectors, the inverse square law and field lines, this sets a foundation for the students to build their understanding of Coulomb's law, electric field and potential difference. The students will be able to apply their understanding to these topics to help develop their understanding of electrostatic force, and field. Vectors and the inverse square law are central concepts underpinning Coulomb's law, in determining the forces acting on charged particles, in collinear and non-collinear settings. Vectors and field lines can be used to represent electric field and the behaviour of charged particles in these field, whilst the inverse square law can be used to mathematically quantify the variation in the electric field strength of a charge. When a charged particle moves in an electric field, the use of vectors and field lines can be used to identify whether positive, negative or zero work occurs, which can be used to discuss the variation of potential or define the potential difference between two points in an electric field.

Chapter 5. Coulomb's law and electric fields.

5.1. Introduction

This chapter discusses the development of the student's understanding of Coulomb's law and electric fields, by using inquiry tutorials that employ a multi-representational approach. The tutorials embed vector concepts, exploration of the inverse square law and field line representations in electrostatics. Combined with their prior learning of mechanics, forces and charge, this allows for the students to generate links and their own understanding of the topics and develop the ability to transfer between all representations. This chapter identifies instances in which the students (a) used their understanding of the three element concepts to develop their understanding of Coulomb's law and electric fields, (b) used their experience in developing their understanding of Coulomb's law and electric fields to better their understanding of the target elements, or both.

The following research question is addressed in this chapter:

- To what extent does the use of a multi-representational structured inquiry approach develop student understanding of electric fields?

The following points were considered when addressing this research question:

1. The student's ability to demonstrate that Coulomb's law is an example of an inverse square law.
2. To what extent the students demonstrate their understanding of electric fields and the interaction of charged objects with fields using vector representations.
3. To what extent the students demonstrate their understanding of electric fields and the interaction of charged objects with fields using the field line representation.
4. To what extent the students demonstrate their ability to transfer a depiction of an electric field from one representation to another representation.

Figure 5.1 depicts how the concepts discussed in chapter 4 and the concepts covered external to the project are prerequisites to learning electrostatics. As before, the colour purple denotes topics completed, at this time, during the project; it shows that the students have studied all prerequisite concepts.

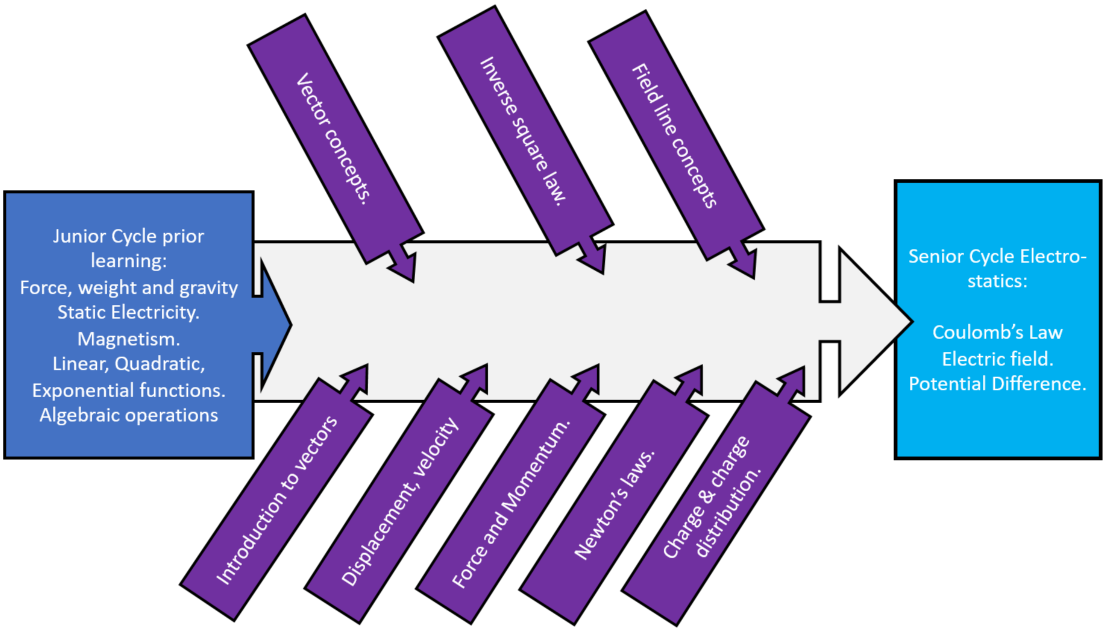


Figure 5.1. Flowchart depicting the topics completed by the students, prior to developing their understanding of Leaving Certificate electrostatics.

The timeline of this section is shown in Table 5.1. Sections in bold refer to materials covered are directly related to the research.

The chapter presents a narrative of the student's use of vectors concepts, the inverse square law and field lines, as they were applied to Coulomb's law and the electric field concept. The student's conceptual development is presented by comparing pre-test and post-test results for the different topics, as well as snapshots of their tutorial worksheets and excerpts of recordings of their conversations during the tutorial sessions.

Section 5.2 reviews difficulties encountered by students in their understanding of vectors, the inverse square relationship and field lines, and their application to Coulomb's law and electric fields, as described in section 2.3.1. Section 5.3 discusses the lessons learned from the research presented in chapter 4, in terms of the difficulties observed in the students understanding after the tutorial lessons, and the implementation of the tutorial lesson format. Section 5.4 discusses how students apply their understanding of vector concepts to the electric field in the context of Coulomb's law. This section focuses on student's use of vector constructions, representing the relationship between magnitude of field strength and distance, and consideration of horizontal and vertical components in vector addition. Section 5.5 looks at the student's application of the inverse square law, and how they apply it to Coulomb's law and electric fields. Students apply the law using tables, graphs, algebra and a scale model, in which they discuss field lines passing through various frames. This approach adapts the model used in Conceptual Physics (Hewitt, 2009). Section 5.6 looks at the student's understanding of the field line representation in electric fields. Here they apply the convections of field line density to represent relative strength, the direction of a field being tangential to a field line. They should understand that the path taken by a body in a field does not follow a field line but the

trajectory of the body is influenced by the field, and the field lines give an indication of the direction and strength of force acting on a body in the field. The students are further introduced to the conventions that field lines do not overlap, and only start and end on electric charges. In the discussions, the results are reviewed to present the impact of the approach on student learning, particularly their ability to transfer representational tools to the electric field context and their reasoning skills.

Week 1, Class 1 (35 mins)	**Presentation to introduce to Coulomb's law.** **Pre-test**
Week 1, Class 2 (80 mins)	**Practice class: Qualitative problems involving Coulomb's Law.**
Week 1, Class 3 (76 mins)	**Research lesson: Coulomb's Law worksheet. Homework assignment given.**
Week 2, Class 1 (35 mins)	**Presentation to introduce to Electric field.** **Pre-test**
Week 2, Class 2 (80 mins)	**Practice class: Qualitative problems involving Electric field.**
Week 2, Class 3 (76 mins)	**Research lesson: Electric field worksheet. Homework assignment given.**
Week 3, Class 1 (35 mins)	**Practice class: More difficult qualitative problems involving Electric field. Homework assignment given**
Week 3, Class 2 (80 mins)	Presentations and demonstrations: Charge distributions and charging by induction.
Week 3, Class 3 (76 mins)	Practice of Leaving Certificate past paper questions.
Week 4, Class 1 (35 mins)	Review of Topics with students.
Week 4, Class 2 (80 mins)	**Post-test.**

Table 5.1. Timeline of the Coulomb's law and electric field tutorial lessons.

5.2. Vectors, inverse square law and field lines in electric fields

This chapter presents a narrative and analysis of the development of the student's understanding of Coulomb's law and the electric field, in terms of their ability to apply vectors, the inverse square law and field lines to this domain. As the students have developed an understanding of these topics, as discussed in chapter 4, the tutorial lessons for this part of the research embed these topics throughout. This gives the students the opportunity to review the material covered in the electric field context, and develop a deeper understanding of the electric field through using multiple representations (Ainsworth, 2006).

The Coulomb's law and Electric field tutorials were designed to provide the students with opportunities to address the difficulties encountered by learners in their understanding of vector concepts, the inverse square law and field line representations detailed in Section 2.1.3. detailed difficulties. The following learning objectives for this section of the research ensued: upon completion of the teaching and learning material, the students would be able to:

- Construct and interpret a uniform and/or non-uniform electric field represented by vector arrows, consider both vector magnitude and direction (Maloney, *et al.*, 2001)
- Apply the principle of superposition to two vector field representations (Maloney, *et al.*, 2001; Nugyen and Meltzer, 2003).
- Students can discuss electric force and field superposition in terms of vector component addition (Furio and Guisasola, 1998; Cao and Brizuela, 2016).
- Demonstrate that Coulomb's law and the electric field follow an inverse square law using a variety of representations (Maloney, *et al.*, 2001; Hewitt, 2009; Moynihan, *et al.*, 2015)
- Apply proportional reasoning involving scaling to inverse square law problems (Arons, 1999, Marzec, 2012).
- Construct and interpret a uniform and / or non – uniform field using field line representations (Törnkvist, *et al.*, 1993; and Galili, 1993; Cao and Brizuela, 2016).
- Apply the principle of superposition to field line diagrams (Törnkvist, *et al.*, 1993; Galili, 1993).
- Accurately predict the behaviour of charged particles under the influence of an electric field (Cao and Brizuela, 2016).

5.3. Lessons learned from previous research

In chapter 4, it was seen that the use of tutorial lessons promoted the development of student understanding of vectors, the inverse square law and field line representations. This approach, patterned after Tutorials in Introductory Physics (McDermott and Shaffer, 2003), breaks concepts

and topics down into a series of lower and higher order questions, designed to elicit student thinking, identify difficulties and provide opportunities to help students overcome difficulties. The emphasis is on students working in groups to think and reason their way through the worksheet. This allows them to apply what they know, organise their thoughts and make judgements and evaluations about physical phenomena. The approach adopted produced various degrees of gains in understanding of vector concepts, though some conceptual difficulties with vector addition remained as it was observed that numerous students did not consider vector addition in terms of component addition. On completion of the inverse square law tutorial most students could recognise and represent an inverse square proportional relationship graphically, explain the variation in the area model using scaling and could apply inverse square proportional reasoning to mathematical questions. Finally, students made some gains in their understanding of field line representations such as relative field strength being represented by field line density, field vectors pointing tangentially to field lines at a point and field lines representing the direction of force experienced by a body, and not the path taken by a body under the influence of a field. However, difficulties related to the superposition of field lines persisted. These difficulties are addressed in the electric field tutorial, discussed in section 5.6.2.

These representations can be used as an aid for students developing conceptually accurate understanding of the electric field, at both second level and during further study at third level. Field theory replaced the preceding model of action at a distance and employs the use of both vector mathematics and mathematics involving the use of calculus. While the application of calculus to electric fields would be beyond the capabilities to the average second level student, an understanding the basic and slightly sophisticated vector concepts can underpin the foundations for future development of further advanced studies into electromagnetism. The inverse square law links to the model of a Gaussian sphere and electric flux density. In Ireland, students typically encounter this model until they have completed second level physics education. If they have not developed a complete understanding of the inverse square law, any developed difficulties would need to be overcome at third level. Field lines are a simple model than can be used to represent a field, and when interpreted correctly, can display a lot of information about a field that would be cumbersome to represent in another format. In developing understanding of these three "pillar" concepts, students can be guided to transfer their reasoning to representations they can struggle with, such as the mathematical formula for Coulomb's law, or problems that employ the use of unfamiliar contexts.

5.4. Student's use of vectors in electric fields

This section presents the student's use of vectors in Coulomb's law, and electric fields. Section 5.4.1 details the Electric field pre-test results, in which it was observed that students struggled to transfer and apply the vector concepts they developed in chapter 4, such as vector magnitude and

superposition. Section 5.4.2 narrates the use of the electric field tutorial lesson, in which the students review their understanding of magnitude, presented as uniform and varying electric fields. The students then apply the concept of superposition to finding the net electric field at various points using vector addition, and discuss the magnitude of the resultant vector in terms of its horizontal and vertical components. Section 5.4.3 discusses the post-test results, in which positive gains were seen in the student's understanding of vector magnitude. Sections 5.4.4 and 5.4.5 detail research on the student's understanding of horizontal and vertical components through the analysis of a student homework assignment, and a teaching and learning interview involving the concept. Section 5.4.6 presents a comparison of the pre-test/post-test results and discusses student gains observed during the tutorial lesson and teaching and learning interview, as well as illustrating persistent difficulties in student understanding of vector concepts and their transfer to the Coulomb's law and electric fields.

5.4.1. Pre-test: Student's use of vectors in electric fields

In the Electric field pre-test, the students were given a question where they were asked to construct an electric field using vector arrows at various points, based on their relative position around (a) a positive charge, (b) a negative charge, and (c) and positive and negative charge. In this way it was determined to what extent students could depict the direction of the field of the charge correctly, display the relative magnitude at different distances from the charges, and apply vector addition for the fields produced by a positive and negative charge at given points. The diagrams are presented in Figure 5.2, and a summary of student responses is presented in Table 5.2.

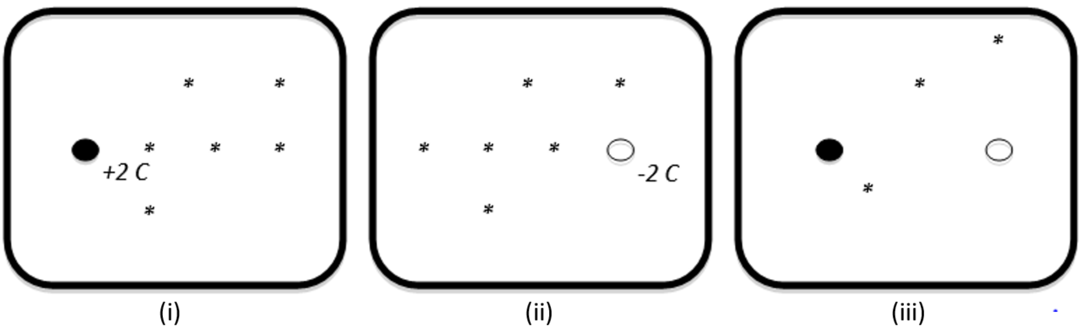


Figure 5.2. Pre-test question applying vectors to electric field context.

The pre-test result shows that a common difficulty was a reversal of the direction in which the vectors should point, (towards positive charge / away from negative charge) to represent the electric field. The students were familiar with using vectors to represent the gravitational field of a planet, so for them to apply the same directions for the vectors in the diagram for the first question was not

unreasonable. It followed that the next question, which asks them to use vectors to represent the field of a charge of an opposing sign, which resulted in the opposite direction being plotted.

Responses	Students
Direction point away from positively charged particle.	**4L.**
Vectors point towards positively charged particle.	4A, 4B, 4D, 4E, 4H.
Attempted field lines.	4F, 4G.
Non-discernable.	4C, 4J.
One vector, originating from charge.	4K, 4M, 4N.
Direction points towards negatively charged particle.	**4A, 4H, 4L, 4G**
Vectors point away from negatively charged particle.	4B. 4C. 4D, 4E. 4F, 4K, 4M, 4N
Vectors originate from charge.	4B, 4C, 4K, 4M, 4N
Attempted field lines.	4D, 4E. 4F, 4G
Non-discernable	4J

Table 5.2. Student responses to vectors and electric field pre-test question.

It was also observed that four students attempted to use field lines instead of vectors to represent the field, and a further five students drew their vectors originating from the charge. The latter suggests that the students have combined elements of the two representations or have confused the field vector for a displacement vector, representing the perceived path to be taken by a charge at a given point.

Table 5.3. presents summary of the vector representational errors made by the students in representing the magnitude of the electric field at the various points. The pre-test showed the students also had difficulties in representing magnitude of the electric field at various points. Only two students drew vectors in which the strength decreased as the distance from the charge increased. Two other students drew vectors that suggested the field strength increased at distances further from the charge, and while one student did not show any variation in field strength. The remaining students results showed difficulties that were not typical of those found in literature, such as vectors of varying magnitudes with no discernible patterns, or displacement vectors between randomly chosen points.

Responses	Students
Magnitude of vectors decreases with distance from charge.	**4H, 4L**
Magnitude of vectors increases with distance from charge.	4A, 4G
No variation in vector magnitude	4M
Magnitude variation follows no discernable pattern.	4C, 4D, 4E, 4F
Vectors magnitude is relative to distance to next point.	4B, 4J
N/a	4K, 4N

Table 5.3. Students responses to variation of field strength with distance.

In the final diagram (Figure 5.2, (iii)), students were required to draw the resultant electric field at three points. A summary of the results to this question is presented in Table 5.4.

Reasonable superposition of vectors demonstrated.	**N/a**
No superposition of vector demonstrated.	4A, 4B, 4C, 4D, 4F, 4H, 4J, 4L, 4M, 4N.
Superposition of field lines attempted.	4E, 4G,
Not attempted	4K

Table 5.4. Students use of superposition with electric fields.

This pre-test question showed that many students did not consider the superposition of their vector arrows from the previous two parts of the question to be appropriate, when drawing the electric field at the points highlighted. Figure 5.3 illustrates some of these difficulties. Ten of the students represented the field but showed no indication of vector addition. In some cases, students drew two vectors acting at the points (4A – Figure 5.3, i), displacement vectors from both charges (4C, 4F), vectors pointing to one charge only (4H, 4J, 4L – Figure 5.3, ii) or force vectors representing attraction between the charged bodies (4B, 4D, 4M, 4N – Figure 5.3, iii).

Two of the students attempted the use of field line representations instead and produced patterns consistent with their observations of the field between two planets (4E), as seen in the field lines tutorial lesson or a pattern consistent with the field lines of a bar magnet (4G).

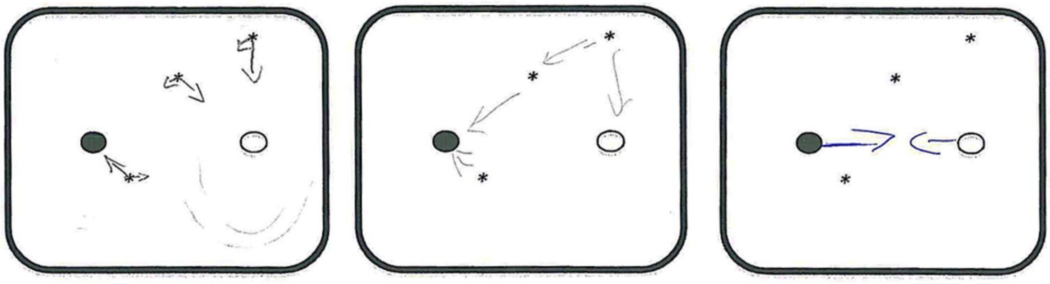


Figure 5.3. Student responses to applying the superposition of vectors to an electric field.

This pre-test showed poor performances by the students to apply their understanding to the electrostatic field context despite previous gains in their understanding of vector magnitude and vector addition. This inability to extend their understanding of vectors to apply it to an electrostatic context indicates that the approach in the tutorial lesson wherein the students are guided through the use of vectors to describe electric fields is justified. This can help promote their ability to transfer their understanding between these two domains.

5.4.2. Tutorial lesson: Student's use of vectors in electric fields

The electric field tutorial lesson was designed for students to transfer the concepts they developed in the tutorials outlined in chapter 4 to electrostatic fields, and further their understanding of electric fields. The students were presented with opportunities to represent electric field of 2 charge systems using both methods. This section will discuss the first half of the tutorial lesson, in which the students explored the use of vectors to show the superposition of an electric field, for a two charged-particle system. The latter half of the tutorial lesson, which looks at the use of field lines, is discussed in section 5.4.3.

The tutorial introduced students to a uniform electric field represented as vectors. The tutorial worksheet defined a uniform field as a field the is equal in magnitude and direction at all points. The students were then required to observe a uniform field and were asked to explain how the vector representation indicated the presence of a uniform field, as shown in Figure 5.4.

The students were required to justify that the vectors are of equal length, and all point in the same direction. Additionally, they were required to rank the electric field strength from strongest to weakest at various given points. While an understanding of vector magnitude should produce a ranking of A=B=C=D, there were discussions within group as to whether D>C>B>A was correct. The reasoning initially used to justify this was that as the vectors move from left to right, more vectors pass through each point. When asked to clarify what was moving, the students would reply that the

field was moving, as indicated by the direction of the arrows. However, when asked to recall a demonstration experiment for lower second level, in which they sprinkled iron filings over a bar magnet to show the field. They recalled that the field pattern they demonstrated did not move from North to South and determined the field. These students were then invited to revisit their notes from the presentation about how the electric field represented the force at an individual point, and with the aid of other students in their groups, they generally volunteered that neither the field or vectors move. The students generally came to the correct ranking during these discussions. In the questions of this initial section, the students were required to represent an increasing electric field, and decreasing electric field, which all students successfully completed.

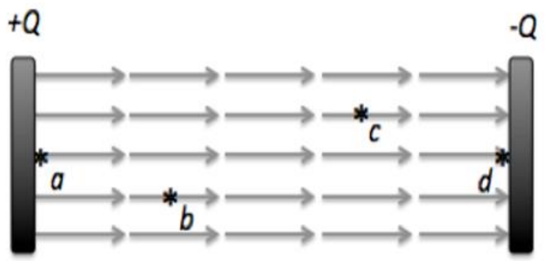


Figure 5.4. Uniform electric represented using vector arrows.

The tutorial then presented a two-dimensional vector addition exercise, in which the students were presented with the steps required to determine the combined field at a point, by utilising a vector construction to determine the superposition of two electric field vectors at a point, as shown in Figure 5.5.

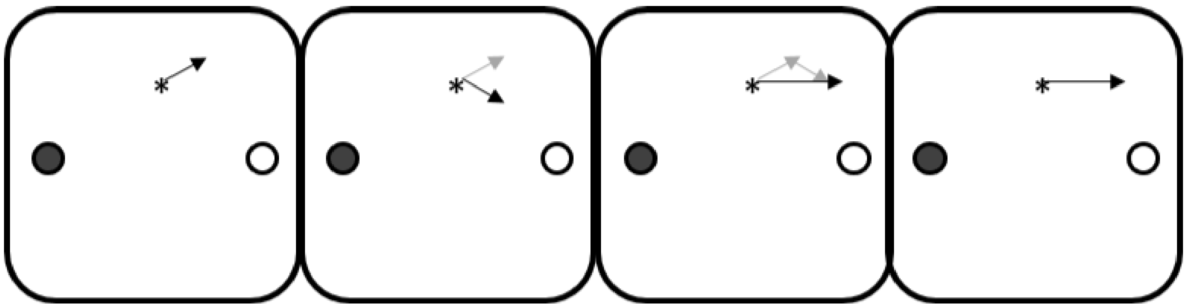


Figure 5.5. Demonstration of the superposition of two vectors representing an electric field.

The students were required to explain each of the steps, demonstrating and applying their understanding of adding vector components. By presenting the steps, and getting them to justify them, it was expected that the students could explain why the electric field is horizontal, and be able to explain the cause of the change in the resultant electric field magnitude, compared to the individual electric field vectors initially presented.

Student 4D: The vertical component cancel out, leaving only horizontal vectors. The vertical components cancel out, because they go in opposite directions, leaving only horizontal components to add to get the magnitude.

The students then were presented with another diagram like that shown in Figure 5.5, in this case where the two particles were positively charged, as opposed to positively and negatively charged. The students were required to use the either the "tip to tail" or parallelogram construction to represent resultant vector. The students were also asked to explain why and how the direction and magnitude of the resultant vector was affected in terms of the horizontal and vertical components. Again, the student groups produced reasoning in which the horizontal vectors cancelled each-other out, leaving only the vertical vector components to combine to produce the resultant electric field vector in a vertical direction.

In the last section of the tutorial lesson that addressed student's understanding of vectors in an electric field, students were presented with the a positively and negatively charged pair of particles, and various positions were highlight for the students, in which they were required to construct the superposition of the electric field at various points. Errors were caused by students initially not considering the magnitude of their vectors to be different, regardless of their distance from the charges. When prompted to consider the field strength based on their distances, the students were quick to realise their errors, and redrew their vectors accordingly. An example of student 4L applying the parallelogram rule to the electric field is presented in Figure 5.6.

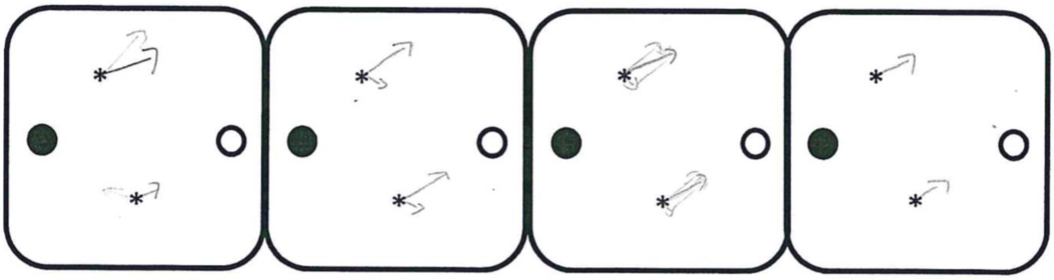


Figure 5.6. Student 4L applying the principle of superposition to represent the electric field.

This section narrated the tutorial guidance used to encourage the students to transfer their understanding of vector concepts to representing electric fields. The student's ability to consider direction and magnitude of vectors was discussed in the initial section of the tutorial, student's understanding of vector addition and how component addition affects the direction and magnitude is illustrated and the student's ability to combine vectors using vector constructions was shown.

5.4.3. Post-test: Student's use of vectors in electric fields

In an Electric field post-test question, the students were presented with a diagram in which they were given the electric field surrounding a charged particle of unknown sign, as shown in Figure 5.7. The students were asked to use the vector representation to identify the sign of the charge and explain why the length of the vectors decreases as the distance from the charge increases. A summary of the student responses is shown in Table 5.5.

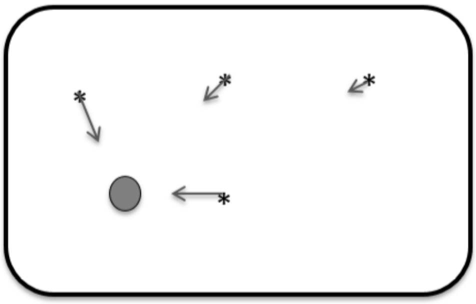


Figure 5.7. Diagram used in Electric field vector post-test question.

Response	Students
Charge is negative	**4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4L, 4M, 4N.**
Charge is positive	4A, 4F, 4K.
Associates direction with negative charge	**4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4L, 4M, 4N.**
Associates vectors with electrons.	4A.
Positive pulls in all arrows	4F.
Associates colour with charge	4K.

Table 5.5. Student responses to Electric field vector post-test question.

Upon completing the post-test, it was clear that most of the students could identify the sign of the charge based on the information in the diagram. Students 4A and 4F associated a negative charge to the arrows and treated them as electrons. This was a persistent difficulty for 4F, as discussed in section 5.4.5, in which they associated a gravitation force to field lines. This would suggest they consider the field to be a tangible construct (Galili, 1993), regardless of whether it is represented with field lines or vectors. Another student, 4K, associated a positive charge to diagrams in which the body is represented as a dark circle, as was done in the tutorial. In the tutorial, this was done to

help students differentiate the two charges with ease but was never intended to set a convention to indicate charge.

The students were then asked to construct the resultant vectors when particle with opposite charge was placed in close vicinity to the original charge, as shown in Figure 5.8.

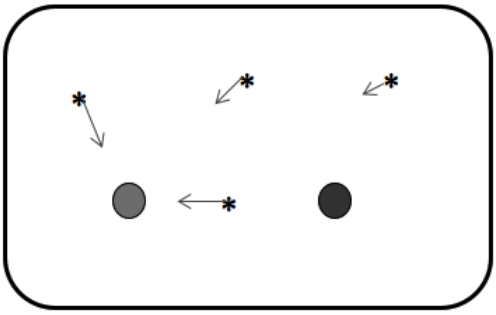


Figure 5.8. Post-test electric field question in which student sketch arrows to represent field components due to positive charge

This would allow me to determine if the students could represent the vectors using the correct direction, display reasonable relative magnitudes and use of the principle of superposition to determine the resultant field vectors. The results are presented in Table 5.6.

All students drew a second set of vectors representing the electric field due to the second charge. Twelve students drew these vectors with a length that varied reasonably with distance from the charge; ten of these correctly drew vector pointing away from the second charge, while two drew vectors pointing towards it, .

The answers of students 4J and 4M are shown in Figure 5.9. Student 4J drew vectors of equal magnitude perpendicular to the original vectors Student 4M drew field lines resembling a dipole field. This strategy could have worked, but the student used superposition to add a vector derived from the dipole field line pattern they drew (i.e., the correct answer) to the original vectors.

Seven students correctly used either the superposition principle, including two students (4A and 4F) who had difficulties in identifying and representing the direction for electric field for the different charges. Additionally, three students (4B, 4C and 4D) correctly applied the principle but for reasons unclear did not apply it to all the points. The remaining students did not apply the superposition principle at all.

Across the three questions in the post-test, it is observed that only two students, 4G and 4I, produced answers which showed they could completely transfer their understanding of drawing and adding vectors to the domain of electric fields. The other students showed errors in some aspect in representing the resultant electric field at the various points, in which case eight students only made one error (4B, 4C, 4D, 4E, 4F, 4H, 4K and 4N) in using vector representation for the electric field of the two charges. The remaining students made two or more errors in the representational transfer.

Responses	Students
Second vector arrow point away from positive charge.	**4B, 4C, 4D, 4E, 4G, 4H, 4I, 4K, 4L, 4N.**
Second vectors point towards positive charge.	4A, 4F.
Vectors are forced to be 90° to previous field vectors.	4J.
No pattern to the vectors	4M.
Reasonable variation of vector length with distance	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4K, 4L, 4N.**
Unreasonable variation of vector length with distance.	4J.
Field line representation used	4M.
Appropriate use of superposition	**4A, 4F, 4G, 4I, 4J, 4L, 4N**
Appropriate use of superposition for some / one of the positions.	4B, 4C, 4D
No superposition of vectors found.	4E, 4H, 4K, 4M.

Table 5.6. Student's application of vector concepts to electric field context.

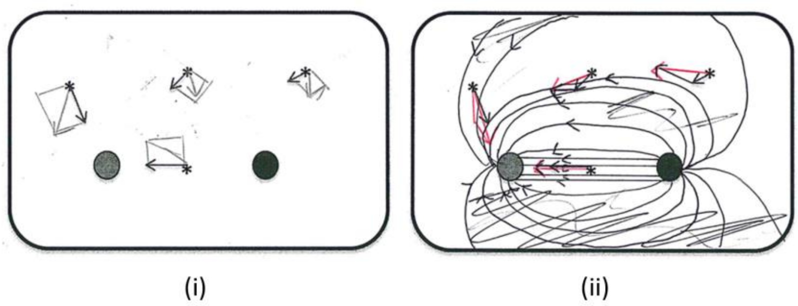


Figure 5.9. Errors in electric field vectors by (i) student 4J and (ii) student 4M.

5.4.4. Homework: Student's use of vectors in Coulomb's law

The students also completed a homework question, in which they could apply their understanding of vector components to a conceptual force question, presented in Figure 5.10. The students were asked to compare the net force acting on the -1 C charged body in (a) with that of (b), and then with that of (c). The question invited the students to use whatever reasoning they deemed appropriate and suggested vector reasoning, calculations or any other reasoning deemed fit by the students. While the vector nature of Coulomb's law was discussed in the class discussion before the tutorial, the tutorial itself did not directly look at the vector nature of electrostatic forces. This question tested if students could transfer their reasoning of vector component directly to the electrostatic context without explicitly exploring it in the tutorial. A summary of the student's responses is shown in Table 5.7.

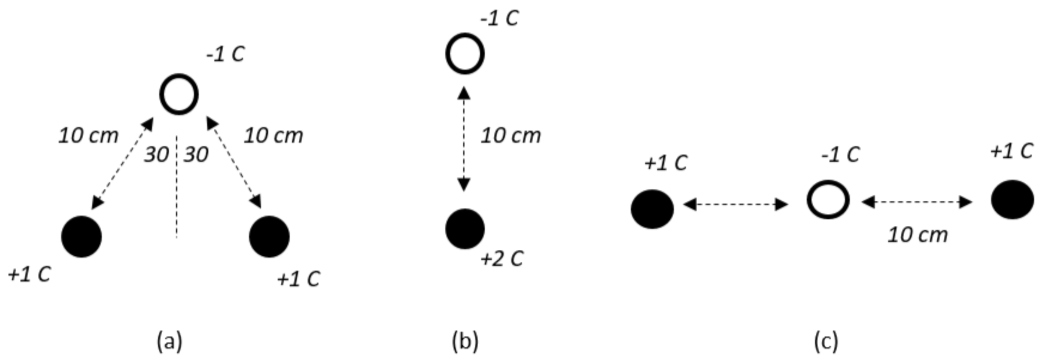


Figure 5.10. Coulomb's law vector concept question

Student Reasoning	Students.
Applied vector component reasoning	**N/a.**
Applied incomplete vector component reasoning	4K
Applied scalar reasoning.	4G, 4M
Described forces in terms of attraction and repulsion.	4C
N/A	4A, 4B, 4D, 4E, 4F, 4H, 4I, 4J, 4L, 4N

Table 5.7. Student's application of vector components to electric field context.

None of the students gave a complete answer: the horizontal components in (a) cancel, resulting in a net magnitude less than (b), and that the horizontal forces in (c) would sum to zero. Student 4K, when describing the forces in diagram (a), acknowledged that there is a combination of horizontal

and vertical vector components acting on the negatively charged particle, while only vertical and horizontal vectors act on negatively charged particle in (b) and (c). However, they stated that the net force is stronger in both cases (b) and (c), and did not consider that the horizontal net force is zero in (c).

Both students 4G and 4M used the equation $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$ in all the setups, and in setup (c) treated the two positively charged bodies as one charged body with a magnitude of +2 C. Student 4M also mentioned that the force would be slightly reduced due to the repulsion between both positive charges, but did not explain this in any detail. Student 4C interpreted the question to explain whether attraction or repulsion existed between the different combinations of charged particles. The remaining students, bar three who were absent and unable to complete the homework, did not make headway with the question. They stated that they were unaware how to approach the question. This indicated that the suggestion to use vector reasoning or calculations did not prompt them to use the understanding they had previously developed in the context of electric fields.

5.4.5. Interview: Student's use of vector components in Coulomb's law

There was no post-test question developed to elicit student's thinking about vector components in Coulomb's law, or electric fields. Instead three of the students were interviewed. They were asked to revisit the homework question discussed in section 5.4.4. The students were told they were permitted to ask questions during the interview to help them along, but they were not permitted to ask directly for the solution.

At the beginning of the interview, students 4A, 4B and 4H stated that the forces would be equal in all cases as the -1 C charged particle was being attracted by a net charge of +2 C, in all cases. However, upon being informed that their reasoning was incorrect, and the net force on the particles was not equal in all cases, they considered the use of vectors to analyse the question. The following interview extract illustrates the student's reasoning.

Student 4H:	The distance is there [a] cause that one will be pulled down the centre line. That is just as strong as charge [b], but it is is the most [strongest force], cause it is direct. And that one will cancel out [c], so it'll be zero. That one [b] will be twice as much if that was one [c].
Teacher:	So, C = zero. Why did you say that?
Student 4B:	Cause it cancels out.
Teacher:	What cancels out?
Student 4H:	The horizontals.

Teacher:	Ok… so now we have horizontal vectors. What type of vectors do we have acting here [b]?
Student 4A:	Vertical vectors.
Teacher:	And everything is vertical? [Students nod in agreement] Ok, so let's just say here [one horizontal vector is sketched on c] is 10 N, and this [vector sketched acting in opposite direction] is 10 N, now what's the force acting on this [b]?
Student 4H:	20 N.
Teacher:	Ok, so look here [a] and ignore this [right positive charge]. What force acts on the negative charge?
Student 4B:	10 N.
Teacher:	Now ignore this [left positive charge] What's the force?
Student 4B:	10 N.
Student 4H:	And we can add them tip to tail now.
Teacher:	We can, but also, consider, you mentioned horizontal and vertical components earlier. Keep the idea of components in your head. Do you think the 10 N and 10 N will sum to 20 N?
Student 4B:	The horizontal components in that one [a] will cancel out.
Teacher:	So, you're only left with what?
Student 4H:	Just vertical components.

When the students had to consider alternative reasoning they resorted to vector reasoning without much prompting. The extract shows the teacher did not volunteer any reasoning the students did not mention themselves but guided them to use the reasoning in the three different layouts. The student's reasoning was based on how the component vectors combined. Based on this, they produced an accurate ranking of B > A > C = 0, to represent the net force on the negative charge in each layout.

5.4.6. Discussion

This section presents a discussion of the student's use of vectors in Coulomb's law and the electric field. It presents a comparison of the student's ability to represent the variation in field strength using vectors, and their ability to apply the principle of superposition of electric fields using vectors. Figure 5.11 presents a comparison of the student's responses in the pre-test and post-test, looking at student's representations of the electric field strength at various points in an electric field.

In the pre-test, it was observed that two students accurately used vectors to represent the magnitude of the electric field at points at various distances around a charged particle. As the students

showed good understanding of vector magnitude, as discussed in section 4.2.5, the following reasons could explain the student's difficulties:

- The students consider that a form of proportional relationship exists between electric field strength and the distance from the charged particle.
- They do not consider electric field strength to vary with distance at all.
- They do not consider electric field strength to be a vector quantity.
- The students were unable to differentiate between electric field and other vector quantities, such as displacement.

These reasons are suggested as they are based on the interpretations of conversations with the students during the tutorial lessons and interpretations of discussions overheard between the students combined with interpretations of the student artefacts which were scanned.

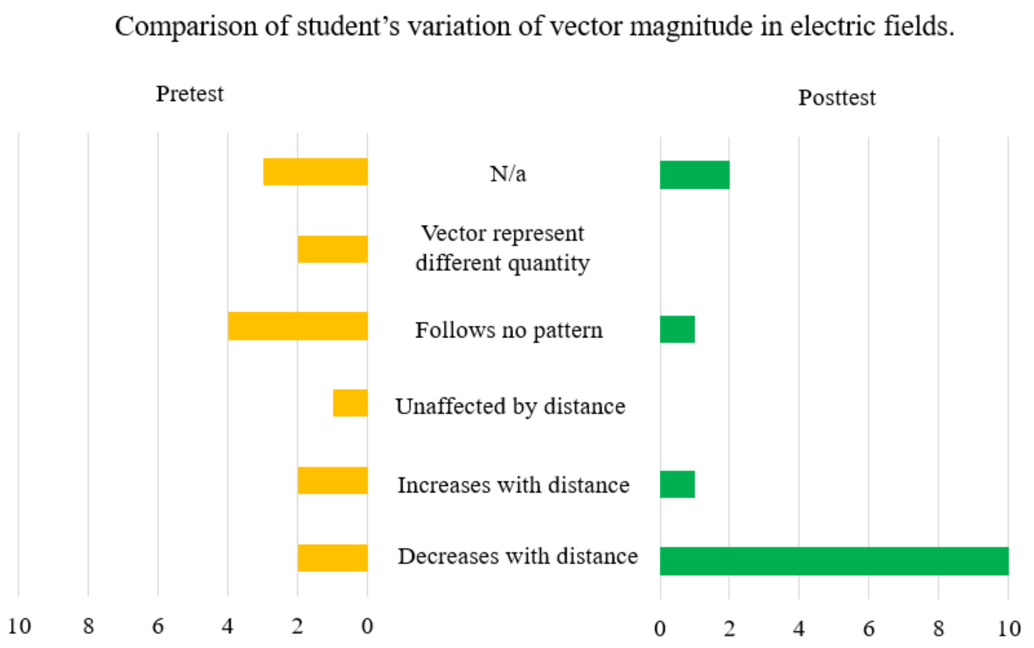


Figure 5.11. Comparison of student's representations of vector magnitude for an electric field.

The post-test showed considerable gains by the students. Most of the students (10/14) correctly represented the change in field strength with distance using vectors of different lengths. The shift in student's abilities to correctly represent the field lines indicates that conceptual exchange occurred (Hewson, 1992), with the comparison of the pre-test and post-test indicating that moderate conceptual change occurred. Ten of the students applied the magnitude convention to their vectors in the post-test, and no longer showed the difficulties seen in the pre-test. Through completing the tutorial lessons, the students applied the reasoning they developed in section 4.2 to a new context, utilising the representation as a useful tool to represent and verbally explain a vector field pattern in an unseen context (Posner, *et al.*, 1982). Not all the difficulties were overcome however, as two of

the student's presented difficulties in which they used field lines and then drew vectors to match the field line pattern, or used vectors previously presented on the diagram as a scale to determine the magnitude of the vectors.

Figure 5.12 presents a comparison of the pre-test and post-test results for the student's use of the superposition principle. In the pre-test, none of the student's applied it to electric fields. This suggests difficulty in transfer to this context, as all the students demonstrated the ability to construct resultant vectors in section 4.3.4. The difficulties reflect those seen in literature when learners struggle to apply vector concepts to the electric field context (Maloney, *et al.*, 2001).

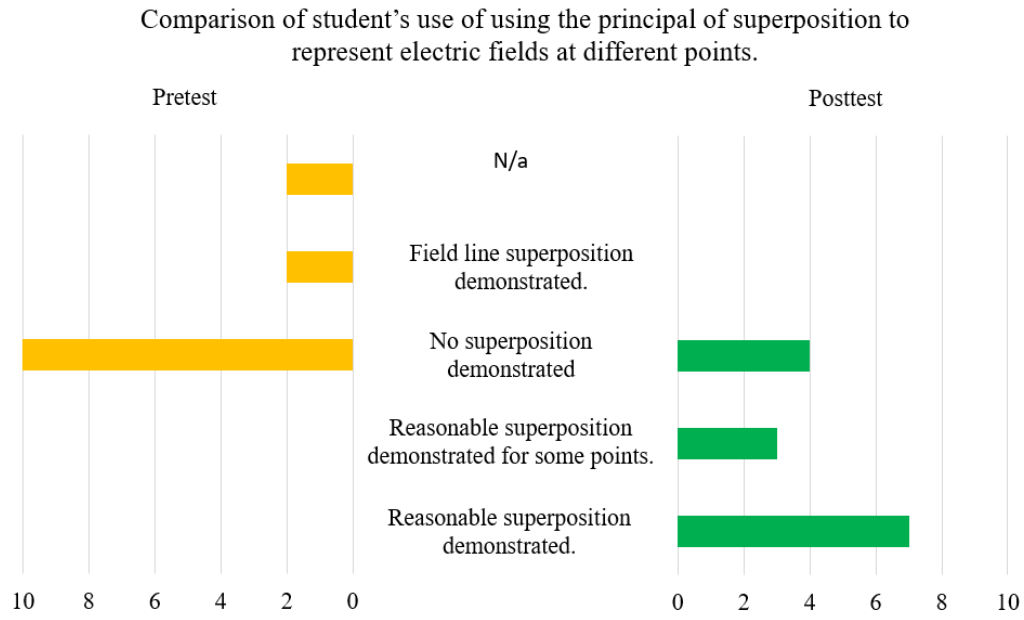


Figure 5.12. Comparison of student's use of superposition to draw an electric field using vectors.

After the tutorial lesson, 10/14 students applied the principle of superposition to the electric field in the post-test. This indicates that students did not initially transfer the skills they developed, as discussed in section 4.2, to the context of electric fields. The students demonstrated this skill in the vectors tutorial but did not transfer this skill to the electric field context until after they had completed the electric field tutorial. This suggests that conceptual extension occurred from completing the tutorial lesson (Hewson, 1992), with a moderate conceptual change observed from comparing the pre-test and post-test results. As discussed in section 5.4.2, the tutorial gave the students the opportunity to practice the vector constructions they previously developed in the context of electric fields, furthering their overall conceptual understanding by applying it to new contexts (Konicek-Moran and Keeley 2015).

Marzec (2012) suggested that learners require multiple opportunities to develop understanding of the inverse square law. The pre-test/post-test comparison and discussion of the tutorial lesson suggest that this could also apply to using vectors in multiple contexts. The students did not

experience any significant difficulties in applying the vector constructions in the tutorial lesson, which indicates that the central difficulty in the pre-test was students not realising the application of their vector understanding. This was also observed in the Coulomb's law homework, in which the students were unable to apply the reasoning they developed in the vectors tutorial question described in section 5.2.4. Apart from one student, the students did not recognise that vector components were applicable to the question given. However, during the electric field tutorial, the students had little difficulty in applying vector component reasoning when drawing the electric field at various points around two charged bodies. The student interview, in section 5.2.5, also indicates that with minimal guidance, the students applied vector reasoning to the homework question. Initially the students based their equal ranking on the net positive charge in each case, instead of the positioning on the charges. This indicates that students value magnitude as the most important aspect of force. This, in turn, suggests that for the transfer of vector concepts to electric fields to be complete, students require the opportunity to develop their reasoning in this context in a classroom setting, such as a tutorial and/or a class discussion, as was the case in this research.

5.5. The inverse square law applied to electric fields and Coulomb's law

The following sections discuss student's understanding of the inverse square law and how they applied it to Coulomb's law and electric fields. Section 5.5.1 discusses results based on student's answering a mathematical problem in a pre-test question, and a problem involving the use of vector representation to show their understanding of the inverse square law. Section 5.5.2 discusses the Coulomb's law tutorial lesson, which focused on the student's understanding of proportionality and the use of tabular data, graphs and mathematical methods to explore the inverse square law. Section 5.5.3 discusses a homework assignment applied the scale model, adapted from Conceptual Physics (Hewitt, 2009) to the electric field. A difficulty in student's understanding of the scaling of area is identified, and a section of a teaching and learning interview is presented to show how students overcame the difficulty. Section 5.5.4 presents the results of a post-test that looks at the student's ability to represent the inverse square law on a graph, how students differentiate between an inverse and inverse square law graphically, student's use of the inverse square law in a mathematical question, and their understanding in change of an area based on a scale model.

5.5.1. Pretest: Coulomb's law and inverse square law

This section discusses the results from the Coulomb's law pre-test, in which the students had to use vector arrows to demonstrate their understanding of a directly proportional relationship, and a

relationship that follows the inverse square law. The latter part of this section presents students graphical representations of a directly proportional relationship, and the inverse square law.

In the first question, the students were asked to state the relationship, between the magnitude of the force, and the distance between them, when presented with the Coulomb's law equation. This allows for gauging the student's ability to recognise relationships in algebraic form and transfer them between tabular, graphical, diagrammatic and mathematical symbolic representations. The students were familiar with the general structure of the Coulomb's law equation, as they had studied Newton's gravitational law and they were formally introduced to Coulomb's law in a presentation and class discussion preceding the tutorial lesson. The equation was presented to the students in the form $F = k\frac{q_1 q_2}{d^2}$. This form presents Coulomb's law as a scalar equation, as the Leaving Certificate Physics course does not employ the use of vector algebra. The students were made aware that this equation can only be used to determine the magnitude of the force between two charges, and when the students are required to determine the directions of the forces acting on the charges, other appropriate methods are employed. A summary of the student's results are presented in Table 5.8.

Inverse square relationship	**4H, 4K, 4M**
Inverse relationship	4G, 4E
Increase distance, decrease the force.	4A, 4C, 4I, 4J
Directly proportional relationship	4B, 4D
N/a	4F, 4L, 4N

Table 5.8. Student pre-test responses to transferring from equation to verbal relationship.

The results show that three of the students could glean the formal relationship between both quantities from the law equation. Two students stated that the force was inversely proportional to the distance, suggesting the students did not observe the index of distance variable, or did not consider its relevance in defining or naming the relationship. A further four students could relate the position of the distance variable as a denominator to determine that increasing the distance from the charges reduces the magnitude of the force between them. Two students were unable to relate the positions of the variables in the equation and determined the variables were directly proportional to each other, while three students did not give an answer.

The students were then presented with a mathematical question, in which they had to apply the inverse square law. The students were not provided with a value for *k* in the Coulomb's law equation, so they would have to employ proportional reasoning to answer the question. The question is provided in Figure 5.13. The student's results are presented in Table 5.9.

Two +8 C charges are held a distance of 10 cm from each other, and the force acting on both charges is 90 N. The charges are moved so the distance between then is now 30 cm. What is the new force acting between the charges? Explain how you know what the change in force is.

Figure 5.13 Pre-test question seeking to elicit student's ability to mathematically apply inverse square law

As the distance between the charges was increased by a factor of 3, the magnitude of the force was reduced by a factor of 9. Only one student, 4C, completed this. It is interesting to note that 4C responded to the previous question by stating the force would decrease without quantifying this decrease, but provided the correct outcome in this question. Three students (4D, 4G and 4K) reduced the force by a factor of 3. Two of these responses were surprising considering student 4D previously stated there was directly proportional relationship and student 4K previously stated there was an inverse square relationship between the variables. This indicates the students did not understand the nature of the relationships or did not apply them in this context. Four students attempted to use the formula to attempt the question, but were unable to complete their calculations since they did not know a value for *k* and could not determine how to tackle the question. This difficulty echoes that shown by Arons (1999). Two students (4E and 4M) respectively stated that the force would increase and decrease, but they did not quantify their answers. This answer was not consistent with 4E's previous question, in which they stated there was an inverse relationship, and 4M did not apply the inverse square relationship they stated in the previous question. The remaining students did not answer this question.

Responses	Students
Reduces force to one ninth original	**4C**
Reduces force to one third original	4D, 4G, 4K
Attempts calculation	4B, 4F, 4L, 4N
Increases force	4E
Decreases force	4M
N/a	4A, 4H, 4I, 4J

Table 5.9. Student's pre-test responses to applying the inverse square law mathematically.

In the final question of this pre-test, the students were asked to use vector arrows to show the change in magnitude of the force with distance. The students were presented with two charges, with vector arrows showing an attractive force between the two charges. They were asked to determine

the effect of doubling the distance between the charges and representing this using vector arrows. The question is presented in Figure 5.14, and the student's results are presented in Table 5.10.

As can be seen in Table 5.10, none of the students correctly represented the vectors by reducing the magnitude by a factor of four. The two more prevalent vector representations showed that the force would reduce by a factor of two or would not be affected at all. This highlights inconsistencies from the four students that previously defined or applied the inverse square law, and the students who applied an inverse relationship, or explained the increasing the distance would decrease the force felt by the charges.

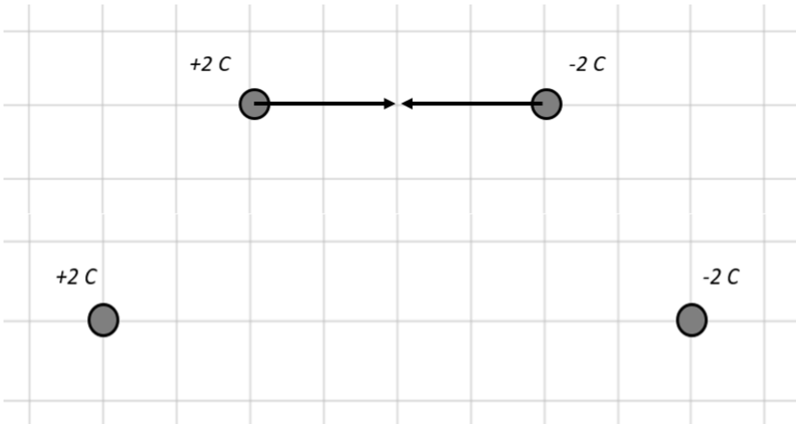


Figure 5.14. Pre-test question utilising the inverse square law and vector representations

Responses	Students
Quarters both vectors.	N/a.
Doubles both vectors	4B
Halves both vectors	4A, 4C, 4D, 4M, 4N,
Increases vectors	4J
No change to vectors.	4E, 4F, 4G, 4H, 4I, 4K, 4L

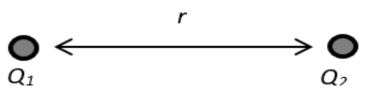
Table 5.10. Student responses to pre-test question that looked at student's application of the inverse square law and vector representations.

These pre-test results indicate that the students encountered difficulties in transferring their understanding of the inverse square law to Coulomb's law. Only a small number of students recognised the inverse square law in the Coulomb's law formula, and only a single student applied the law mathematically. None of the students presented the inverse square law using vector representation, and most of the students displayed reasoning consistent with an inversely proportional relationship.

5.5.2. Tutorial lesson: Coulomb's law and inverse square law

During the Coulomb's law tutorial lesson, the students were introduced to a formal definition of Coulomb's law, in which the magnitude of the force between two point-charges is directly proportional to the product of the magnitude of the charges, and inversely proportional to the square of the distance between them. The lesson aimed to allow students to verify these relationships themselves, using tabulated data, graphs and calculations based on the formula. In the first half of the tutorial, the students were guided through this process for a directly proportional relationship, and in the latter half, they had to apply the same skills to show an inverse square proportional relationship.

The students were presented with tabulated data for the force between two charges, and 3 columns with values for the magnitude of the first charge only, the magnitude second charge only and the values for the product of magnitudes of the two charges, as shown in Figure 5.15.



q_1 (C)	q_2 (C)	$q_1 q_2$ (C^2)	F (N)
1	1	1	2
1	2	2	4
2	1.5	3	6
4	1	4	8
1	5	5	10
2	3	6	12

Figure 5.15. Data set from Coulomb's law tutorial, relating force to product of charges.

The students were to identify which column, when coupled with the force values, provided a familiar pattern, being linear, quadratic, exponential, inverse or inverse square. The students then rewrote these two columns in a separate table and were guided to show that the ratio of force to the product of the charges was constant, by dividing the values for F by the values for q_1q_2. This would allow the students to demonstrate that the pattern followed the general form $y = mx$. The students were then asked to graph their data and explain how the shape of the graph showed a directly proportional relationship. They were required to use their graph to determine the magnitude of the force when the product of the charges was 2 C^2 and 6 C^2. From this, they explained that tripling the product of the charges has the effect of tripling the magnitude of the force.

The students were then required to complete calculations to show the effect of tripling the produce of the charges. They were provided with a sample calculation between a 6 μC charged

particle and a 3 μC charged particle that were placed a distance 1 cm apart, as shown in Figure 5.16. The mathematical operations were completed in the sample and they were required to identify which operations took place. This ensured they were familiar with the nuances of completing the calculations and aimed for students to avoid errors in performing the calculations. They then completed similar calculations, between two charges of magnitudes 3 μC and 9 μC.

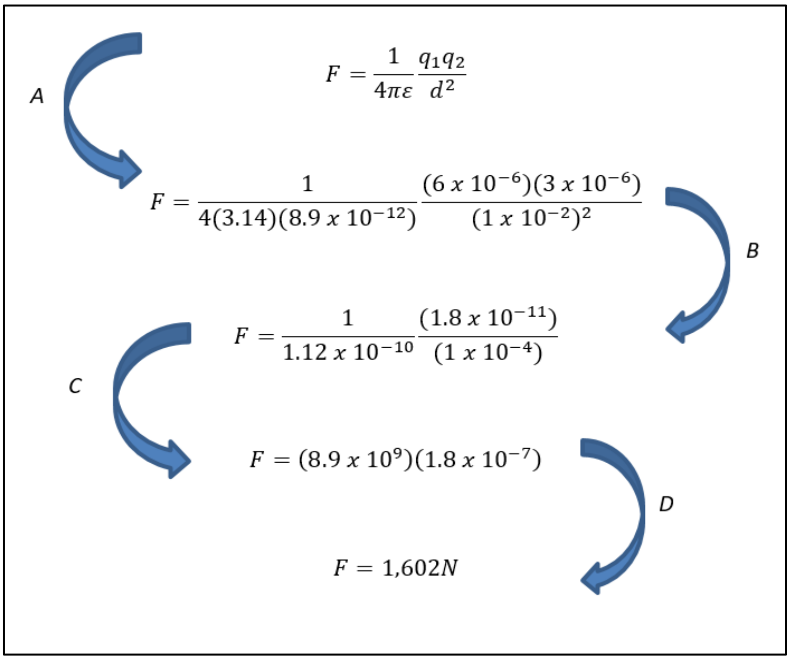


Figure 5.16. Coulomb's law tutorial extract, in which students identify the operations in the calculation.

In the second section of the tutorial lesson, the students were asked to develop the reasoning to show the relationship between the force between two charges and the distance between them was an inverse square relationship. Students were given the table of data shown in Figure 5.17, and they were required to either use the data directly to show it followed an inverse square relationship or place it on a graph and use the graph to do so. During the inverse square law tutorial, as discussed in section 4.3.2, the students can pick x-values from the domain of the graph and read the corresponding y-values for these, they can calculate what factor the values change by and determine if an inverse square law was observed. For instance, choosing the values 1 and 3 from the x-axis would produce a decrease by a factor of 9 in the two y-axis values, if it followed an inverse square law.

To complete the tabular section of the tutorial lesson, the students were presented with the general forms of linear equations, quadratic equations, inverse equations and inverse square equations, in which they were shown what operations between the variables produced a constant. The general forms of $y = \frac{k}{x}$ and $y = \frac{k}{x^2}$ were less familiar to the students, as they generally picked either the quadratic or the inverse relationship initially to use to manipulate the data. However, it was clear that the ratios of force to the square of the distance or the products of force and distance values

did not produce a constant. When the students picked the inverse square proportional equation, the students determined that the product of the square of distance and the force produced a constant. Overall, this section proved to be the most challenging for the students, and took all groups approximately twenty minutes to complete, as they picked the different relationships and explored them.

The students were also required to show the inverse square law on a graph. It was observed that the students plotted the data on the graph, which produced the characteristic curve typical of an inverse square law. However, the students were satisfied that this graph demonstrated an inverse square law, based on its shape, even though it was similar in shape to an inverse graph of the form $y = \frac{k}{x}$. As the students had data points to follow, they did not repeat the errors of drawing quadratic patterns, or linearly decreasing patterns, discussed in section 4.3.4. All groups had to be prompted to pick two points on the graph and show, in this case limited by the domain and range of the data, that double the distance would result in one quarter the force.

Directly proportional general equation: $y = mx, \quad \frac{y}{x} = m, \quad \frac{y}{x} = constant$

Directly proportional to square equation: $y = ax^2, \quad \frac{y}{x^2} = a, \quad \frac{y}{x^2} = constant$

Inverse proportional general equation: $y = \frac{k}{x}, \quad xy = k, \quad xy = constant$

Inverse square proportional equation: $y = \frac{k}{x^2}, \quad x^2y = k, \quad x^2y = constant$

y	F (N)	100	25	11.11	6.25	4	2.78
x	d (m)	1	2	3	4	5	6

Figure 5.17. Coulomb's law tutorial extract, using data to demonstrate the inverse square law.

In the last section of the tutorial, the students were presented with the Coulomb's law equation, with a set of data, and asked to determine the relationship, as shown in Figure 5.18. While this section took time for the students to complete due to the many steps involved in the calculations, no groups found it overly challenging. They did need to be prompted use the values of the two forces to make a ratio at the end of the calculations, in which they could determine the factor decrease observed when the distance between the two charges was doubled. This method is similar in nature to that described on the previous page, involving the use of graphs.

At the end, the students were asked which method they found to be the most effective and simplest to use. Six of the students mentioned that they preferred to use the equation method, as they stated there was a structured approach to follow, in which the ratio was easy to determine. Four students preferred the graphical method, in which the students read the values for the forces directly from the graph and developed the inverse square relationship from their values. They found the method easier than the others, and the values can be easily obtained. Teacher observations from this lesson stated that initially, the students only used the shape of the graph as the reference to the inverse square relationship, and the values were initially ignored. This observation is considered in the post-test question utilising a graph, see section 5.5.4.

This section illustrated that students encounter difficulties in analysing tabular data and graphical representations, when seeking to determine the mathematical relationship displayed. Difficulties seen in using tabular data was determine the steps required to manipulate the data to show that $x^2y = k$, even though they were guided through the process for a linear pattern. This indicates difficulty in both their mathematical ability and the physics context in which the pattern is applied. While it may have been preferable to do this mathematical work during the tutorials completed in chapter 4, time constraints did not allow for this and instead, it was integrated into the electric field tutorial lesson. The student's unfamiliarity with the approach presented in the tutorial is likely to contribute to the difficulty observed, as the method used for analysing data in a table, for graphical analysis in their math course, is not applied to this function (LC Project Maths syllabus, NCCA, 2013). The prevalent difficulty involved in the student's use of analysing the graph was an over-emphasis on the shape of the graph itself. Students tended to ignore the values from the graph and did not analyse the values to show an inverse square pattern, suggesting they would be unable to differentiate between an inverse graph, and an inverse square graph.

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

$\varepsilon = 8.85 \times 10^{-12}\ F\ m^{-1}$, $q_1 = 6 \times 10^{-6}\ C$, $q_2 = 4 \times 10^{-6}\ C.$

$d_1 = 4\ cm$, $d_2 = 8\ cm$

Figure 5.18. Coulomb's law tutorial extract, to demonstrate inverse square relationship mathematically.

5.5.3. Homework: Electric field and inverse square law

This section presents an analysis of the electric field homework tutorial. This was developed to apply the scale model adapted from Conceptual Physics (Hewitt, 2009) and the use of field lines in lieu of spray paint droplets, as discussed in the previous chapter, in section 4.3.2.

The students were presented with the diagrams shown in Figure 5.19. They were told that 100 field lines diverging from a charged particle placed at the left of the diagram pass through the first frame, as shown in Figure 5.19 (i). The model was designed to give students an appreciation of concept of an electric field integrated over the surface area of a sphere, without formally introducing electric flux or Gauss' Law for closed surface contexts. The students were required to explain why the area increased in the pattern shown in the diagram, and to explain what effect this had on the lines passing through each frame as they moved from left to right, as shown in Figure 5.19 (ii).

Five students qualitatively described the increase in area as the frames moved from left to right. This reasoning revolved around the distance from the charge being greater results in a bigger area, without articulating that field lines diverge as the distance increases. One student described the field lines diverging further apart as the distance increase as a reason for the increased area. Three students explained that increasing the distance from the charge increases both the length and width of the frame, increasing the area as shown in the diagram.

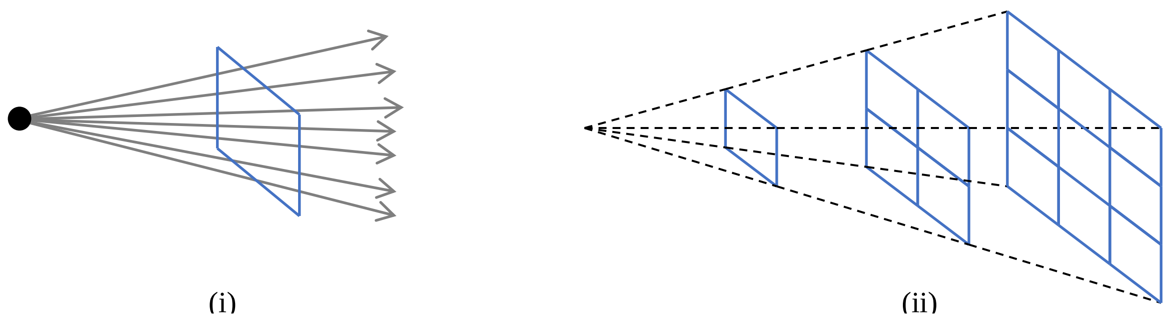


Figure 5.19. Electric field line homework extract, applying the scale model to electric field and field lines.

The remaining students used mathematical calculations, in which they squared the values of two to get four. It is unknown if this represented squaring the length of the square frames, squaring the factor in which the distance from the charge to the frame, or both. The students consistently repeated applied their reasoning to the 9 frames, and all students determined that the number of field lines passing through each frame would be 25 lines and 11.1 lines respectively. When asked to explain the behaviour of the field lines passing through the frames, eight of the students qualitatively explained that the field lines were spreading out more the distance increased, resulting in a lower number of

field lines passing through the individual frames. Only four of these students added that this models an inverse square law. The remaining students did not answer this question.

Explaining the change in area proved to be difficult for some of the students. During an interview with students 4C and 4E they resorted to relating the change in area to the distance from the charge, whilst not considering the change in the lengths and widths of the frames.

Student 4C: If that distance is 1, [distance to first frame] 1^2 is 1, which is 1 frame. Then this distance is 2 [distance to second frame], so 2^2 is 4, which is four frames.

It was clear that the students could use the model presented to describe the inverse square law but did not consider the source of the change in the area, in terms of scaling up the dimensions of the frames. The teacher engaged the students in a discussion to focus on the length and width of the frames, and to link these back to the increase in distance, to show the overall increase in the area.

Student 4E: The length of the second frame is 2, and the width is also 2.

Student 4C: The distance increases by 2, and so does the length and width.

Student 4E: And therefore, the area by 4.

This suggests that students need to be guided to focus how the change in distance affects the length of the frame and width of the frame separately. They can use the increases in these dimensions to explain the overall increase in the area of the frames.

This section has illustrated the student's application of scale model adapted from Hewitt (2009) to electric field lines. The context used displayed field lines passing through various frames and the students analysed this context to explain the relationship between the distance from a point to a charged body and the magnitude of the electric field strength at the point. Using the scaling model, they demonstrated the electric field follows an inverse square law mathematically. However, some students still struggled to articulate why the area increased in a quadratic pattern, and their final explanations did not reference the inverse square but did reference the divergence of field lines through different frames, showing a conceptual understanding of the behaviour of the model. The interview with two students illustrated the need to be guided to focus how the change in distance effect the length of the frame and width of the frame separately. They can use the increases in these dimensions to explain the overall increase of the area of the frames.

5.5.4. Post-test: Coulomb's law, electric fields and the inverse square law

The first question presented in this section was completed by the students not immediately after completing the Coulomb's law tutorial lesson (which focused on the inverse square law) but as part of the electric field pre-test. As this was given after the Coulomb's law tutorial lesson, the question presented can pragmatically be considered a post-test question.

In the question, the students were presented with the formula that relates the magnitude of the electric field strength at a point to the magnitude of the charge, and the distance of the point to the charged particle; $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2}$. The students were asked to explain the relationship between the electric field strength and distance based on the equation, and to graph the pattern that represents the mathematical relationship. The question is presented in Figure 5.20, and the results are summarised in Table 5.11 and Table 5.12.

The results indicate that 10 of the students associate the asymptotic pattern based on the relative position of the electric field strength and the distance variables in the equation. However, only two students, 4E and 4G, could determine the relationship from the equation. Five students stated an inverse relationship existed between the variables, which would also account for general shape of the graph. Student 4D was consistent in their explanation that doubling the distance would result in one quarter the electric field strength. Student 4F was consistent in graphing a directly proportional relationship, indicating they were unable to determine the relationship from the equation. Both 4J and 4N stated that increasing the distance would increase the field strength, but drew graphs contrary to this reasoning, also indicating they were unable to determine relationships based on the equations. The results suggest that students require more attention in overcoming difficulties in recognising and articulating relationships presented in equations, but progress was made in student's ability to transfer relationships from one representation to another.

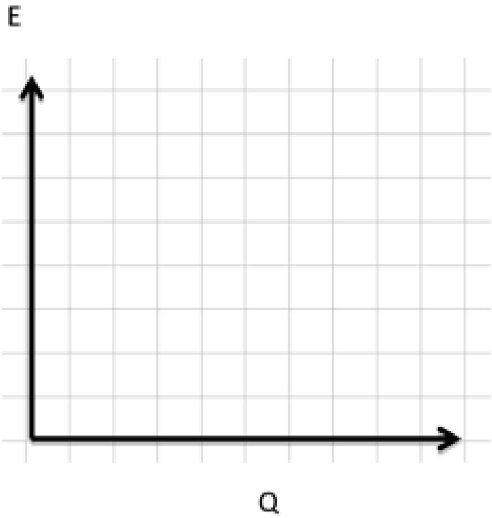
4.. The electric field strength around a charge is given by the formula $E = \frac{1}{4\pi\varepsilon}\frac{Q}{d^2}$.

What is the relationship between the electric field strength and (i) the magnitude of the charge causing it, and (ii) the distance from the charge.

(i)

(ii)

Draw these relationships on the graphs below.



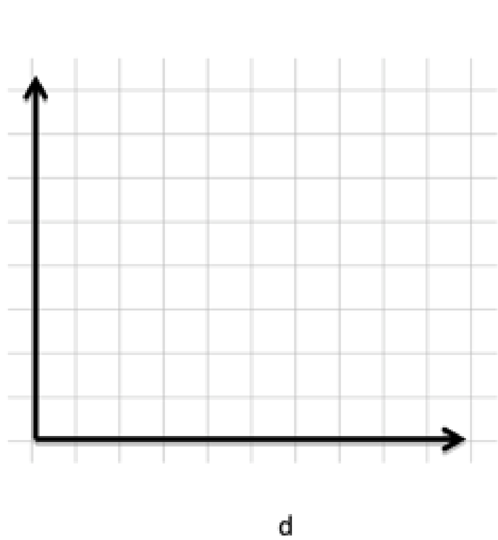


Figure 5.20. Electric field question in which students transfer inverse square law from symbolic to word and graphical representations.

Responses	Students.
Inverse square proportionality.	**4E, 4G,**
Inverse proportionality	4A, 4B, 4C, 4H, 4M
Illustrates inverse square relationship with examples.	4D
Directly proportional	4F, 4N.
Qualitative answer only.	4J, 4L, 4K
N/A	4I

Table 5.11. Student's responses from electric field pre-test, determining student's ability to transfer from equation to verbal relationship.

Responses	Students.
Asymptotic decreasing graph.	**4A, 4B, 4C, 4D, 4E, 4G, 4H, 4J, 4M, 4N.**
Directly proportional relationship.	4F.
Increasing quadratic curve.	4L
N/a.	4I, 4K.

Table 5.12. Student's responses from electric field pre-test, determining student's ability to transfer from equation to graphical representation.

The previous questions indicated that the students related an asymptotic pattern to an inverse and inverse square relationship. However, it did not determine if the students could recognise the difference between the patterns on the graph, or if they had just memorised the general shape of the graph. The first question presented in the Coulomb's law and Electric field post-test tested for this. The students were asked to pick out and justify the correct shape of both a directly proportional relationship and inverse square relationship in the graph shown in Figure 5.21. The students were also asked to determine which patterns showed the relationship between the force between two charges and (i) the product of their magnitude and (ii) the distance between the charges, to see if they could transfer the formal definition to a graphical representation. A summary of the student's responses is presented in Table 5.13.

The following graph shows 3 patterns for different types of functions, *A*, *B* and *C*.

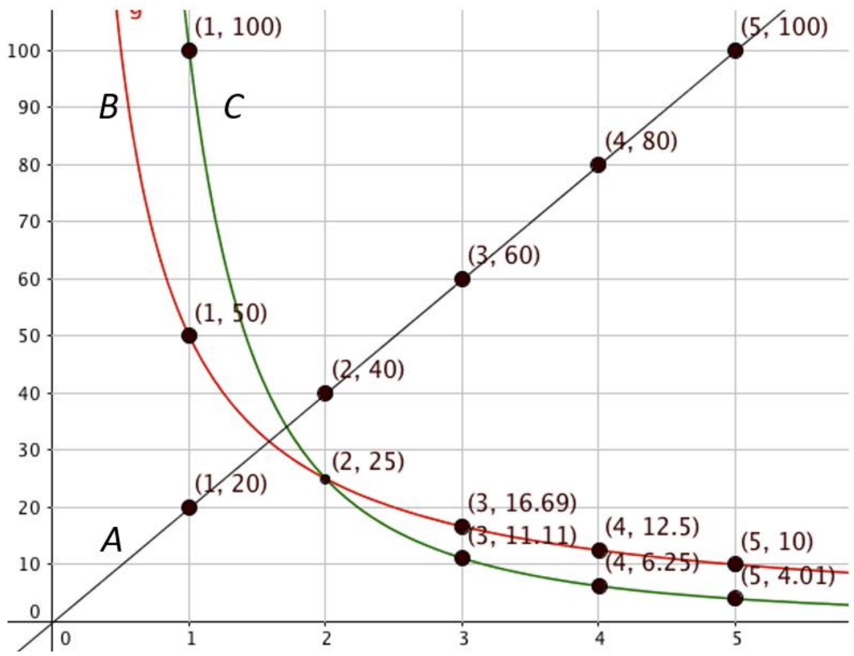


Figure 5.21. Graphical representations of a directly proportional, an inverse and an inverse square relationship.

Responses	Students.
Picks A as directly proportional.	**4A, 4B, 4C, 4E, 4F, 4G, 4H, 4I, 4J, 4L, 4M.**
Picks B as directly proportional.	4N
Picks C as directly proportional.	4D
N/a	4K
Picks C as inverse square.	**4B, 4E, 4G, 4H, 4J, 4I, 4L, 4M**
Picks B as inverse square.	4D, 4N
Picks B and C as inverse square.	4C, 4F
N/a	4A, 4K

Table 5.13. Student's post-test responses in determining relationships based on graphical data.

For both relationships, the results show that eleven and eight students respectively determined the correct graphical pattern for the directly proportional and inverse square relationships in Coulomb's law. The students primarily referenced the shape of the graph, being linear with a y-intercept of zero, to explain the direct proportion relationship. One student, 4E, also employed the use of a table to demonstrate the relationship, although they had apparent issues articulating the final reasoning.

Student 4A: They form a straight line… product (of charges) are proportional.

Student 4E: *y* = *mx*, is a straight linear function. *y* is directly proportional to *x*, it has to get bigger.

Student 4K: A. Because *y* = *mx* is directly proportional, which tells us it is a straight line.

However, only 8 students correctly determined which of the graphs represented the inverse square relationship Typically students substituted values from the graphs into the two equations and saw which set of results fit the equation. Some students determined a value for "k" in the manner they were guided to use in the Coulomb law tutorial lesson (section 5.5.2). Mistakes commonly made by the students were that they looked at the pattern and recognised an inverse relationship, but could not differentiate between the inverse and inverse square pattern shown on the graph, in which they picked either B or both B and C.

Student 4B: (1, 50)
$50 = 100\left(\frac{1}{1^2}\right)$
Not curve B.
(2, 25)
$25 = 100\left(\frac{1}{2^2}\right)$
Curve C is of the form $y = k\,\frac{1}{x^2}$. As the distance increases, the force decreases

Student 4F: C is $y = k\frac{1}{x^2}$, as it does not decrease at a constant rate.

B. It is inversely proportional to the distance squared.

Student 4G: Coulomb's law states that the force between 2 point-charges is directly proportional to the product of the magnitude of their charges and inversely proportional to the distance squared. If x = 1, and y = 100, when the distance is doubled, it is 2 (x) and $100\left(\frac{1}{2^2}\right) = 25$ (y), and that point is on C.

Students 4N and 4D consistently picked B as the graph which represented both $y = k\frac{1}{x^2}$, and the magnitude of the force between two point charges as a function of the distance between them. Student 4C picked both B and C to represent the mathematical equation and the relationship in Coulomb's law, suggesting that they focus on the shape, but not the values produced by the pattern. Student 4F correctly determined that C represented $y = k\frac{1}{x^2}$, but chose pattern B as the representation of the inverse square relationship in Coulomb's law. As they quoted the inverse square law in their response, this would also indicate they focused on the shape over the values produced by the pattern. Students 4A and 4K did not complete this question, which suggests the tutorial was ineffective for these students in aiding their development of transferring the inverse square law to this context.

The results of the last two questions would indicate that 8 of the students were able to graphically represent the inverse square law on a graph and differentiate it from a pattern that represents an inverse relationship. Some students had persistent difficulties in which they ignored the data and based their responses on the recollection of the shape of the graph, focusing on the mathematical implication that as one variable increased, the other decreased. Additionally, while the students were able to apply their understanding of the inverse square law in this graphical form, they struggled to transfer the relationship from an equation into written form.

A later question on the post-test was designed to determine the student's ability to apply the inverse square law in an electric field context. The students were presented with a positive charge held in a fixed position, with 3 points around the charge, as shown in Figure 5.22. The students were required to rank the electric field strength around the electric field, and determine the ratio of the electric field between points A and C. The student's responses are summarised in Table 5.14.

The results from this question suggest that the students are clearly aware that as distance from the charge increases, the electric field strength decreases. Nine of the students showed that the electric field strength at *c* was one quarter the strength at *a*, whilst another two students (4E and 4J) produced a ratio that had a value of 1:4, but not in its simplest form.

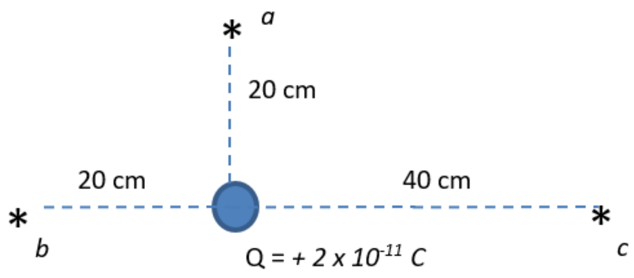


Figure 5.22. Post-test electric field question, testing understanding of inverse square law.

Student's reasoning was predominantly based on calculations (seven students) using $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2}$ or developing reasoning that doubling the distance would produce an electric field strength that was reduced by a factor of four (six students). All students who used the latter reasoning produced the correct ratio. In some cases, they substituted values for the charge and distance variables but ignored the $\frac{1}{4\pi\varepsilon}$ term in the equation. Two of the students made calculator errors, but they did not attempt to revise their work to develop alternative values for the magnitude of the charges. Two of the students simply halved the magnitude of the field strength when comparing *a* to *c*, as the distance was doubled.

Responses	Students
c < a = b	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M.**
Ratio of c to a is 1/4	**4B, 4C, 4D, 4G, 4H, 4I, 4K, 4L, 4M.**
Ratio of c to a is 1/4, not in simplest form	4E, 4J
Incomplete due to error in using calculations to determine ratio.	4F.
Ratio of c to a is 1/2	4A, 4N

Table 5.14. Student's post-test responses applying the inverse square law mathematically.

From looking at the examples of students work in Table 5.15, there were numerous errors in the student's calculations, which could still lead the students to develop the correct ratio. Difficulties that stand out are the use of scientific notation and prefixes, use of the $\frac{1}{4\pi\varepsilon}$ terms in the electric field equation, and misinterpreting electrostatic force for electric field strength.

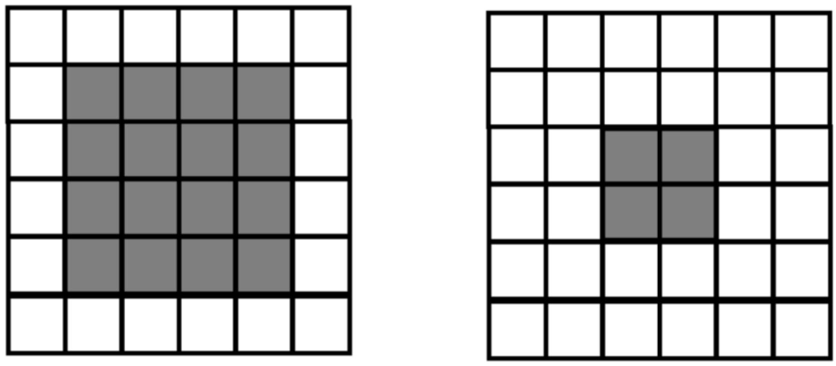
<table>
<tr><td colspan="3">Student 4L:
The distance between the charge and particle is doubled. This means that the magnitude of “c” is ¼ that of “a”.</td></tr>
<tr>
<td>Student 4C:
$\frac{Q}{d^2}:\frac{Q}{d^2}$
$\frac{2\,x\,10^{-12}}{0.2^2} : \frac{2\,x\,10^{-12}}{0.4^2}$
$5\,x\,10^{-11}$
$:\ 1.25\,x\,10^{-11}$
$4 : 1$</td>
<td>Student 4J:
$E = \frac{Q}{d^2} \qquad E = \frac{Q}{d^2}$
$E = \frac{2\,x\,10^{-6}}{0.2^2} E = \frac{2\,x\,10^{-6}}{0.4^2}$
$E = 1.25\,x\,10^{-5} \;\; E = 5\,x\,10^{-5}$
$1.25\,x\,10^{-5} \ : \ 5\,x\,10^{-5}$</td>
<td>Student 4A:
$\frac{1}{4\pi\varepsilon}\,x\,\frac{q}{d^2}$
$\frac{1}{4(3.14)(8.89\,x\,10^{-12)}}\,x\,\frac{2\,x\,10^{-11}}{0.2^2}$
$1.118\,x\,10^{-12}\,x\,\frac{2\,x\,10^{-11}}{0.04}$
5.59
$5.59\ \div 2 = 2.80\,N$</td>
</tr>
</table>

Table 5.15. Examples of responses from student 4L, 4C, 4J and 4A.

This reliance on calculations, while useful for students to generate their own evidence / data to interpret, can lead to a habitual approach where the students unknowingly make careless mistakes, but as they were familiar with what the answer should be, from their tutorial exercises, they did not revise their calculations and are satisfied to submit their answer. For example, in Table 5.15, it was seen that student 4J made errors in their calculation using the electric field formula and also mathematically demonstrated the 1:4 ratio, with non-simplified values. Conversely, six of the students demonstrated the ability to correctly develop the ratio using no formulae, overcoming difficulties presented by Arons (1997) and Maloney, *et al.*, (2000).

In the final question, the students were presented with a shaded grid, similar to the one used in the inverse square law paint can “intensity” homework. They were presented with the diagram in Figure 5.23 (i) and told that 100 field lines pass through the shaded region when the charge is held 10m from the charge. They were then asked to determine at what distance the 100 lines would only pass through the shaded region, as seen in Figure 5.23 (ii).

This question was designed as a manner to engage their conceptual understanding of the area model employed in the homework, as discussed in section 5.5.3. A similar question was presented in section 4.3.3, which the students did well on. However, here the distance values do no related to the area covered in the diagram, as like previous questions, this allowed me to determine to what extent the students were focusing on the area of the shaded squares and/or the distance from the charge. A summary of the student results is presented in Table 5.16.



(i) (ii)

Figure 5.23. Post-test electric field question, utilising the area / scale model.

The results showed that five of the students could correctly identify the distance from which the charge would produce one quarter the field line intensity. There was a variety in responses from students who determined the correct distance that the distance was *5 m*, in which the they sketched an isometric view of the charge and the frames, demonstrated partial or complete reasoning based on reducing the length / widths of the frames, and qualitatively referencing the related increase and decrease of the variables involved. Some examples of this reasoning are shown in Figure 5.24.

It was also seen that students submitted the correct answer but give the wrong reasoning, based on an inversely proportional relationship, such as 4A and 4F. Both these students stated that as the intensity is doubled, the distance is halved. It is not clear how these students determined the intensity doubled, when considering going from sixteen frames to four frames. However, this error allowed them to produce the correct final distance from the charge to the frame.

Responses	Students.
r = 5 m	**4A, 4D, 4E, 4F, 4G, 4L.**
r = 2.5 m	4C, 4K, 4N.
r = 2 m	4B, 4I
r = 20 m	4H, 4M
r = 40 m	4J
References quadratic link between distance and area – Scaling	**4E, 4G, 4H, 4I, 4M.**
References inverse relationship	4A, 4C, 4F, 4K, 4N.
Closer to charge puts lines through less boxes.	4B, 4D
Uses alternative mathematical ratio.	4L.

Table 5.16. Student responses to scaling model, relating distance to area covered by spray can.

<table>
<tr><td colspan="3">Student 4E:

</td></tr>
<tr><td>Student 4D:
The closer to the charge, the less spread out the field lines are.</td><td>Student 4G:
The length of the width / length is half the original ($4 \div 2 = 2$). This means the distance from the charge must also be half the original ($10\ m \div 2 = 5\ m$).</td><td>Student 4L:
If $4\ x\ 4\ \rightarrow 10\ m$
Then $2\ x\ 2\ \rightarrow 5\ m$</td></tr>
</table>

Figure 5.24. Inverse square law reasoning provided by students 4E, 4D, 4G and 4L.

Another difficulty was presented by some students determined that the area of the second frame was one quarter the area of the first, and that the distance reduced by the same ratio. Other students determined that the distance reduced by a factor of 2, but instead of charging the 10 m distance by this factor, they suggested new distance was 2 m. One student attempted to use an inverse square law, but instead increased the distance from the charge to the frames, instead of decreasing it, suggesting they did not consider the effect of diverging field lines passing through multiple frames, as displayed in Table 5.17.

The post-test results show that students gained understanding of how the inverse square law applies to Coulomb's law and the electric field. Most students could draw the shape of an inverse graph from the electric field formula. The post-test question showed that eight of the students could differentiate between an inverse and inverse square pattern, primarily using a form of data analysis of values obtained from the graph to verify their choices. Prevalent difficulties in the remaining students involved interpreting Coulomb's law as an example of an inverse law. When analysing graphs, it was observed that without intervention, the students focused solely on the shape of the graph, and did not consider obtaining values from the graph and analyse them, as discussed in section 5.5.2.

Student 4B:	Student 4M:
$$\frac{4}{16} = \frac{1}{4}$$ $$\frac{16}{4} = 4$$ $$\sqrt{4} = 2\,m$$ *If I move the particles closer, the number of field lines that pushes through each box increases. To have 100 field lines pass through the grid, the number of boxes must decrease.*	$$16\,x\,\frac{1}{4} = 4$$ $$\frac{2}{1} \rightarrow \frac{1}{2^2} = \frac{1}{4}$$ *Double the distance. – 20m.*

Table 5.17. Erroneous reasoning produced by students 4B and 4M.

When completing the ranking question, and asked to develop a ratio, it was seen that six of the students applied the inverse square law while seven attempted calculations. Some errors in mathematical operations caused difficulties for some of these students, preventing them from showing the ratio. In two cases, the students produced an accurate ratio, but did not reduce it to its simplest form. A small number of students demonstrated that they still applied inverse proportional reasoning in this question. The final question indicates that just under half of the students could applied the inverse square law correctly using a scale model to determine the distance from a charged particle to the frames presented. Prevalent difficulties presented themselves as students applying inverse proportional reasoning, using qualitative reasoning instead of quantitative, or applying the inverse square law incorrectly.

5.5.5. Discussion

This section presents the discussion on the student's application of the inverse square law to Coulomb's law and the electric field. The student's pre-test and post-test results are compared to indicate instances of conceptual change that occurred and references to the tutorial lesson and homework discussion (section 5.5.2 and 5.5.3) are highlighted as examples of evidence that conceptual change occurred. The discussion mainly focuses on the student's application of the inverse square law mathematically, while issues and examples of understanding of scaling are referenced (Arons, 1999; Marzec, 2012).

The pre-test indicated that students encountered difficulties when required to transfer their understanding of the inverse square law to the Coulomb's law and electric field contexts. As seen in Figure 5.25, in the Coulomb's law pre-test question presented, it was observed that only three students could recognise that Coulomb's law followed contained an inverse proportional relationship. Additionally, only one student could successfully mathematically apply the inverse

square law to the pre-test. The remaining students either qualitatively, randomly attempted some manner of calculations or were unable to attempt the question. This suggests that the students required instruction that would result in conceptual extension, to promote the students to apply the reasoning and understanding they demonstrated in section 4.3, to an electrostatics context (Hewson, 1992; Konicek-Moran and Keeley, 2011).

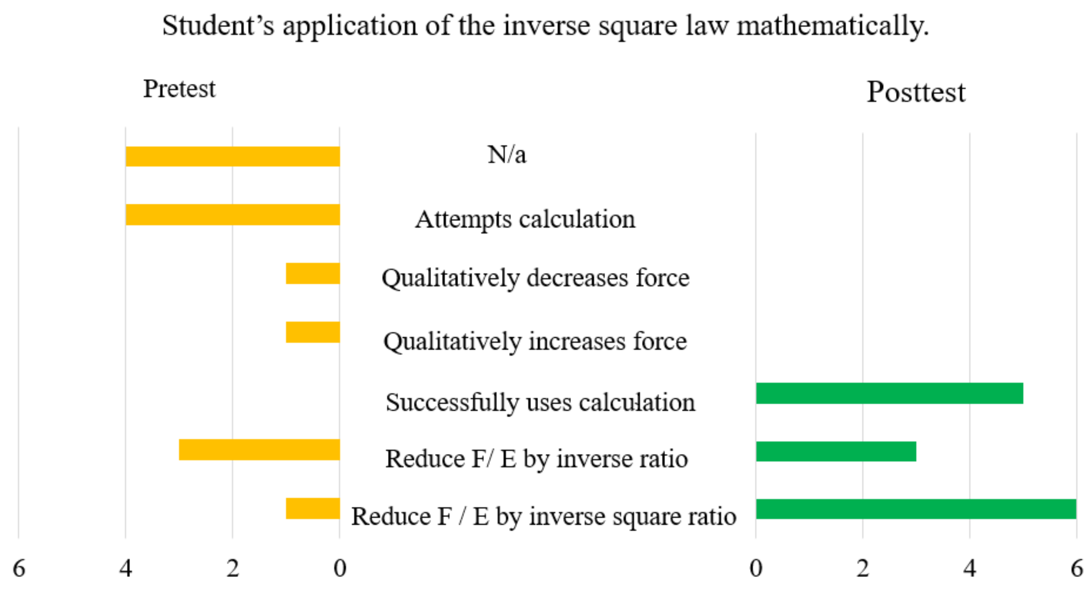


Figure 5.25. Comparison of students use of inverse square law in mathematical problems.

Gains were observed in the student's understanding and application of the inverse square law in the post-test results, as discussed in section 5.5.3. In the post-test six students applied inverse square law reasoning, without relying on formulaic substitution and evaluation, whilst a further five students did rely on the formula. Two students relied on use of the formula but used inverse proportional reasoning, while one student produced a ratio based on the distances presented in the question. This suggests that half of the students are reliant on the use of formulae, whilst the other half consider their understanding of the proportional nature of Coulomb's law and the electric field sufficient to approach the problem. The shift in results from the pre-test to post-test indicate that conceptual exchange (Hewson, 1992) occurred, as more students are observed correctly applying the inverse square law to the post-test contexts than was observed in the pre-test, and the students with persistent errors made more progress before encountering difficulty in the post-test, whereas they were unable to attempt the questions in the pre-test. Based on the shift in student responses shown in Figure 5.25, partial conceptual change was observed over through employing the use of the tutorials in the Coulomb's law and electric field contexts.

The student's reliance on formulae in the post-test is unsurprising. In tutorial the use of formulae was the most preferred method to explore the inverse square law. The students used a multiple representational approach to explore the inverse square law mathematically, and modelled the inverse

square behaviour of electric fields in the homework exercise. This homework activity was intentionally written in the same style as the spray paint model, discussed in section 4.3, as the students were familiar with the model, to allow for ease of transfer. The homework responses indicated that the students were proficient in the use of the frame model to explain the behaviour of the electric field lines. The interview indicated that the approach was unsuccessful in promoting student's consideration of mathematical area scaling, which underpins learner difficulty (Arons, 1999; Marzec, 2012). During the interview, the students demonstrated that they did not think in terms of the concept (Konicek-Moran and Keeley, 2011) of varying dimensions as they used the frame model to explore the variation of intensity through the unit areas at different distances from the charge, as discussed in section 5.5.3. However, the presentation of the frame model itself may reduce the requirement to rely on the understanding of the dimensional scaling, as students can consider the area of the frame directly.

Difficulties were also observed in the final question of the post-test using the area model, as depicted in Figure 5.23. Some students struggled to correctly show that reducing the distance from the can to the frames by a factor of a half would decrease the area by a factor of four. Students referenced the inverse square law in this question, suggesting they did not consider the quadratic nature that links the area of the spray to the distance from the can. Whilst the students attempted to approach the problem without using formulae and utilise their understanding, their application of the inverse square law directly instead of considering the dimensional scaling of the area. It suggests that they incorrectly apply the wrong relationship when considering the scaling itself but can determine the distance related to the given area, as both distance and area are familiar quantities to the students.

The electric field graphing pre-test question asked students to represent the relationship between the magnitude of the electric field at a point, and the distance from the charge. Ten of the students correctly represented the relationship using a characteristic decreasing asymptotic curve, while the remaining students erroneously produced linear of quadratic graphs or did not answer the question. However, in the post-test question, in which the students were presented with both an inverse and inverse square pattern, it was observed that difficulties were prevalent in six student's abilities to determine which pattern followed an inverse square law. Student difficulties related to the shape of the graph, not analysing the data on the graph by mapping the data into equations of the form $y = k\frac{1}{x}$ and $y = k\frac{1}{x^2}$, or analysing the data in terms of the behaviour of the reduction factors. This indicates the difficulties presented in section 4.3.3 were persistent at the end of this study, and that conceptual extinction of them did not occur.

5.6. Student's use of field lines to represent electric fields

Literature informs us that there are many representational difficulties that students can have when using field lines to represent an electric field (Galili, 1993; Törnkvist, *et al.,*1993; Maloney, *et al*., 2001). The following representational conventions for field lines were identified as target areas for the students to learn during the tutorial lessons, as they are accessible to the student's level of cognitive capability and could be used in application for other parts of the course, such as explaining the photoelectric effect, thermionic emission, production of x-rays and the use of particles accelerators.

- The closer field lines are, the stronger the field.
- When a field line curves, the direction of the force is tangential to the field lines.
- The field line represents the direction of force acting on a body, not the path taken by a body in the field.
- Field line patterns do not overlap or intersect. Two bodies that cause a field both contribute to one overall field surrounding both objects.
- Field lines only terminate on electric charges. They should extend to infinity / off the page / to the end of the diagram boundary.

Section 5.6.1 discusses the pre-test results, illustrating the student's understanding of the direction of force on a charged particle in a field, student's representations of the path taken by a body in a field, student's determination of relative field strength, and using vectors to represent the field at various points. Section 5.6.2 details the tutorial lesson in which students applied field line conventions to electric fields. Section 5.6.3 reviews the student's responses from the post-test, in which the student's gains in their understanding of field line conventions are illustrated.

5.6.1. Pre-test: Electric field

In the pre-test, the students were presented with the field pattern presented in Figure 5.26. While this field pattern shows some errors, due to poor design, from the learning outcomes for fields described in section 5.6, the questions asked of the students generally do not require the reasoning that field lines extent to infinity to correctly answer the pre-test questions. This field pattern is also not representative for a single point charge, before the students commenced the pre-test, that were informed that this diagram was a snapshot of a bigger field diagram with other charges elsewhere affecting the shape of the field lines. They were to only focus on the information they could use from the pattern they observed in the pre-test question.

They were asked a series of questions to determine the student's transfer of conceptual understanding from the field lines tutorial, and gauge what new concepts would be developed when transferring from gravitational fields to electrostatic fields. The students were asked the following questions:

- To draw in the path taken by an electron placed at the white circle.
- To rank the field strength of the point *a, b* and *c*.
- Draw vectors to represent field at the points *a, b* and *c*.

The first two of these three questions are discussed in this section. The last question, which involves the transfer of field lines to vector representation, is discussed in section 5.7.

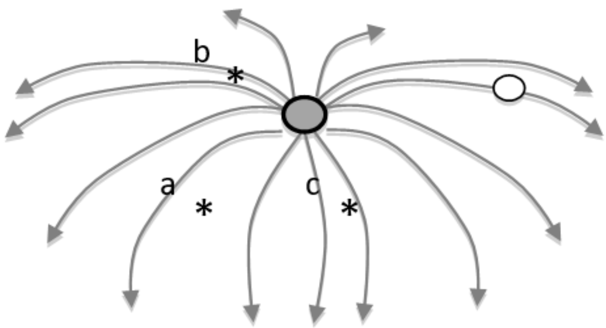


Figure 5.26. Pre-test question electric field pattern.

When representing the path taken by a negatively charged particle in this question, the students needed to consider the direction of the force acting on the charged particle, and how to represent the path under the influence of the field. Expected errors would be that the force on the negatively charged particle would be in the direction of the field line and that trajectory of the particle would follow the field line. A summary of the student's results for this question is given in Table 5.18.

The results show that three of the students determined that the direction the charge would follow was against the field line, while seven of the students determine it would go with the field line. In the class presentation to the students, the direction of the field line indicating the direction of force acting on a positive test charge was emphasized, and the opposite direction for a negatively charged particle was referenced. However, only three students recalled and applied this to their pre-test responses. This is not unreasonable as, except for the class discussion, the students did not spend any time exploring this concept.

Responses	Students
Electron moves against field lines.	***4A, 4J, 4M.***
Electron moves with field lines.	4B, 4C, 4E, 4F, 4G, 4K,4L.

Electron ignores field lines.	4D, 4H, 4N
N/a	
Electron diverges from path.	***4A, 4B, 4D.***
Electron takes linear path	4H.
Electron take unrealistic divergent path.	4E, 4K.
Electron sticks to field line.	4C, 4F, 4G, 4M.
Circular path	4N.
Electron ignores field and moves towards charge directly.	4D, 4J.

Table 5.18. Students pre-test results in determining the path taken by a negatively charged particle in an electric field.

Two of the remaining students (4D and 4J) drew paths in which the particle ignored the field and moved directly to the positively charged body. This is not considered a valid answer for this concept, even though the direction of the force acts against the direction of the field, due to the interpretation that the students did not consider the shape of the field lines when determining the path taken by the negatively charged particle. The final student (4N) ignored the field lines and drew a circular path for the electron around the positively charged particle. As there was no initial velocity for the charged particle, it is reasonable to speculate that student 4N is recalling aspects of the field line tutorial involving gravity. The production of a circular path would reflect the curved orbital paths referenced in the earlier tutorial.

Table 5.18 also shows the students results for using the field line as a guide for the path taken by the electron. Four of the students drew paths that reasonable diverged from the field lines. Errors were seen in one student (4H) drawing a path in the correct direction, but linear instead of curved. This indicates the student was not considering the force acting on the charge as it moves, and instead considered it a “once only” interaction between the field and the charged particle. Four students (4C, 4F, 4J and 4M) draw paths along the field line, indicating that the field line represented not on the force acting on charged particle, but the trajectory taken by the path. Two students (4E and 4K) drew an unreasonable diverging path, in which the electron moved away from field line, in a direction that does not correspond to direction of the force at its initial point. This indicates that while they are aware that field lines do not represent path taken, they are unclear how to apply the concepts of force, acceleration and velocity to determine the path taken. A sample of these paths are depicted in Figure 5.27.

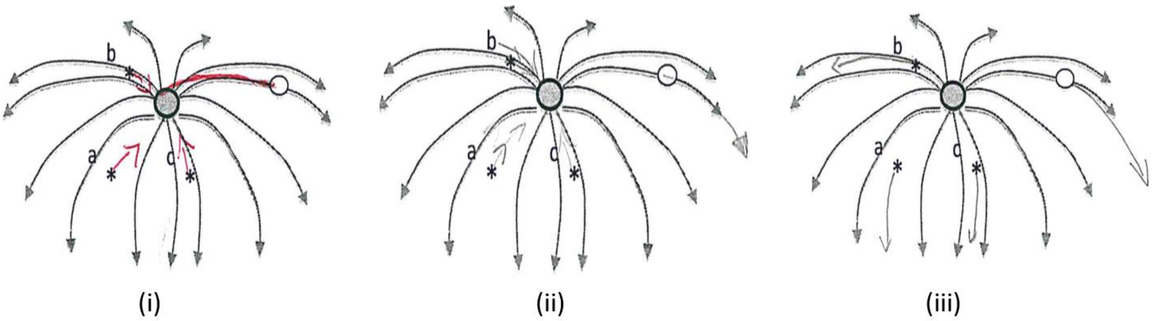


Figure 5.27. Student depictions of path of charged body which reasonably diverges from field lines (i), follows field line (ii) and diverges unreasonably (iii).

The next question on the pre-test looked at student's ability to rank the field strength at various points in a field, based on the diagram they were given. It was envisaged that successful students would use the field line density to determine the relative field strength, with a small number of students relying on the distance from the positively charged particle. A summary of the student's results is shown in Table 5.19.

Responses	Students
B > C > A.	**4A, 4B, 4D, 4E, 4F, 4G, 4H, 4J, 4K, 4L, 4M, 4N.**
B > A > C.	4C.
Field line density reasoning.	4E, 4G.
Distance from charge reasoning.	4A, 4B, 4C, 4D, 4E, 4F, 4H, 4L, 4N
Distance from field lines	4J, 4K, 4M.

Table 5.19. Summary of student's pre-test ranking of electric field strength and reasons used.

Many students were able rank the field strength surrounding the group of charges accurately. However, the reasoning utilised by nine of the students was based on the distance from the charge, whilst only three of the students attempted to use field line density as the evidence to justify their rankings. Additionally, three students used erroneous reasoning, utilising the distance from the points to the field line to represent the strength, i.e., the closer to a field line, the stronger the field. This suggests that most of students did not transfer this skill from the field line tutorial, or they did not warrant it as important over the reasoning based on distance from the charge. This could be highly likely, as this pre-test followed the tutorial on Coulomb's law, in which the inverse square relationship between force and distance was stressed as an important relationship.

Student 4G: b > c > a.
b is between 2 lines that are closest together and c is between 2 that are slightly further apart and a is between 2 lines that are very far apart.

Student 4D: b, c, a.
Because the closer you are [to the charge], the stronger the strength.

Student 4M: b, c, a.
B is closed to the long field line, c is slightly further away and a is much further away.

In summary, the pre-test results show that the student gains discussed in section 5.4 did not transfer to electric field as much as one would have hoped. A notable number of thought the field lines represent the trajectory of a body under the influence of a field. Most of the students assumed a negatively charged particle would follow a field line, although this is reasonable for the students at this point, as they had not yet explored the behaviour of a negatively charged body in a field. Moreover, in gravitational fields, there is no equivalence for this behaviour. In ranking electric field strength, most of the students relied on distance from the charge to determine strength, as opposed to using the field line density to justify their rankings.

5.6.2. Tutorial lesson: Electric field

As mentioned in section 5.4.2, the electric field tutorial lesson addressed both vector representations and field lines representations. This section looks at the latter half of the tutorial lesson. In this section, the students recapped previously covered concepts in their study of field lines, such as (a) the closer field lines are, the stronger the field, (b) when a field line curves, the direction of the force is tangential to the field lines, and (c) the field line represents the direction of force acting on a body, not the path taken by a body in the field. From the previous tutorial on field lines, as mentioned in section 4.4.5, it was observed that students made errors that the electric field line tutorial addresses. Those errors were as follows:

- Field line patterns do not overlap or intersect. Two bodies that cause a field both contribute to one overall field surrounding both objects.
- Field lines only terminate on charges. They should extend to infinity / off the page / to the end of the diagram boundary.

These rules were presented to students in an introductory paragraph on the tutorial sheet, and students were prompted to refer to them during course of the tutorial lesson. The students utilised these rules to explain the variation of an electric field of a positive charge and construct the electric field for a negative charge, whilst explaining the similarities and differences of both patterns.

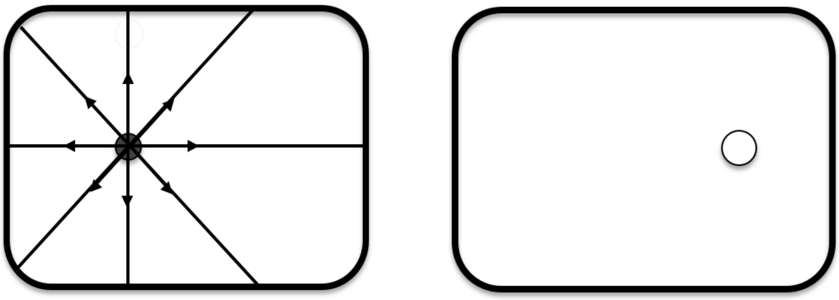


Figure 5.28. Tutorial setting where students represent electric fields using lines.

The students were required to use the field lines seen in Figure 5.28 to identify the sign of the charge as positive, and explain how the variation of field strength was represented. All students were consistent in referring to the direction of the field line pointing away from the charge as the indicator for the positive charge, in some cases referring to the fields moving to infinity. The students were also consistent in using the field line density as an indicator for the field strength. All students successful represented a negatively charged particle using the same field line pattern, with the directions placed towards the negatively charged body, as opposed to away from it, highlighting the differences between the two patterns.

Student 4E: Positively charged because field line are pointing outwards to infinity. [Field strength] decreases, field lines are further apart from each-other. [Differences] D = Arrow changes direction because negatively charged. [Same] S = field line still stretch to infinity.

The next section of the tutorial required students to apply the superposition principle to construct field lines for a system of two charges. In the first case, the students were required to represent the field of two dissimilar charges, and in the second case, the field of similar charges. In both cases, the students were required to draw the path taken by a positively charged particle under the influence of the field at a point. There were no errors shown in the student's representation of field lines for these cases. Some groups represented the path taken by the charge along the field lines. Questioning from the teacher or discussions with other students about the initial acceleration of the particle helped these students confront the errors in their initial responses. The prompt questions asked students to consider the direction of the force at the initial moment and consider the position of the particle at the next moment, having moved under the influence of the force. Student's revised their paths taken, using this approach, but still produced reasoning that was indicative of some confusion about the field line representation. For instance, one group of students, consisting of students 4D, 4E, 4F and 4N submitted the reasoning shown in Figure 5.29.

The use of the term "off track" in this case is still indicative of the error that the field lines represent a path or trajectory of some type, but the forces acting on the charge have caused it to move away from the track, akin to a car sliding off the road on a turn. Students 4G, 4H and 4I also submitted

the correct path, but gave reasoning based on the force interaction of the charges, as opposed to the field. This indicates the students were considering the interaction of the forces between the charges over the interaction of the positive particle and the field. Using this force orientated reasoning, the students appear to use the field to guide the trajectory, that's influenced by the charges generating the field.

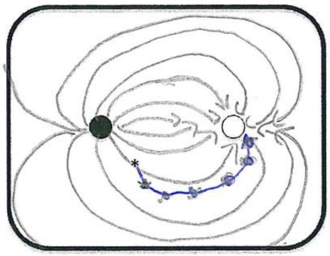
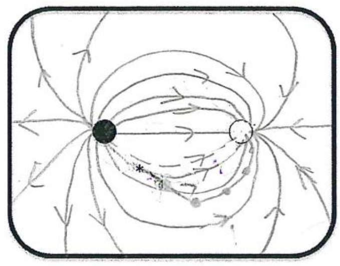
Student 4D:	Student 4G:
No velocity at the start. It accelerates a little and begins to move off track. 	*As the particle is repelled by the positive charge, it will gradually be more affected by the negative charge than the positive charge, as it moves further away from the positive charge.* 

Figure 5.29. Students 4D and 4G's depiction of path taken by charged particle in an electric field.

This tutorial narrative presented evidence that the students applied their understanding of field lines to electric fields, to some degree of success. The students identified and represented the electric fields for single charges and use the field line density to identify the variation of electric field strength for the charges. The students could also represent the superposition of similar and dissimilar charges, using field lines. While further work appears to be required to further develop the student's reasoning for the interactions of charged particles in an electric field, the students responses showed, with prompting, they accurately predicted the behaviour of a charged particle in an electric field and draw its trajectory accurately.

5.6.3. Post-test: electric field

In the post-test, the students were presented with the electric field of two positive charges shown in Figure 5.30. The students were asked a series of questions to determine their understanding of the information provided in the diagram. They were required to:

1. determine the charges on P and Q,
2. determine which of the two charges had the greatest magnitude,

3. explain the variation of field strength as a field line is followed from P to the boundary of the diagram,
4. draw the path a negative charge would take, if placed at R,
5. represent this electric field using vector arrows at different points. The results are shown in the following table.

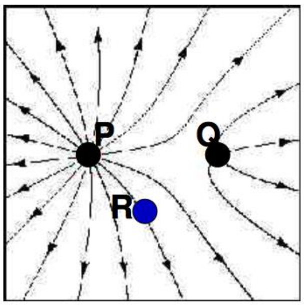


Figure 5.30. Diagram from the electric fields lines post-test question.

The first four of these questions are presented in this section, while the fifth question is addressed in section 5.7, as it specifically deals with transfer between representations. Table 5.20 presents a summary of the student's responses for the first two questions, including the reasoning provided by the students.

The post-test results indicate that the students were successful in applying their understanding of field line representation for positive and negative charges to this question. Except for one student (4F), all the students correctly identified that both bodes were positively charged, using the field line direction as justification for their responses. All the students recognised that the number of field lines coming out of the bodies could be used to determine the relative magnitude of their strength, which is an application of the field line density concept.

Student's 4A and 4G also incorporated the field line density into their responses. Student 4N referred to the number of field lines, but showed an error in their reasoning, alluding to the field lines of P pushing the field lines of Q away, indicating the field lines themselves exhibit their own tangible charge like property, or force-like behaviour.

Student 4E:	[The magnitude of P is] greater. The electric field lines are closer together.
Student 4G:	[The magnitude of P is] greater than [Q], because there are more field lines that are closer together from P, meaning that it is stronger.
Student 4N:	[The magnitude of P is] greater, because it has more field lines, and it pushes the field lines of Q away.

Responses	Students
P and Q are positive	**4A, 4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**
P and Q are negative	4F
Based on field lines leaving the charge.	**4A, 4B, 4C, 4D, 4E, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**
No reasoning.	4F
Magnitude of P is greater than Q.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**
Answer recognized the number of field lines is greater for P over Q.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N.**

Table 5.20. Student responses of the charges on P and Q, and relative charge magnitude between the bodies.

The third question asked students to place their finger on the charge P and follow one line to the boundary of the diagram. They were required to explain the variation of field strength as their finger moved and justify their explanation. Student responses are shown in Table 5.21.

The results clearly indicate that the students correctly identified the variation of the field strength using the field line density as justification. Only one student, 4M, used the distance from the charge as an indicator for field strength. Neither of these examples of reasoning are incorrect, but the results show a shift by the class to using the representational conventions of field line density, from relying on the relationship between the field strength and distance from the charge.

Student 4J: It decreases cause the field lines are more separated.

Student 4M: It gets weaker because the further you go from a charged particle, the weaker the electric field strength.

The fourth question required students to draw the path taken by a negatively charge particle in the field, when placed at the point R, depicted in Figure 5.29. This question was like the pre-test question, looking to determine if the students would draw a path that reasonably diverted from the field line, and whether the force acting on the negatively charged particle would go against the direction of the field. A summary of the student's responses is presented in Table 5.22.

While most students were able to show a reasonable path diverging from the field lines in a direction that moved against the field lines, minor errors were still observed in some student responses. Students 4L and 4M both submitted the correct path, but used naïve reasoning based on the attraction of the positive charge P, and the negatively charged particles placed down. These students failed to explicitly articulate their reasoning for the shape of the path. By presenting a reasonable curved path that diverges from the field line, they indicated that they understood the

velocity (or inertia) of the particle moves it from the field line. Student 4B drew a path in which the initial force was in the correct direction and was tangential to the field line. However, as the particle moved, the direction of the net force on the charge would have changed, but 4B did not take this into account and drew a linear path that followed the direction of the initial force. Student 4C also drew a linear path, but in this case, they ignored the shape of the electric field, and draw a path directly to the charge, basing their path on the attraction between the charges.

Responses.	Students.
Field strength decreases.	**4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N**
Field strength increases.	N/a
Field strength remains unchanged.	N/a
Justified by using field line density.	4A, 4B, 4C, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4N
Justified by distance from charge.	4M

Table 5.21. Student's post-test responses for the variation of field strength, and their justifications.

Responses.	Students.
Negative charge moves against field line.	**4A, 4B, 4C, 4D, 4G, 4H, 4I, 4K, 4L, 4M, 4N**
Negative charge moves with field lines.	4F
Reasoning based on attraction	4L, 4M
N/a	4E, 4J
Reasonable path off the field lines produced.	**4A, 4D, 4F, 4G, 4H, 4I, 4K, 4L, 4M, 4N**
Paths was linear	4B
Path ignore field lines, direct to P.	4C
N/a	4E, 4J

Table 5.22. Summary of student's post-test responses to drawing a negatively charged particle moving in an electric field.

Student 4F had incorrectly identified that P and Q were negative charges instead of positive charges and drew their path accordingly. While the path did leave the field line, in what is considered a reasonable path, the student did not use information based on the field line direction to determine what direction the force on the charge would act. They used the interaction between the negative charge placed down at R and the force it would experience based on its repulsion from P and Q. Student 4F did however explain that the increasing velocity would move it from the field line, and

drew a reasonable curved path that indicated repulsion on the mobile charged particle from both P and Q. A sample of the students correct, and incorrect paths are displayed in Figure 5.31.

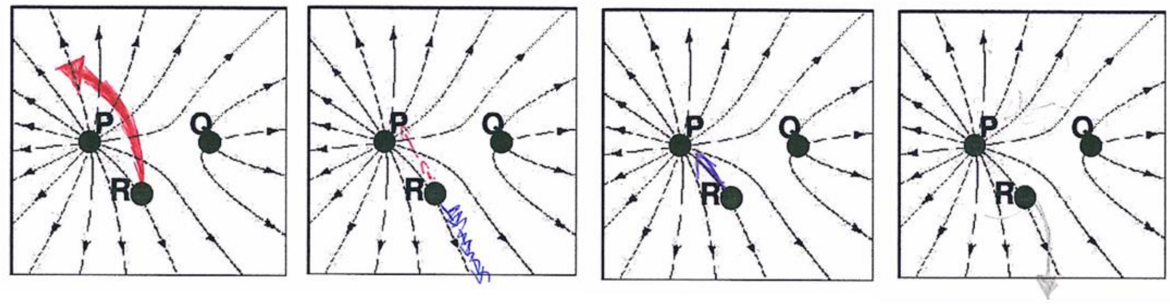


Figure 5.31. Paths taken by negative charge in field, from students 4H, 4B, 4C and 4F.

This section discussed the post-test results. The results indicate that, across the whole group of 14 students, gains occurred in the student's understanding and application of field line concepts. The students demonstrated they could identify a charge, based on the field line pattern, and determine the relative magnitude of a charged body based on the number of field lines going into or out of it. All but one of the students used the field line density to explain the variation in field strength as an indicator for relative field strength. The final question discussed in this section showed that the majority of students could accurately draw the path taken by a negatively charged particle in a field accurately but errors were persistent for a small number of students, such as the direction of the force acting on the negative charge in a field, the continuous nature of the force acting on the negative charge, and students ignoring the field and drawing the path based on attraction interaction between the negative charge and P or Q.

5.6.4. Discussion

This section discusses the student's pre-test and post-test results, focusing on their understanding of the direction of force acting on a charged object, using the field lines as a guide to draw a reasonable trajectory, and the used of field line density as an indicator for field strength. When appropriate, the student's ability to transfer their reasoning from the tutorials in Chapter 4 is also discussed in each section. The first part of this discussion focuses on the student's understanding of the direction of force acting on a charged particle, under the influence of an electric field. Figure 5.32 shows the pre-test – post-test comparison for the student's understanding that the force on a negative charge acts in a direction that goes against the field.

The pre-test results indicate that over half of the students were unaware of how a field line can be used to determine the direction of force acting on a charged negatively charged particle or ignored the field itself and focused on the interaction of charges. Both difficulties are commonly observed in

learners understanding of electric fields (Furio and Guisasola, 1998; Cao and Brizuela, 2016), and both were identified as targets for conceptual extension (Hewson, 1992).

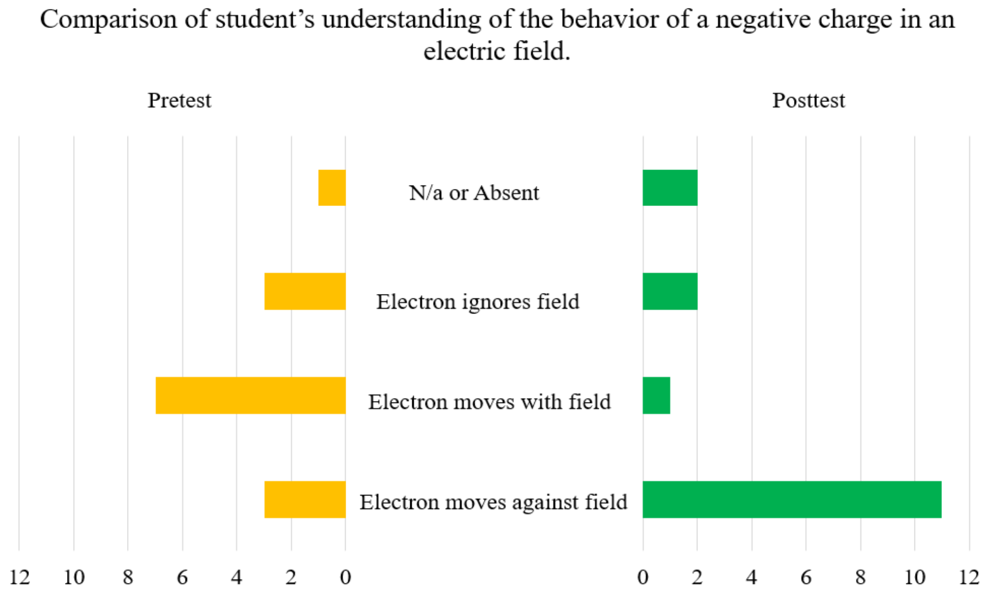


Figure 5.32. Comparison of results regarding the force on a negatively charged particle in an electric field.

Section 5.6.2 narrated the student's development during the tutorial. The latter part indicated that when both a field and charges are presented to the students, the students reasoning is based on the interaction of charges, as opposed to the interaction of the field with the charged particle. However, this was not observed with high frequency in the post-test. The post-test showed a shift in the number of students that determined an electron would be influenced by a force to move against the direction of the field using reasoning based on the charged particle and the field. Figure 5.32 displays the student's responses to this question and indicates that conceptual extension occurred, with a jump of 3 students to 11 students using field-based reasoning. Reasoning based on the interaction between charges was rarer in the post-test with only two students (4L and 4M) using the reasoning. The paths drawn by these two students were correct and suggest that they were aware of how the electron would behave, but they appeared to value the interaction between the charges as a stronger indicator to determine the path than the interaction with the electron and the field. Both types of reasoning, charge interactions and field interactions, were observed in the tutorial and post-test, and this suggests that the students consider the scenarios in terms of both styles of reasoning and applied them as they saw fit to the task presented to them. This suggests conceptual exchange did occur in the student's models, but conceptual extinction did not occur. As the comparison of the results demonstrates an increase in eight students utilising the correct reasoning, the extent to which conceptual change was observed was moderate.

The next section compares the paths drawn by a charged particle under the influence of an electric field. Figure 5.33 shows the pre-test – post-test comparison, in which it is clearly observed that there was a reduction in the number of difficulties encountered by the students.

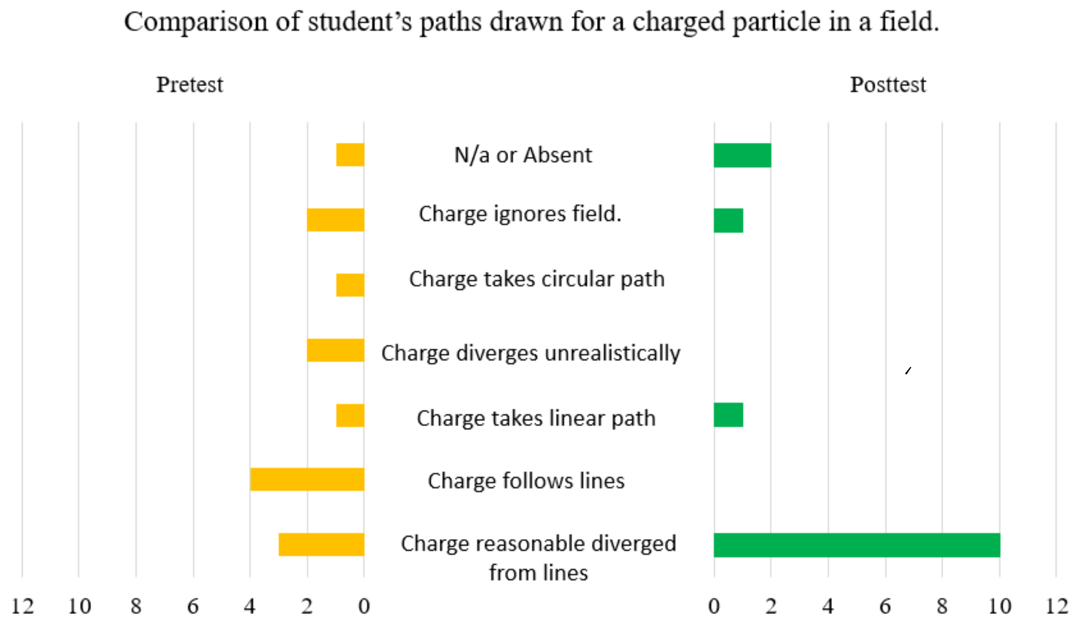


Figure 5.33. Comparison of results regarding the path taken by a charged body in an electric field.

While only a small number of students showed the path following the field line in the pre-test, which was expected to be the most prominent error, the results from Figure 5.33 suggest that students struggled to construct a reasonable path during the pre-test (Törnkvist, *et al.*, 1993; and Galili, 1993). In section 4.4.4, it was seen that 12 of the students constructed a reasonable path taken by a body under the influence of a gravitational field. When this is compared to the electric field pre-test, it is clear the students did not transfer their understanding to the electric field context. An inability to transfer a concept to multiple contexts is an indicator of the limit of the student's conceptual understanding (Konicek-Moran and Keeley, 2011) and the tutorial was developed to aid student extend their understanding to the electric field (Hewson, 1992), to which partial conceptual change was observed.

In the electric field tutorial lesson, it was seen that students required time and guidance to consider how the force acting on charged particles affects their acceleration and velocity to construct a reasonable path. When difficulties occurred, the motion diagrams were utilised as prompts by the teacher. This allowed the students to consider how the charged particles moved from moment to moment, whilst the students explained how the force at every moment would cause the particle to change its motion. The teacher provided the dissatisfaction in the explanatory power of considering the field lines as indicative of the path taken by a charged body and presented a motion diagram which allowed the students to develop intelligible reasoning based on their understanding of forces, acceleration and velocity (Posner, *et al.*, 1982). As the students to focus on applying the outcome of

the interaction and the concept involved, as opposed to focusing solely on the outcome in the electric field context (Konicek-Moran and Keeley, 2011). The post-test results showed that 10 students constructed reasonable field line trajectories, overcoming most misconceptions presented in the pre-test. This suggests an extension in the student's understanding to generate a reasonable path in a gravitational field to an electric field context (Hewson, 1992)

Figure 5.34 illustrates the pre-test post-test comparison for the student's justifications in determining relative field strength in an electric field.

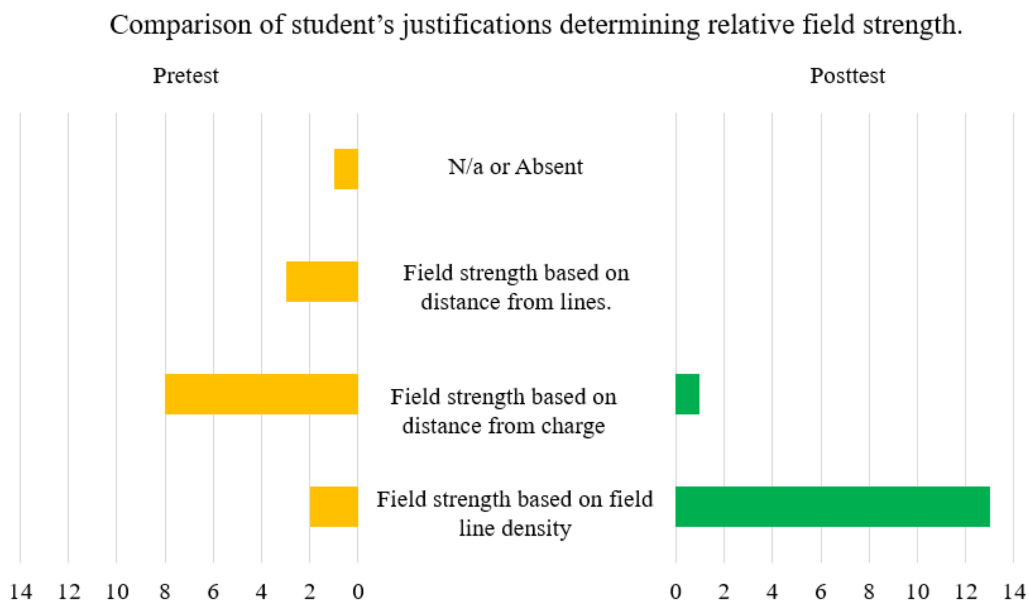


Figure 5.34. Comparison of results regarding the relative field strength in an electric field.

In chapter 4, section 4.4.5, it was observed that 10 of the students could rank field strength based on field line density. In the electric field pre-test, it was seen that only two students extended their understanding field density to determine the relative field strength of different points to the electric field context (Hewson, 1992; Törnkvist, *et al.*, 1993; and Galili, 1993), whilst eight students relied on distance from the charge. It is unclear whether the students were unable to extend and transfer their understanding to from gravitational lines, or whether they did not consider this concept to be more appropriate to employ over using distance from charge to justify their ranking. As the students previously completed the tutorial on the inverse square law and Coulomb's law, it is reasonable they considered the qualitative relationship first as it was recently used in their memory. During the tutorial lesson, the students demonstrated that there was no difficulty in apply the convention and apply it in the electric field context (Konicek-Moran and Keeley, 2011), and in the post-test, it was observed that all but one of the students applied it in the post-test. As this question could be answered using both distance reasoning and field line density reasoning, with both concepts recently in their memory, most students opted to use field line density as their justification. This indicates that the students were able to extend the concept of field line density to represent the relative field strength

between points, but they may opt to using alternative, but valid, reasoning when the task they were completing allowed them to do so. Therefore, the level of conceptual change observed in the student's understanding of relating relative field strength to field line density was moderate.

This section discussed the student's understanding of the force acting on a negative charge and showed the tutorial lesson allowed for conceptual change to occur. Post-instruction, some minor errors were persistent such as students ignoring the field focusing on the interaction between the charged particles, or directing the force acting on negatively charge particle following the field line direction. Ten of the students showed that the path taken by a charge would reasonably diverge from field lines, with only one student considering the force to act on the charged particle at a single instant, and one student ignoring the field. Finally, the students demonstrated that they could utilise the field line density to rank the field strength at different points, instead of relying on the relationship between electric field strength and distance, when the concept was recently employed in their classwork.

5.7. Student's use of vector and field lines representations in electrostatics

Thus far, this chapter has illustrated and discussed the student's use of vectors, the inverse square law and field lines, as they applied them to Coulomb's law and the electric field. This section details how students transferred between field line representations and vector representations (Törnkvist, *et al.*, 1993; and Galili, 1993). The ability to transfer isomorphically, that is transfer between both directions without difficulty either way, is demonstrated as a trait of expert problem solvers (Kozma, 2003). The first half of this section discusses an example of students representing vector arrows from a given electric field, and the latter half discusses an example of students representing a vector field using field lines.

5.7.1. Student transfer from field lines to vectors

In the electric field post-test, the students were presented with an opportunity to construct a vector field plot from a field line diagram. The field line diagram was shown in Figure 5.30, at the beginning of section 5.6.3. The students were presented with a diagram that presented the charges P and Q and six points indicated with asterisks shown in Figure 5.35.

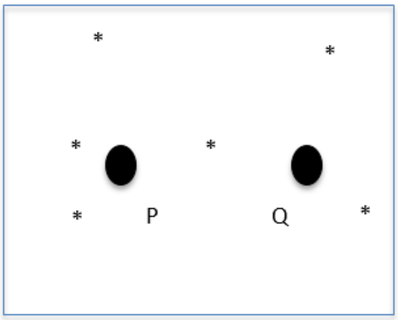


Figure 5.35. Points to represent vector arrows from field lines diagram.

When asked to represent the electric field using vector arrows at the points shown, most of the students encountered difficulties in transferring between the two representations. The student's results are summarized in Table 5.23.

Responses	Students.
Vectors are correct direction and correct relative magnitude	**4B, 4D, 4E, 4G**
Magnitudes are not relatively in scale.	4C, 4H, 4I, 4J, 4L
Directions of vectors are mildly non-tangential.	4M
Tangential, but points towards P and Q.	4C, 4J, 4K
Vectors from both charges shown, no super-position of vectors attempted.	4A, 4F
Attempted superposition shown	4N

Table 5.23. Student's attempts to transfer from field line to vector representation.

Only four of the students could correctly represent the electric field at various points around the two charges using vectors. The most prominent error shown by the students was sketching vectors of the correct orientation but incorrect relative magnitudes. Section 5.6.3 showed that the students were aware of how the field line density represented the field strength, and during section 5.4.2, it was noted in the tutorial lessons that the students tended to forget to represent the magnitude of the field correctly before applying the superposition principle. Although the students were prompted to the error and they rectified it in the tutorial, it was clearly not sufficient to produce a long-term change in their vector representation, when constructing vector fields.

Three students, 4C, 4J and 4K, reversed the direction of the vectors relative to the direction of the field at the point. All these students previously identified the sign of the charge on P and Q, based on the direction of the field lines, so these vectors are not consistent with their previous reasoning. It is possible that these students drew these vectors to represent the force acting on a negative charge at the given points, as the previous question required them to draw the path acting on a negative

charge. The relative magnitudes of these student's vectors were reasonable and accurately represented the field strength at the points shown.

Another difficulty in transferring to vector representation was encountered by students 4A and 4F. This their case, they drew vectors surrounding each individual charge, but attempted no superposition. This is not considered an attempt at transfer between the representations, but an attempt at construction of the vectors from scratch, suggesting that the student's do not comprehend the link between the two representations.

Student 4N had similar difficulties in which they attempted to construct the vector field based on the positions of the charges, without using the direction of the field lines as a guide for the direction and relative magnitudes of the vectors. Student 4N also overlaid the electric field on their diagram and produced reasonable resultant vectors. Whilst their vectors were drawn reasonably, it is clear they did not ground their construction in transferring between field lines, but instead relied on the principle of superposition. Examples of the student's constructions are presented in Figure 5.36.

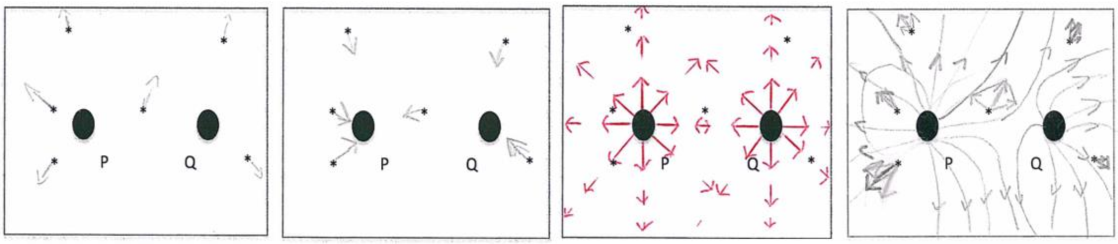


Figure 5.36. Vector fields transferred from field line representations, from students 4D, 4C, 4A and 4N.

This section showed that the approach undertaken in this study did address the student's ability to transfer from field lines to vector representations. Difficulties about representing vector magnitude qualitatively using the length of the arrows were the most persistent difficulty presented by the students, whilst most students appeared to grasp that the direction of the field was tangential to the field lines at the points shown. In some cases, students did not transfer the direction of the arrows correctly, even though they were depicted on the original field lines diagram. This difficulty may have been influenced by the previous question in which the students was asked to consider a negative charge, indicating they do not consider the field independent of the charges it interacts with. Other students resorted to rote learned vector patterns without drawing the superposition and one student was able to represent the field drawing vectors but relied using both superposition between the charges and overlaying the field line pattern to scaffold their construction.

5.7.2. Student transfer from vectors to field lines

In a separate question in the post-test, the students were presented with a vector field, as shown in Figure 5.37 (i), and asked to identify the sign of the charge on the particle. They were told an oppositely charged body was nailed down next to the first charge and they were asked to construct the superposition of the field at the points shown in Figure 5.37 (ii). The student's responses to these questions were discussed in section 5.4.3. As an extension to these questions, the students were required to draw electric field lines to match the vector field they constructed, as seen in Figure 5.37 (iii). A summary of the student's responses is presented in the Table 5.24.

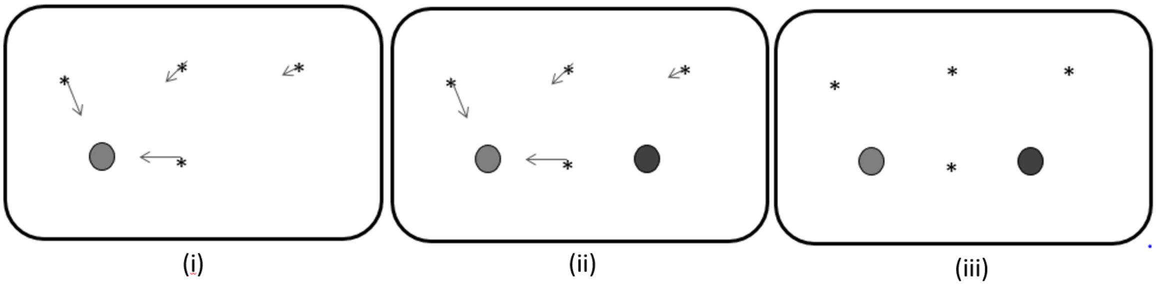


Figure 5.37. Post-test question in in which students apply vectors to an electric field.

Responses.	Students.
Field lines consistently represent vectors	**4D, 4E, 4H, 4J, 4L, 4M.**
Field lines not consistent with vector arrows	4A, 4B, 4F, 4G
Incorrect superposition – Not all vectors consistent with lines.	4C, 4I, 4K,
No transfer demonstrated – used both representations	4N

Table 5.24. Student's attempts to transfer from field line to vector representation.

The results indicate that half of the students could construct a field line representation that was consistent with their vectors, while the other half of the students could not. The six students who correctly transferred their vectors to field lines aligned field lines reasonably well with the directions and magnitudes of the vectors, as shown in the example of student 4H's response in Figure 5.38. In all cases, minor errors were seen, and it was reasonable if the student's diagrams accurately represented 3 of the vectors, and reasonably represented the fourth.

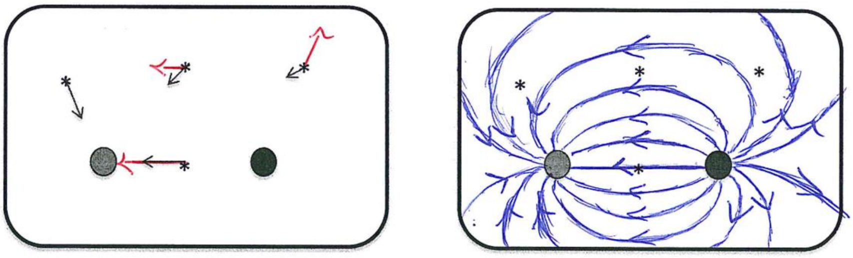


Figure 5.38. Example of Student 4H representing vectors using field line representation.

As both charges were equal in magnitude, the field line pattern should have been symmetrical, but in cases where the student's vectors were inaccurate with magnitude, this error was transferred to their field line diagrams. These errors are considered to be reasonable as the diagrams are qualitative in nature, and the consistency in the error suggests the students were employing representational transfer. For instance, student 4E did not find the superposition of the vectors, and when representing the field lines, skewed the symmetry of the diagram to produce the shape of the field in line with the strongest vectors in their field, except for the top right point, as seen in Figure 5.39.

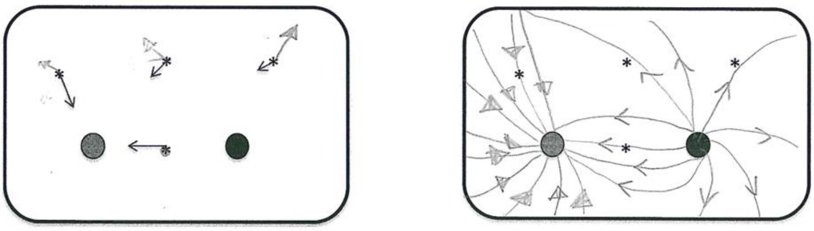


Figure 5.39. Example of Student 4E transferring error consistently to field line representation.

Four students drew field patterns which were correct and symmetrical but did not represent the vectors. This indicates they may have produced a rote learned model but did not consider the direction of the field at all points. For instance, Figure 5.40 shows student 4G showed the superposition at various points, but when drawing the field lines, three of the points did not align to their vectors. Furthermore, their field line diagram suggests only the field in the space between the charges is where the superposition principle applies, ignoring the areas on the left and right of the diagram. This contradicts the vector superposition they represented in the vector field diagram.

This error was also seen in the responses of students 4A and 4B. Student 4F gave incorrect vector directions. They interpreted the question to be two negatively charged particles. However, the field line pattern they drew was indicative of oppositely charged particles. This indicates the student was resorting to rote learned patterns, as their diagrams are inconsistent.

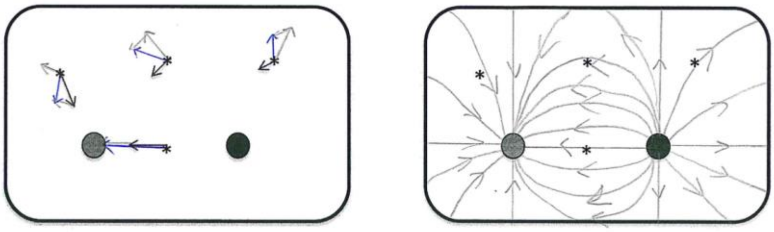


Figure 5.40. Inconsistencies in the vector transfer to field line representation, from student 4G.

Students 4C, 4I and 4K drew patterns that also did not transfer from their vector arrows. As seen in Figure 5.41, the two points in the centre of the diagram are not represented for their field. They drew a pattern that indicates repulsion and is inconsistent with their vectors. However, an example of transfer can be seen on the points shown to the top right and top left of their pattern, in which their patter follows the direction of the vector of highest magnitude, an error also seen in student 4E's submission. Student 4K also displayed this error. Student 4I drew overlapping lines of the field lines pattern for both attraction and repulsion and display the field line direction to be in the opposite direction of that which they drew directions of the vector arrows. This is depicted in Figure 5.42.

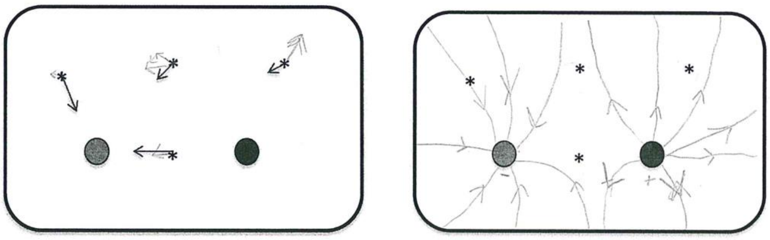


Figure 5.41. Inconsistencies in the vector transfer to field line representation, from student 4C.

This section showed that only a small number of the students represented a field line pattern using field lines. The most persistent difficulty in students was maintaining consistency in the direction in the diagrams drawn from vector arrows to field lines. A likely issue in the responses was students resorting to rote learned patterns for field lines of oppositely charged particles. The students appeared to demonstrate an overreliance on position of the charges to produce their field line pattern, and not on the vector arrows they drew.

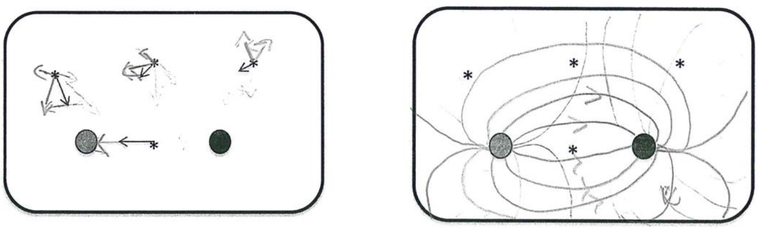


Figure 5.42. Examples of errors in the vector diagram transfer to field line representation, from student 4I.

5.7.3. Discussion

This discussion reviews the student's understanding of concepts related to vector and field lines representations and provides a commentary on student gains observed in the previous two sections. Section 5.4 illustrated the student's progress in extending their understanding of vectors to applying it in an electric field diagrams, when required to represent the field using arrows at various points around a charged particle (Maloney, *et al.*, 2001). Section 5.6 illustrated the students understanding of the relative field strength and the direction of force acting on a charged body at various points in an electric field (Galili, 1993; Cao and Brizuela, 2016). As seen in section 5.7.1, the gains that the students displayed in the post-test for these representational contexts individually were not as frequent when the students were required to transfer between the two representations.

Section 5.7.1 showed that the prevalent difficulty in student's ability to transfer between representations was representing vector magnitude that was consistent with a given electric field line diagram (Törnkvist, *et al.*, 1993). This difficulty was not observed to the same degree in the post-test question, seen in section 5.4, as in this section where the students were required to transfer between field line representation to vector representation. As section 5.4 indicates that the students could apply the vector convention to the electric field context, this suggests that conceptual extension did not occur effective (Hewson, 1992), to enable the students to apply the concept in a task that involved transfer between the two representations. As nine of the students were able to correctly represent the directions of the vectors, this indicates that some level of transfer occurred, in which the students could recognise the tangential nature of the electric field at positions represented along a field line.

Section 5.7.2 detailed the post-test question, which elicited the student's ability to transfer from vector representation to field line representation. It was observed that half of the students were able to construct a field line pattern that was consistent with their own vector constructions (Törnkvist, *et al.*, 1993). The most common difficulties in the post-test were that students drew vector fields that were suggestive of both charges having the same charge, but the fields indicated their charges were opposite or vice-versa. This suggests these students did not extend their understanding to interpret their vector and apply their field line reasoning but constructed their field line patterns from patterns that were memorised, or results to guessing.

From reviewing both section 5.7.1 and 5.7.2, it was observed that only students 4D and 4E consistently answered the post-test questions without error and demonstrated the understanding to allow them to transfer between the representations isomorphically. Students 4B, 4G, 4H, 4L and 4M answered one of the two questions without error, and tended to make minor errors in the other question. This suggests that conceptual extension (Hewson, 1992) occurred for these students during the tutorial, as these students displayed proficiency in the vector and field lines tutorials, as seen in chapter 4, and showed progress in transferring their understanding to the electric field context. The

remaining students demonstrated errors in both transfer questions. The errors displayed by these students are reflective of the errors seen in chapter 4 and/or the difficulties observed when applying these concepts to the electric field, as seen in section 5.4 and 5.6, such as terminating field lines, or lines overlapping. As these students displayed errors in their understanding of both representations in chapter 4, it is reasonable that difficulties in transfer between the representation occurred.

5.8. Conclusions

This section of the chapter presents the conclusions based on the studies presented in this chapter in two separate subsections. They address the impact on student learning, and the student's ability to transfer the representational tool to the electric field context. The conclusion addressed the overall research question that was introduced at the beginning of this chapter:

- To what extent does the use of a multi-representational structured inquiry approach develop student understanding of electric fields?

This question is addressed in the impact of student learning, which addresses the following considerations related to the research question:

1. The student's ability to demonstrate that Coulomb's law, and the electric field, is an example of an inverse square law?
2. To what extent the students demonstrate their understanding electric fields and the interaction of charged objects with fields using vector representation.
3. To what extent the students demonstrate their understanding electric fields and the interaction of charged objects with fields using field line representation.

The student's ability to transfer representational tools to the electric field context addressed the following consideration related to the research question:

4. To what extent the students demonstrate their ability to transfer a depiction of an electric field from one representation to another representation?

5.8.1. Impact on student learning

The approach adopted in this study focused on developing student's ability to transfer and their understanding of vectors, the inverse square law and field lines to Coulomb's law and the electric

field. The vector nature of forces is a fundamental pillar of electrostatics, and further electromagnetism, in which vector mathematics is utilised, along with calculus. The inverse square law is one of the fundamental relationships studied by students, which is seen in optics, sound, gravitational and electrostatic forces, and radiation. The concept of a field underpins the interactions of particles and field lines are an efficient manner to represent them, as they can convey various information about a field, in a simple manner.

The first of the research considerations looks at student's application of the inverse square law to Coulomb's law and electric fields. The approach adopted utilised two methods for students to apply their understanding of the inverse square law to Coulomb's law and the electric field. The first method involved mathematical data analysis in various forms, as suggested by Hestenes and Wells (2006), and the second method was adapted from the spray paint model, presented by Hewitt (2009). The pre-test results for the student's application of the inverse square law to Coulomb's law and the electric field suggested that the students did not incorporate the ability to recognise Coulomb's law as an equation of the form $y = k\frac{1}{x^2}$, and apply mental mathematical ratios to determine a reduction in force, or did not remember how to apply inverse square proportional reasoning. This suggests the students did not transfer what they learned during the inverse square law tutorial lessons, as discussed in section 4.3, where nine students successfully used the law or made reasonable attempts in applying it. However, in the Coulomb's law pre-test, this number dropped to four across different manners of representing the inverse square law. Upon completing the Coulomb's law tutorial and electric field homework exercise, it was seen that approximately half of the students still require formulae to apply the inverse square law, and struggle to fully conceptually grasp the mathematical scaling that applies to the inverse square law. However, they were able to apply the scaling to diverging field lines. The tutorial also showed the students prefer to utilise algebraic substitution and evaluation to apply the inverse square law, due to their perceived ease of use. This is also an interesting observation to note, as there are less mathematical operations involved in using the graphical or tabular analysis that the students completed. This may be due to the student's familiarity with solving quantitative problems, which rely on algorithmic problem-solving strategies. The post-test results showed that repeated exposure to the inverse square law promoted student's ability to perform proportional reasoning operations to show the ratio of reduction for 2 points, but over half the students relied on the use of mathematical formulae with substitution and evaluation to complete the task.

Additionally, the electric field homework interview and post-test scaling model question results indicate that the students do not consider the dimensional change in width and height of the scale model when using area problems, and do not link this to the change in distance. This error in student thinking can lead to them apply directly proportional thinking to the model, which in turn leads to inverse proportionality when applying concepts such as intensity and field strength to the model. Further development of these concepts, in a mathematical or physics context, could employ active learning strategies focusing on constructions of geometrical regular shapes such as squares and

rectangles which involve increases in dimensions of lengths and width, and recording and analysing the patterns between the variables could help students with these concepts.

The second research consideration addresses the student's ability to transfer their understanding of vectors to the electric field context. Section 5.2 detailed the student's progression in learning vector concepts in isolation of any physical context, and the discussions showed that progress was made by the students with regards to representing vector magnitude using the lengths of the arrows, accurately using vector constructions to show the superposition of two vectors and consideration of horizontal and vertical components of vectors. The electric field pre-test suggests that the students struggled to recognise and transfer this representation to the electric field context. Errors were seen in which students did not demonstrate the reduction in magnitude with distance and could not consistently apply the principle of superposition. It is unlikely they were unaware of the relationship between the distance from a point to electric field strength as the preceding Coulomb's law tutorial and class discussion introducing electric field reviewed the relationship. The electric field tutorial lesson afforded students the opportunity to apply vector concepts and vector addition to an electric field context. This explicit application was justified as the students required prompting to consider both the magnitude and direction of the field at different points, and then apply the principle of superposition. The student gains in transfer manifested in the post-test results, in which most of the students demonstrated the relationship between distance and electric field strength using vector arrows, and reasonable apply the vector constructions to find resultant vectors. This indicates that students learning vector concepts in isolation in not effective for contextual transfer, and the concepts require explicit application in a variety of contexts for students to utilise vectors as a tool to explain and represent those contexts.

The third research consideration addresses student's use of field lines in representing the electric field. Section 5.4 showed progression in student's understanding of field line density representative of relative field strength, the force on a body acting tangentially to a point on a field line and reasonable deviations of a body from a field line. The students did not demonstrate transfer of these concepts as much as was expected to the electric field pre-test. Few students could reasonably draw the path taken by a charged body in a field and field strength was ranked by students using qualitative reasoning based on distance instead of the field line density. The tutorial lesson allowed students to apply the field line conventions to field line contexts and gains were seen in the post-test results. As was seen with the vector concepts, explicit lessons in which field lines are applied to electrostatic contexts are required for them to correctly apply it to the context. This is justified as the results indicate the field line tutorial using a gravitational context alone were not sufficient in helping students develop and transfer the understanding of basic field lines to electrostatics. Additionally, the behaviour of negatively charged particles in field lines does not have an equivalent concept in gravitational field lines. Further explicit use of vectors and field lines in context is appropriate in magnetostatics and electromagnetism, to explain the variation of magnetic field strengths, application

and understanding of the right-hand grip rule and Fleming's left-hand rule and determining the direction of induced current in electromagnetic induction.

The extent to which the student's developed their understanding of Coulomb's law and the electric field varied during the tutorials. Figure 5.43 presents a line plot to illustrate and compare the extent of the conceptual change recorded in these studies. A legend of the codes used in Figure 5.43 can be found in Appendix F.

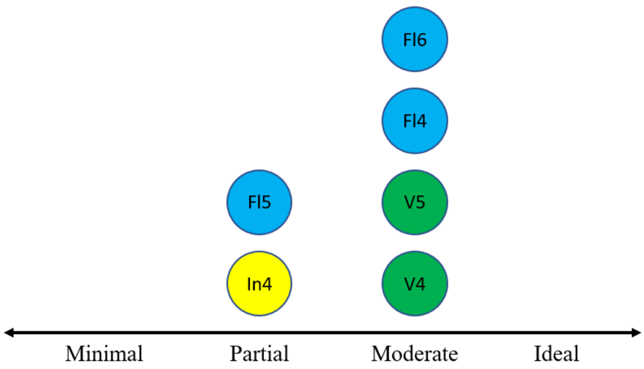


Figure 5.43. Line plot of extent of conceptual change in student's understanding of Coulomb's law and the electric field.

Unlike the results shown in section 4.5, the range to which the extent of conceptual change occurred was not as large between the student's transfer of vectors, the inverse square law and field lines in this section of the research. The most gains were observed in field line and vector concepts, but this is unsurprising as the most gains were observed in with these representations in chapter 4. The student's development and application of the inverse square law in an electro-static context was the most challenging, as seen by only one instance of partial conceptual change observed in Figure 5.43. As discussed in section 4.5, the tutorials in the electrostatic context also employed both mathematical reasoning and scientific reasoning when the students were applying the inverse square law to Coulomb's law, and when using the scale model applied to field lines, as opposed to field lines and vector concepts, when the students were primarily engaging with scientific reasoning. This requirement to employ dual reasoning may once again have overloaded student's working memory resources (Reid, 2009) and impede the student's progress.

5.8.2. Student's transfer between representations in electrostatics

This section addresses the last of the research considerations, in which present the conclusions for the student's ability to transfer. Section 5.7 outlines the student's ability to transfer between vector and field line representations. In transferring from field lines to vectors, persistent difficulties generally related to either students ignoring the magnitude of the vectors, relative to the field strength,

or drawing the direction of the arrows opposing the field lines. The former was a difficulty persistent seen in the students use of vectors, but suggests these students only considered the direction of the field lines when transferring between the representations. The latter difficultly it likely due to a previous question which involved the presence of a negatively charged particles influencing their rationale. This also indicates the students consider the field to be influenced by a charge that interacts with it, instead of being independent of the field.

In transferring from vectors to field lines, it was observed that just under half of the students drew the correct field diagram consistent to their vector representation. The most common difficulty observed was the student's reliance on drawing a field based on the position and types of charged bodies present, instead of being consistent with their vector diagrams. One student utilised the superposition principle to find the resultant vectors are various points, but also sketched field lines, in which their resultant vectors were consistent with their field lines, suggesting the student did not transfer directly from one representation to another, but utilised both representations to accurately show the field.

In comparing both transfers, it was seen that only two students iso-morphically transferred between vectors and field lines, i.e., they could transfer from field lines to vectors accurately, whilst transferring from vectors to field lines. The results suggest that the students were more comfortable with transferring from field line representation to vectors, with minor errors in the transfer present. When transferring from vector to field line representations, slightly more students completed this task effectively, but the errors shown by the remaining students suggested they did not consider the vectors but the setup of the charged particles, which is a major hindrance in their ability to transfer between representations.

One aspect of the difficulties observed in the student's transfer could relate to the mathematical underpinnings required to fully construct a mental model of electric fields. The mathematical courses completed by the students do not contain vector mathematics, nor have the students completed the calculus section of the mathematics course required to completed build a more accurate model of electric fields, in line with the scientific consensus. Issues of transfer between the representation were presented as rules, as opposed to delving into the reasoning underpinning them. For instance, the equation $\vec{E} = \frac{\vec{F}}{q}$ is treated as an algebraic equation as opposed to a vector equation. Reference to the direction of the force being tangential to the field is referenced by the sign of the variable is the same when the charge used is positive, and they're opposite when they're negative. However, without a deeper understanding of vector mathematics, the students unable form coherent models and instances of the students operating with incomplete models were observed in the tutorials sections of this chapter. These issues like have contributed to the relatively low numbers of students being able to transfer between vector and field line representations iso-morphically.

Chapter 6. Work and potential difference

6.1. Introduction

This chapter discusses the development of the student's understanding of the concepts of work and potential difference, as applied to electric fields. The Work tutorial employs the use of vector concepts and field lines to get students thinking about positive, negative and zero work. The potential difference tutorial employed verbal, mathematical and graphical representations, to promote student understanding of the concept, and ability to predict the behaviour of charges acting under the influence of a potential difference. The students also apply their understanding of work and potential difference, along with concepts covered in chapter 5, to explain the behaviour of current and potential difference in a simple circuit.

The following research question is addressed in this chapter:

- To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

The following points are considered when addressing this research question:

- To what extent does the use of vectors and field lines, representing force and displacement, enable students to identify positive, negative and zero work?
- What affect does the use of graphs and diagrams have on students understanding of potential difference?
- What difficulties are encountered by the students during this transfer to a potential difference context?

The timeline of this section of the project is shown in Table 6.1. Sections in bold refer to materials covered as they related to the research. Sections that are not presented in both are required to be covered for completion of the required syllabus for Leaving Certificate Physics. As the Coulomb's law, electric field, work and potential difference tutorial lessons were run concurrently, Table 6.1 starts on Week 5, which chronologically followed Week 4 of the Coulomb's law and electric field tutorial lessons, as was shown in Table 5.1.

The last two subsections of section 2.1.3 detailed difficulties encountered by learners in their understanding of work and potential difference. Based on the difficulties typically encountered by learners, as discussed in these sections, the Work and Potential Difference tutorials were designed

provide the students with opportunities to overcome these common difficulties. These difficulties influence the drafting of the learning objectives for this section, as upon completion of the teaching and learning material, the students would be able to:

1. Identify and explain instances of positive, negative and zero work (Lindsey, *et al*., 2009; Doughty, 2013)
2. Identify work and displacement, based on electric field line diagrams (Doughy, 2013)
3. Associate relatively high and low potential to positively and negatively charged particles respectively (Hazelton, 2013).
4. Explain the behaviour of charged particles, under the influence of a potential difference (Guisasola, *et al.*, 2002; Maloney, *et al.*, 2003; Hazelton, 2013).

Week 5, Class 1 (35 mins)	Presentation to introduce to review work **Pre-test**
Week 5, Class 2 (80 mins)	**Research lesson: Work worksheet.**
Week 5, Class 3 (76 mins)	Presentation to introduce potential difference. Practise class: qualitative problems.
Week 6, Class 1 (35 mins)	Further qualitative problems involving potential difference, work, potential energy and kinetic energy.
Week 6, Class 2 (80 mins)	Practice class: Qualitative problems involving Electric field.
Week 6, Class 3 (76 mins)	**Research lesson: Potential difference. Homework assignment given.**
Week 7, Class 1 (35 mins)	Review of topics.
Week 7, Class 2 (80 mins)	**Post-test.**

Table 6.1. Timeline of implementation of work and potential difference tutorial lessons.

This chapter presents a narrative of the students use of various representations as they were applied to the context of work and potential difference, whilst targeting the 4 learning objectives. The student's development is presented by comparing pre-test and post-test results for the different topics, as well as display the development of students understanding during the tutorial lessons, with both snapshots of their tutorial worksheets and extracts of recordings of the student's conversations that occurred during the tutorial sessions. Section 6.2 discusses the use of student's application of their understanding of vector and field concepts to develop their knowledge of work, and the use of various representations to develop their understanding of potential difference as a mathematical ratio of work done per charge, their association of relative high and low potential to charged bodies and

how charges move under the influence of a potential difference. Section 6.3 compares student difficulties before and after instruction, and comments on expanded contexts that were used in the post-test, that draw on concepts seen in chapter 5, the pre-test and the tutorial materials. The chapter closes with conclusions, which address the research question and considerations under the headings of the tutorial's impact on student learning, and how the use of various representations helped develop student understanding.

6.2. Work and potential difference tutorials

This section of the chapter discusses the implementation of the work and potential difference tutorials. Section 6.2.1 discusses the pre-test, which focused on student's understanding of work done as a charge is moved from in an electric field, how force and displacement vectors are used to determine several types of work, the behaviour of charged objects in the presence of a potential difference. It also focuses on the association of relative high and low potential to regions surrounding positively and negatively charged particles. This concept is illustrated in Figure 6.1 using a pHet simulation, in which the positive and negative charge are held fixed and equipotential lines are displayed. Any positively charged mobile test charge placed in the space between the charges held fixed would be influenced to move from left to right in this figure, while a negatively charged mobile test charge would move in the opposite manner. This concept would be more commonly applied to the relative potentials of the terminals of a battery, or the plates of a capacitor.

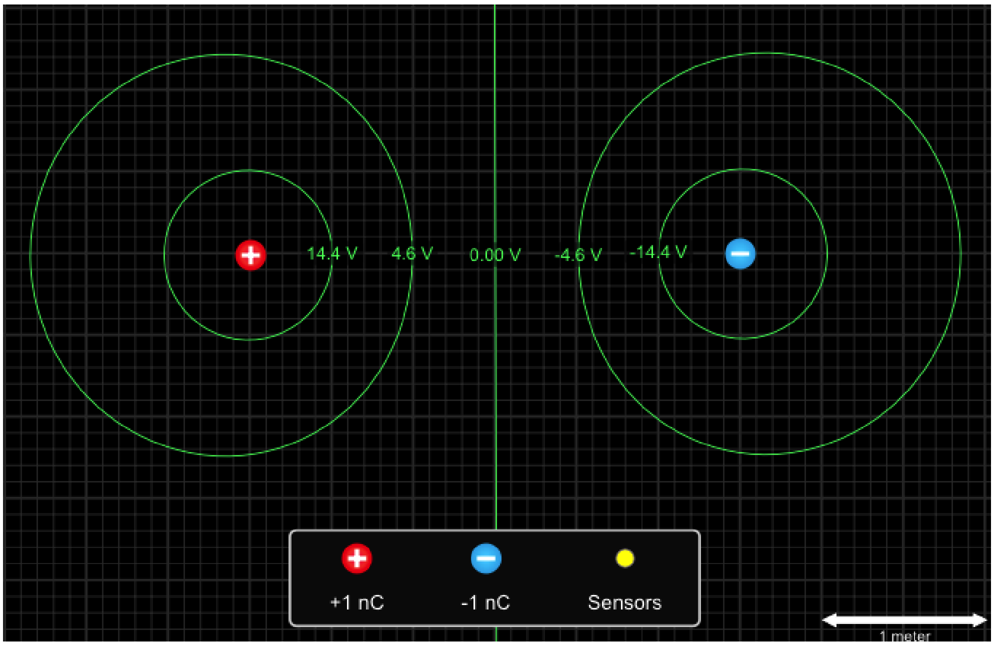


Figure 6.1. pHet simulation displaying the relative high and low equipotential lines due to the presence of positive and negative charges.

Section 6.2.2 and 6.2.3 narrate the Work and Potential Difference tutorial lessons, which identify instances of difficulties encountered by the students. Section 6.2.4 discusses the homework, which advances the students understanding of potential and applies a graphical approach to thinking about the variation of potential due to positive and negative charges. Section 6.2.5 analyses question given to students in the post-test, that either directly relate to questions asked in the pre-test, or present contexts that are extensions from questions seen in the pre-test, tutorials and homework. The tutorials were written in the same style as those seen in Tutorials in Introductory Physics, with some ideas and contexts taken from Conceptual Physics (Hewitt, 2009).

6.2.1. Pre-test: Work and Potential difference

The pre-test question was designed to test student's understanding of work, focusing on their understanding of displacement vs distance travelled when a body moves from one point to another in a field. Figure 6.2 shows the diagram, in which the students were to rank the work done in moving from A to B, along the three paths. Correct reasoning would show that the work done is the same in all cases. This could be argued by the displacement in all cases being the same, or discussing the contribution of positive, negative and zero work along all the paths taken. Expected incorrect reasoning would be the student's relying on the distance travelled along the different paths, instead of displacement. A summary of the student's responses is presented in Table 6.2.

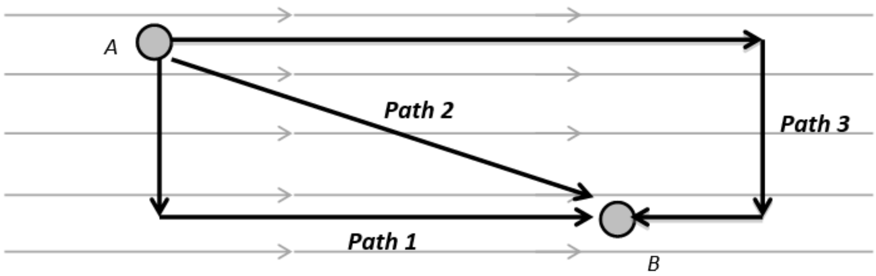


Figure 6.2.Extract from pre-test question in which student's rank work done in 3 paths.

The pre-test results indicate that the students did not consider positive, negative and zero work reasoning to the problem, and predominantly focused on the distance travelled along the paths as the justification of their ranking. Furthermore, five of the students referenced that in path 3, the path moves against the field which leads to more work, with two of the student referencing the ease taken along the other paths and the resistance encountered in going against the field. This suggests these students still conceptualise the field with tangible properties, in this case, as contributing to a resistance that must be overcome, similar in nature to swimming against the current in a river. Similar reasoning was also observed in student 4H's responses, in which they said the path was deflected by the field. They reasoned that path 2 had only one deflection at an acute angle, while path 1 deflected

at ninety degrees leading to more work involved in this path and as there was two perpendicular turns in path 3, there was more work in that path. Another student, 4C, referenced both the distance taken in the paths, but also noted that path 2 was the resultant of the components vectors for path 1, and equated this as contributing to more work in path 2.

Responses.	Students.
$\boldsymbol{W_1 = W_2 = W_3}$	**N/a.**
$W_2 < W_1 < W_3$	4A, 4D, 4E, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M.
$W_2 < W_1 = W_3$	4B.
$W_1 < W_2 < W_3$	4C.
$W_2 < W_3 < W_1$	4N.
Displacement / Positive, negative, zero work reasoning.	**N/a**
Reasoning based on distance travelled.	4B, 4G, 4I, 4J, 4K, 4L, 4M.
Reasoning based on "resistance" encountered.	4A, 4C, 4F, 4G, 4I.
Reasoning based on vector resultant.	4C.
Reasoning based on field "deflection" of paths	4H.
No clear reasoning	4D, 4E, 4N.

Table 6.2. Student's responses and reasoning to pre-test work ranking question.

In the second pre-test question, the students were required to rank the magnitude of the work done, for various pairs of force – displacement vector pairs. Christensen, *et al.*, (2004) noted ~75% of undergraduate students in their study could mentally apply the dot product to rank vector pairs when the angles were in the range $0^{\circ} \leq \theta \leq 90^{\circ}$, but when angles greater than 90° were introduced, the percentage dropped to ~60%, noting students had difficulties in recognising vector layouts that would produce negative work. Their research specifically looked at vector mathematics, whilst the pre-test question uses a work context and as the magnitude of negative and positive work are equal in this question, it was decided that the sign of the work would not have to be observed to consider a correct answer, if the students referred to the scalar nature of work. Figure 6.3 shows the question presented in the pre-test, and Table 6.3 summarises the student responses.

Rank the magnitude of the work done for the following pairs, (a) to (d), of Force – Displacement vectors.

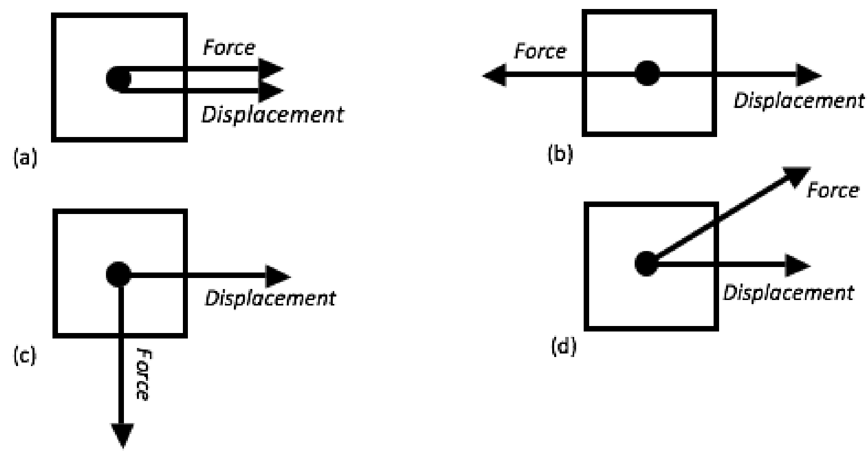


Figure 6.3 Pre-test question in which students use vectors to rank work done.

The results show that none of the students ranked the magnitude of the work done correctly in any of the setups shown. The most common incorrect ranking, produced by 9 of the students, was $W_A > W_D > W_C > W_B$, and the reasoning produced by these students varied, with three students (4C, 4D and 4E) using vector addition reasoning with the force and displacement. Two students (4F and 4M) referenced the force as adding resistance to the displacement when they are anti-parallel. These students may think force adds a thrust to the displacement when they are parallel, and does not affect displacement when perpendicular. Thus, the angle between the force and displacement would determine the work done (student 4G) and the relative displacement between the arrowheads of the force vector and displacement vector indicates the amount of work done in the system. Of the other rankings, only student 4H provided reasoning, suggesting as the displacement in all cases did not change, the force was indicative of the work and used this to produce their ranking.

The next question was designed to determine the student's understanding of the behaviour of charged objects in a potential difference. They were required to predict the movement of the two boxes, shown in Figure 6.4, and justify their predictions. A summary of their responses is shown in Table 6.4.

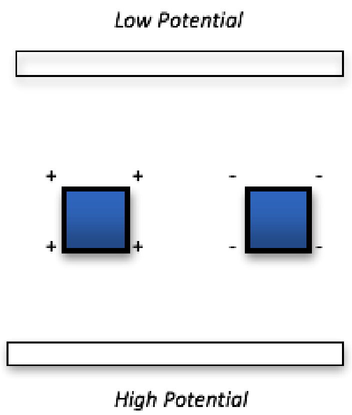
Responses.	**Students.**
$\boldsymbol{W_A = W_D > W_C = 0 > W_B}$	**N/a**
$\boldsymbol{W_A = W_B = W_D > W_C = 0}$	**N/a.**
$W_A > W_D > W_C > W_B$	4A, 4C, 4D, 4E, 4F, 4G, 4K, 4M, 4N.
$W_A > W_B > W_C > W_D$	4B.
$W_D > W_A = W_B = W_C$	4H.

$W_C > W_B > W_D > W_A$	4I, 4J.
$W_D > W_C > W_A > W_B$	4L.
Force and displacement cancel out.	4C, 4D.
Force decreases with distance.	4A.
Determines resultant of force and displacement.	4E.
References resistance.	4F, 4M.
Wider angle results in more work.	4G.
Force magnitude determines work.	4H.
Distance between arrowheads / vectors indicates work.	4K, 4N.
N/a.	4B, 4I, 4J, 4L.

Table 6.3. Student's responses and reasoning to pre-test work questions, based on force and displacement components.

A positively charged box and a negatively charged box are suspended between two charged plates, one which has high potential and the other has low potential.

(i) When the positively charged box is released, will it move (a) towards the high potential or (b) towards the low potential. Explain



(ii) When the negatively charged box is released, will it move (a) towards the high potential or (b) towards the low potential. Explain

Figure 6.4. Pre-test question about charged objects under the influence of a potential difference.

While the students correctly predicted the behaviour of the charged boxes in a potential difference, they did not apply correct reasoning to do so, based on the information they had. The most common reason used to justify the student's predictions was to associate a positive charge with the plate labelled high potential and negative charge to the plate labelled low potential. Once they had determined the charges on both of the plates, they predicted the motion of the boxes based on the force interactions between the boxes and the charge of the plates. While such reasoning is valid,

the students did not come across this in the tutorials nor the class discussions and may have (a) guessed this was how the potential of the plates can be used as a reference to determine the charge on them or (b) assumed that positive indicates high and negative indicates low due to positive integers having higher values than negative integers, which are relatively lower in value. Two students (4J and 4K) referenced the effect of gravity on the boxes, although student 4K's responses were not consistent with their predictions, in which the bodies would move in anti-parallel directions. Students 4C and 4D referenced that the boxes had less and more protons for their respective charges and moved to the low potential and high potential because of this, without clarifying why they referenced the protons as they did or explain how the behaviour was dictated by protons. Student 4B referenced that the bodies become unstable due to having potential energy. As they stated that both boxes would move from high to low, this may be a reference to gravitational potential energy.

In the final question on the pre-test, the students were given a graphical representation of a profile for potential difference and asked to sketch the positions of charges to produce the low – high – low potential variance, as shown in Figure 6.5. This question was designed to elicit student's thinking about the relative potential associated with positive and negative charges, positive being high potential and negative being low potential. A summary of the student's responses is found in Table 6.5.

Responses.	Students.
Positive moves from high to low.	**4B, 4C, 4D, 4E, 4F, 4G, 4M, 4N**
Positive moves from low to high.	4A, 4K, 4L 4J
N/a	4I, 4J
Negative moves from low to high.	**4A, 4C, 4D, 4E, 4F, 4G, 4H, 4J, 4L, 4M**
Negative moves from high to low.	4B, 4K
N/a	4I
References behaviour due to potential difference.	**N/a.**
Assumes charges on plates.	4E, 4F, 4G, 4M, 4N.
Use gravitational field as for reasoning.	4J, 4K
References protons.	4C, 4D
Instability due to potential energy.	4B
Reasoning unclear / no reasoning submitted.	4A, 4I, 4L

Table 6.4. Responses and reasoning to pre-test question involving the movement of charges bodies acting under the influence of a potential difference.

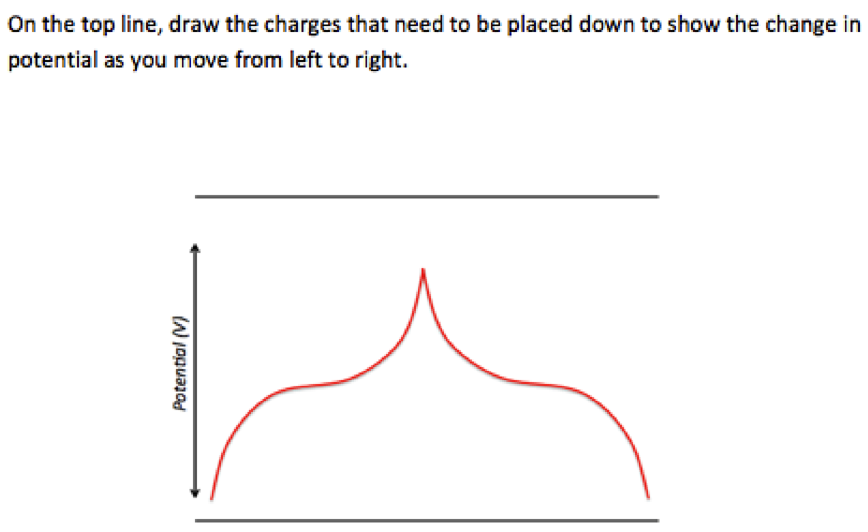


Figure 6.5. Pre-test question eliciting student's association of high and low potential to charges.

Responses	***Students.***
Associate high potential with positive charge.	**4B, 4G, 4I.**
Associate low potential with positive charge.	4E, 4F, 4L, 4M.
Does not associate charge with potential.	4A, 4C, 4D.
N/a.	4H, 4J, 4K, 4N.
Associate low potential with negative charge.	**4G, 4I.**
Associate high potential with negative charge.	4E, 4F, 4L, 4M
Does not associate charge with potential.	4C, 4D.
N/a.	4A, 4B, 4H, 4J, 4K, 4N.

Table 6.5. Responses to pre-test question involving the association of high and low potential of charged bodies.

A correct response to this question would place negatively charged particles at the two positions where the graphs are at the minimal values. A positively charged particle would also be expected to be placed at the position where the graph was at the maximum value. This would show the student applying the association of relative high and low potential to the areas around positively and negative charged particles, as described in section 6.2 and illustrated in Figure 6.1. The pre-test results show that most of the students did not associate relative high potential to regions with positively charged bodies and regions negatively charged bodies with relative low potential. Only two students, 4G and

4I, submitted the correct charges for the high and low potentials, with only student 4G submitting reasoning. Student 4G was consistent with their assumption shown in the last question, in which they associated high potential to positive and low potential to negative, whilst 4I gave no reasoning in either this question or the last question. Surprisingly, students 4E, 4F, 4M and 4N correctly guessed the association in the last question, but the first three of these students reversed their association in this question, while student 4N did not attempt the question. As the remaining students provided no reasoning for their responses, it is unclear as to why they chose the charges as they did, but it is probable that it was due to guessing.

This section detailed the student's initial misunderstandings and limits of their understanding of work and potential. It was seen that when considering work in a field, when different paths can be taken from one point to another, the prevalent difficulties encountered by the students considering the distance travelled over the displacement travelled, or the considered the field to act with a tangible property providing a resistance for the work to overcome in moving a charge against the field. It was seen that the students could not rank the work done in moving a box when the force and displacement vectors were aligned in various directions, due to an inability to identify positive, negative or zero work, or consider the absolute value of the work done, regardless of whether it involved increasing or decreasing the energy in the system. The third question showed the students could predict the movement of positively and negatively charged objects under the influence of a potential difference, but several students associated positive and negative charge with high and low potential, which was not consistently observed in the last pre-test question.

6.2.2. Tutorial lesson: Work

The tutorial lesson on work began with a twenty-minute class discussion and presentation of the concept of work. The initial context used was the same used in the mechanics section of their physics course, in which a car moves from one point to another and the students were asked to explain what effect a force has on the car when pointing (i) in the direction of the displacement, (ii) against the displacement and (iii) directed towards the ground, as shown in Figure 6.6. Using a think-pair-share strategy, the student groups were able to explain that when the force and displacement were parallel, the velocity of the car would increase; when in opposite direction, the velocity would decrease, and when perpendicular the velocity would not change. The teacher reintroduced the terms positive, negative and zero work and applied explained them in terms of the student's responses, as these terms were used when they initially studied mechanics. The students were then presented with a formula for work, $W = Fs\cos\theta$, and required to substitute values in for the three force-displacement diagrams shown on the car. The students were not presented with vector notation due to their unfamiliarity with the notation in their mathematics course, and because the aim to develop their

understanding in the tutorial was primarily qualitative and conceptual in nature. The discussion repeated for contexts such as a ball on the end of a compression spring pushed from its equilibrium position to a compressed state, and a charged particle being pushed towards a positively charge dome. These contexts were taken from Conceptual Physics (Hewitt, 2009).

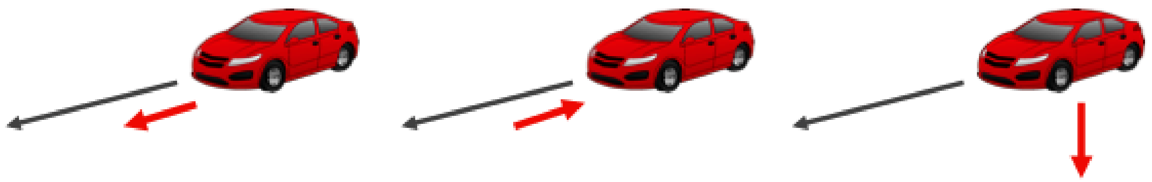


Figure 6.6. Initial context used to illustrate the concept of positive, negative and zero work.

The tutorial worksheet involved the students considering the path taken in getting from two points using different paths, as shown in Figure 6.7 (i). The students were required to explain the different between the distance travelled and the displacement. The students were then presented with a block of weight 100 N, that is pushed a total displacement of 6 m with 50 N of force. They were required to calculate the work done by the person pushing the block, and determine if any work was done by gravity, and if so, how it contributed to the movement of the block. The students were then required to consider the block being pulled with a force of 50 N through the 6 m, with the rope making an angle of 30° to the horizontal, as depicted in Figure 6.7 (iii). The students were required to resolve the force vectors, with acted along the 30° diagonal, its horizontal and vertical components, and determine which components contributed to the positive work and zero work. The students were required to calculate the net work done on the block, and were verbally asked to explain why the net work done, from the scenario presented in Figure 6.7 (iii) did not equal the work done when the force was applied horizontally, as presented in Figure 6.7 (ii), even though both blocks were moved with 50 N through a distance of 6 m.

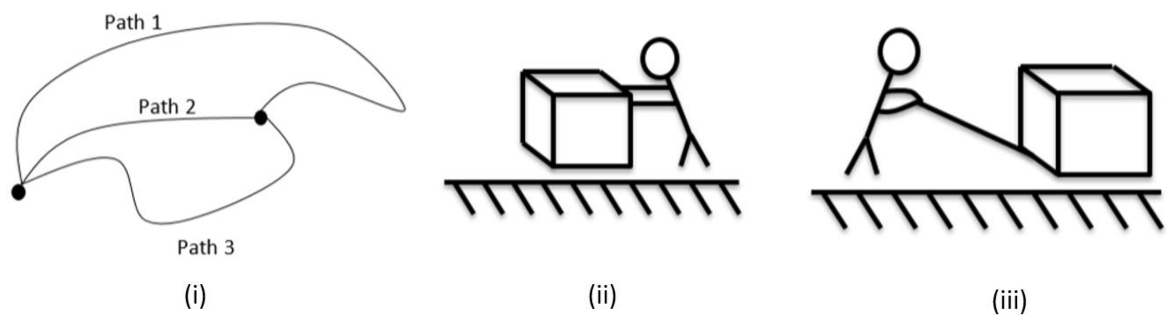


Figure 6.7. Diagram extracts from the work tutorial involving displacement and forces.

The students were then presented with the second pre-test question (see Figure 6.3), in which they had to rank the magnitude of the net work done in all four cases. They were encouraged to use

rulers to record relative magnitudes and, in some cases, resolve the force vectors into their horizontal and vertical components. Having discussed the concepts of positive, negative and zero work, as well as completing the previous questions in the tutorial, the students tended to produced rankings of $W_A = W_D > W_C > W_B$, or $W_A = W_B = W_D > W_C$. The students were encouraged to measure values from the diagrams by using a ruler and applying a scale of 1cm : 1N and 1cm : 1m, to determine the force and displacement respectively. In cases of the first ranking, students explicitly stated that the negative work in diagram B was less than zero, in cases of the last ranking, students stated that although the work was negative, it would have the same value as diagram A and D. One group displayed a unique error, consisting of students 4J, 4K, 4L and 4M, in which they added the force and displacement vector magnitudes, instead of multiplying them. This error is inconsistent with the work they completed in the previous section of the tutorial, in which all students multiplied the force and displacement magnitudes. One possible source of this error is the student's interpretation of the boxes Figure 6.3, in which they considered and applied vector addition when they observed the vector arrow pairs. If this is the case, this error came from the students misinterpreting the representation used, and were thus unable to correctly transfer the information to the mathematical symbolic representation, W = F.s.

Student 4H: 5.29 [J] is greater than 0 [J], which is greater than -5.29 [J]

Student 4K: The [horizontal] components in A, B and D result in 4 [J] because work is 2 + 2 which is 4, but because C has a vertical vector, the magnitude = 0.

In the last section of the work tutorial, the students were presented with the diagram shown in Figure 6.8, in which they were informed that a 1 kg mass was moved between different points.

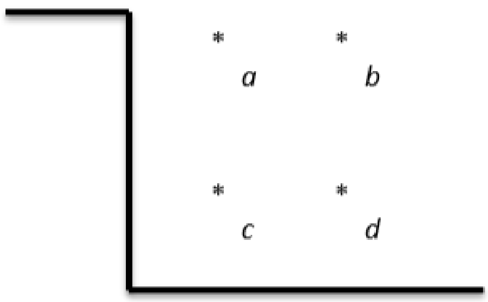
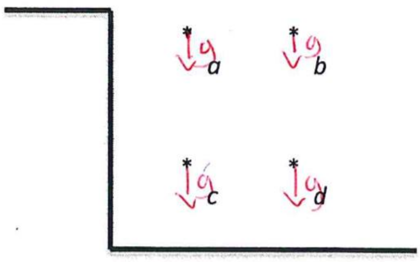


Figure 6.8. Diagram from work tutorial question focusing on positive, negative and zero work done by gravity.

They were required to identify the direction of the gravitational field, and determine if the direction of the displacement from *a* to *c* was parallel, anti-parallel or perpendicular to the gravitational field. This allowed the students to determine if the work done in moving from *a* to *c* was positive, negative or zero. From identifying the sign of the work done, the students had to

comment on the potential energy at the top and the kinetic energy at the bottom. The students then had to identify the work done in moving from *d* to *b* and *a* to *b*, and upon completion, were verbally asked to comment on the potential energy changes for these types of work done. Each group of students stated that negative work, with respect to the gravitational field, occurred when the body moved *d* to *b*, reasoning that the force of gravity was anti-parallel to the displacement travelled. They also reasoned that zero work occurred when a body moved from *a* to *b*, as the vectors were perpendicular. Having identified this, the groups verbally articulated that, with respect to the influence of the gravitational field on bodies between the points, positive work resulted a decrease in gravitational potential energy and an increase in kinetic energy, negative work resulted in an increase in gravitational potential energy and a decrease in kinetic energy and zero work resulted in no change in the gravitational potential energy. An example of a student's written response that displays most of these points is shown in Figure 6.9.

Student 4B:



The displacement is in the direction of the gravitational force. Positive [work moving from a to c] because the force is in the same direction of the displacement. The potential energy increases from c to a. The potential energy at a = to the kinetic energy at c.

Figure 6.9. Student 4B's response for section on work due to gravity in tutorial lesson.

This section discussed the student's progress throughout the work tutorial. It was observed that the students could identify instances of positive, negative and zero work. Having been guided through how to analyse and perform calculations to determine a value for work for instances in which force and displacement vectors diverge at an acute angle, all but one of the groups were able to apply this skill to a similar context. I suggest that the group's error was their interpretation of the information presented on the diagram, in which they decided to apply vector addition to the arrows, instead of multiplication of their magnitudes. The last section displayed that students could identify, in a field setting, positive and negative work, which was targeted so (a) they could apply it the concept of potential difference in an electric field context and (b) they could relate work the maximum potential and kinetic energies so they could apply conservation of energy calculations in electric fields, such as electrons moving in a cathode ray tube or x-ray tube.

6.2.3. Tutorial lesson: Potential difference

This section discusses the potential difference tutorial. The tutorial employs the use of field lines and displacement vectors to identify positive, negative and zero work. The students were then guided through developing the formal definition of potential difference, i.e., the negative of the work done by the field per unit charge when a charge is moved from point to another, through analysing a diagrammatic and mathematical scenario. The students were then given an electric field and were asked to complete calculations involving work, potential difference, and potential and kinetic energy.

Figure 6.10 presents the first electric field with various points marked. The students were required to identify the work done in moving through various combinations of the points, similar in nature to the exercise at the end of the work tutorial.

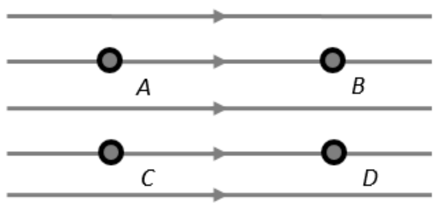


Figure 6.10. Diagram from potential difference tutorial focusing on positive, negative and zero work done on a charged body.

Having identified instances of positive, negative and zero work, the students were then required to consider paths taken in which involved a combination of positive and zero work, through analysing a dialogue between two hypothetical students.

Person 1: When we move the charge from A to D directly, there is less work done than moving it from A to C to D as we add up the work done moving from A to C directly to the work moving from C to D directly.

Person 2: When the charge if brought from A to C and C to D, the displacement has a vertical component which gives zero work. This makes the work done independent of the path taken.

By thinking about this dialogue the students would consider both the displacement between the initial and last point and how combinations of positive and zero work, or negative and zero work, can simultaneously contribute to a mobile test charge moving in a field. When the students were comfortable with applying the concepts of work to the electric field, the tutorial shifted to applying mathematics to moving charges in an electric field to develop their understanding of potential difference. They were presented with Figure 6.11, (i) and (ii), in which they initially considered the potential energy of a body lifted 3 m into the air, dropped to the ground.

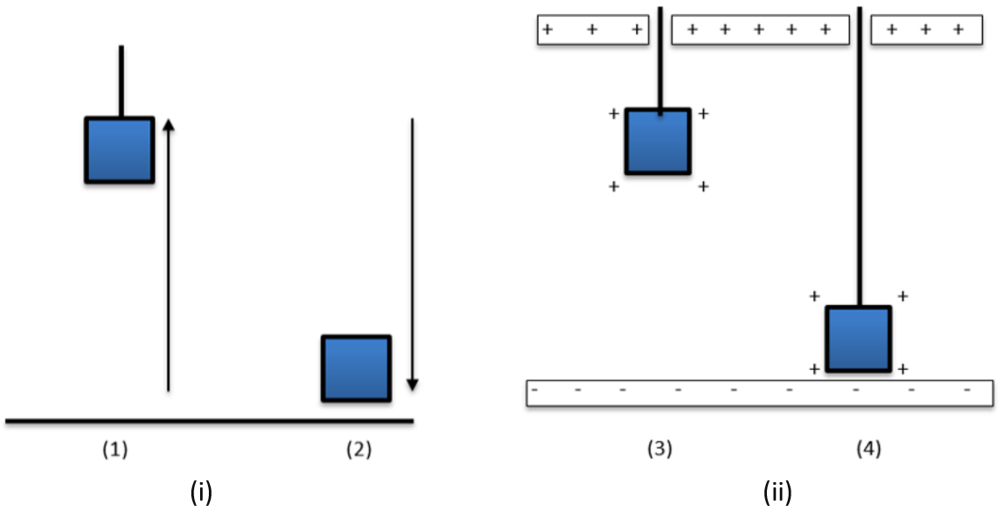


Figure 6.11. Diagram from potential difference tutorial comparing gravitational work with electrostatic work.

The students calculated the work done by gravity when 1 kg, 2 kg and 3 kg were dropped, and determined the ratio of work done per mass of the box. While there is no measured quantity for gravitational potential difference, this allowed the students to observe the fixed quantity nature of potential difference, in a context that is not abstract, in this case, the work done in moving 1 kg of a body. The students then compared the potential energy of the box at position (1) and the kinetic energy of the body at position (2) and expressed this in written form, and mathematically.

This set of activities was repeated using an electric field and a charged body, as shown in Figure 6.11 (ii). The students worked out the work done in moving charge bodies of +1 C, +2 C and +3 C a distance of 3 m. The students calculated the work done per unit charge in each case, observing that potential difference was a fixed quantity in the setup. The students had to summarize their finding by defining the ratio in their own words.

Student 4L: The work done is directly proportional to the charge → the ratios are constant.

Student 4B: Equal and proportional. The work divided by the charge is equal to 6/1, and is always constant.

Student 4I: 6 J of work is needed per +1 C of charge.

In the last section of the tutorial, the students were again presented with an electric field, and two points, A and B, as seen in Figure 6.12. The students were introduced to the formal definition of the potential difference, and the formula, $V = \frac{W}{q}$. The students were required to calculate the potential difference in moving a -1 C charge along path 1, and then path 2, and explain similarities between the two values. Most students said the potential difference was the same in each case, favouring explanations that reference the displacement between the points, instead of commenting on positive,

zero, and negative work reasoning, although one student, 4F, did identify the work moving from B to A would be positive but did not explicitly link it to either of the paths taken.

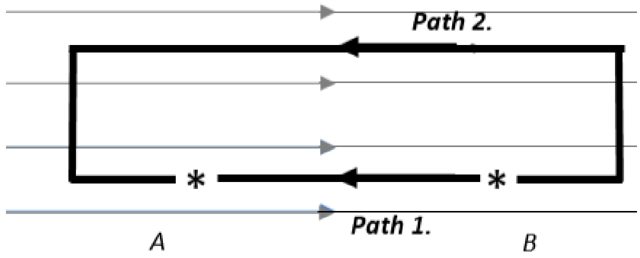


Figure 6.12. Diagram extracted from tutorial in which students apply work, potential difference and energy to different paths.

Using the potential difference value, the students were required to determine the work done in moving a -4 C charge and determine the magnitude of a charge if the field produces 36 J of work in moving the charge. They then applied their understanding of the conservation of energy to calculate the velocity of a body with mass 0.5 g and charge of magnitude -3 C as it moves from B to A. This allowed them to show that a body moving under the influence of a potential difference will convert potential energy, based on its position in a field, into kinetic energy. A sample of calculations is shown in Table 6.6. This demonstrates how the last section of the tutorial guided the students to relate the potential difference between two points in a field to apply the conservation of energy. By completing this, they demonstrate how to complete calculations involving potential and kinetic energy in electric fields. This is typical of the style of calculations the students complete when studying x-ray tubes, cathode ray tubes and particle accelerators, in which they need to utilise the potential difference of the device to calculate the energies and velocities of particles that move in the device.

Student 4H:	
$6 = \frac{w}{3}$ $w = 6\,x\,3 = 18\,J$	$w = \frac{1}{2}mv^2$ $18 = \frac{1}{2}(5\,x\,10^{-4})v^2$ $v^2 = 72{,}000$ $v = 268.33\,m/s$

Table 6.6. Example of calculations produced by student 4H.

6.2.4. Homework: Potential difference

The homework involved students using their understanding of attraction, repulsion and electric fields to build up an understanding of the behaviour of charged particles acting under the influence of a potential difference. As seen in the potential difference tutorial, in section 6.3.3, the students

initially explained the behaviour of a body acting under a gravitational field of high and low potential, and applied the reasoning to an electric field context, as seen in Figure 6.13 (i). They then extended their model, as seen in Figure 6.13 (ii) to incorporate the behaviour of negatively charged bodies in a potential difference, a phenomenon which has no equivalent behaviour in mechanics.

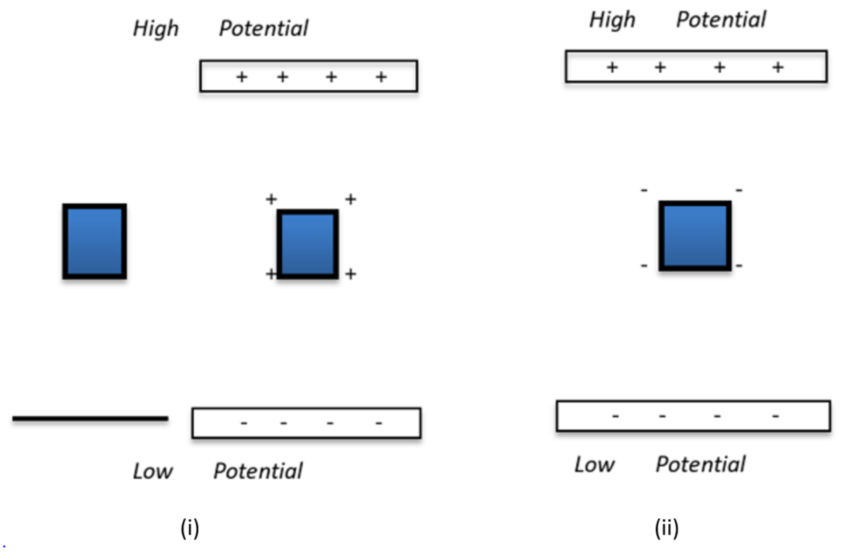


Figure 6.13. Diagram from homework for charged bodies moving under the infleunce of a potential difference.

It was seen that all students who completed the homework assignment explained that the positively charged box moves from high to low potential, generally noting how the box is attracted to the negative charged plate and reflects the behaviour of a body falling under gravity. They did not reference the interaction of charged box and the positive plate, and exclusively based their reasoning on the force of attraction they predicted. All the students acknowledged the negatively charged body would move up towards the positively charged plate and most could use this behaviour to determine that the negatively charged bodies would move from regions of low to high potential. However, student 4I and 4K contradictorily stated that it would move towards areas of low potential. When asked to explain this in feedback, they acknowledge that if the body is moving up towards the positive plate, it must move to high potential, and behave in the opposite fashion to the positively charged box. Student 4F explained that the body would build up potential energy, as it moved under the attraction to the positive plate. When asked to explain this, the student suggested that since the body was higher, it would have more potential energy. The student was asked to consider the phenomena as displaying the opposite behaviour to the positively charged box and asked to consider which of the two mimics gravitational fields and which acts in the opposite manner.

In the last section of the homework, the students were given a set of graphs with a charge layout, as seen in Figure 6.14. These graphs were to help students visualise and associated positively charged regions as areas of high potential and negatively charged regions as areas of low potential. The shapes of the graph were intended to show how relative potential decreases as the distance from a positive

charge increases, and the relative potential decreases as the distance as the distance from a negative charge decreases, as previously illustrated in Figure 6.1. Initially, the student had to explain the shapes of the graph, as the position from moves from left to right.

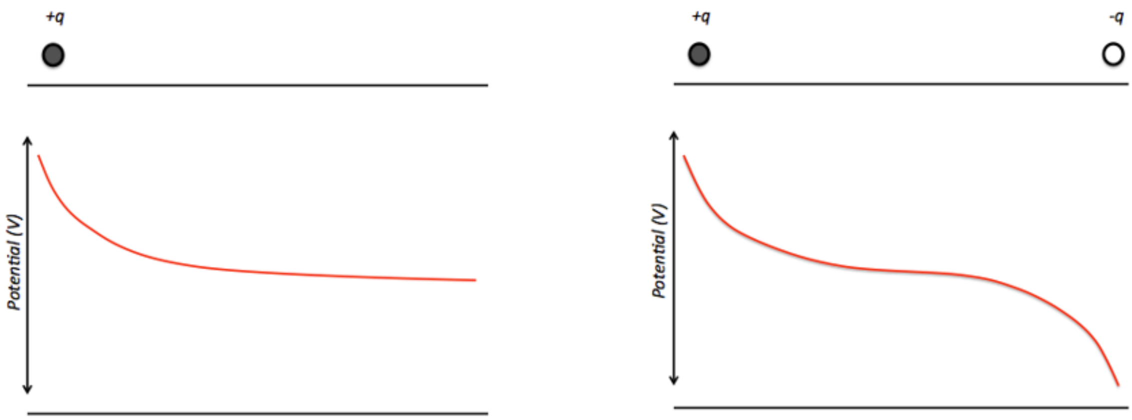


Figure 6.14. Graphs from potential difference homework.

The students had many difficulties with this section, and only two students (4B and 4G) explained that the first diagram showed variation from high to low potential while the second graphed showed variation from high to even lower potential, due to the negatively charged particle. Student 4K also indicated that moving from left to right showed moving from high to low potential but did not qualitatively compare how the presence of the negatively charged particle affected the potential.

A prevalent difficulty observed in the remaining seven student's responses was that they associated the negative particle as repelling the potential downwards or focusing on the shape of the graph and stating that the potential displays an inverse pattern that was stronger near the positive charge and weaker near the negative charge. The initial difficulty indicates a failure to understand potential as a property of an electric field produced by a charge. There is also an indication of attributing a tangible property to potential, which is commonly seen with field lines, but unexpectedly, this difficulty was translated onto a graphical representation. The latter difficulty shows an example of a student mathematically analysing the pattern shown, which accurate models the pattern seen in inverse relationships. The use of the terms weaker and stronger indicates a property of dominance between the two regions, typically seen in the superposition of vector quantities, such as force and motion. In both cases, students are attributing qualities and properties to the potential that the tutorial lesson and homework failed to address.

The students were then asked to graph the potential energy for the first set of charges in Figure 6.15, and then to sketch what types of charges could produce the graph seen the second half to the diagram.

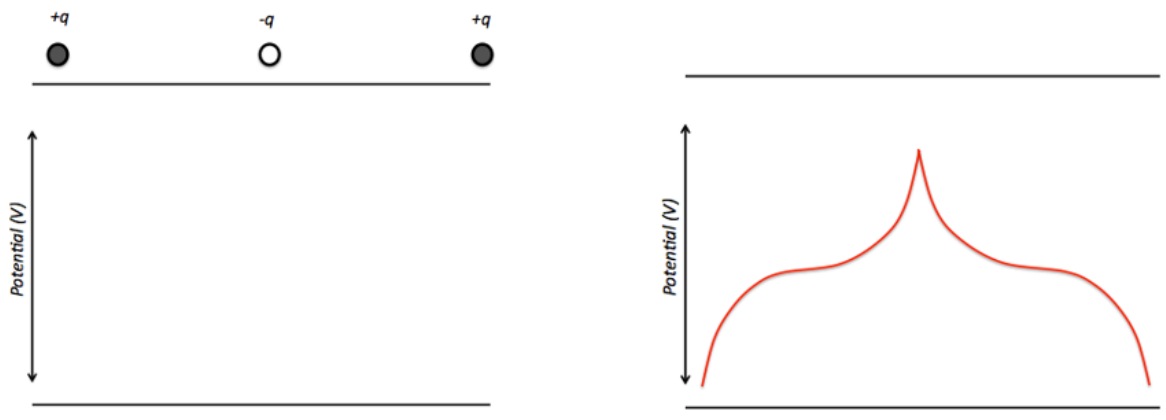


Figure 6.15. Graphs from potential difference homework, representing the variation of potential and charge layout.

All the students drew graphs of the correct shape for the first half of the diagram, with the two positive charges being regions of high potential, and the negative charge as a region of low potential. Additionally, all the students placed negative charges to the ends of the region and a positive charge in the middle, for the second diagram. However, the difficulties of attraction, repulsion and stronger and weaker potentials were also encountered in this section.

This section showed that using diagrammatic representations and drawing comparisons to between a body's behaviour in a gravitational field and a charged body's behaviour in an electric field can help student's construct their own understanding of how charged behave in a potential difference. It was also seen that the use of graphical representations can be employed, but difficulties about the nature of potential were seen and further instructional design would be required to address these difficulties.

6.2.5. Post-test: Work and potential difference

This section outlines the post-test results from the students, in which they apply their understanding of work in an electric field setup. The students also had to apply their understanding of the behaviour of charge under the influence of conducting spheres, connected by a conductor under the influence of an externally charged rod. The students were then required to use their understanding of vectors, forces, electric fields and potential difference to explain the behaviour of current in a closed circuit containing a battery and a wire. Two students were absent during the period this post-test was administered (N=12).

In the first post-test question the students were presented with an electric field, as shown in Figure 6.16, and asked to identify the direction of force acting on a negatively charged body placed at point *O*. A correct response for this question would be the student identifying that the force acts against

the field, due to the field pointing in the direction of a positive test charge. From this the students were asked to determine what type of work was done in moving a negatively charged particle to the points *A*, *B*, *C* and *D*. Correct reasoning would indicate the students were consider where the force and displacement vectors were parallel, anti-parallel or perpendicular to each other. The student results are presented in Table 6.7.

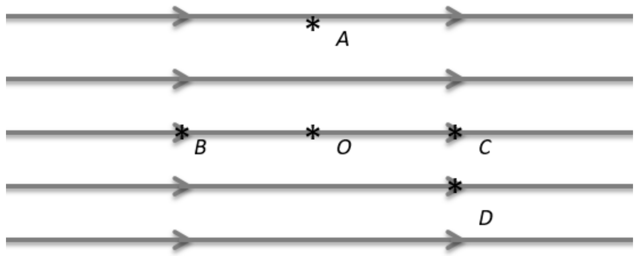


Figure 6.16. Diagram from post-test question involving work done in an electric field between various points.

Four of the students (4F, 4K, 4L, 4N) stated the stated that the negatively charged particle would follow the field line, not considering the convention that the field line points in the direction of force acting on a positively charged body. The remaining students correctly depicted the direction of the force acting on the negatively charged particle was anti-parallel to the field at the point *O*. Accounting for the error presented by the four students, it was observed that all the students consistently identified the work done was positive when the force and displacement vectors were parallel, negative when they were anti-parallel and zero when they were perpendicular. A summary of student's comparisons of the work done in the path (i) *O* to *C*, (ii) *O* to *D* and (iii) *O* to *C* to *D* is presented in Table 6.7.

The results show that only six of the students determined that the work done in moving the designated paths were equal. Four of these students (4D, 4G, 4M and 4N) produced explicit reasoning, in which they referred to the horizontal displacement producing non-zero work and vertical displacement producing zero work, or the horizontal displacement and force acting on the body being the same in all cases. Four students (4C, 4H, 4I and 4L) stated that only horizontal vectors need to be considered in this question, which alludes to their understanding of non-zero and zero work being done without being explicit about it.. Two of these students, 4C and 4H, produced the correct ranking with this reasoning. Students 4I and 4L stated that there is more work done in moving directly from *O* to *D* than there is in moving to *D* via *C*, using the same reasoning as students 4C and 4H. This suggests that these students applied the concept of non-zero and zero work when the vectors are explicitly parallel, anti-parallel and perpendicular, but struggle to apply this when the angle between the displacement and force vector is acute. This was seen in the case of the work done in moving from *O* to *D*, in which the displacement vector can be resolved into horizontal and vertical component, which contribute non-zero and zero work respectively, which was not considered by these students.

Responses.	Students.
$W_C = W_D = W_{CD}$	**4C, 4D, 4G, 4H, 4L, 4M.**
$W_D > W_C = W_{CD}$	4B, 4I, 4K, 4N
$W_D = W_{CD} > W_C$	4F
$W_D = W_{CD}$, W_C not defined.	4J
Perpendicular vectors produce zero work.	**4D, 4M, 4N**
Same force and same displacement in each path	**4G**
Only need to consider horizontal vectors.	4C, 4H, 4I, 4L
Displacement reasoning.	4B, 4F, 4I, 4K, 4N
Other	4J
N/a	4A, 4E

Table 6.7. Responses to post-test question involving ranking the work done in moving between different points in an electric field.

Two other students, 4K and 4N explicitly referenced that the displacement from *O* to *D* was the greatest but equated the other two paths. This suggests they also considered to apply the concept of non-zero and zero work to those paths but did not consider the displacement for *O* to *D* as a combination of horizontal and vertical vectors. Student 4F based their ranking on the absolute displacement for the start and end points of the paths. In this case, they ranked that *O* to *D* and *O* to *C* to *D* as having the same work done, but more than *O* to *C*, due to the smaller displacement in the last path. Student 4J also used the same reasoning for the first two paths but did not explain how the work done in the last path compared to the first two.

Another post-test question looked the student's ability to explain the behaviour of electrons moving under the influence of a potential difference, in a depiction of two metal spheres connected by a wire, with a galvanometer. The students were told that a rod of different charge is placed close to the spheres and that negative charge moved in the direction as shown on the galvanometer, as shown in Figure 6.17.

The students were asked to compare the potential on the two spheres in each of the three cases and explain their rationale. This question only references the initial potentials, just as the charges are being made to move, not the final potential when the charges have stopped moving and the potential is equal in all cases. A summary of the student's answers is presented in Table 6.8.

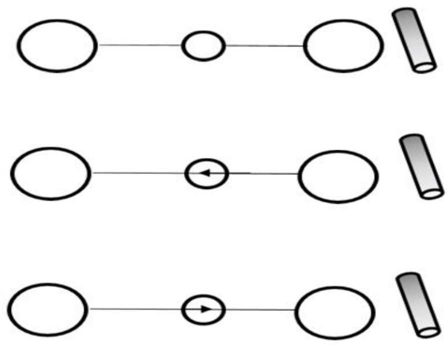


Figure 6.17. Diagram from post-test question eliciting understanding of the movement of charge in a potential difference.

Table 6.8 shows that the seven students stated the right dome had lower potential than the left when the charge moved to the left, and higher potential than the left when the charge moved to the right. This indicates that the students explained the direction of current moving is influenced by areas of low to high potential, in the initial moments of the scenario shown. One of the students (4G) used the movement of charge and considered how the charge built up on each sphere as time went on. For instance, in the middle diagram of Figure 6.17, their reasoning suggested that the right sphere was positively charged as it lost electrons. As it became positively charged, it developed a relative high potential. The opposite was true of the leftmost dome, in which it built up an excess negative charge and developed a relative low potential due to this build up.

Student 4C: The potential on the left is stronger than the potential on the right.

Student 4G: Low potential in the left [sphere] because it is negatively charged. High potential in the right [sphere] because it is positively charged.

Four students (4B, 4D, 4L and 4N) stated the opposite potentials for the spheres in the pictures, with reasoning that indicated they were considering the potential of the spheres are time moved on. For instance, in the last picture, in which the current moves to the right sphere, they indicated that as the charge builds up on the sphere to the right, it develops a negative charge and its potential lowers, whilst the sphere on the left builds up a positive charge and its potential rises. In the second case, they stated the opposite to be true. This suggests the students were considering the effect of potential at the spheres as the charges build up over time, although the question asked them to consider the potential at the start as the current just starts to move.

Student 4L: There is a higher potential at "A" [Left sphere] as the negatives have just travelled to "B." [Right dome].

Responses	Students.
Both domes have equal potential	**4B, 4C, 4D, 4F, 4G, 4H, 4J, 4I, 4J, 4L, 4M, 4N.**
Right dome has low potential, left dome has high potential.	**4C, 4F, 4G 4H, 4J, 4I, 4J, 4M.**
Right dome has high potential, left dome has low potential.	4B, 4D, 4L, 4N.
Right dome has high potential, left dome has low potential.	**4C, 4F, 4G, 4H, 4I, 4J, 4M.**
Right dome has low potential, left dome has high potential.	4B, 4D, 4L 4N.
N/a	4A, 4E, 4K.

Table 6.8. Responses to post-test question involving the movement of negative charge under the influence of a potential difference.

All students who attempted this question could indicate that the potential of both sphere was equal when no current was observed on the ammeter, when a charged rod was placed beside the spheres for a long time. The reasoning was based on the observation that if no current was flowing, no potential difference exists between the two bodies. None of the students however mentioned how the potential difference observed in the first two setups reduced to zero as time moved on.

Student 4C: The potential is equal. Meaning that the electric charge wo not move.

Student 4L: They're equal, as there is no electricity moving.

The students were also presented with a graphing question, in which they were required to sketch the variance in potential for a series of positive and negative charges in various positions. The two graphs setups the students were presented with are shown in Figure 6.18, and the student results are summarised in Table 6.9.

The results clearly show that the students associated high and low potentials with positive and negatively charged objects, as guided in the homework exercises described in section 6.2.4. However, the general shape of the graphs drawn by the students were crude and did not fully represent the behaviour of potential for point charges, in which the inverse relationship was notable missing or badly represented. Only two students, 4G and 4H, produced at least one graph which were considered to reasonably show the relationships, with 4H producing two graphs which showed the inverse pattern that potential around a point charge displays. The remaining student made errors in

which the potential only changed between positive and negative charges but did not change before or after the charges had moved between the spheres. In these cases, when the potential dropped when close to a negative charge, the potential remained low and did not increase unless a positive charge was close. There were also instances in which this was shown for the positive charge. The third graph in Figure 6.19 illustrates this, where it is clear student 4I sketched a constant value for the potential until their graph moved into the region that represented the space between positive and negative charge, where the shape of their graph showed a decrease. However, as the potential decreases as the distance from a charged particle increases, the constant potential shown should not have been sketched.

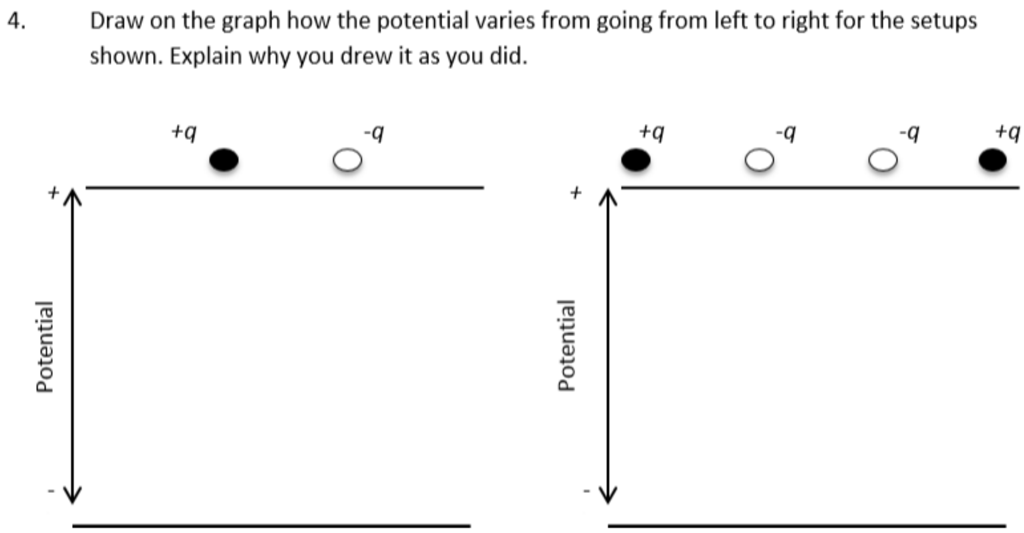


Figure 6.18. Post-test question, utilising graphical representations for potential.

Figure 6.19 shows the graphs produced by students 4G, 4H and 4I. Reasonable graphs can be seen by 4G and 4I, where they display not only the drop in the potential between the positive and negative charges, but also the increase and decrease in potential before and after the charges (i), and the increase in potential between the two negative charges (ii). In the final diagram, (iii), errors can be seen in which the potential was constantly high and drops between the positive and negative charge. In this case, an error is also seen in which the potential continues to drop as the graph moves to the left past the position of the negative charge. These errors were typical of those made by the remaining students. This indicates that students associate the positive and negative charges with high and low potential, but do not correctly consider the variation of potential around point charges.

Responses	Students
Positive is high potential	**4B, 4C, 4D, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N**
Negative is low potential	**4B, 4C, 4D, 4F, 4G, 4H, 4I, 4J, 4K, 4L, 4M, 4N**
Positive is low potential	N/a
Negative is high potential	N/a
Reasonably correct shaped graph	**4G, 4H**
Notable errors in graph shape	4B, 4C, 4D, 4F, 4I, 4J, 4K, 4L, 4M, 4N
N/a	4A, 4E

Table 6.9. Responses to post-test question involving the association of high and low potential with charged bodies.

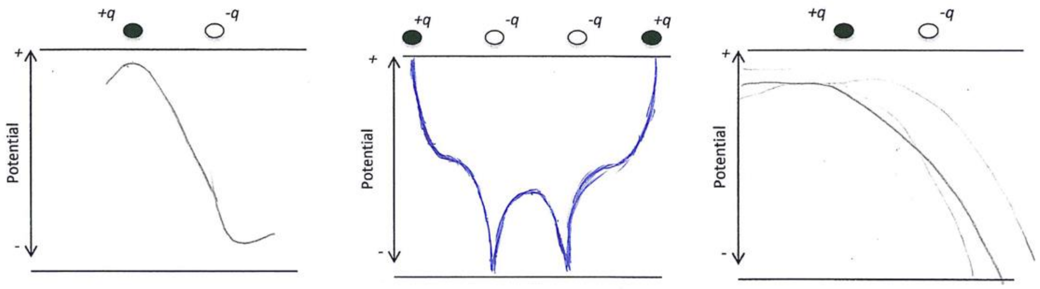


Figure 6.19. Examples of responses from students 4G, 4H and 4I.

In another post-test question the students were presented with the illustration of a battery connected by two wires from the positive to negative terminal shown in Figure 6.20.

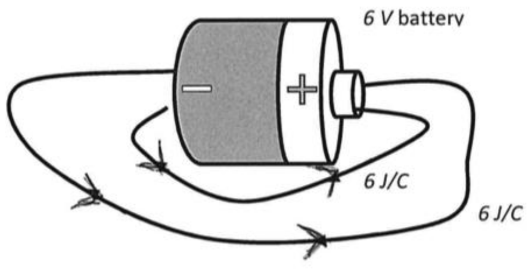


Figure 6.20. Diagram from post-test question requiring students to explain behaviour of current.

The students were required to explain the following two behaviours in the circuit, using the concepts they had learnt in the tutorial lessons:

1. Current (which is moving negative charge) flows from the negative to the positive terminal.

2. The *work done per unit charge* in moving charge from one terminal to the other is constant, regardless of the length / layout of the wire.

The students were given a list of key concepts, and asked to use their understanding of some, or all, of them to explain the two observations. The key concepts were work, potential, electric fields, electric field lines, vectors, the behaviour of negative charges in electric fields and the behaviour of negative charges between a potential difference. The forces of attraction and repulsion were explicitly omitted from this list, as to gauge how the students could apply the concepts mentioned in the list to explain the behaviour of current in a simple circuit. Although reasoning related to the attraction of the electrons in the current to the positive terminal, and repulsion between the electrons and the negative terminal are valid, the aim of the question was to elicit the other manners in which students could explain the behaviour of current. A summary of the student's concepts used in the first question is presented in Table 6.10. Students marked with an asterisk signify they referenced the concept but applied it incorrectly in this context.

Concepts referenced	**Students**
Attraction / Repulsion.	4B, 4C, 4D, 4G, 4H, 4J, 4M
Electric field.	4G, 4M, 4N*
Potential of plates.	4G, 4H, 4K*
N/a.	4A, 4E, 4F, 4I

Table 6.10. Responses to post-test question explaining the movement of current in a circuit.

The results show that seven of the students used reasoning based on the attraction of the mobile electrons to the positive terminal, and repulsion between the electrons and the negative terminal, of the battery. Six of these responses were exclusively based on these force interactions, without any further reference to the electric field or the relative potential of the plates suggested by the students. Student 4G expanded their explanation to include the behaviour of the negative charges in an electric field, in which they stated the direction of the field points from positive to negative and stating how the charge behaves in a field. They also stated how negative charges moves due to reasoning based on potential difference, stating that they flow from an area of low potential to high potential. Student 4H also submitted this reasoning based this potential difference, in addition to using attraction and repulsions reasoning. However, their explanation included a minor error, in which they referred to "potential" as "potential difference." The source of this error is unclear. Student 4M correctly used both attraction / repulsion reasoning, and reasoning based on the behaviour of the electrons in an electric field.

Student 4G: The negative current is repelled from the negative terminal of the battery and is attached to the positive terminal. Because it is negative, it is also attracted to the

high potential at the positive terminal of the battery. Negative charges always act in the opposite direction to the electric field.

Student 4H: Because the moving negative charge is attracted towards the positive terminal of the battery. Also, the negative charge goes towards the higher potential difference at the positive terminal of the battery.

Student 4M: Current flows from negative to positive because only negative charge moves and is attracted to the positive charge. The electric field lines go from positive to negative, but the charges go against the field lines. The is, the current flows from negative to positive.

Only two students, 4K and 4M, used reasoning that did not involve force. Both these students incorrectly used reasoning based on electric field lines and potential difference. Student 4M incorrectly stated that the field would point from the negative plate to the positive plate, which suggests they believed the electrons would move in a parallel direction to the field lines. Student 4K associated a high potential to the negative plate, and low potential to the positive plate, and indicated that charge would move from high to low potential. In both cases, it is observed the student's errors are rooted in incorrectly reversing the conventions of electric field and potentials, as they are applied to positive and negatively charged objects.

Student 4K: It travels from the high potential energy area to the low potential.

Student 4N: The field lines go from the negative to the positive.

The second part of this question required the students to explain why the work done in moving a unit of charge from one terminal to another in the circuit was constant. Again, the students were encouraged to use the key concepts listed in the question and apply them to the circuit to produce their explanation. A summary of their responses is presented in Table 6.11.

Concepts referenced	Students
References displacement between the plates	**4B, 4C, 4D, 4G, 4H, 4I, 4K, 4M**
References the force exerted on the current.	4G
Uses other reasoning.	4J, 4L, 4N
N/a.	4A, 4E, 4F

Table 6.11. Responses to post-test question explaining why the length of a wire does not affect the potential difference in the circuit.

As seen in the last part of this question, most of the students picked a concept with which they could explain the observation. The most prevalent response was that the terminals have a fixed displacement between them, so the work done in moving between them will be constant. Student 4G added to this by stating that the force acting on the charges would also be constant in both wires, presented in Figure 6.20. Although it is not clear what reasoning the student used to ascertain that

the force would be constant, as the net work done in both cases would be equal, this was considered a correct answer. Student 4J appeared to reference a model of current, referring to a constant flow in the wire, regardless of length. If referring to current by the flow, then the student infers that a constant current would associate constant work between the positive and negative terminal. Student 4L and 4N produced responses that reworded the observation of the direction of the current but provided no reasoning.

Student 4B: The work done is constant, because no matter how long the wire is, the displacement between the positive and negative terminal is constant.

Student 4G: The work done per charge is constant because the charge experiences the same force from the battery and moves a uniform displacement (straight line distance).

Student 4J: The work done is constant no matter the length of wire or layout cause it going to be the same flowing from 1 terminal to another terminal.

This section presented and displayed the results of post-test questions undertaken by the students that help elicit their thinking of work and potential difference. The results indicate that the students can identify instances of positive, negative and zero work using field line and vector representations. However, in instances in which the force and displacement vectors produce acute / obtuse angles in which combinations of zero and non-work are presented, students can encounter difficulties. It was seen that the students can generally compare, and justify, the potential difference between two points based on the movement of current, and display that when electrons have travelled through a system, the potential will equalise. When using their understanding to explain the behaviour of current in a circuit, it was observed that the students generally focused on explanations that involved using only one concept, instead of trying to apply multiple concepts to model their observations.

6.2.6. Discussions

This section compares pre-test and post-test data for student's understanding of work and potential difference. It reveals student's gains in both reasoning and being able to either apply various representations to the different concepts or discern and interpret information from the various representations. The concepts addressed in this section are the ability to identify positive, negative and zero work, identify work with regard to the direction of displacement using an electric field line diagrams (Doughy, 2013), associate higher and lower potential to positively and negatively charged particles respectively (Hazelton, 2013), and explain the behaviour of charged particles under the influence of a potential difference (Guisasola, *et al.*, 2002; Maloney, *et al.*, 2003; Hazelton, 2013).

The first of these concepts is the student's identification of positive, negative or zero work. The first question in both the pre-test and post-test probed the student's understanding of work. A comparison of the student's responses is shown in Figure 6.21.

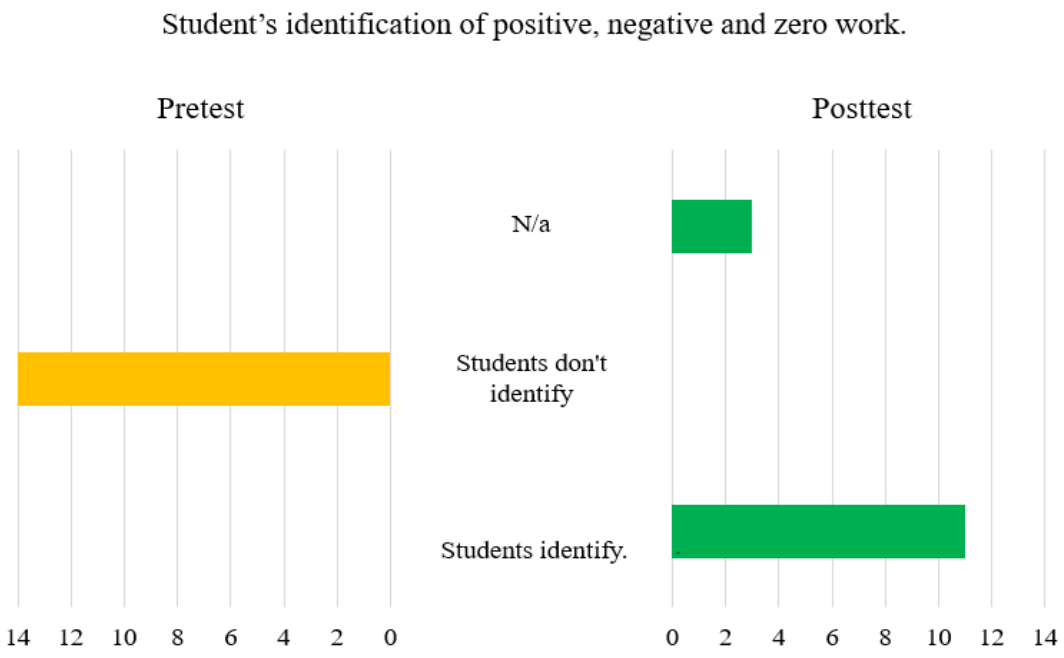


Figure 6.21. Comparison of student's ability to identify positive, negative and zero work.

In the pre-test, it was observed that none of the students identified instances of positive, negative and zero work. These students relied on using distance reasoning to determine the work done in moving from one point to another. In the post-test, all the students in attendance identified instances of positive, negative and zero work. Student reasoning focused on the displacement and force vectors being parallel, anti-parallel, zero or a combination of both. This progress reflects that observed by Doughty (2013). This shift in reasoning suggests that the tutorial was effective in enabling students to link their understanding of force, displacement and vectors and extend them to conceptually identify instances of positive, negative and zero work (Hewson, 1992; Konicek-Moran and Keeley 2015). The gain of 11 students developing their understanding indicates that moderate conceptual change occurred. Section 6.2.2. discussed the introduction of the concepts of positive, negative and zero work, and how the students applied this concept during the tutorial. The section illustrates an instance in which students were incorrectly applying the concept, but upon realising their error, they readdressed their understanding and figured out the application of work to produce the correct ranking (Posner, *et al.*, 1982). Section 6.2.3 presented the initial section of the potential difference tutorial, in which they applied this concept of work to an electric field context. Whilst there was a notable gain in the student's understanding from pre-test to post-test, it was seen that combinations of two types of work (work that has components that are both zero and positive/negative) caused difficulty for several students.

When students had to consider the absolute value of the work done, student's beliefs about the direction of the displacement of a charge in a field influenced their reasoning. Figure 6.22 shows students who considered work to be based on the total distance travelled by a charged particle in an electric field, and students who considered the work to be based on the net displacement of the charged particle.

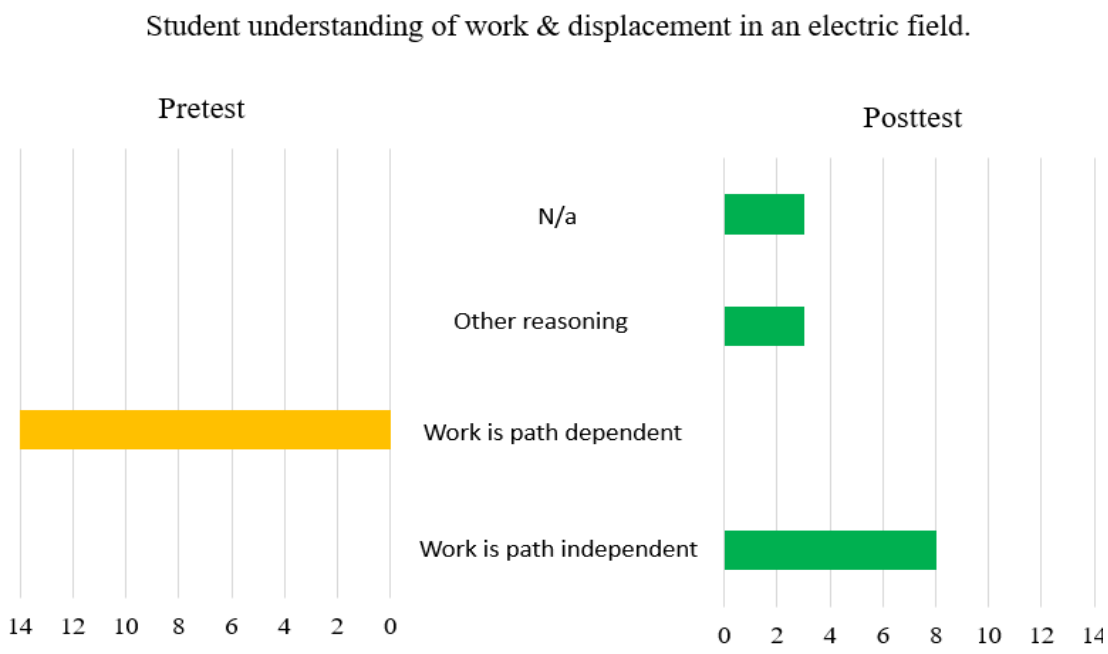


Figure 6.22. Comparison of student's understanding of the use of displacement in determining work done.

Even though the students had previously covered quantitative questions involving force, displacement and work during the mechanics section of their physics course, the pre-test results indicated they considered that the distance travelled affected the work, as opposed to the net displacement. To challenge this difficulty, the work tutorial addressed the quantities of distance and displacement using diagrammatic and verbal reasoning, as discussed in section 6.2.2. This allowed the students to apply both concepts to work, and become dissatisfied with the distance concept, and develop confidence to apply the displacement concept in an intelligible manner (Posner, *et al.*, 1982). Section 6.2.3 then illustrated how these concepts were applied to the potential difference context.

In the last post-test question, as discussed in section 6.2.5, it was seen that eight of the students shifted their thinking to consider the displacement between the start and end-point of a path in an electric field to determine the work done when current flows in an electric circuit. This indicates that conceptual exchange occurred and their conceptual understanding improves, as these students did not reference the errors observed in the pre-test and they applied the concept to an unseen context (Hewson, 1992; Konicek-Moran and Keeley 2015). The gain in eight students developing their understanding indicates that the extent to which conceptual change occurred was moderate. One difficulty that was persistent post-instruction was that some students reasoned that a constant current

in the circuit requires a constant voltage, regardless off the path taken. This reasoning would be erroneously transferred to electric circuits and does not account for variation of current caused by difference resistances in various branches of combinations of parallel and series components in circuits.

In the first question of the post-test, discussed in section 6.3.5, it was also observed that several the students did not consider the displacement vectors that were parallel and perpendicular to the field. Difficulty arose when students were required to consider displacements that were combination of parallel and perpendicular components. There was little difficulty in student identifying a path with two stages, the first being negative work and the second behind zero work. There was also no difficultly in students equating this work to a path in which only the first stage is taken. However, in a path which combined the positive and zero or negative and zero work, it was seen that students suggested the work would be greater than the two stages separately. This indicates that when considering parallel, antiparallel and perpendicular paths, the students analyse the problem in terms of positive, negative and zero work. However, when the paths taken make acute or obtuse angles to the field, the students shift their reasoning to think in terms of absolute displacement. The sources of difficulty with this thinking is the student do not consider parallel, anti-parallel and perpendicular displacement components separately, and relate them to the work concept. This difficulty is not directly reflected in the work done by Lindsey, *et al*., (2009), but in both cases, it was seen that a lackof understanding of whether displacement or distance travelled is the relevant concept when considering work can lead to student confusion.

The next section of the discussion associate relatively high and low potential to positively and negatively charged particles respectively. Figure 6.23 shows how the presents the students associations of potential to bodies of different charge, which was tested for using graphical representations.

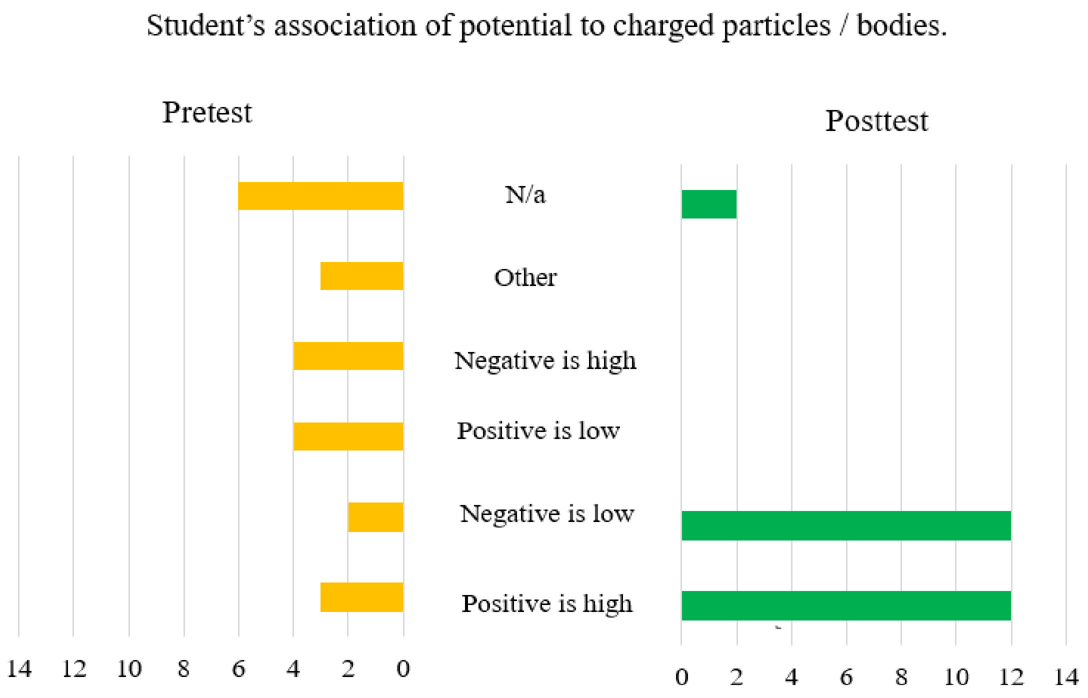


Figure 6.23. Comparison of student's association of potential with charged particles, using graphical representations.

The pre-test results clearly indicate that the students were unaware of the association of positive and negative charges to potential, with only two students successfully answering the question. In the post-test, there was a clear shift in student's associations to correctly apply the association to point charges. These results suggest that moderate conceptual exchange occurred, insofar there was a total gain of nine students correctly associating high potential with a positive charge relative to a low potential with a negative charge (Hewson, 1992). However, errors were observed in both the pre-test and post-test where students struggled to correctly represent the shape of the graphs correctly. However, the shape of the inverse patterns was not a target of this study and was not explored by the students during the tutorial.

One prevalent error in student's responses was that they appeared to represent the potential as constant until another charge increases or decreases it, or that the increase / decrease in potential continues past the point where the charges are placed, as shown in last student response of Figure 6.19. These students may have been considering a uniform electric field, which does not vary with distance, and applying this property to potential. As sections of the tutorials used contexts involving uniform electric fields, using field lines, more so than varying ones, this is not unlikely, but further work would need to be completed to help students separate these two types of thinking.

The last section of this discussion discusses the student understanding of the movement of a charge body under the influence of a potential difference. Figure 6.24 compares the student's pre-

test and homework results for this concept. In this case the homework assignment was used in lieu of a post-test question.

In the pre-test, it was observed that the students could predict the behaviour of charged bodies in a potential difference, and a drop in the student's correct responses was observed in the homework assignment. However, in the pre-test, the predominant strategy employed by the students was to assume the charges of the high and low potential plates. This is in line with difficulties presented by Guisasola, *et al.*, (2002). The homework exercise aimed to address this, by initially asking students to answer in terms in charges, and then answer in terms of potential. This appeared to enable some of the students to develop their understanding, but gravity-like thinking, in which all bodies move from high to low potential, was persistent in some of the student's responses, even when they directly contradicted their previous responses.

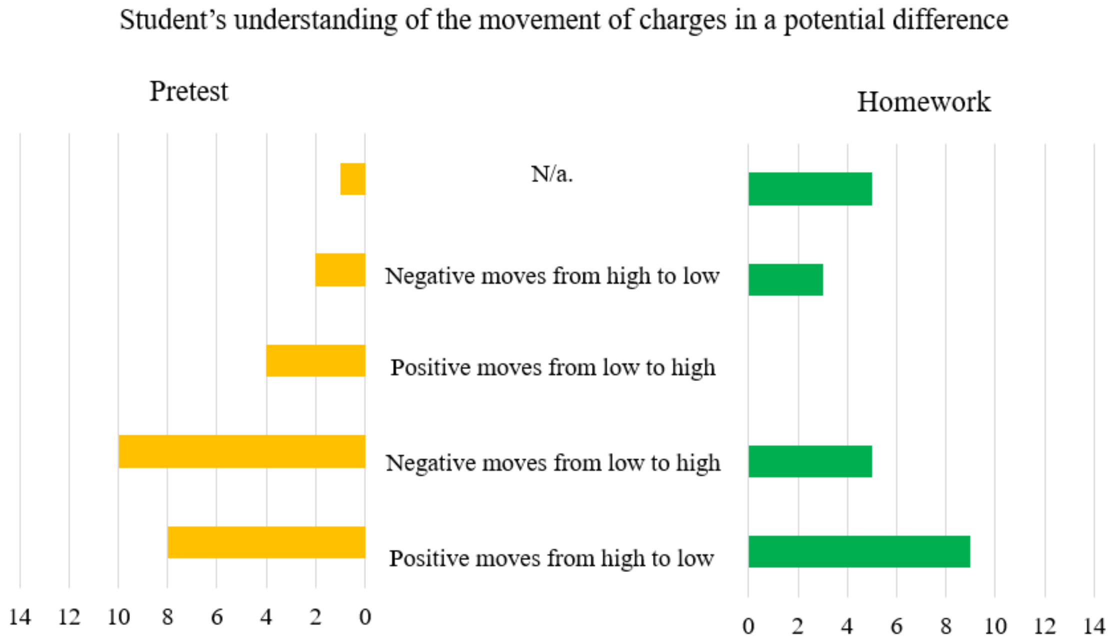


Figure 6.24. Comparison of student's understanding of the movement of charge under the influence a potential difference.

The reasoning used in the homework indicated that more students were thinking in terms of the potential difference of the scenario they were presented with, than the relative location of positive and negative charges in the presented scenario. This suggests that the tutorial helped promote conceptual exchange (Hewson, 1992), as there was a shift in the focus from considering charges to considering the potential difference of the setups presented in the pre-test homework assignments. Further time to develop this concept may be required, using a combination of static and current electricity contexts, to encourage the remaining students to focus on thinking and applying potential difference.

In the final pre-test question shown, the students were asked to explain the behaviour of charge in a field. The most common responses by the students involved the interaction of the charge with the charged plates, referring to the attraction and repulsion force experienced. Guisasola, *et al.,* (2002) stated that student's resort to using charge models to explain interactions when possible, even then other manners such as potential are appropriate or required. Attraction and repulsion was not listed on the key concepts to answer the question with, but the students favoured it. However, other responses in the student's homework and post-test questions suggest they could explain the behaviour under the influence of potential difference, when a question prompts them to directly. The student's familiarity with attraction and repulsion, and their interpretation for the apparent ease in using this reasoning may explain their reliance on using it. As the students demonstrated they could associate high and low potentials in line with the learning expectations of the tutorials, but their application of the reasoning was typically rooted in attraction and repulsion, the extent to which conceptual change occurred was determined to be partial.

This section presented the discussions on student's development of work and potential difference. Whilst the approach adopted in the tutorial helped the students develop understanding, there is still space for development. The use of field lines and vectors showed gains in the student's ability to reason situation of positive, negative and zero work, but difficulties were seen in correctly applying the displacement vector in situations where the force and displacement were neither parallel or perpendicular. The use of graphs and diagrams help students develop understanding of potential and the behaviour of charges in potential difference, but the use of graphs without the context of a mathematical formula produced errors for the student's understanding of potential and led to difficulties indicative of thinking about uniform electric field. The use of diagrams to enable students to develop understanding of the behaviour of charged bodies in potential difference opened the opportunity to use charge-based reasoning to explain the student predictions, and not focus on the potential difference concept.

6.3. Conclusions

This chapter seeks to address the following research question:

- To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

This question is addressed by addressed the following considerations:

- To what extent does the use of vectors and field lines, representing force and displacement, enable students to identify positive, negative and zero work?
- What affect does the use of graphs and diagrams have on students understanding of potential difference
- What difficulties are encountered by the students during this transfer to a potential difference context?

The approach developed used tutorial lessons that employed the use of vector and field lines to help students develop their understanding of work. Mathematically, work is a scalar concept that employs a dot product to produce a scalar from two vector quantities. This mathematics is not covered by the students and instead, the approach adopted looked to develop conceptual understanding. The work tutorial employed the use of vectors to enable the students to consider the relationship between force and displacement and produced an opportunity for them to deconstruct parallel and perpendicular component vectors, so they could be considered to produce non-zero and zero work respectively (Doughty, 2013). This skill was seen in the student's use of work in the potential difference tutorial, in which they could identify the sign of the work done in an electric field. However, component deconstruction was observed to be a difficulty in answering conceptual questions in the post-test and requires focus in future lessons for the students to correctly apply the skill. Additionally, how the work done in electric fields, as completed in the research, did not address the concept of how work increases or decreases the energy of a system.

The use of graphs and diagrams was seen to promote conceptual understanding of potential difference. The graphical method provided ease for students to develop an association of high and low potential to positive and negative charges. By students constructing their own sketches in line with the initial examples in the homework, they developed an intelligible method to apply the association of relative potential to positively and negative charged bodies in similar contexts, to allow them to apply and engage with the concept (Posner, *et al.,* 1982; Konicek-Moran and Keeley 2015). The use of diagrams provided a simple model for students to consider when thinking about the movement of charged bodies in a potential difference. Guiding student's reasoning to consider attractive and repulsive forces employed their prior knowledge and extended it by getting them to consider the potentials involved in the positively and negatively charged regions. This approach also reinforced their association of high and low potential to charged plates.

The representational approach highlighted difficulties in the student's models of potential difference. The use of graphs in the post-test questions elicited that students thought that potential drops remains constant along a path unless another charge increases or decreases it. This model is in error for point charges, although it is useful in explaining and representing potential along conducting wires in circuits with components (Reeves, 2003). The use of diagrammatic representation presents

the opportunity for students to be over-reliant on the use of charge interactions to explain processes that can be explained in terms of potential difference (Guisasola, *et al.*, 2002). In combining the association of potentials to charges, and the movement of charge under a potential difference, this model could be employed to help students understand the behaviour of a capacitor discharging, combining diagrammatic models involving potential and current, and graphing data of potential difference vs. time and current vs. time, showing many processes occurring using the different representations.

The extent to which the student's developed their understanding of work and potential difference is displayed in Figure 6.25. A legend of the codes used can be found in Appendix F.

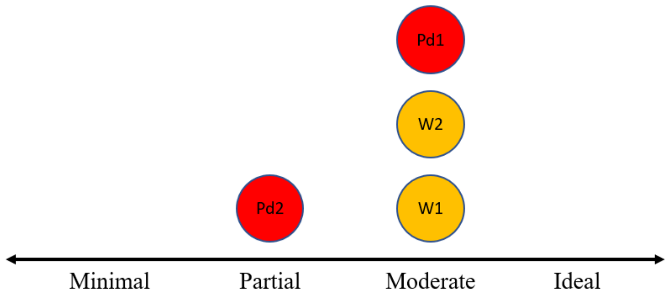


Figure 6.25. Line plot of extent of conceptual change for student's understanding of work and potential difference.

It was seen that the student gains in understanding from completing the tutorials were mostly moderate. The student's understanding of work and potential difference was rooted in enabling them to explain the behaviour of charged bodies under the influence of a potential difference. As the student's mathematical abilities are limited, calculus and vector mathematics was not employed in the tutorials and limited the depth of understanding developing of the students. Therefore, the developed student reasoning was phenomenological in nature, which impeded the mental models constructed by the students.

Overall, the use of multiple representations through structured inquiry tutorials encouraged conceptual change in the student's understanding of work and potential difference. There were persistent student difficulties observed that could be addressed in future research, and extensions to other topics to use multiple representations to explain the behaviour of different processes. Examples of such an extension study student's application of vectors and field lines to various processes in electromagnetism, applying tabular data, graphs, strobe diagrams and mathematical symbolic representations to developing students understanding in mechanics topics such as motion and the conservation of energy.

Chapter 7. Conclusions and implications.

This section summarises the main findings from chapters 4, 5 and 6, and discusses these findings in the light of the research questions presented in the last section of chapter 1. Section 7.1 presents the conclusions for the student's understanding of vector concepts, the inverse square law and field lines. Section 7.2 presents the conclusions related to the use of multiple representations to promote student's understanding of Coulomb's law and the electric field. Section 7.3 presents conclusion related to the use of field lines, vectors, graphs and diagrams in developing student understanding of work and potential difference, section 7.4 discusses implications for learning and section 7.5 presents further conclusions.

This thesis addressed 5 research questions in relation to student development of conceptual understanding in electrostatics. The 5 research questions were as follows:

RQ 1. To what extent does the use of a structured inquiry approach develop student understanding of vector concepts?

RQ 2. To what extent does the use of a structured inquiry approach develop student understanding of the inverse square law relationship, by employing multiple representations?

RQ 3. To what extent does the use of a structured inquiry approach develop student understanding of the field, when utilising the field line representation?

RQ 4. To what extent does the use of a multi-representational structured inquiry approach develop student understanding of Coulomb's law and electric fields?

RQ 5. To what extent does the use of multiple representations (vectors, field lines, graphs, diagrams) enable the students to demonstrate a conceptually correct understanding of work and potential difference?

The approach adopted in these research studies was structured inquiry tutorials. A small number of individual questions were directly taken from Tutorials in Introductory Physics (McDermott and Shaffer, 2003), but otherwise, the suite of pre-tests, tutorials, home-works and post-tests developed in this research are of original design. As discussed in Chapter 2, the literature that references student difficulties primarily revolves around research in third level, typically with undergraduates studying introductory physics. Published research that utilised the approach of Tutorials in Introductory Physics (McDermott and Shaffer, 2003) also revolves around undergraduates, while there is little

research published that looks at adopting the approach to the second level context. One piece of research that uses the approach at second level adopted two tutorials in a current electricity context and reports the gains by students by comparing the pre-test and post-test results (Benegas and Flores, 2014). Their study presents quantitative data in terms of student gains between pre- and post-tests but gives little indication of how the student's developed their understanding or what difficulties they encountered during the tutorials. In contrast, the studies in this research used specifically designed tutorials, along with many methods of data collection. This approach allowed me to probe, develop and assess the student understanding, by analysing and interpreting the qualitative data collected. The tutorials in this research were of original design to ensure the targeted concepts were accessible to the students at an appropriate level for their ability. The use of this approach enabled the collection of evidence to identify instances where conceptual change did/did not occur in the student's understanding of vector concepts, the inverse square law, field lines, Coulomb's law, the electric field, work and potential difference. The findings of these studies allowed for the determination of the type of conceptual change occurred, identifying instances of conceptual extinction, exchange and extension (Hewson, 1982). The approach adopted also allowed the extent to which conceptual change was achieved, ranging from minimal, partial, moderate to ideal.

7.1. Vector concepts, the inverse square law and field lines

When attempting to discern student difficulties when developing their understanding of Coulomb's law, the electric field, work and potential difference, it can be difficult to determine if the difficulties are rooted in the topics themselves or in prerequisite concepts. To address this, the approach that was taken in this research allowed the students to first develop their understanding of vectors, the inverse square law and field lines in a mechanics context, in the first set of tutorial lessons as discussed in chapter 4. Research questions 1, 2 and 3 address the development of the student's understanding for these concepts.

The inquiry tutorial focused on developing student's understanding of vector magnitude, their application of vector constructions and their conceptual understanding of vector addition in terms of superposition of the vector components and how it can affect the resultant magnitude of two, or more, vectors. The results indicated that moderate conceptual exchange was observed over the course of the tutorial lessons. The tutorials focused on students developing a conceptual understanding of vector concepts, without utilising specific vector notation or operations. The student's mathematical ability, or lack thereof, impeded the depth of understanding to which the tutorial lessons could target. Had the students been more familiar with vector mathematics, the opportunity to develop a richer understanding of vectors in a physics context may be been possible. Considering this limitation to construct the boundaries of the vector concepts targeted in this research, the results indicate that the

tutorials approach was effective in promoting conceptual change in the student's understanding. Such an approach would be fruitful for other teachers to adopt into their practise, to develop students understanding of vectors at upper secondary level.

The tutorials which focused on the inverse square law employed a multi-representation approach, which used a diagrammatic model involving scaling, tabular data and graphical analysis. The diagrammatic model which utilised area scaling enabled students to explain the behaviour of phenomena that follow an inverse square law. The use of multiple representations during the tutorials enabled the students to explore the inverse square law relationship both qualitatively and quantitatively. The extent to which conceptual change was observed in the tutorial lessons was in the minimal and partial range. This indicates that the students encountered difficulties in transferring between the representations and relating their qualitative reasoning to their quantitative findings. Two possible explanations and implications from this study are presented here. The first implication is that the student's unfamiliarity with inverse quadratic functions in their mathematical education could have hampered their learning. In lower secondary level mathematics, the student's encounter linear, quadratic and exponential functions and briefly explore inverse proportionality. Joint planning between mathematics and physics teachers in a second level setting could provide a better opportunity for students to develop a more consistent understanding of inverse relationship and mathematical operations related to them. This cooperative planning could also be extended to include understanding of inverse square relationships. The second implication relates to the use of the number of representations and the dual reasoning between quantitative and qualitative required during the tutorials. As there is a lot of information transferred between representations, it is plausible that the student's working memory was saturated with information and therefore they were unable to develop a coherent understanding of the inverse square relationship (Reid, 2009). Marzec (2003) states that multiple exposures to contexts that involve the inverse square law may be required before learners can develop an intelligible understanding of the relationship. The implementation of tutorials that allow students to encounter this relationship in various contexts would support upper secondary students develop their understanding. Over time, this could allow the students to cluster the information from the multiple representations and would allow them to free up their working memory capacity. This would enable students to incrementally form a coherent understanding of the inverse square law.

The findings from the studies involving field lines showed moderate and ideal gains in the student's conceptual understanding and their ability to use field line representation. The tutorials facilitated students to associate the field line density with relative field strength. Students were able to determine the direction of the field at various points, and to show that the path taken by a body in a field is influenced by, but not identical to, the pattern of the field lines (Törnkvist, *et al.*, 1993; Galili, 1993 and Cao and Brizuela, 2016). The tutorials focused on students transferring from vector to field line representations and explore the behaviour of bodies acting under the influence of the

field. As the students have limited knowledge of vector mathematics, transferring between vectors to field lines involved students primarily applying rules, such as the force is tangential to the field lines, without developing an appreciation for why this is based on mathematical equations using vector notation. In line with this constraint, moderate and ideal change was observed in the student's understanding of the field line representation. The students developed clear reasoning to explain the behaviour of bodies under the influence of a field, linking the representation to relevant vector concepts such as velocity, momentum, acceleration and force (Konicek-Moran and Keeley, 2015). Overall, the approach adopted in these tutorials was effective in promoting the students understanding of the basic behaviour that bodies display under the influence of fields, in a manner that is appropriate for teaching and learning at upper secondary level.

7.2. Coulomb's law and electric fields

Research question 4 addressed the development of the student's understanding of Coulomb's law and the electric field, by employing different representational tools for the topics. This section presents the conclusions of how the students revisited vectors, the inverse square law and field lines as they applied them to Coulomb's law and the electric field. The use of structured inquiry tutorials not only enabled me to gather evidence of the students as they developed their understanding of these electrostatic topics, but also to gather evidence of successful transfer of vectors, the inverse square law and field lines to the electrostatics context.

The prior learning of vector concepts developed in completing the tutorials presented in chapter 4 was not initially demonstrated in the electrostatic context, based on the pre-test results presented in chapter 5. This suggests that the students struggled to apply their understanding in a new context and/or the status they associated with the conflicting concepts was not in line with their initial understanding of these concepts. This is not surprising, as the students working memory must process concepts relating to vectors and simultaneously interpret information about electrostatics. If their initial understanding of vector concepts, discussed in chapter 4, were not clustered together, it is plausible the students working memory was full and therefore they were unable to apply the vector reasoning to new contexts (Reid, 2009). Revisiting these concepts in the tutorial lessons gave the students an opportunity to develop clusters in their working memory. Over the course of the Coulomb's law and electric field tutorials, the students revisited the concepts from the initial vector tutorial and transferred them to this context. Moderate instances of conceptual change were observed in the student's application of vector concepts to Coulomb's law and the electric field.

When students were applying the inverse square law to Coulomb's law and the electric field, partial conceptual change was observed in the students understanding. Using both qualitative and

quantitative reasoning in the application of the inverse square law in the electrostatics context, it was observed that there was a lack of coherence between the student's understanding and their ability to use the inverse square law mathematically. This suggests the students were operating without complete mental models but were effective with using the inverses square law to approach and solve quantitative mathematical problems. As discussed in section 7.1, multiple exposures to the inverse square law in context in tutorial classes could allow the students to cluster information regarding the inverse square law, freeing up working memory capacity to develop a deeper understanding of the relationship. The reasoning displayed by the students during teaching and learning interviews, discussed in section 5.5.3, indicated that when the students were prompted to focus on the change in dimensions in the area model, they displayed deeper conceptually coherent reasoning than was previously recorded in the initial tutorial, as discussed in chapter 4. This indicates that exposure to multiple explorations of the inverse square law across various contexts can incrementally develop student's understanding.

Difficulties were recorded with the student's transfer of their understanding of field line concepts to the electrostatics context. Gains recorded in the field lines tutorials and post-tests in the initial section of the research, as discussed in chapter 4, were not observed in the electrostatic field lines pre-test with similar frequency. Having completed two tutorials revolving around these representational tools, moderate conceptual change was observed to have occurred, which allowed them to effectively apply the concepts to electrostatic contexts. The Coulomb's law and electric field post-test results indicated that the students were proficient in interpreting field lines and predicting the behaviour of charged bodies in electric fields. As mentioned in section 7.1, the students lack understanding of using vector mathematics and this hindered the depth to which the students could explain the conventions they applied to using field lines. The direction of force being tangential to the field was referenced considering the definition of electric field strength, but the student's application of this was primarily phenomenological. Taking this into account, the tutorial lessons were effective in enabling the students to use field line representations to describe simple systems of charge particles, and could be adopted into teacher practise, not only for electrostatics, but also electromagnetism.

The difficulties targeted for conceptual change were chosen from reviewing literature, as discussed in section 2.3.1. The findings of the research indicate that the tutorial lessons were effective in addressing these difficulties, although persistent difficulties did remain in some cases. While not generalizable, the findings of this section of the research could be used by teachers who wish to develop the structured tutorial approach in their own practise. The findings presented could be used to inform the development in teaching and learning materials for other teachers who would wish to address these difficulties with their own students.

7.3. Work and potential difference

Structured inquiry tutorials were used in the teaching and learning of work and potential difference. In these lessons, the structured tutorial approach employed field lines, vectors, symbolic and graphical representations. This allowed for multiple collections of evidence to gauge the student's conceptual understanding across the different representations. The results can also be used to illustrate instances of conceptual extension, evidenced by the successful transfer of vectors concepts and field lines to work and potential difference.

The pre-test and tutorials on Work and potential difference highlighted that initially the students had conceptual difficulties with the implications of the dot product and did not consider the relative parallel / perpendicular components of the displacement and force vectors. The post-test results discussed in section 6.2 indicate that conceptual change occurred in the student's understanding of the relationship between force, displacement and work, but some difficulties persisted post-instruction. Conceptual extension occurred in which the students applied vectors and field lines to a mechanical work context, and exchange occurred as the tutorial was effective at developing student's understanding of positive, negative and zero work. However, scenarios which involve combinations of zero and non-zero work by parallel and perpendicular components of forces proved difficult for the students. Further development of student's understanding of displacement in terms of components, and its application to work could help alleviate this difficulty in students understanding.

The use of multiple representations in the context of potential difference allowed students to develop the concepts targeted in the research. The use of diagrammatic models allowed for a relatively easy manner for the students to determine the behaviour of charged bodies acting under the influence of a potential difference. However, some students continued to rely on reasoning based on the force of attraction and repulsion between a charged body and oppositely charged stationary plates. This suggests that conceptual extension did occur, but not in a manner that was envisioned during the design of the potential difference tutorial, as the majority of the student's relied on reasoning based on force interactions, as opposed to reasoning potential difference which the students only referenced if explicitly asked.

The employment of graphical representations was successful in helping students associate the high and low potential to point charges. In this manner, the graphical representation enables the students to engage in conceptual exchange, through their interpretation of the graphs. The sketches of the student's graphs provided evidence to indicate the student's association of the relative high and low potential of areas surrounding positively and negatively charged particles. However, some difficulties persisted, as a number of the students could not accurately represent of the variation of potential with distance for simple systems of two, or more, point charges and suggests further revision in this area would be warranted to complete the student's conceptual exchange. Their

understanding could be further developed by employing tabular data for various arrangement of charges and guiding the students to explain the variation of potential, before constructing graphs of their own design. This approach has proved fruitful in the inverse square law and Coulomb's law tutorials.

Prior to, and during, the carrying out of these studies, the participating students had not completed calculus mathematics - which is a necessary tool required to develop a complete understanding of potential and potential difference in an electrostatic context. Electrostatic potential is an abstract topic, with very little accessible examples that upper secondary physics students can relate to. Mathematics is an important tool in helping students develop an understanding of such abstract physics topics. Therefore, this study was limited to students developing a phenomenological understanding of work and potential difference in the electrostatic context. Partial and moderate conceptual change was observed in the students understanding of work and potential difference, but it is acknowledged that their mental models are likely incomplete due to the lack of possession of the required mathematical tools to fully explore these concepts.

As the difficulties targeted for work and potential difference were recorded in research literature, the findings of this research could be used to aid in the design of teaching and learning sequences in other classrooms. This could allow other students to address the common difficulties when learning about work and potential difference.

7.4. Implications for classroom teaching and education policy.

The use of the inquiry-based learning approach in this work enabled the students the opportunity to overcome difficulties related to electrostatics. As discussed in section 2.1.2, Mestre (1991) and Roth (1990) suggest this opportunity would likely not have been provided in traditional lesson sequences, and it is likely the difficulties in student understanding would have been unknown to both the students and teacher as the students progressed through their physics course. The approach adopted allows for the specific identification of student difficulties in upper secondary level electrostatics topics, and difficulties in student's ability to transfer conceptual understanding between representations. When these difficulties were addressed, there were gains in student understanding evidenced, that varied from conceptual extinction to conceptual exchange and conceptual extension.

Having covered the various representational tools in lesson before electrostatics, it allowed for discussions involving these concepts and tools, to help students develop deeper understanding than it typically afforded in traditional approaches. The use of multiple representations in inquiry learning, as presented in this research, allowed multiples dimensions to collect evidence of student understanding of electrostatics. Comparisons of the student's responses in the different

representations can gauge if they were consistent in their reasoning when addressing the same concept in different ways, or if the representation influenced the reasoning used by the students. This also has implications for assessing student's understanding in other topics in physics, and some aspects of mathematics.

The use of inquiry-based learning that employ multiple representations is not limited to developing student understanding of electrostatics concepts. The student's understanding of vectors, field lines and the inverse square law and utilisation of graphs, tabular data, diagrams and mathematics can be applied to other domains of secondary school physics, such as mechanics, optics, radioactivity, sound and electromagnetism. By employing inquiry-based learning in these contexts, a teacher may gather evidence of student learning, such as pre-test and post-test data, as well as insights from the tutorials themselves, to make a judgement as to the efficacy of the teaching and learning that occurred. By determining the overall impact that the approach had on student learning, they could adjust their lesson planning for future iterations to tackle difficulties they notice with their own students. The tutorials not only allow for the teacher to address difficulties in student learning, but also to explore how students to engage in tasks, in which they develop reasoning to address difficulties and engage in conceptual change.

In addition to the use of this approach in teaching upper secondary physics, this research highlighted other issues with student's development of their understanding of Coulomb's law, electric fields, work and potential difference. There were several instances in which students lacked the appropriate mathematical knowledge and understanding to develop a complete understanding of the electrostatic concepts targeted and this impeded their ability to form complete mental models, as discussed in the conclusions of chapters 5 and 6. The student's ability to perform calculations based on the inverse square law, vector and calculus mathematics and understand the mathematical concepts underpinning these is necessary to develop a coherent model for understanding electric fields, work and potential difference. Numerous actions could be considered to address this. Educational Policy on curriculum and assessment could align the learning outcomes of mathematics with the outcomes of science, technology and engineering subjects to ensure that the students also explore these concepts in their mathematics courses, enabling them to develop and apply their understanding. This coherence in policy would enable consistency in teaching and learning in classrooms, and promote students developing links across different subjects. Another action could be to require physics teachers to dedicate lessons to focus on the development of mathematical understanding and continuously build on student's understanding over the implementation of the physics course. A final proposed action would be for education policy makers to review the level of coherence between the mathematics and the physics syllabi. For example, in Ireland there is a shift towards learning outcomes-based curricula and this provides an opportunity to align the two emergent specifications. In the cases where this cannot occur, there is an argument to make that certain topics in physics may not be appropriate for secondary level and it would be more appropriate

to be taught in third level. At this level, the students would have acquired the necessary mathematics to develop conceptually accurate mental models of these concepts. While these actions have their advantages and drawbacks, they are worth considering by policy makers and educational stakeholders.

Another implication arising from this research addresses the role of assessment in physics education. Assessment can be a strong influencer on what pedagogical approach teachers use in their classrooms. As shown in Figure 1.1, the assessment of upper secondary physics in Ireland relies heavily on content that can be memorised and uses qualitative problems that can be solved using algorithmic procedures (SEC, 2015). The inclusion of a broader range of questions, such as qualitative conceptual questions, two-tier ranking questions or diagnostic questions in physics could enhance the assessment of student understanding and influence teacher's classroom practice. This approach could provide evidence of the student's development of mental models and problem-solving strategies.

7.5. Implications for research.

Possible extensions to this work would be to conduct the research with a bigger sample size. As these research studies were completed with a small group of students, the findings of this research would not be generalizable to the wider population of students learning physics at upper secondary level. However, a series of research studies in which gathers data from a larger group would be informed by both the student gains and student difficulties presented in this thesis and could be used to reliably determine how frequent each of the gains and difficulties occur with different groups of students. If multiple teachers were to adopt the approach, the research could also gauge teacher attitudes to the use of structured inquiry tutorials and multiple representations in their own practise, and gauge how teacher implementation of the approach affect's student's understanding. Another extension would be to gauge the efficacy of adopting structured inquiry tutorials and multiple representations in other domains of Physics at second level. The topics covered in this research would allow for extensions into electromagnetism and mechanics topics.

Research regarding the use of tutorial lessons primarily focuses on the development of conceptual understanding in third level, but it is less studied at secondary level. One example of such research is Benegas and Flores (2014), who implemented tutorial lessons with upper second level students in Argentina, in which they presented quantitative analyses of pre-test and post-test results, with little insight into the student's conceptual development. The research presented in this thesis presents qualitative findings with upper second level students, developed using the tutorials approach, which is relatively unreported in the literature. This research represents a novel approach

in employing the use of the structured tutorials at secondary level and highlights the opportunity for further research in this area. A coordinated research study that uses both qualitative and quantitative findings would validate the efficacy of this approach and illustrate how this approach can enable students to develop their understanding and form coherent mental models of various topics of physics. Tutorials in Introductory Physics (McDermott and Shaffer, 2003) address multiple topics in Physics and they could also be used as a guide to draft and develop tutorials that adopt the tutorial approach in the second level context, as they were for this research. The findings of this research could also be used to guide future research at lower secondary level science and upper secondary chemistry and biology. If the approach were to be used at lower secondary level, it may be advisable to limit the representations to diagrammatic, graphs and tables, to help students understand how a process occurs, but remove any complex symbolic representations, as their mathematical ability to interpret them effectively would likely be underdeveloped. At upper secondary chemistry and biology, a focus for the structure tutorials on visualising complex processes could help students link observations between the atomic-scale, micro-scale and macro-scale.

A conceptual change model was employed as the underpinning theoretical framework employed in this research. This is not the only framework that could have been utilised. A framework revolving around developing and assessing the student's mental models could have been employed. The various types of evidence collection used in this research and use of multiple external representations could be used as indicators as to the mental models the students possess. However, unlike the closely related topics of magnetism (Borges and Gilbert, 1998) and current electricity (Borges and Gilbert, 1999), there is not an abundance of literature of naïve models of electric fields in which to compare to compare the student's mental models to. A future extension to this research could be to probe students understanding and establish descriptors of naïve mental models of electric fields, in a similar manner to that used by Borges and Gilbert (1998, 1999). Upon completion of the tutorial lessons, the analysis of the collected evidence could be used to determine what initial mental models the students were operating with, and how these mental models changed over the course of the lessons. The student's overall conceptual understanding would still be developed and assessed, without any critical changes to the tutorial lessons required to take place. Employing a modelling framework to the approach taken in this work could extend the research presented in this thesis and align it with trends in modelling research currently taking place in the physics education research sphere.

Chapter 8. References

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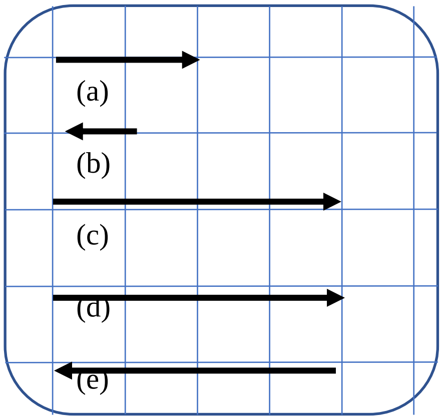
Appendix A

Vectors tutorial materials.

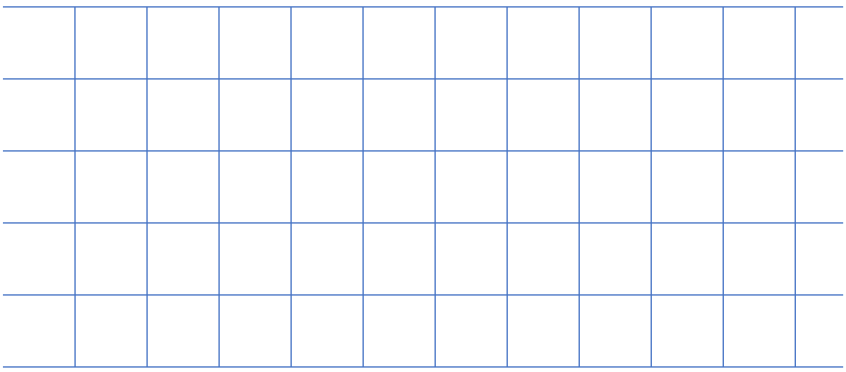
1. Rank the following vector arrows (a – e) from weakest magnitude to strongest magnitude.

Ranking:

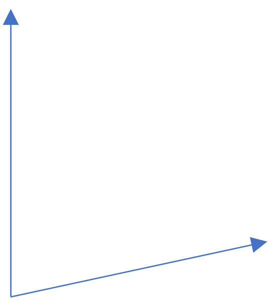
Explanation:



2. In the space below, Construct the resultant vector of (a) and (c) from question 1.

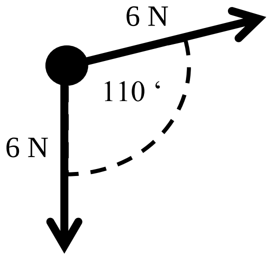


3. Construct the resultant vector of the following two vectors.



4. A charge experiences a force as shown in diagram (i). An equal charge experiences 2 forces as shown in diagram (ii).

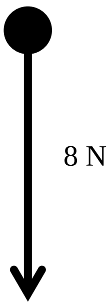
(i)



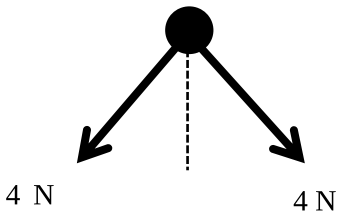
(ii)

Which charge experiences the most force? Explain your choice. You may draw on the diagrams if necessary.

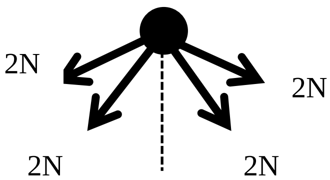
5. The following set of diagrams shows charges experiencing multiple forces. Which, charge, if any, experiences the most force? Explain. If they are the same, state so explicitly and explain why. (Angles shown are in (iv) are 45’ and in (v) are 45’ and 75’)



(iii)



(iv)

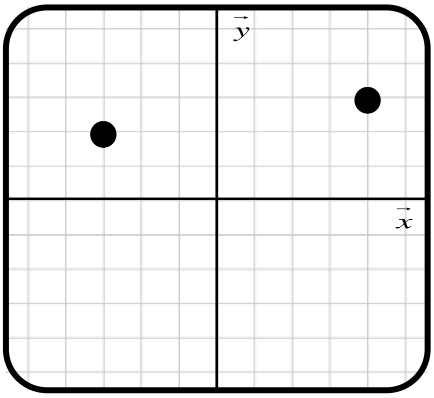


(v)

Writing vectors in terms of components using vector notation.

(i) Starting from the origin, explain to get to point "*a*" as shown.

(ii) Write how to get to "a" in terms of steps in the x direction and y directions. (e.g. 2 steps in the x and 1 step in the y is written as "$2\vec{x} + \vec{y}$")



(iii) Circle which of the following correctly maps how to get from the origin to the point labelled "*b*."

(a) $3\vec{x} + 2\vec{y}$ (b) $2\vec{x} - 3\vec{y}$ (c) -$3\vec{x} + 2\vec{y}$ (d) $2\vec{x} + 3\vec{y}$

Explain why you picked the answer you did. (It may help to explain why the other answers are incorrect)

(iv) On the diagram above, draw arrows from the origin to the points "*a*" and "*b*." We will call these vector arrows $\vec{a}$ and $\vec{b}$ respectively.

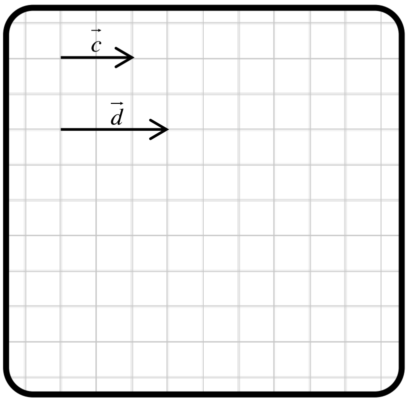
(v) On the diagram above, draw $\vec{a}$ as a combination of arrows along the $\vec{x}$ and $\vec{y}$ axis. Do the same for $\vec{b}$.

(vi) Using either trigonometry or co-ordinate geometry, explain how you could find the magnitude (length) of the vectors $\vec{a}$ and $\vec{b}$.

(vii) Find the length of $\vec{a}$ and $\vec{b}$.

II. Adding vectors.

Two vectors, $\vec{c}$ and $\vec{d}$, are shown to the right.

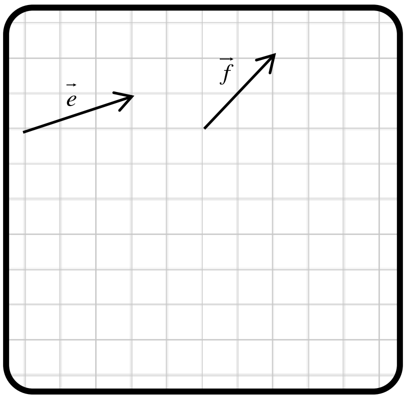


(i) Write both vectors in terms of their horizontal and vertical components. (eg, $\vec{a} = 4\vec{x} + 3\vec{y}$)

(ii) By connecting the tail of one vector to the tail of the other vector, show the resultant vector of $\vec{c} + \vec{d}$, on the diagram.

(iii) Add the horizontal components of $\vec{c}$ and $\vec{d}$ together. Add the vertical components of $\vec{c}$ and $\vec{d}$ together. Do the horizontal and vertical components define the resultant vector you drew in part (ii). Explain.

Two vectors, $\vec{e}$ and $\vec{f}$, are shown to the right.



(iv) Write both vectors in terms of their horizontal and vertical components. (eg, $\vec{a} = 4\vec{x} + 3\vec{y}$)

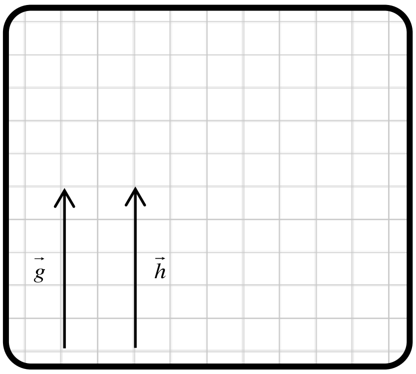
(v) By connecting the tail of one vector to the tail of the other vector, show the resultant vector of $\vec{e}$ +
→

(vi) Add the horizontal components of $\vec{c}$ and $\vec{d}$ together. Add the vertical components of $\vec{c}$ and $\vec{d}$ together. Does the horizontal and vertical components define the resultant vector you drew in part (ii). Explain.

(vii) Explain how adding vectors head to tail is the same as adding vectors by adding their components.

III. Determining how horizontal and vertical vectors affect the resultant vectors.

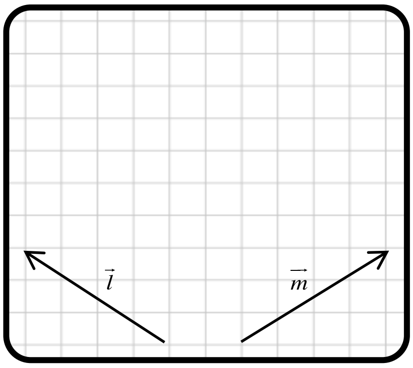
Two vectors, $\vec{g}$ and $\vec{h}$, are shown to the right.



(i) What is the magnitude of $\vec{g}$ and $\vec{h}$?

(ii) Draw in the resultant vector of the $\vec{g}+\vec{h}$. What is the magnitude of the resultant vector you drew?

Two vectors, $\vec{l}$ and $\vec{m}$, are shown to the right.



(iv) Show the magnitude of $\vec{l}$ and $\vec{m}$ is 5 units for each vector.

(v) Draw in the resultant vector of the $\vec{l}+\vec{m}$. What is the magnitude of the resultant vector you

(vi) Write both vectors in terms of their horizontal and vertical components.

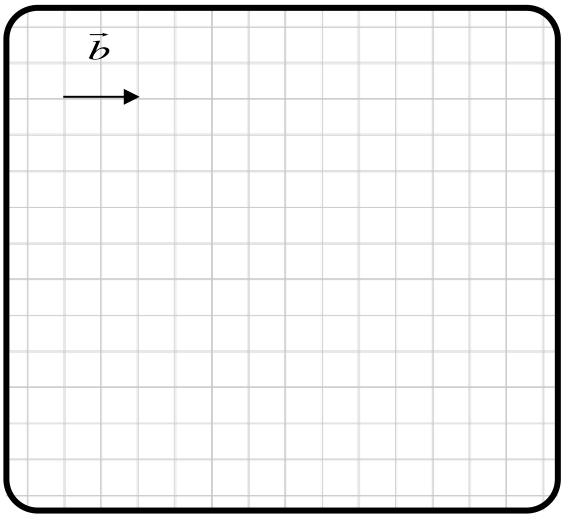
(vii) What is the result when the horizontal components are added together?

(viii) What is the result when the vertical components are added together?

(ix) From your results in (vii) and (viii), explain why the magnitude for the resultant you got in (v) is less than 10 units. (i.e. directly adding the magnitude of both vectors; 5 + 5 = 10)

I. **Drawing vectors.**

The arrow in the diagram to the right shows a vector, $\vec{b}$. In the extra space, draw the following vectors.



(i) $2\vec{b}$

(ii) $4\vec{b}$

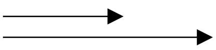
(iii) $-\vec{b}$

(iv) $-3\vec{b}$

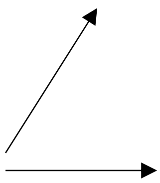
→ →

I. Adding vectors.

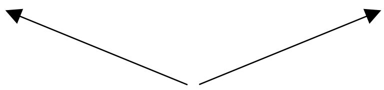
Use the tip to tail, or the parallelogram rule to add the following pairs vectors. Clearly show any construction lines you draw.



(i)

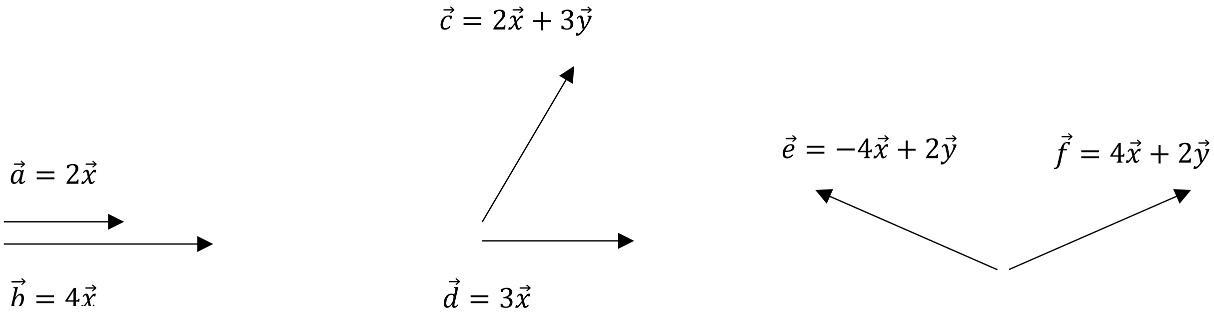


(ii)



(iii)

II. **Adding vectors using components.**



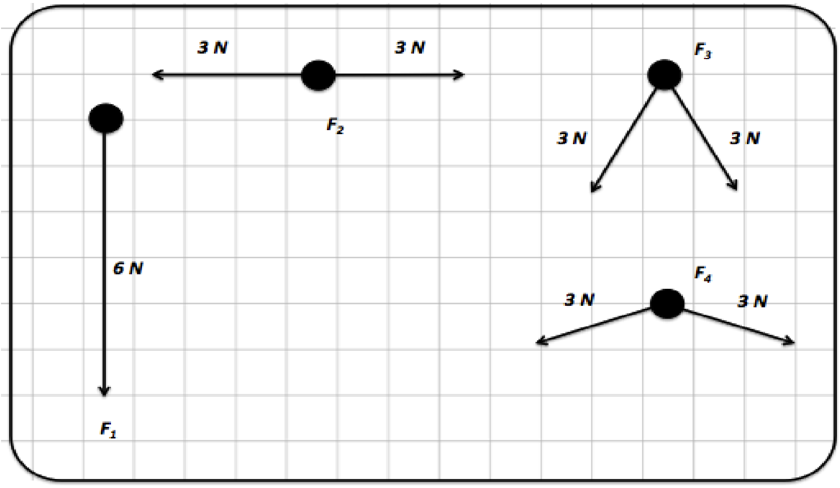
(i) Using the vector notation given, add the following vector pairs:

$\vec{a} + \vec{b}$, $\vec{c} + \vec{d}$, $\vec{e} + \vec{f}$

(ii) Use Pythagoras' theorem to find the magnitude of $\vec{c} + \vec{d}$ and $\vec{e} + \vec{f}$.

(iii) Explain, referring to the addition of horizontal and vertical components, explain why the magnitude of $\vec{c} + \vec{d}$ is greater than $\vec{c}$ and $\vec{d}$, individually, but the magnitude of $\vec{e} + \vec{f}$ is less than $\vec{e}$ and $\vec{f}$ individually.

III. Looking at how horizontal and vertical components affect the resultant.



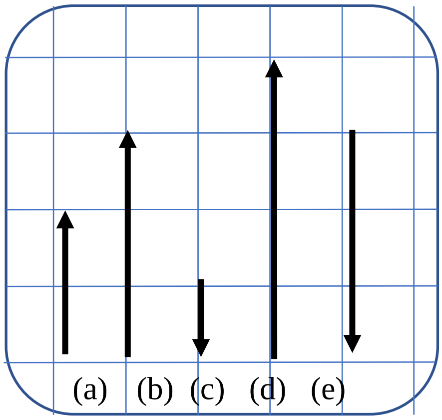
A small ball has a force or numerous forces acting on it as shown in the diagram to the right. The net force (resultant force) is labelled as F_1, F_2, F_3 and F_4.

(i) Rank the net forces acting on the ball, from highest to lowest. Explain your ranking. (Refer to either the tip to tail / parallelogram rule or refer to horizontal or vertical components, or both to give a full answer)

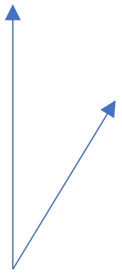
1. Rank the following vector arrows (a – e) from weakest magnitude to strongest magnitude.

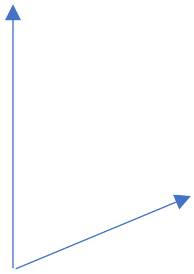
Ranking:

Explanation:

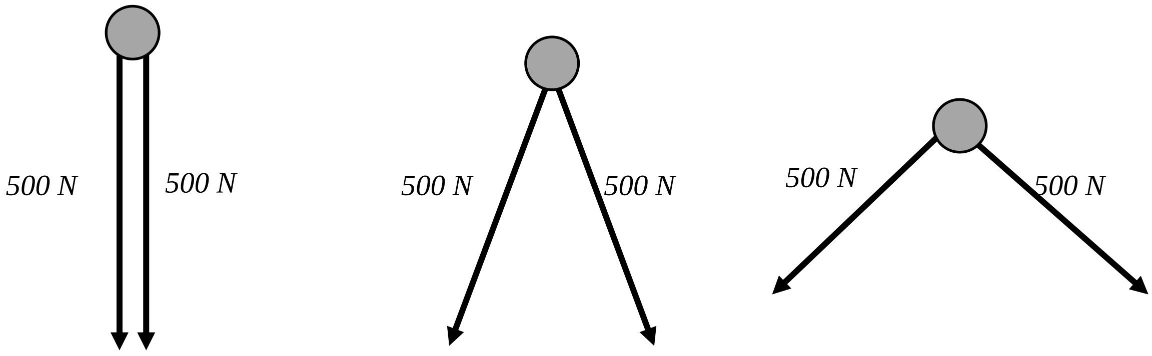


2. Show how to construct resultant of the following vectors.





3. A truck is broken down and needs to be pulled. A number of different cars can be used to pull the truck to a safe spot. All cars have the same pulling strength as shown buy the force vectors in the 3 diagrams below. The circle represents the centre of mass of the truck.



(i) Which diagram represents the vectors that will result in the strongest magnitude of force pulling the truck?

(ii) Explanation for part (i).

(iii) Which diagram represents the vectors that will result in the weakest magnitude of force pulling the truck?

(iv) Explanation for part (iii).

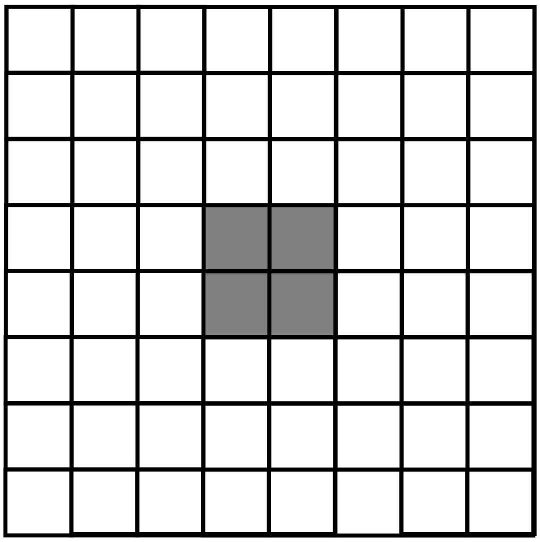
Appendix B

Inverse square law tutorial materials.

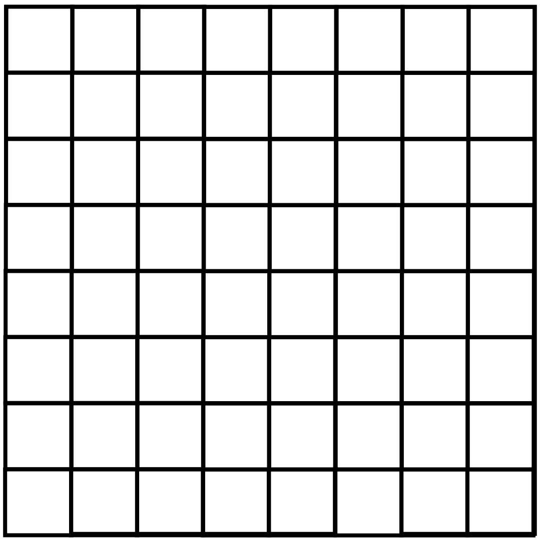
I. On the graph, sketch a graph of the pattern seen when you graph the function $y = k\frac{1}{x^2}$

Explain why you drew the pattern as you did.

II. A bulb shines on a wall from 1 m. The wall has an 8 x 8 grid on it



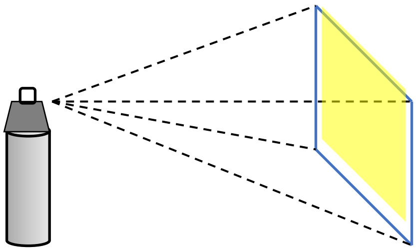
(i) If the bulb were moved to 3 m, shade in the shape would look like on the *8 x 8* grid below. (next page)



(ii) Explain why you drew it at you did.

III. A bulb is held 2 m from a light sensor. The sensor record the light intensity to be 100 Wm^{-2}. If the bulb is moved so that it is 4 m from the light sensor, what reading will the light sensor read?

I. Spray paint "Intensity."



A can of spray emits 100 drops of paint in 1 second.

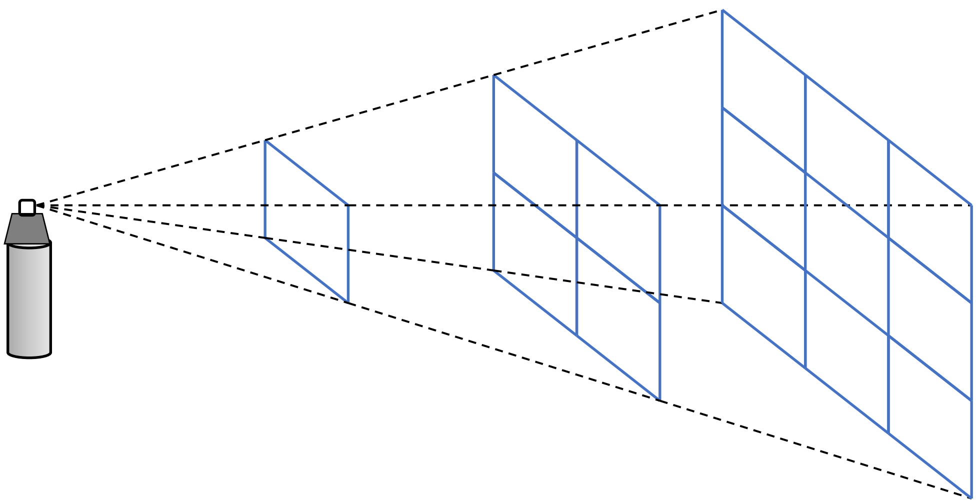
A metal frame, in the shape of a square, is placed in path of the paint spray so that all of the droplets of paint passes through it is area.

The square frame has a width of 10 cm and each of the

(i) Calculate the area, in m^2, of the metal frame

(iii) Determine how many paint droplets pass through the $1m^2$ frame in 1 second. Choose an appropriate unit for your answer.

II. How distance affects spray paint "intensity."



A second metal frame is placed away from the can, so that all it is outer corners are 10 cm from the can of paint. This metal frame is made up of smaller frames that are identical to the frame discussed at the top of the page.

(i) Explain why this second frame has an area that is four times bigger than the original frame used on the last page.

(ii) What is the value of the area of the overall frame that is 10 cm from the can.

(iii) Determine the how many droplets of paint pass through $1m^2$ frame in 1 second, for this overall frame.

A third metal frame is placed away from the source, so that all it is outer corners are 15 cm from the can of paint. This metal frame is made up of smaller frames that are identical to the frame discussed at the top of the previous page.

(iv) Explain why this third frame has an area that is nine times bigger than the original frame used on the last page.

(v) What is the value of the area of the overall frame that is 15 cm from the can.

(vi) Determine the how many droplets of paint pass through $1m^2$ frame in 1 second, for this overall frame.

(vii) As the distance from the paint can increases, the number of droplets of paint passing through a 1 m^2 area in 1 second decreases. Using your answers from the previous questions, explain why this occurs.

The relationship between distance from source and spray paint "intensity" is an example of an inverse square relationship. Inverse square relationships have the general equation:

$$y = k\frac{1}{x^2}$$

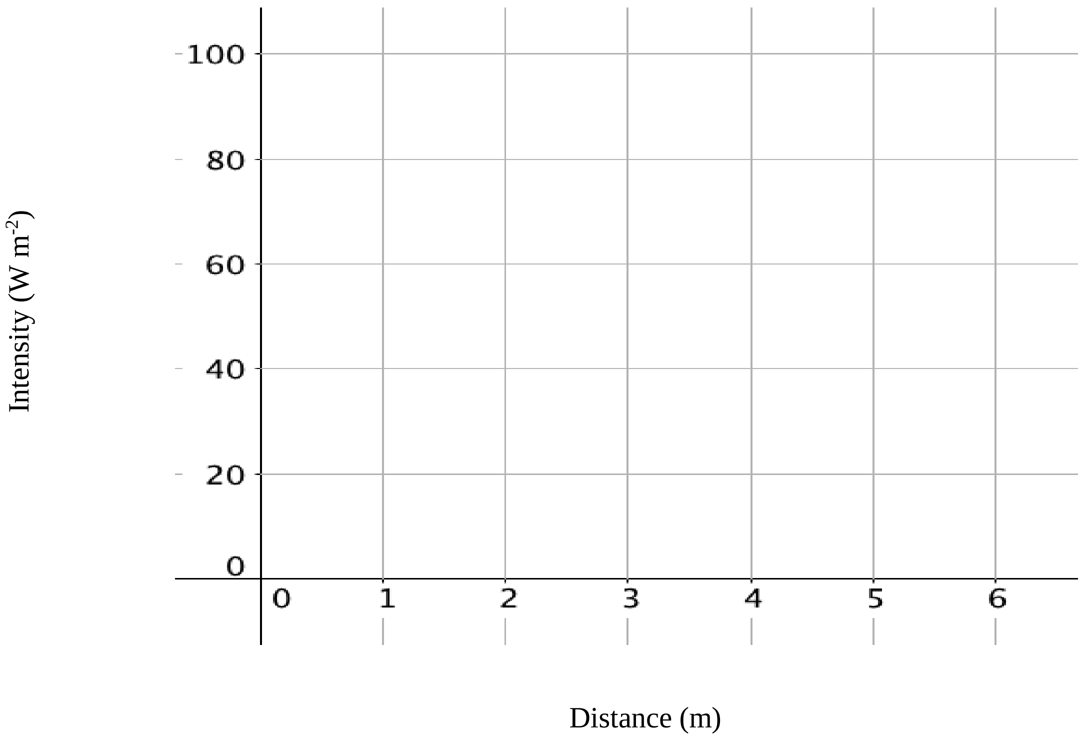
Note that as the *x* values increase, the *y* values decrease. This is due to the position of the x as a denominator on the right hand side of the equation. We also can note that the power on the *x* is x^2, which separates the inverse square relationship from an inverse relationship. We will now explore this mathematically.

III. Exploring the inverse square relationship.

The following table shows spray paint intensity given by a the can that emits 100 droplets of paint per second.

Plot this data on the graph below.

Distance (d)	**Intensity (I)**
1	100.0
2	25.0
3	11.1
4	6.3
5	4.0
6	2.8



(i) Describe the pattern shown using the following criteria: shape, increasing / decreasing, change in the slope. Include others if you think of them.

(ii) As the distance from the bulb increases, does the intensity increase, decrease or stay the same? Explain with reference to the shape of the graph.

(iii) Use your graph to determine the intensity of the light at a distance of 1 m from the bulb, and 2 m from the bulb

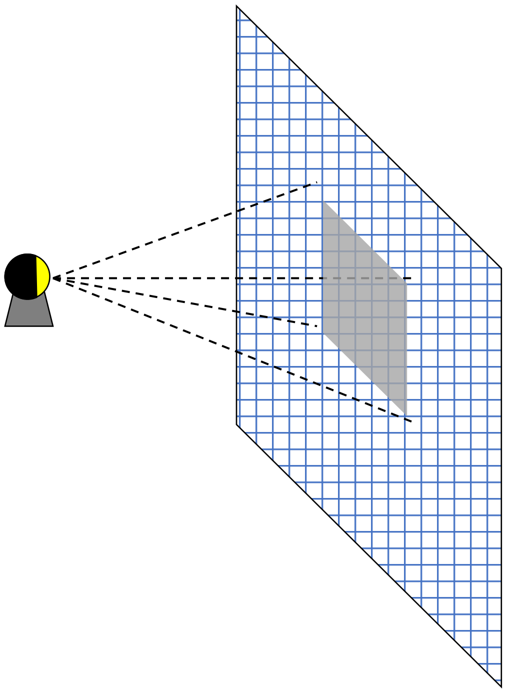
(iv) By what factor is the intensity at 2 m smaller than the smaller/bigger than the intensity at 1 m?

(v) Use your graph to determine the intensity of the light at a distance of 1 m from the bulb, and 5 m from the bulb

(vi) By what factor is the intensity at 5 m smaller than the smaller/bigger than the intensity at 1 m?

(vii) Using your answers, and the reasoning you developed on the first two pages, explain why this is an example of an inverse square law.

I. Light Intensity.

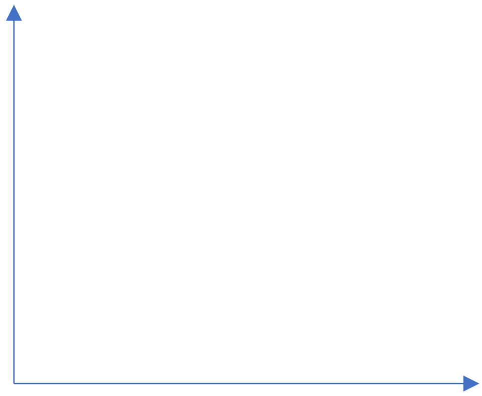


(i) Calculate the light intensity at a point 2 m from the bulb. (Formulae: $I = \frac{P}{A}$, $A = \pi r^2$)

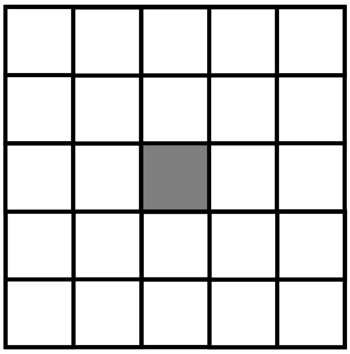
(ii) If you increased the distance from the wall to to the bulb, would the light intensity increase, decrease or stay the same? Explain your reasoning. (review what you covered in the worksheet for spray paint "intensity" if you need to)

(iii) Sketch a graph to show the relationship you explained in part (ii) and explain how it accurately shows the relationship from the formula you used in part (i)

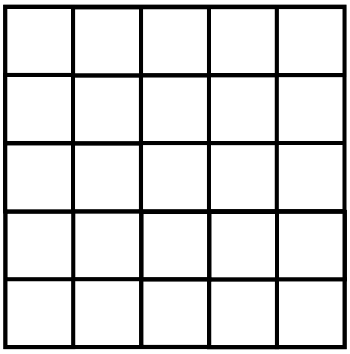
How it shows the relationship:



The photograph of the wall shows the following shape of the light on the wall. This is the shape of the light when the *6 x 6* grid is looked at head on.



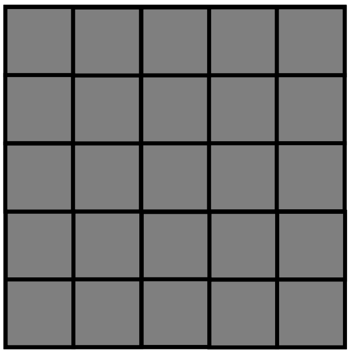
(iv) If the bulb were moved to 4 m, shade in the shape would look like on the *6 x 6* grid below.



(v) Explain why you drew it at you did.

(vi) Has the light intensity on the grid increased, decreased or remained the same? Explain your reasoning (consider the effect of moving the bulb back, in conjunction with your sketch in (iv)).

(vii) The bulb is moved to a distance where the shape of the light is shown on the grid on the next page. Determine how far the bulb is from the wall. Ensure you show how you figured it out.



III. Calculations. (Formulae: $I = \frac{P}{A}$, $A = \pi r^2$)

(i) A 200 W bulb is placed 5 m from a light sensor. Calculate the light intensity that the light sensor.

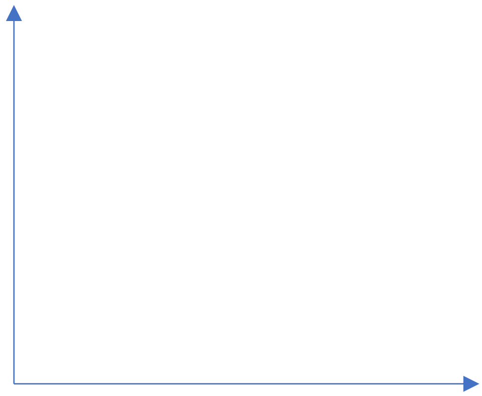
(ii) The 200W bulb is moved to 10m from the light sensor. Calculate the light intensity that the light sensor.

(iii) Use your answers from (i) and (ii) to show that light intensity follows an inverse square law.

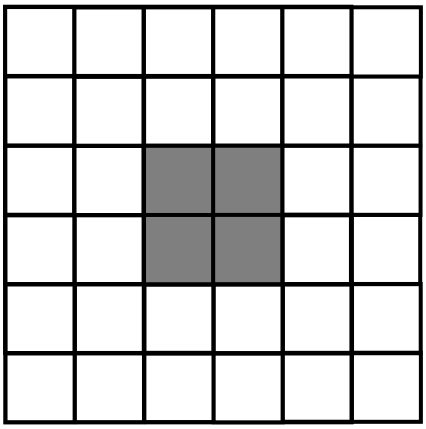
I. A can of spray paint emits 200 droplets of paint per second from the nozzle. The amount of droplets from a can of spray paint that fall on a given area (intensity – I) is given by the formula: $I = \frac{200}{0.125\,\pi r^2}$.

Draw a sketch of the graph that represents the relationship between spray paint intensity (I) and the distance from the nozzle (r), and explain how it shows the relationship.

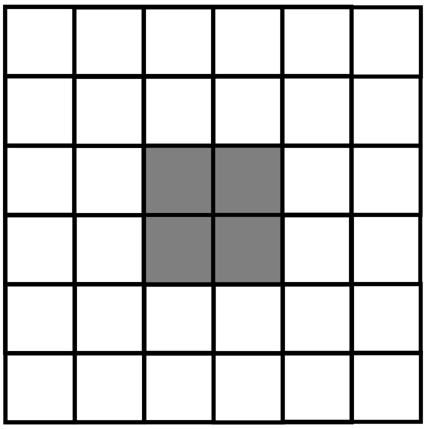
How it shows the relationship:



II. The can is held 2 metres from a wall that has squares marked on it like a grid. The can is then sprayed for 1 second. This is the shape of the paint landing on grid is looked at head on, is shown in the diagram below.



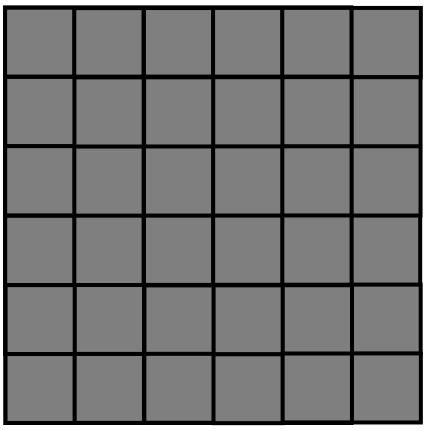
(iii) If the can was moved to 4 m, shade in the shape would look like on the *6 x 6* grid below.



(iv) Explain why you drew it at you did.

(v) Has the intensity of the droplets per square on the grid increased, decreased or remained the same? Explain your reasoning

(vi) The can is moved to a distance where the shape of the paint is shown on the grid below. Determine how far the can is from the wall. Ensure you show how you figured it out.



III. Calculations.

(iv) The can is placed 5 m from a wall. Calculate the paint intensity, using the formula given on the first page.

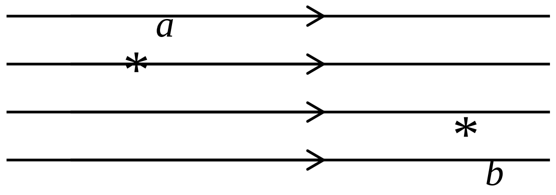
(v) If the distance from the can to the wall is doubled, what is the new “paint intensity?” Explain how you got your answer?

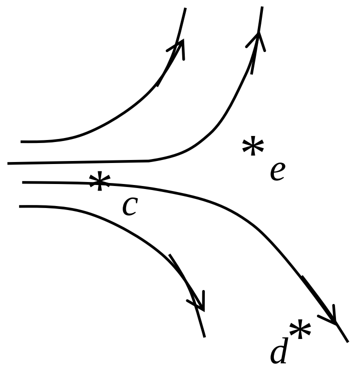
(vi) Explain how your answer from (ii) shows this is an example of an inverse square law.

Appendix C

Field lines tutorial materials.

1. Here are two examples of field lines.

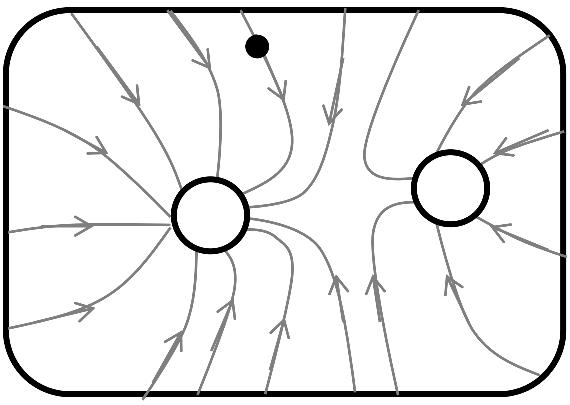




(i) Rank the field strength, from highest to lowest, for the points (a) to (e). Explain your reasoning.

(ii) Use vector arrows to show the direction of force acting on an object if it were to be placed at (a) to (e).

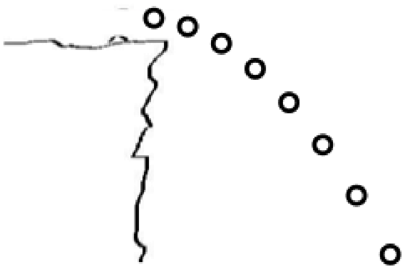
2. A meteor is placed at rest between two planets, that are close to each other. The gravitational field of both planets is shown in the diagram using field lines.



On the diagram, show the path taken by the meteor as it accelerates under the influence of the gravitational field. Explain why you drew the path as you did.

I. Gravitational field (Acceleration due to gravity).

A ball is thrown off a cliff, with an initially velocity *10 ms*$^{-1}$. A camera takes a photo every quarter of a second and all the pictures of the ball are combined into one picture as shown.



(i) Can you tell from the diagram whether the magnitude of the velocity of the ball is changing? Explain.

(ii) Can you tell from the diagram whether the direction of the velocity of the ball is changing? Explain.

(iii) Can you tell from the diagram alone, whether the object accelerates in the horizontal direction or vertical direction. Explain.

The ball has a mass 0.5 kg.

(i) Calculate the force of gravity acting on the ball. (Take mass of earth as 6 x 10^{24} *kg*, radius of earth as 6.4 x 10^{6} *m* and the gravitational constant as 6.7 x 10^{-11} $N\,m^{2}\,kg^{-2}$.

(ii) Is the force of gravity acting on the ball constant during the duration of the fall? Explain.

(iii) Using $F = ma$, determine the acceleration, due to gravity, acting on the ball.

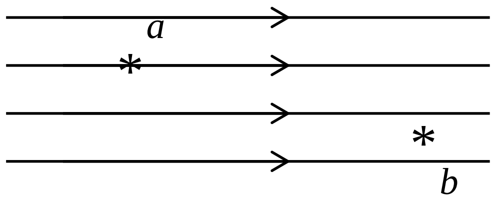
(iv) Is the acceleration, due to gravity, acting on the ball constant during the duration of the fall? Explain.

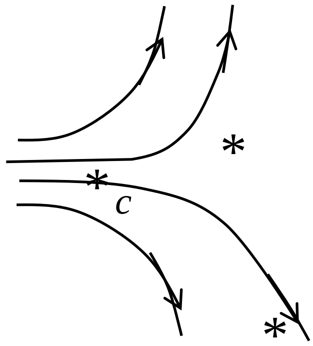
(v) Draw vector arrows on the all the balls to represent your answer from (iv). (8 arrows in total, one from each ball).

II. Gravitational field (lines).

In the last question of section II, you were asked to draw 8 vector arrows to represent the acceleration acting on the ball. Drawing vectors arrows in some cases can be clumber-some, and it is sometimes easier to use field line representations instead. Field lines represent the direction and strength of the force felt by objects that interact with those fields.

These are continuous lines that, when a point in picked, a tangent to the line at that point denotes the direction of force at that point. The closer field lines are together, the stronger the force is and a body does not have to be on a field line to feel a force. Here are two examples of field lines.





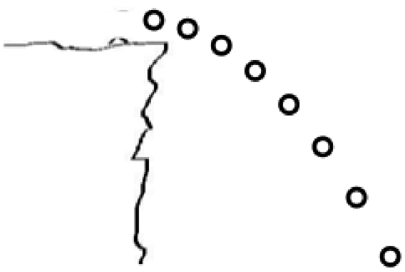
(i) Rank the field strength, from highest to lowest, for the points (a) to (e). Explain how you used the field lines to justify your ranking.

(ii) Use vector arrows to show the direction of force acting on an object if it were to be placed at (a), (d) and (e) (use a ruler).

A uniform field is described as a field that always points in the same direction, and has a constant field strength.

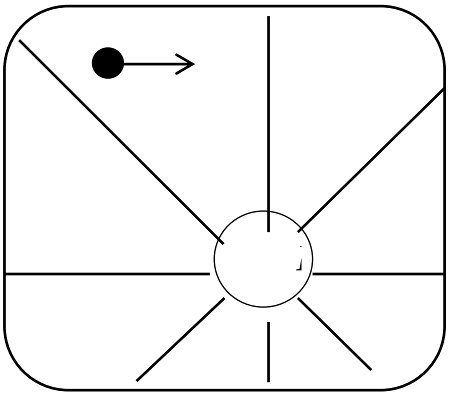
(iii) Which field (left or right) shows a uniform field? Explain how the field shows it is uniform.

(iv) Use field lines to sketch the field that causes the ball to fall. Explain if the field is uniform or not. (refer back to section I)



III. Gravitational field of the earth.

The diagram shown contains a small meteor that is passing close to the earth, shown as a small black circle. It has a velocity of 10 *km/s*, shown by the vector arrow. The earth is shown as a big circle, and it is gravitational field is sketched. At no point, does the meteor collide with the planet.



(i) Is the strength of the gravitation field caused by the ***earth*** the same everywhere? Explain.

(ii) Using $F = mg$, and $F = G\frac{Mm}{r^2}$, show that the acceleration due to gravity is given by $a = G\frac{M}{r^2}$

(iii) Explain how the field lines represent that gravity follow an inverse square law.

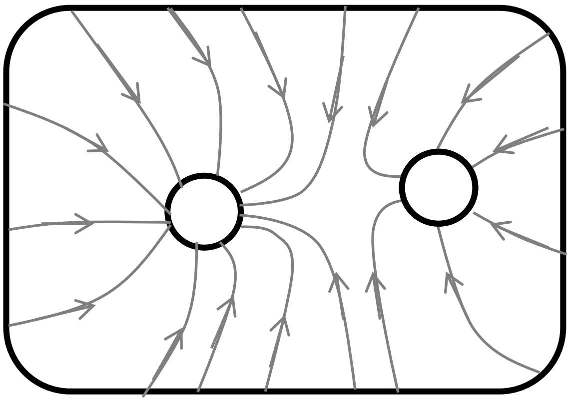
(iv) Use arrowheads to show the direction the field-lines point. Explain why you drew them as you did.

(iii) Draw the path followed by the meteor, under the influence of the gravitational field. Explain your reasoning for drawing it as you did. **(Remember, it has an initial velocity as shown with the vector arrow in the diagram) (Explain why your path follows / does not follow the field lines)**

IV. Gravitational field between two planets, very close together.

The diagram to the right shows the field between two planets.

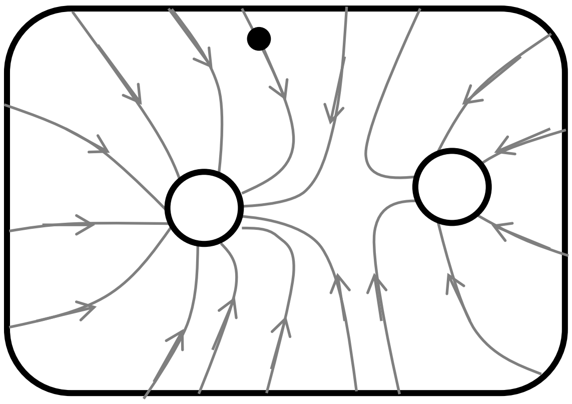
(i) Using the field lines, determine whether the mass of one of the planets is bigger than the other, or if they are the same. Explain your reasoning.



Consider the following student dialogue, between two students (S_1 and S_2) concerning a small meteor initially at rest placed at a location shown by the small black circle.

S_1: *The field lines indicated the direction of the force, so the meteor will be forced along the line until it hits the left planet*

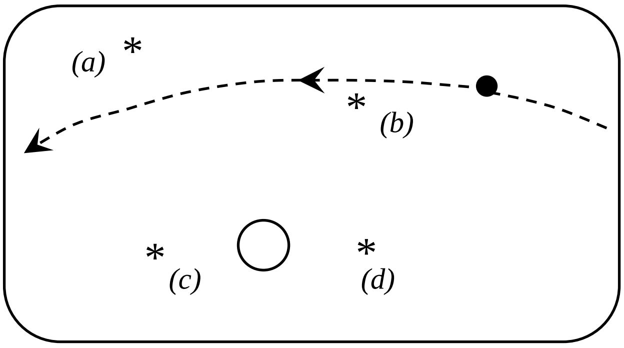
S_2 *As the small meteor begins to accelerate, its gained velocity will make it move away from the field line that it was on originally, so we can be sure it'll*



(iii) Which student, if any, do you agree with. Explain why your reasoning.

(v) Using your answer from (iii), draw in the likely path followed by the meteor on the diagram above.

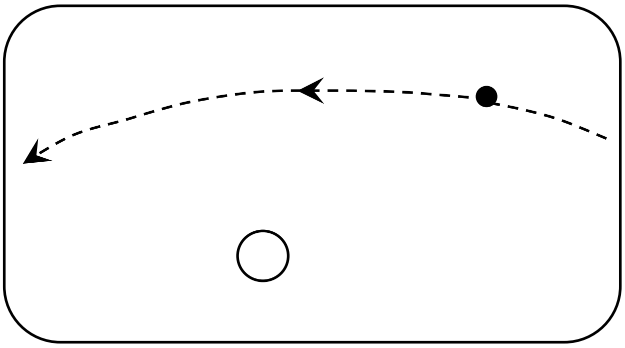
1. The following diagram shows a small planet, denoted by the white circle. A meteor passes by the planet in the path shown. A number of points (a) – (d) are also highlighted. The planet has mass of 3×10^{24} *kg.*



(i) Use field lines to represent the gravitational field caused by the planet.

(ii) Rank the field strength, from lowest to highest, at the points (a) – (d). Justify your ranking, ensuring you reference the field you drew in (i).

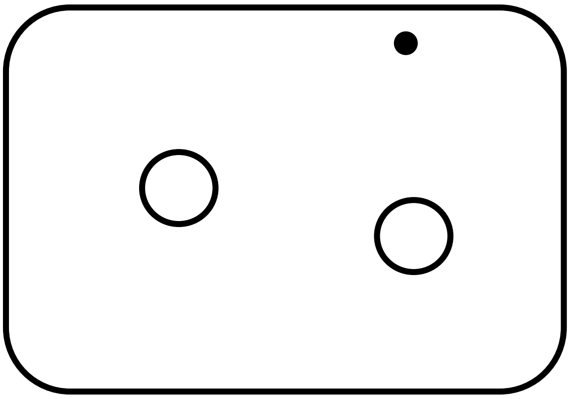
(iii) If the planet had a mass of 6×10^{24} *kg*, draw in the field lines that represent that this planet has an increase in mass on the diagram below.



(iv) Draw in the new path taken by the meteor in this scenario, in which the mass of the planet has increased. Explain why you drew the new path as you did. (If you think it follows the original path, which is shown in the diagram, explain exactly why you think this).

(v) What effect, if any, does increasing the mass of the meteor have on the path taken? Explain. (If you think it does not affect the path taken, explain exactly why you think this).

3. A meteor is placed at rest between two planets of equal mass, that are close to each other.

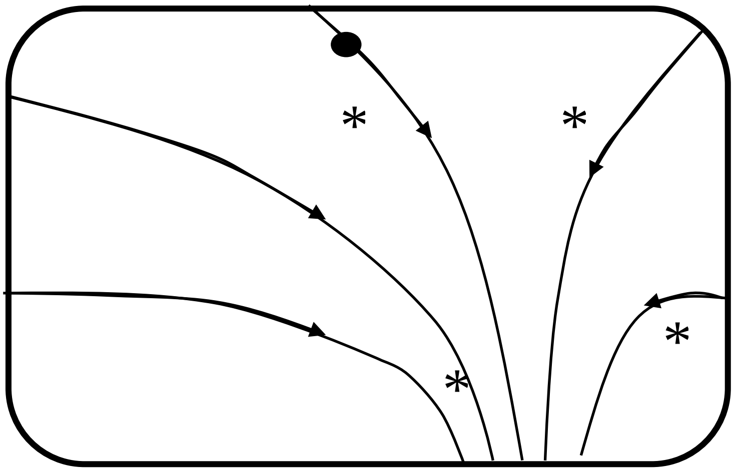


(i) Draw the field lines to represent the gravitation field caused by both planets in the diagram above.

(ii) On the diagram on the top of the page, show the path taken by the meteor as it accelerates under the influence of the gravitational field. Explain why you drew the path as you did.

(iii) Highlight the point between the two planets where the gravitational field is zero using a circle. Explain why this point exists.

1. Here is a small snapshot of a section of field lines.



(i) Trace you finger to the end of any field line, starting where the lines are closest together, so that it travels against the direction of field line. How the field strength varies as you move your finger. Justify your answer.

(ii) Use vector arrows to show the direction of the force the points highlighted.

(iii) A small body is placed at the point marked with a black circle. show the path taken by the body as it accelerates under the influence of the gravitational field. Explain why you drew the path as you did.

Appendix D

Coulomb’s law and electric field tutorial materials.

Coulomb's Law explains the attraction or repulsion between two charges. It is given by the formula

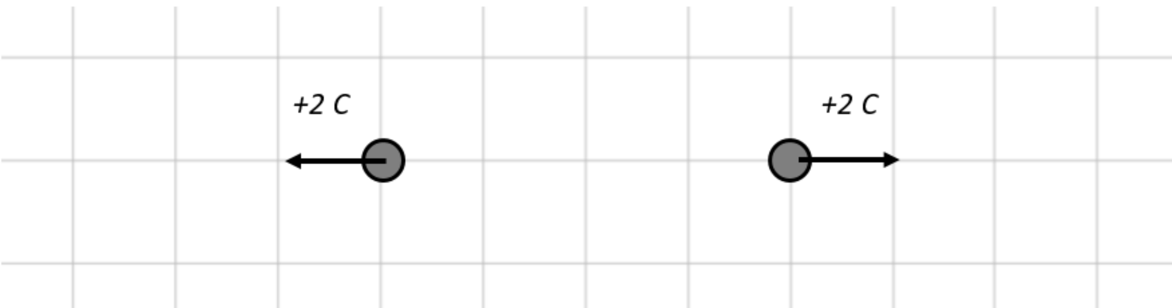
$$F = k\,\frac{q_1 q_2}{d^2}$$

1. Using the formula above, explain the relationship between the force between the two charges and their magnitudes.

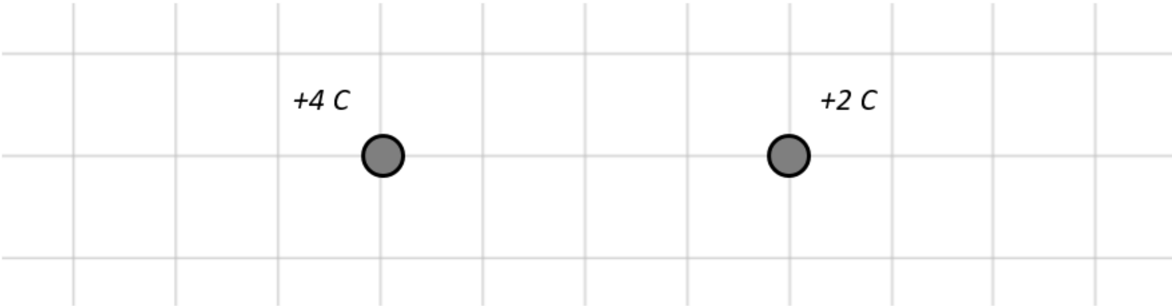
2. Using the formula above, explain the relationship between the force between the two charges and the distance between them.

3. Two *+3 C* charges are held a distance of *10 cm* from each other, and the force acting on both charges is *10 N*. If one of the +3C charges is replaced with a +9C charge, what is the new force acting on the charges? Explain how you know what the change in force is.

4. A *+2 C* and a *+2 C* charge are placed 20 cm from each other. The vectors to show the force are shown in the following diagram.



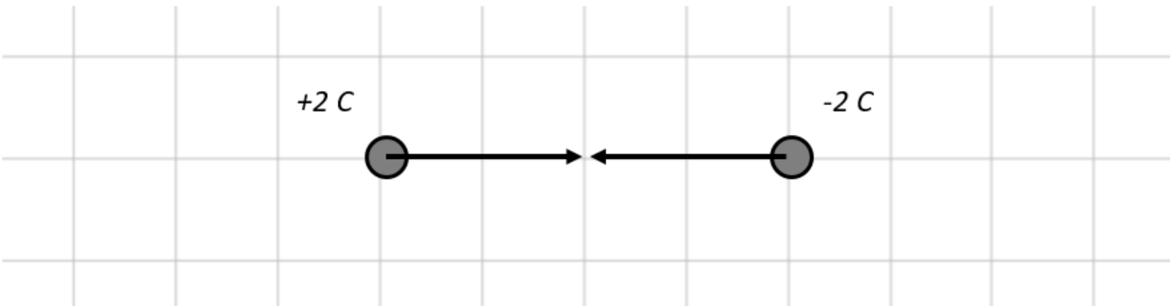
If the +2 C charge on the left is replaced with a +4C charge. Draw the vectors to represent the forces now acting on the charges.



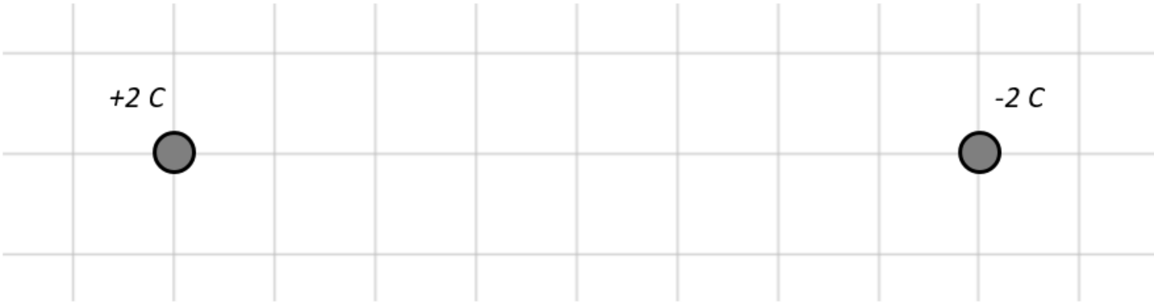
Explanation:

5. Two *+8 C* charges are held a distance of *10 cm* from each other, and the force acting on both charges is *90 N*. The charges are moved so the distance between then is now *30 cm*. What is the new force acting between the charges? Explain how you know what the change in force is.

6. A *+2 C* and a *-2 C* charge are placed 20 cm from each other. The vectors to show the force are shown in the following diagram.



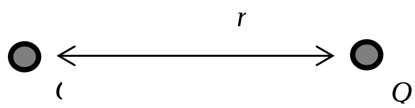
If the distance between the charges is increased to *40 cm*, draw the vector arrows to show the force acting between the charges now.



Explanation:

I. **Looking at the relationship between the charges and the force in Coulombs' Law.**

Two charges are held a distance apart from each – other, a constant distance "r." There is a force exerted between the charges. Each charge, q_1 and q_2, are replaced and with various stronger charges and the forces are recorded as shown. (The product of the charges ($q_1 * q_2$) is shown in the third column)



q_1 (C)	q_2 (C)	$q_1 q_2$ (C^2)	F (N)
1	1	1	2
1	2	2	4
2	1.5	3	6
4	1	4	8
1	5	5	10
2	3	6	12

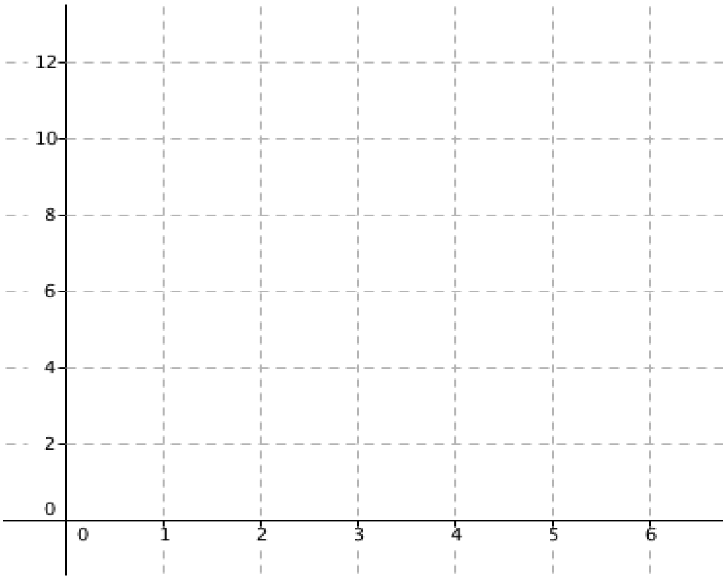
(i) Can you see any pattern between the first column (q_1), the second column (q_2) or the third column ($q_1 q_2$) with the fourth column (F)?

If so, how would you describe this pattern.

(ii) Linear patterns through the origin follow the form y = mx, where m has a constant value. By letting the values for column 4 (F) represent your y values, and which-ever of the other three column you chose to in (i) for the x-values, show that $\frac{y}{x}$ is a constant.

y	F (N)	2	4	6	8	10	12
X							
$\frac{y}{x}$	$\frac{F\ (N)}{}$						

If you identifies any patterns in sections (i) – (iii), graph it / them on the graph shown. (Label the axis)



(iii) Explain how this graph shows a directly proportional relationship.

(iv) What force is exerted between the charges when the product of the charges is 2 C^2?

(v) What is the force exerted between the charges when the product of the charges is 6 C^2?

(vi) From part (iv) and (v), explain what affect does tripling the product of the charges have on the force exerted between the charges?

(vii) Is this in agreement to your answer from part (iii) that this is a directly proportional relationship? Explain your answer.

II. Calculations involving a directly proportional relationship.

The force, F_A, exerted by the two charges shown, can be calculated in the manner shown below, if q_1 has a magnitude of $6\ \mu C$ and q_2 has a magnitude of $3\ \mu C$, and the distance between the charges is 1 *cm*. (Use $F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$, where $\varepsilon = 8.85 \times 10^{-12}\ F\ m^{-1}$)

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

A

$$F = \frac{1}{4(3.14)(8.9\ x\ 10^{-12})}\frac{(6\ x\ 10^{-6})(3\ x\ 10^{-6})}{(1\ x\ 10^{-2})^2}$$

B

$$F = \frac{1}{1.12\ x\ 10^{-10}}\frac{(1.8\ x\ 10^{-11})}{(1\ x\ 10^{-4})}$$

C

$$F = (8.9\ x\ 10^{9})(1.8\ x\ 10^{-7})$$

D

$$F = 1{,}602N$$

(i) Explain the mathematical step that occurs in A. (include explanation as to the use of scientific notation for the values)

(ii) Explain the mathematical steps that occur in B (there are 3).

(iii) Explain the mathematical steps that occur in C (there are 2).

(iv) Explain the final steps that occur in D.

(v) From your steps outlines in (i) to (iv), calculate the force, F_B, exerted between the charges if the 3 μC charge is replaced with a 9 μC charge.

(vi) By what factor is the force between the charges increased?

(vii) How does your answers show that the force experienced by the charges is directly proportional to the product of their magnitudes?

(viii) Explain how there would there have been a quick way for you to determine the new force after the replacement?

III. Looking at the relationship between the distance and the force in Coulombs' Law.

Coulomb's law states that the force between two charges is directly proportional to the product of the magnitude of the charges, and inversely proportional to the square of the distance between them. This is given by the following formula.

$$F = k\,\frac{q_1 q_2}{d^2} \qquad k = \frac{1}{4\pi\varepsilon}$$

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

In the last 4 pages, you learnt how to show the first relationship by, between the force exerted and the product of the charges by using tables, graphs and calculations. Using what you learnt, your task is to prove the relationship between the force exerted between the charges, and the distance between them, using whichever methods you choose. Attempt all of them. If you get stuck with a method, ask for help in using it.

You can use the following to help you.

Directly proportional general equation: $y = mx, \quad \frac{y}{x} = m, \quad \frac{y}{x} = constant$

Directly proportional to square equation: $y = ax^2, \quad \frac{y}{x^2} = a, \quad \frac{y}{x^2} = constant$

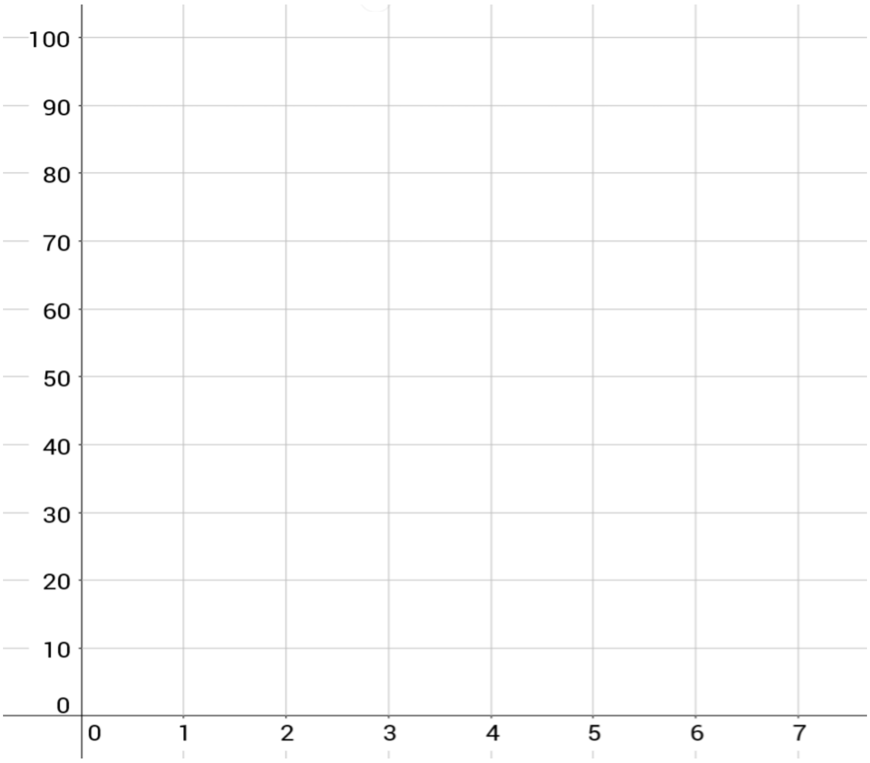
Inverse proportional general equation: $y = \frac{k}{x}, \quad xy = k, \quad xy = constant$

Inverse square proportional equation: $y = \frac{k}{x^2}, \quad x^2 y = k, \quad x^2 y = constant$

Table:

y	F (N)	100	25	11.11	6.25	4	2.78
x	d (m)	1	2	3	4	5	6

Graph:



Equation and values for calculation:

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{d^2}$$

$\varepsilon = 8.85 \times 10^{-12}\ F\ m^{-1}$, $q_1 = 6 \times 10^{-6}\ C$, $q_2 = 4 \times 10^{-6}\ C$.

$d_1 = 4\ cm$, $d_2 = 8\ cm$

Which method do you think is the most effective? Why?

Which method is the easiest do use? Why?

Which method would you use, if you have to choose one? Why?

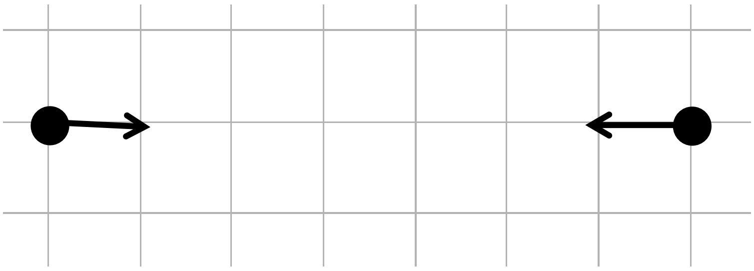
I. **Numerical calculations involving Coulomb's Law. (Formula is in equation tables, as is value for Ɛ)**

A *+3 C* and a *-3 C* charge are placed 10 cm from eachother.

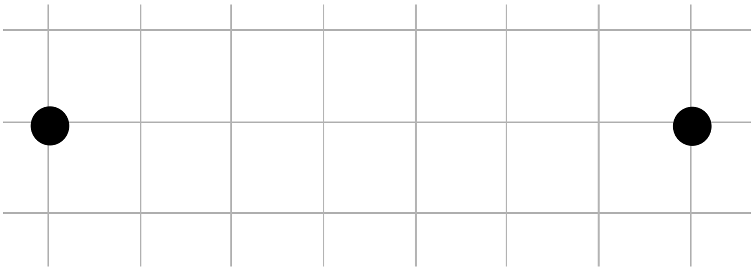
(i) Calculate the force between these two charges.

(ii) If one of these charges is replaced with a *+9 C* charge, what is the new force acting between these two charges.

The force between the two charges from (i) is shown in the following diagram.



(iii) Using your answers from (i) and (ii), draw the vector arrows to how the force acting between the charges from (ii).



(iv) Explanation how your answers from (i) – (iii) show the force between two charges is directly proportional to their magnitude.

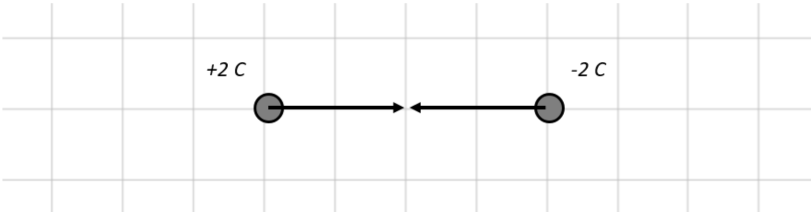
I. **Numerical calculations involving Coulomb's Law. (Formula is in equation tables, as is value for Ɛ)**

A *+2 C* charge and -*2 C* charge are placed 20 cm from eachother.

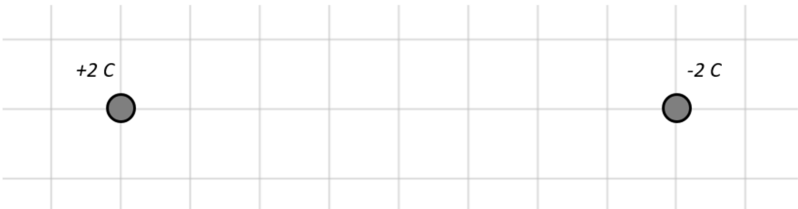
(ii) Calculate the force between these two charges.

(ii) These charges are moved to 40 cm from eachother. Calculate the new force acting between these two charges.

The force between the two charges from (i) is shown in the following diagram.

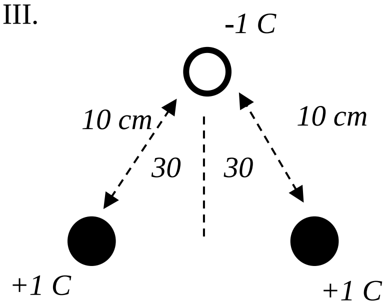


(iii) Using your answers from (i) and (ii), draw the vector arrows to how the force acting between the charges from (ii).

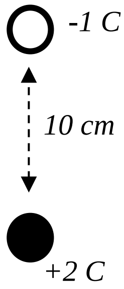


(iii) Explanation how your answers from (i) – (iii) show the force between two charges is inversely proportional to the square of the distance between the charges.

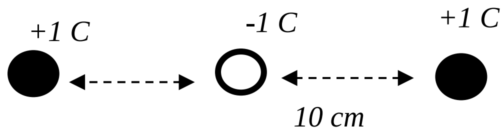
III.



(a)



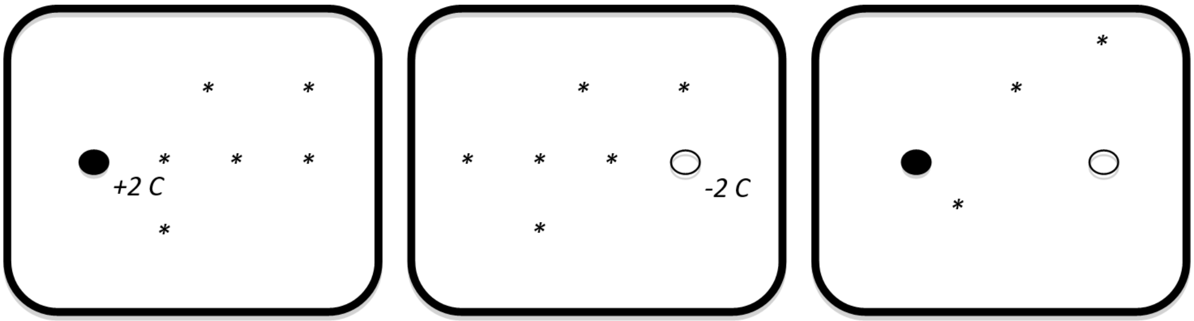
(b)



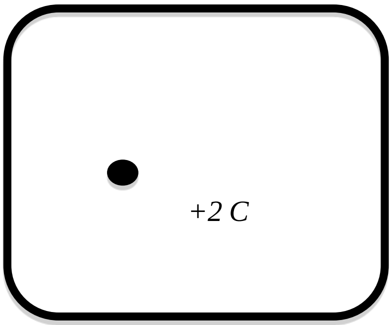
(c)

(i) 3 setups of charges are set up as shown in the above diagram. How does the magnitude of the net force acting on the negative charge (white) in setup (a) compare to the magnitude of the net force acting on the negative charge in (b) and (c). Explain your answer, using vectors, calculations or any other manner you see fit.

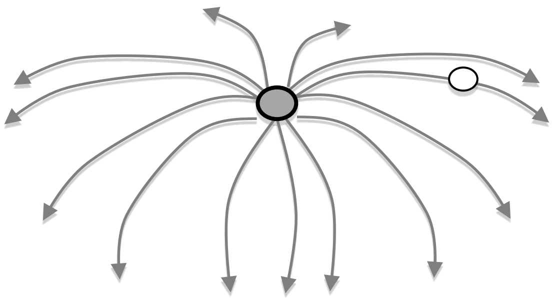
1. Use vector arrows to draw the electric field surrounds charges, at the points highlighted with stars, in the following three diagrams.



2. Represent the first diagram you drew, for the +2 C positive charge, using field lines.



3. A collection of unknown charges are located in the grey circle, which produce an electric field as shown to the right. An electron is placed at the position marked with the while circle.



(i) Draw in the path followed by the electron as a result of it is position in the force field.

(ii) Explain why you drew the direction as you did.

(iii) Rank the magnitude of the electric field strength, from highest to lowest, between a, b and c. Explain your ranking.

(iv) Draw vector arrows to represent the electric field at the points a, b and c on the diagram above.

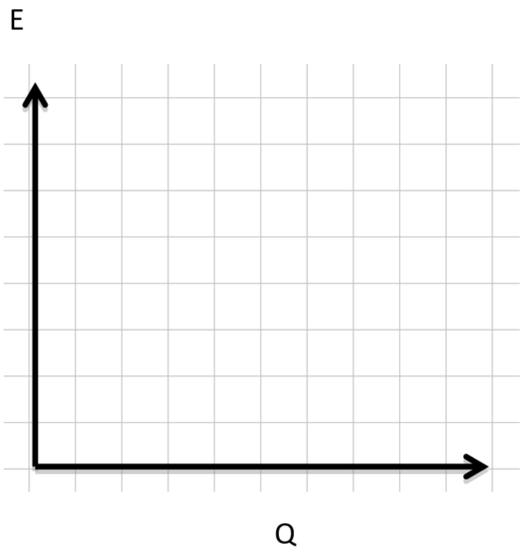
4.. The electric field strength around a charge is given by the formula $E = \frac{1}{4\pi\varepsilon}\frac{Q}{d^2}$.

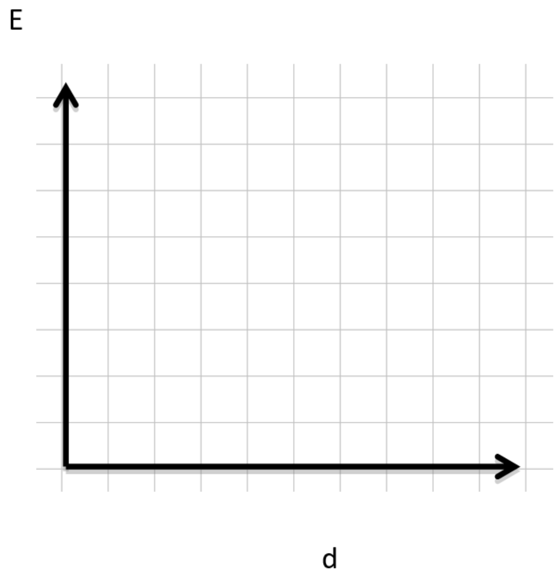
What is the relationship between the electric field strength and (i) the magnitude of the charge causing it, and (ii) the distance from the charge.

(i)

(ii)

Draw these relationships on the graphs below.

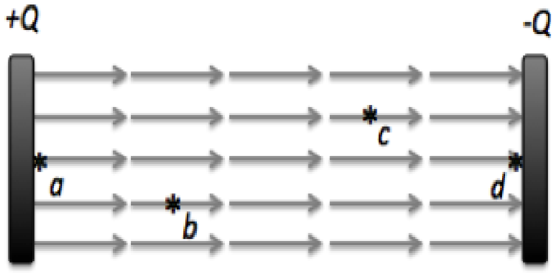




I. The forces experienced by a particle by an electric field – Vector treatment.

An electric field between two charged plates is depicted using vector arrows as shown. 4 points (*a*, *b*, *c* and *d*) are highlighted as shown.

(i) How would you describe variation of direction and strength of the electric field?



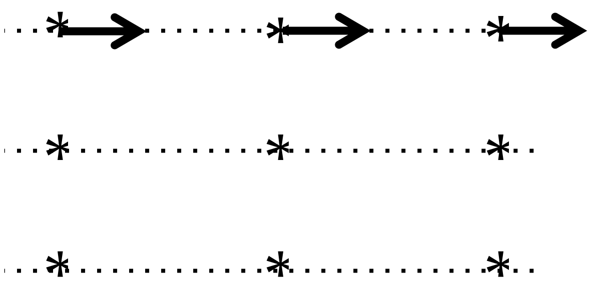
The picture above represents a ***uniform electric field***. This means that the strength of the electric field is the same at all points, and is in always in the same direction.

(ii) How does the representation show that the electric field strength is the same at all points?

(iii) Would the force experienced by a *+4 C* charge placed at *b* be *stronger*, *weaker* or *the same* compared to it being placed at *c*. Explain.

(iv) If the electric field has strength, $E = 20{,}000\ N\ C^{-1}$, find the force experienced by a *+4 C* charge when placed in the field.

(v) On the diagram on the right, the first line shows vector arrows, going left to right, for an electric field that is uniform.

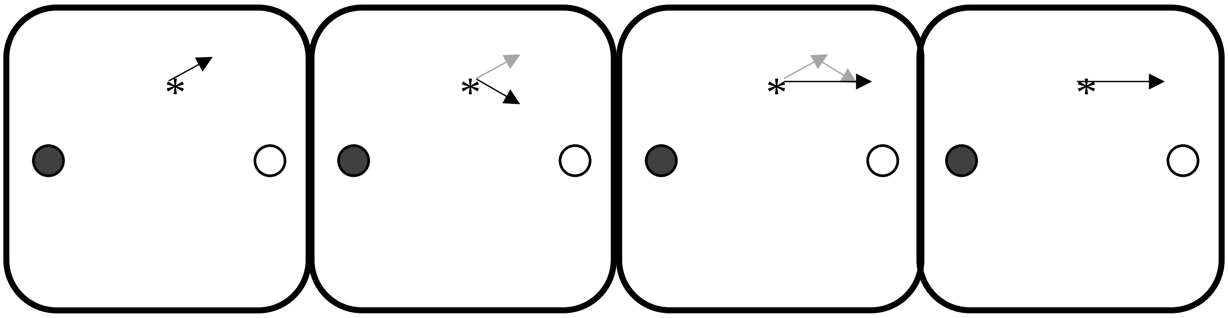


On the second and third line, use vector arrows to show (a) an increasing electric field and (b) a decreasing electric field.

II. Electric field vectors – Principle of Superposition.

One positive and one negative charged particles are placed a small distance from each – other. Both charged particles are have an equal magnitude. The electric field at a point marked with a star (*), are shown using vectors in the following order.

- The first field vector represents the direction of the electric field based on the position of the positive charge only.
- The second diagram represents the direction of the electric field based on the position of the negative charge, but the vector from the first diagram is represented with a grey arrow.
- The third diagram shows the net electric field vector, at this point, based on the two individual electric field vectors shown in the first and second diagram.
- The final diagram shows the net resultant electric field vector without referencing the two vector arrows that were used to construct it.

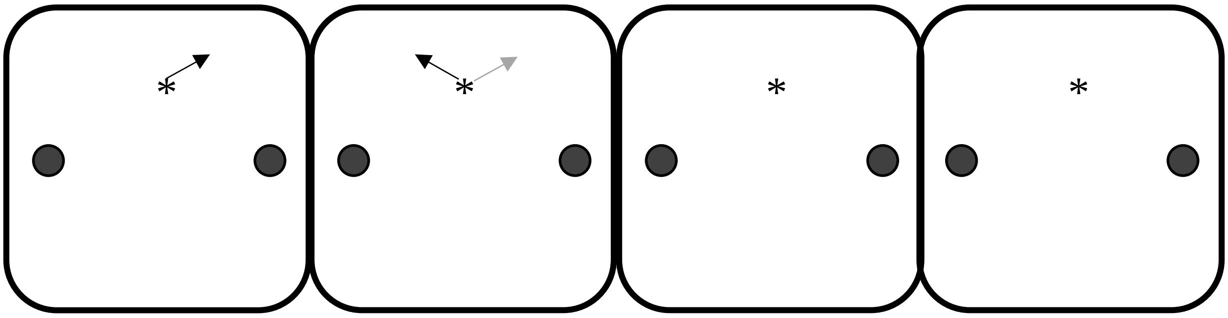


(i) Explain the process in the third diagram that allows us to find the net electric field vector, at this point.

(ii) In the first and second diagram, we can see the electric field vectors point in diagonal directions. However, the net vector in the third and fourth diagram is only horizontal. Explain why this is the case reference the horizontal and vertical vectors of the first two vectors.

(iii) If we measure the magnitude of the vectors in the first, second and third diagram, we will find the net electric field in the third diagram has a weaker magnitude than the sum of the vectors in the first and second diagram. Explain why this is the case, referencing the horizontal and vertical vector components of the first two vectors.

The negatively charged particle is replaced with a positively charged particle of equal magnitude. The first and second diagram represent the electric field of the positive charge on the left and right respectively.

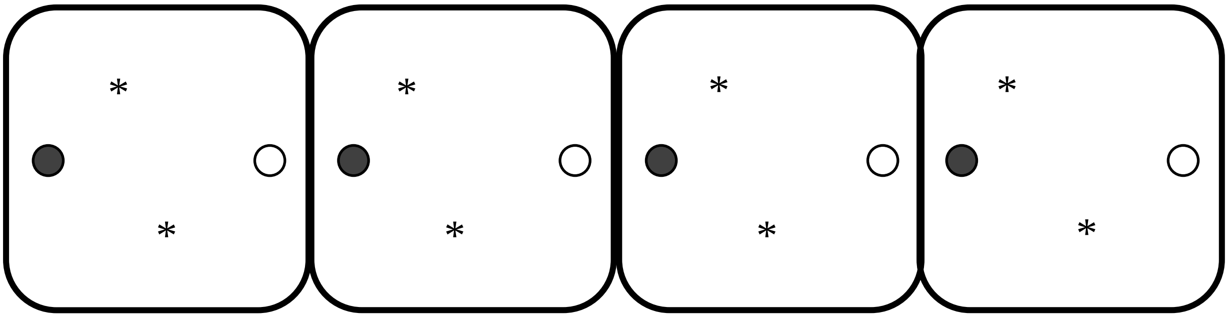


(iv) Draw the net electric field at this point in the third and fourth diagrams using the vector arrows presented in the first two diagrams, in the same manner as shown on the previous page.

(v) Is the net electric field you drew pointing in a horizontal, vertical or diagonal direction? Explain why it points in this direction, referencing the horizontal and vertical components of the first two vectors.

(vi) If we measure the magnitude of the vectors in the first, second and third diagram, we will find the net electric field in the third diagram has a weaker magnitude than the sum of the vectors in the first and second diagram. Explain why this is the case, referencing the horizontal and vertical vector components.

(vii) In the diagram below, use vectors to show the net electric field at the two points points shown, between a positive and negative charge. Show the initial component vectors and how you use them to construct the net electric field in the fourth diagram.



III. The forces experienced by a particle by an electric field – Field treatment.

When representing an electric field, it can be easier to use field lines. We have seen in mechanics that we can use field lines in replacement, or combined with vectors to show the direction of the force at a point, and the relative strength in a field.

The electric field always points in the direction that a small positive charge would feel a force. From this, we determine that an electric field line will always point away from a positively charged object, and towards a negatively charged object.

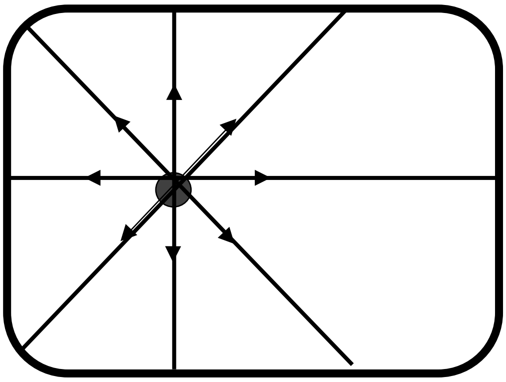
The rules you've already covered about field lines is as follows.

- The closer field lines are, the stronger the field.
- When a field line curves, the direction of the force is tangential to the field lines.
- The field line represents the direction of force acting on a body, no the path taken by a body in the field.

Other rules for using field lines are as follows:

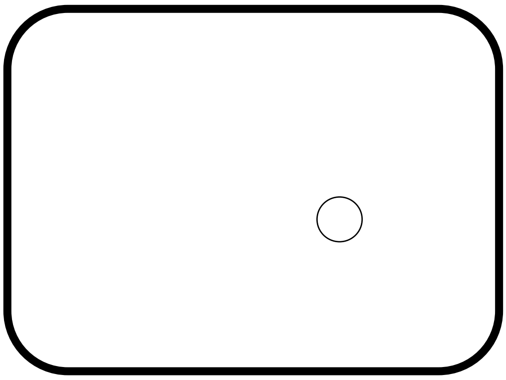
- Field line patterns do not overlap or intersect. Two bodies that cause a field both contribute to one overall field surrounding both objects.
- Field lines do not finish, or terminate. They should extend to infinity / off the page / to the end of the diagram boundary.

(i) Taking this into account, identify the charge on the following particle. Explain how you can tell.



(ii) Does the electric field strength increase, decrease or stay the same as you move away from the charge? How can you tell?

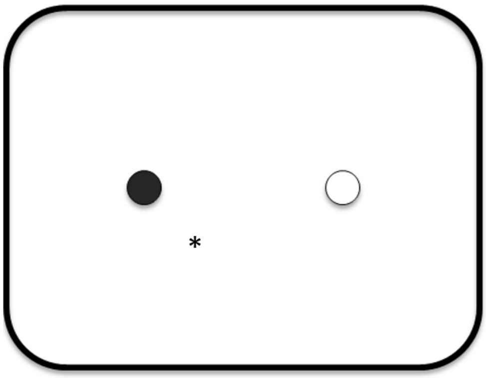
(iii) Draw the electric field for an oppositely charged particle, as shown to the right.



(iv) Explain the differences and similarities for the field for the two charges.

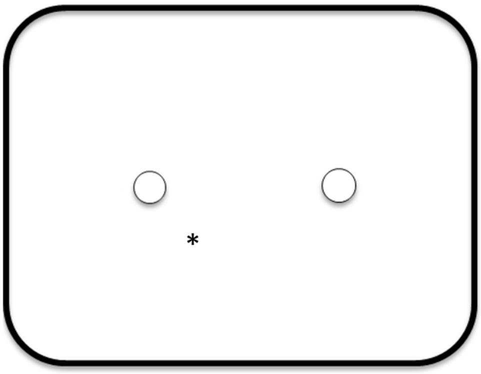
IV Electric field vectors – Principle of Superposition.

(i) Using the principle of superposition (how you determined the net vectors on pages 2 and 3) sketch the electric field between the positive and negative charge of equal magnitude. Ensure that you draw one field line going through the star.



(ii) If a positively charged particle were to be placed at rest at the star, sketch the path it would take. Assume the initial two charges do not move. You can represent the path taken in any manner you see fit (a bold line, a strobe diagram, vectors, etc). Explain why you drew the path as you did.

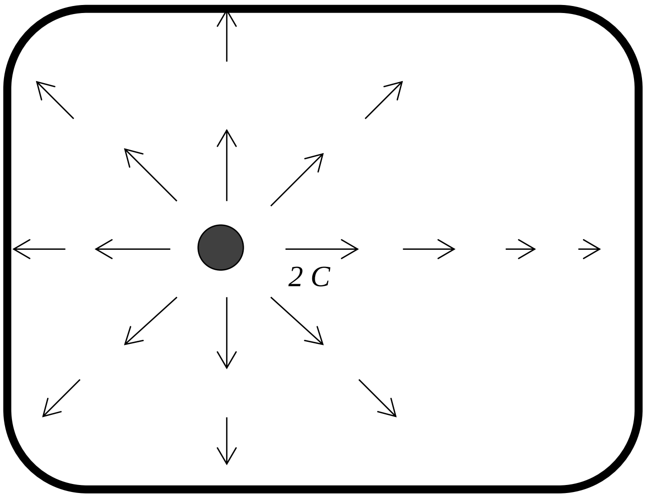
(iii) Using the principle of superposition (how you determined the net vectors on pages 2 and 3) sketch the electric field between two negative charges of equal magnitude. Ensure that you draw one field line going through the star.



(iv) If a negatively charged particle were to be placed at rest at the star, it would take. Assume the initial two charges do not move. You can represent the path taken in any manner you see fit (a bold line, a strobe diagram, vectors, etc). Explain why you drew the path as you did.

I. Electric field of a particle.

The electric field surrounding a *2 C* charge is shown in the diagram to the right, using vectors.

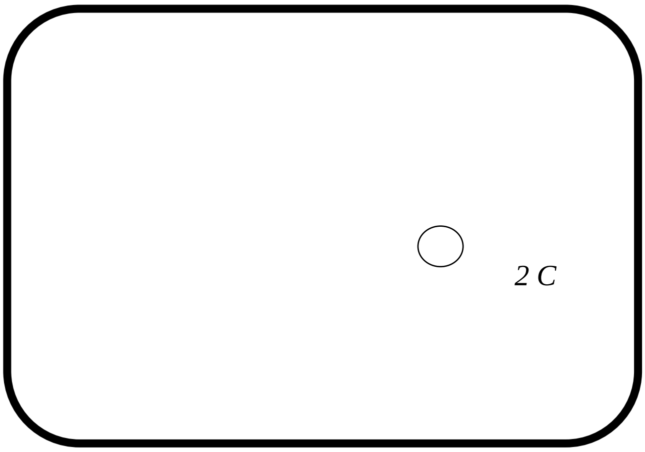


(i) What can you tell about the electric field strength as the distance from the charge increases. How can you tell?

(ii) By using the arrow directions, detemine whether the 2 C charge is positive or negative.

The charged particle is removed and an particle that **is oppositely charged** is placed down, as shown in the following diagram.

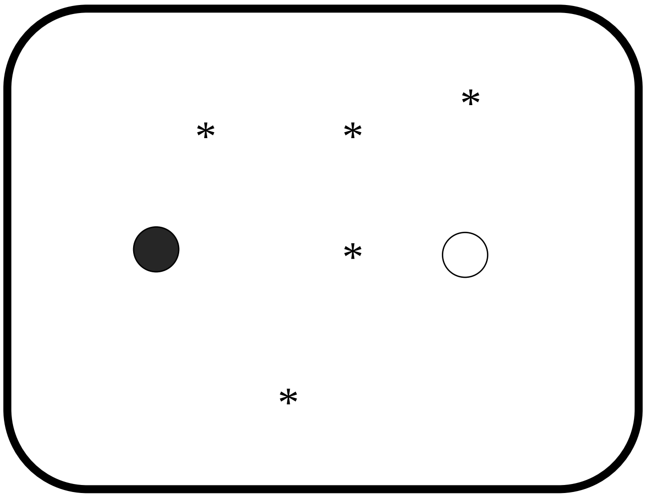
(iii) Sketch the electric field, ***using vector arrows***, the electric field around the new charge.



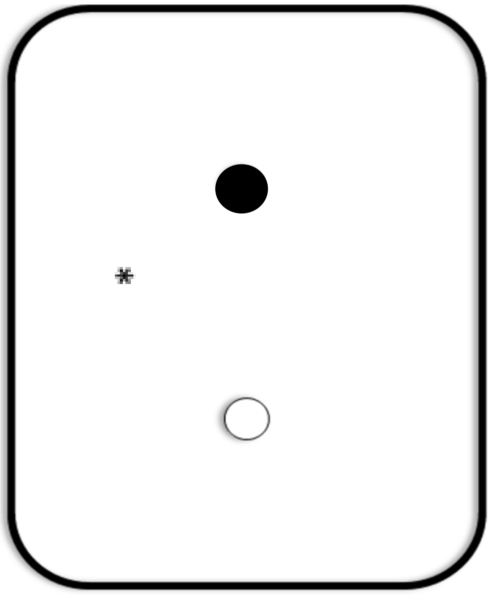
(iv) Explain how your arrows show the change in electric field strength, if any, as you move away from the electric charge.

(v) Explain why you drew your arrows pointing either ***towards or away from*** the negative charge.

(vi) ***Construct vector arrows*** at the points marked with stars to show the electric field around a positive and negative charge. (use the diagrams you did already in ***I(i)*** and ***II(i)*** to help you)



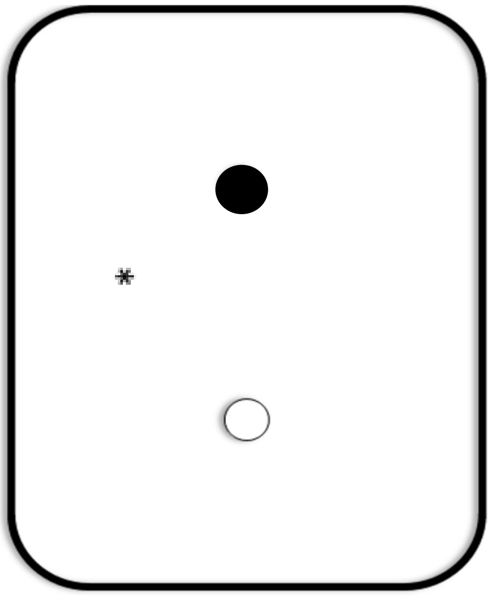
II. Path taken by a charged object in an electric field.



(i) In the above diagram, represent the electric field between the two charged particles, using any manner of your choosing. In this setup, the positively charged particle (black) has twice the magnitude of the negatively charged particle.

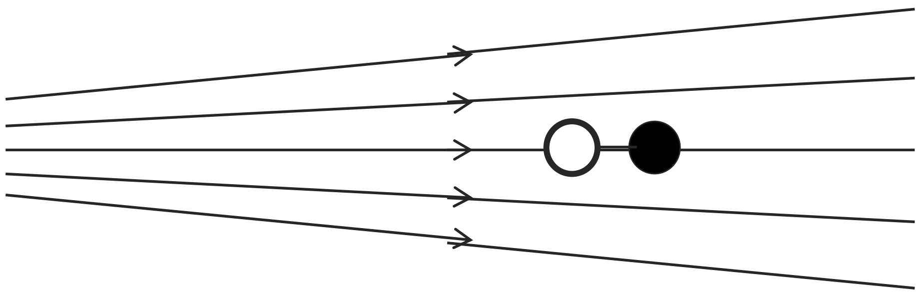
(ii) Draw in the path a positively charged particle would take, if it were placed at the star. Explain why you drew it as you did.

(iii) On the second diagram, draw in the path the particle would take if it has an initial velocity to the right, as shown in the diagram with the arrow. Explain why you drew as you did.



III. Behaviour of charges in an electric field.

An electric field is shown in the following diagram. Within this electric field, a negatively charged particle (white circle) is attached to a positively charged particle (black circle) so they cannot be separated. They are initially at rest.



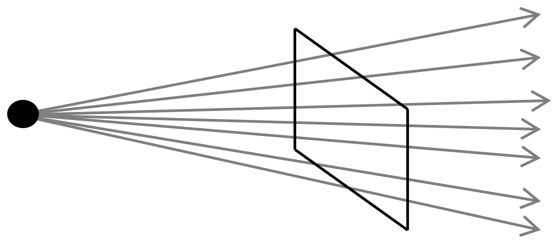
(i) As you go from left to right, does the electric field strength increase, decrease or stay constant? Explain how you determine this?

(ii) What direction will the force on the negatively charged particle act, caused by the electric field? Explain.

(iii) What direction will the force on the negatively charged particle act, caused by the electric field? Explain.

(iv) Using your answers from (i) to (iii), determine whether two charges will move to the left, right or will remain at rest. Justify the outcome you pick.

I. Investigating the variation of electric field strength with charge.



The above diagram uses lines to represent an electric field, of a *+1 C* charge, passing through a *1 m^2* frame. While not accurately sketched, assume there are 100 field lines coming from the charged particle passing through the frame. ***There should be more lines coming symmetrically from the charge in all directions, but for simplicity, we have not drawn on the diagram above.***

(i) If we doubled the magnitude of the charge (*+2 C*), we would double the amount of field lines that are coming from the charge. How many lines pass through the *1 m^2* frame?

(ii) If we had a charge of +4 C, how many lines would pass through the *1 m^2* frame?

(iii) What effect does increasing the charge generating a field have on the electric field strength at a point? Use (i) and (ii) to justify your answer (include the type of relationship observed).

(iv) If we used a *2 m^2*, frame, we would see 200 lines passing through it, as the lines are coming out of the charge symmetrically. Does using a bigger frame change the intensity of how many lines pass through a 1 m^2 frame? Explain your answer (may help to consider how many lines are in a *3 m^2*, *4 m^2*, etc frame)

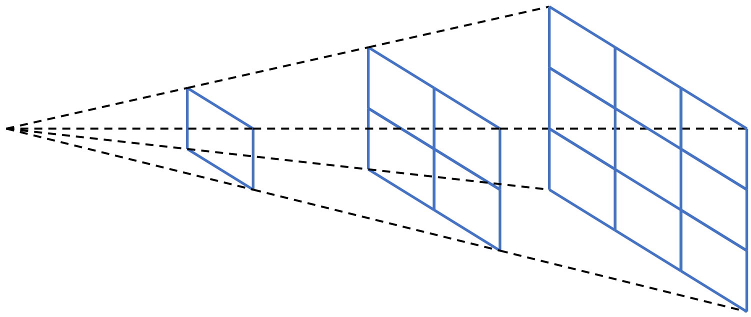
(v) Using your answer from section I (iv) and section I (iv), explain why changing the magnitude of the test charge used to measure electric field strength has no effect on the electric field strength at that point.

(v) If the +1 C charge was replaced with a – 1 C charge, what affect, if any, would you have to make to the field lines?

(vi) How would replacing the positive charge with a negative charge affect the number of field lines passing through *1 m^2* frame?

(vii) Using your answer from (v) and (vi), explain why changing the sign of the test charge used to measure the electric field strength at a point has no effect on the electric field strength.

II. Investigating the variation of electric field strength with distance.



A second frame is placed away from the charge, so that all its outer corners are twice the distance from the charge This frame is made up of smaller frames that are identical to the frame in section II.

(viii) Explain why this second frame has an area that is four times bigger than the original frame used on the last page.

(ix) Determine the how many field lines through each of 1m^2 frames, for this second overall frame.

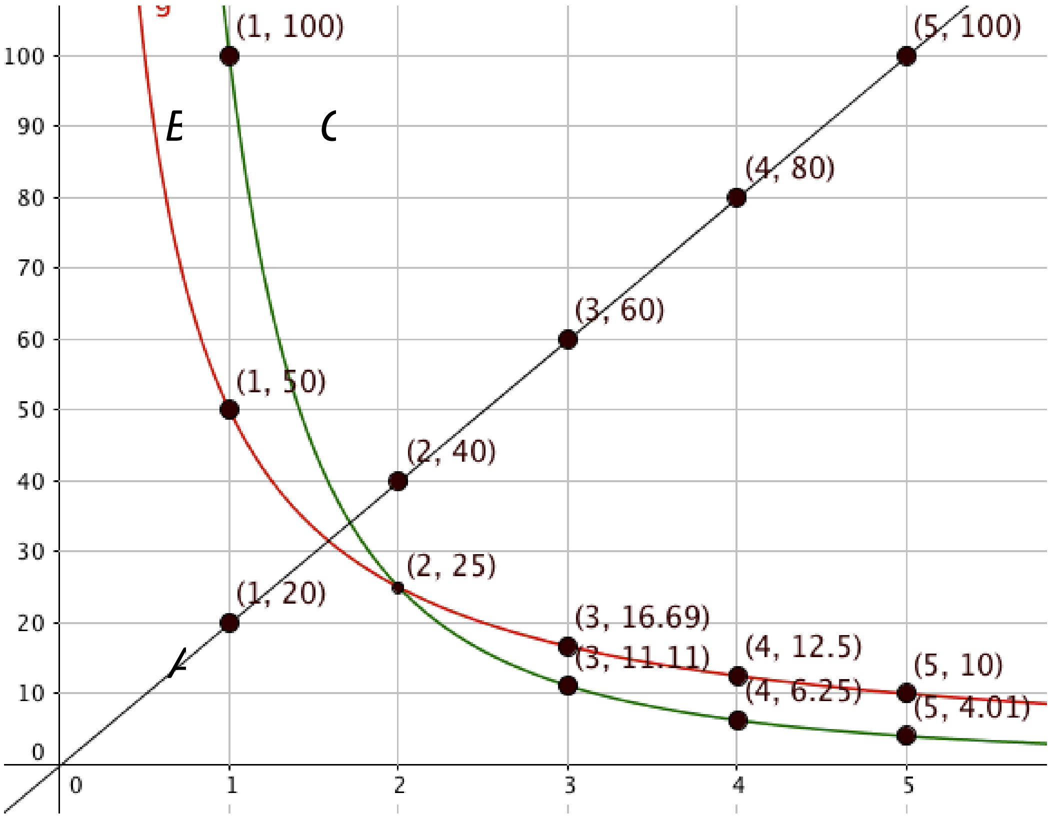
A third frame is placed away from the charge, so that all its outer corners are triple the distance from the charge. This frame is made up of smaller frames that are identical to the frame in section II.

(x) Explain why this third frame has an area that is nine times bigger than the original frame used on the last page.

(xi) Determine the how many field lines through each of 1m^2 frames, for this third overall frame.

(xii) As the distance from the charge increases, the number of lines passing through a 1 m^2 area decreases. Using your answers from the previous questions, explain why this occurs. (Explain the relationship involved)

1. The following graph shows 3 patterns for different types of functions, *A*, *B* and *C*.



(i) Determine which of the three functions is of the form: $y = mx$. Justify your answer using whatever reasoning you wish.

(ii) Determine which of the three functions is of the form: $y = k\frac{1}{x^2}$. Justify your answer using whatever reasoning you wish.

(iii) Which graph represents the relationship between the force (y-axis) and distance (x-axis) between two charged objects following Coulomb's law?

(iv) Explanation:

(v) Which graph represents the relationship between the force (y-axis) and product of two charged objects (x-axis) following Coulomb's laws?

(vi) Explanation:

2. A $+2\ \mu C$ charge and a $+3\ \mu C$ charge are held 20 cm away from each other. The force of repulsions between the two charges is 1.35 N (rounded to two decimal places). (remember, $1\ \mu C = 1 \times 10^{-6}\ C$)

(i) If you replaced the $+3\ \mu C$ charge with a $+\ 12\ \mu C$ charge, what would the magnitude of the force of repulsion be?

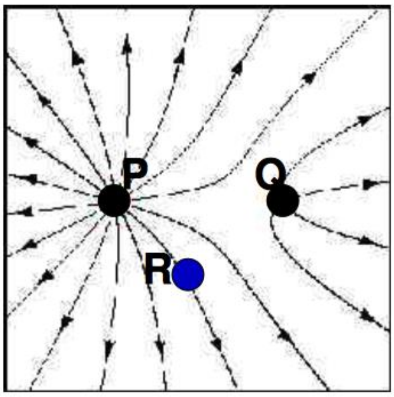
(ii) Use Coulomb's law (equation on last page) to calculate (to 2 decimal places)the force of repulsion between the $+2\ \mu C$ and $+\ 12\ \mu C$ charge at a distance of *20 cm* from eachother. Use your answer to verify your answer from (i) .

The original +2 μC charge and a +3 μC charge are held 20cm away from each-other again.

(iii) If the distance between the charges was increased to 40 cm away from each-other, what would the magnitude of the force of repulsion be?

(ii) Use Coulomb's law to calculate (to 2 decimal places)the force of repulsion between the $+2\ \mu C$ and $+\ 3\ \mu C$ charge at a distance of *40 cm* from each-other. Use your answer to verify your answer from (iii).

3.



Consider the electric field above, where P and Q are charged particles and R is a point in the electric field.

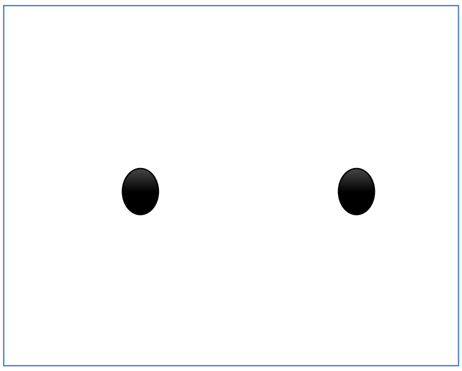
(i) Determine the charge on P and Q. Explain how you can tell.

(ii) Is the magnitude on the charge P greater than, equal to or less than the magnitude on the charge Q. Explain how you can tell.

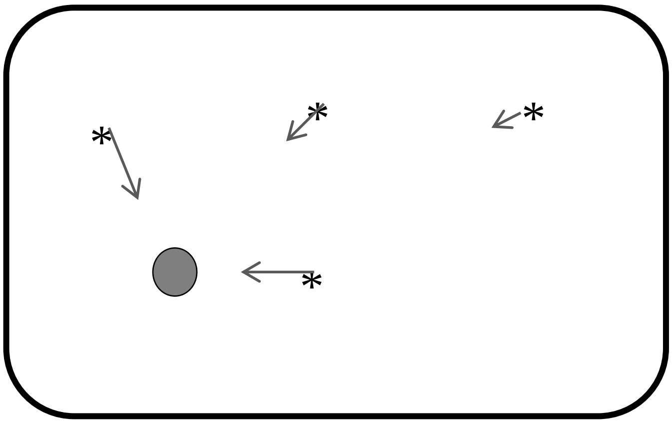
(iii) Place a finger on P and follow one of the field lines coming out of P. As you trace out the path, does the electric field strength increase, decrease of remain unchanged? Explain how you can tell.

(iv) If a negatively charged particle was placed at R, draw on the diagram the path you think the particle would take. Explain why you think it would take this path.

(v) Use vector arrows to above to represent the field shown above, at the following points.



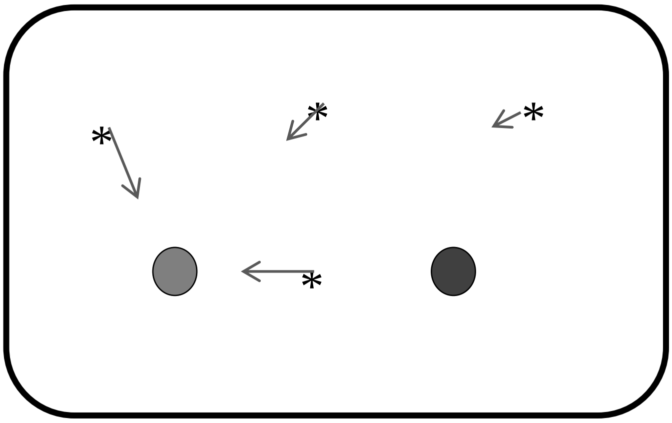
4. An electric charge is nailed down and the electric field at points around it is shown in the diagram, using vector arrows.



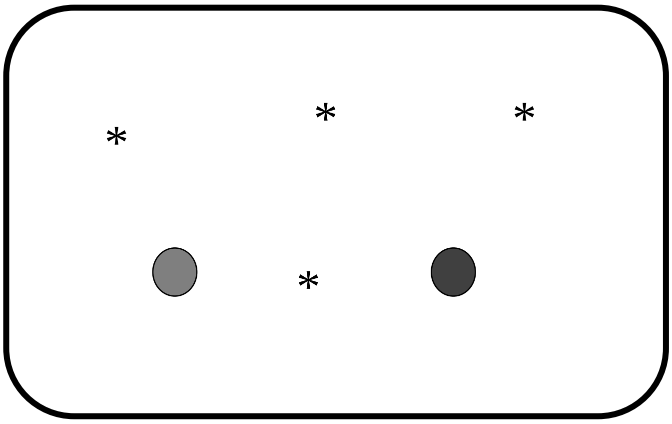
(i) What sign is the charge? Explain how you can tell.

(ii) Explain why the lengths of the vector arrows are not all the same.

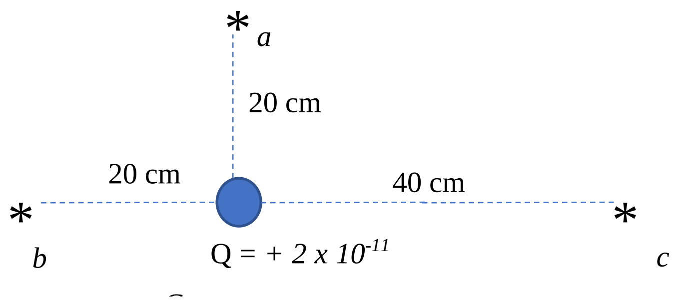
(iii) Another charge, of opposite sign and equal magnitude, is placed at the position shown in the following diagram. ***Construct*** the vectors that show the magnitudes and directions of the electric field of the two charges.



(iv) Use field lines to show the field produced by these two charges.



5. A positively charged object is placed down as shown above. 3 points are marked, a – c, are marked around the charge. The magnitude of the charge, and the distances to the points from the charge are shown on the diagram.

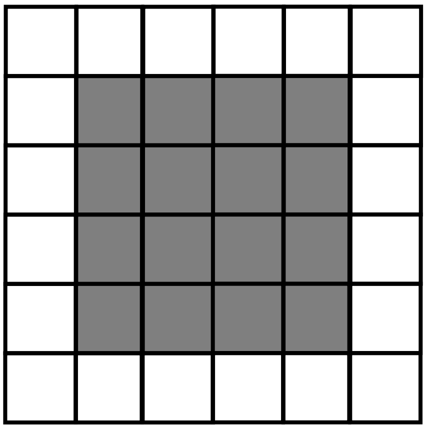


(i) Rank the strength of the electric field, *from lowest to highest*, at the positions *a, b* and *c*. Justify your ranking.

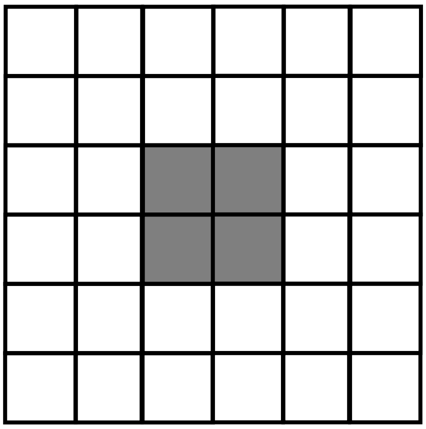
(ii) Show that the electric field at a point around an electric field is given by the formula $E = k\frac{Q}{d^2}$, where $k = \frac{1}{4\pi\varepsilon}$.

(iii) What is the ratio of the magnitude of the electric field at *c* to *a?* Show your workings.

6. A charge is placed down and a square shaped grid is held a distance of 10m from it. 100 electric field lines go through the shaded area of the grid.



(i) At what distance do I need to move the grid from the charge to get the 100 electric field lines to pass through the following shaded area. Explain your answer.



Distance to get 100 lines in this area:

Justification:

Formulae:

You may still want to have your mathematical tables handy, unless I missed something but these are the formulae you should need to complete this test.

$$F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{d^2}$$

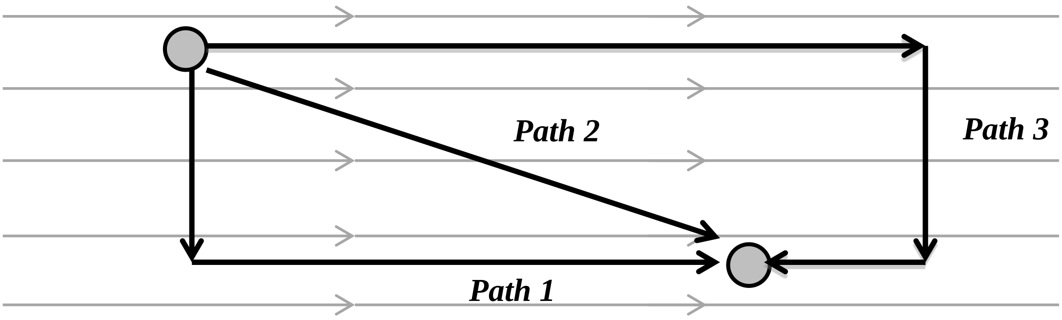
$$E = \frac{F}{q}$$

$$E = \frac{1}{4\pi\varepsilon} \frac{Q}{d^2}$$

Appendix E

Work and potential difference tutorial materials.

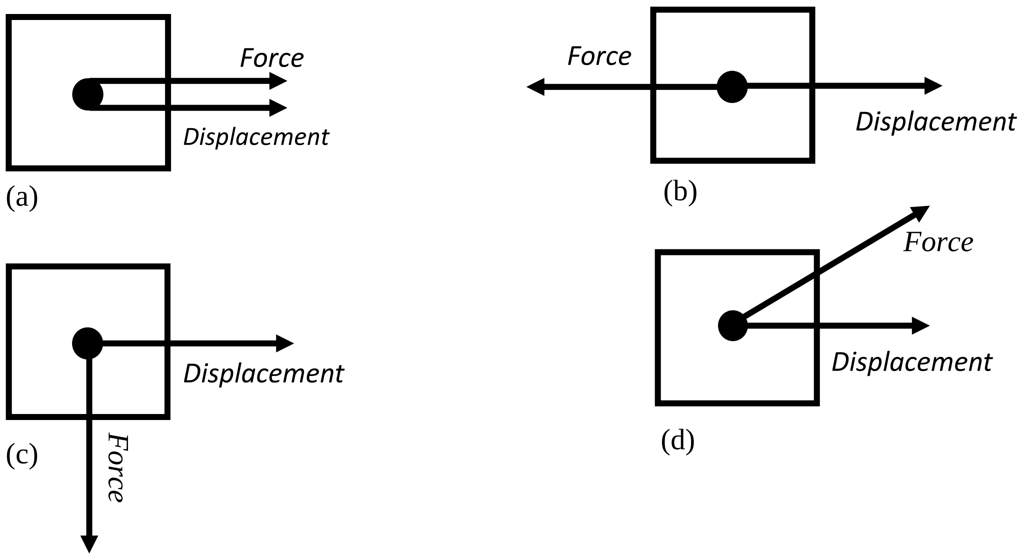
1. The electric field shown is uniform. A positively charged particle can be moved from point *a* to point *b* in one of the three paths as shown. Rank the net work done, by the field, in moving the charge from point *a* to point *b* in the different paths it can take. Justify your ranking.



Ranking:

Justification of ranking:

2. Rank the magnitude of the work done for the following pairs, (a) to (d), of Force – Displacement vectors.

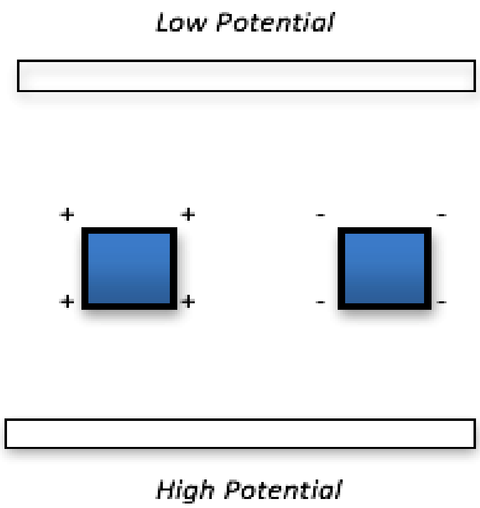


Ranking:

Justification for ranking:

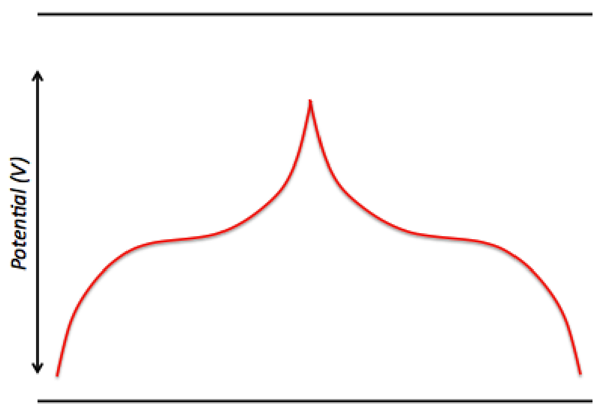
3. A positively charged box and a negatively charged box are suspended between two charged plates, one which has high potential and the other has low potential.

(i) When the positively charged box is released, will it move (a) towards the high potential or (b) towards the low potential. Explain



(ii) When the negatively charged box is released, will it move (a) towards the high potential or (b) towards the low potential. Explain

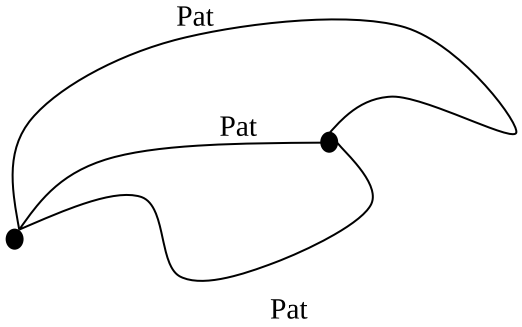
4. On the top line, draw the charges that need to be placed down to show the change in potential as you move from left to right.



Explain why you drew as you did:

I. Distance vs Displacement.

A person can move from point *A* to point *B* using one of the three paths shown.



(i) In which path, if any, is the distance travel greatest?

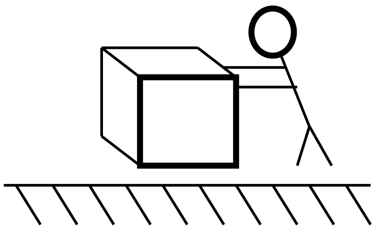
(ii) In which path, if any is the displacement greatest?

(iii) Explain why the distance travelled is not the same as the displacement.

II. Calculating work.

A person pushes a cube shaped block and ground as shown. The block weighs *100 N* and the person is pushes it with a force, F_{Push}, of *50 N*. The block moves a total displacement of *6 m*.

(i) On the diagram, draw vector arrows, to show the direction of the force and the direction of the displacement.



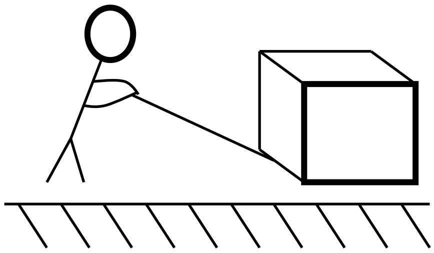
(ii) Calculate the work done by ther person in pushing the block.

(iii) What work is done, if any, on the block by gravity? Explain

III. Resolving a force vector into component vectors.

The person attaches a rope to the block to pull the block, to the left, with a force, F_{Pull}, of 50 N. The rope makes an angle, θ, of 30° with the ground. The block is pulled a displacement of *6 m,* to the left.

(i) On the diagram, draw vector arrows, to show the direction of the force and the direction of the displacement.



(ii) From your answers in (vi), is the work done in pulling the block *positive*, *negative* or *zero*? Explain.

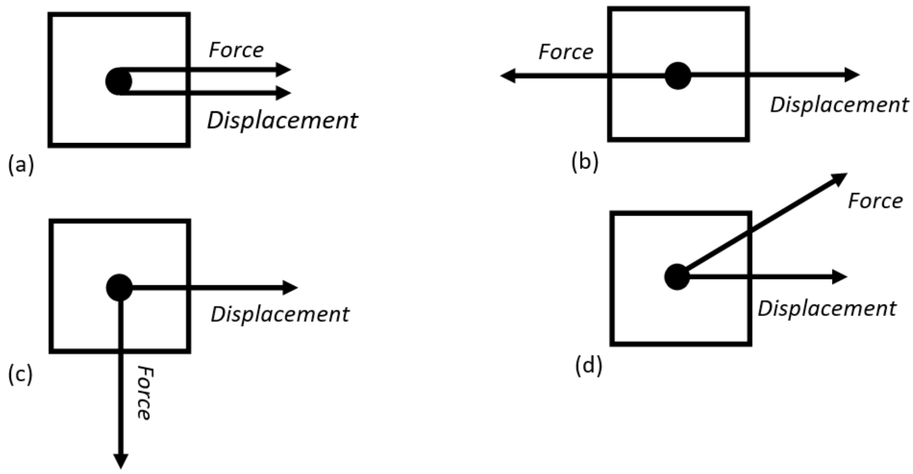
(iii) Copy the force vector into the box to the right. Resolve the vector into it is horizontal, F_x and vectical, F_y, components.

(iv) Which component, if any, contributes *zero* work to moving the block to the left.

(v) Show that the horizontal force moving the block can be given by $F_x = F_{Pull}Cos\theta$

(vii) From part (v), show by multiplying the horizontal force, F_x, component by the displacement, *s*, that the work done on the block is 260 J.

(viii) Rank the magnitude of the work done for the following pairs of Force – Displacement vectors. You may use a ruler to record the relative magnitudes, if necessary. If required, resolve the force vectors in the following into horizontal components.

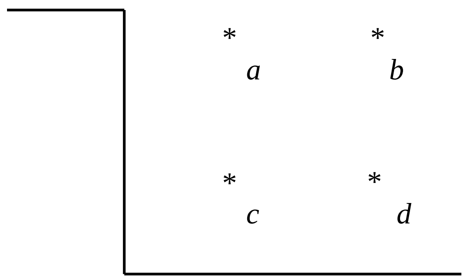


Ranking:

Justification:

III. Work done in a gravitational field.

An object, of mass 1kg, is moved between the points shown in the diagram.



(i) What is the direction of the gravitational force acting on the mass when it is held at *a, b, c* and *d?*

(ii) When the mass is moved from *a* to *c*, is the displacement *in the direction of, against the direction of* or *perpendicular* to the gravitational force?

(ii) Is the work done by the gravitational field, in moving the mass from *a* to *c*, *positive, negative* or *zero*. Explain.

(iii) When the *1 kg* mass is moved from *c* to *a*, does the potential energy of the mass increase, decrease or remain the same.

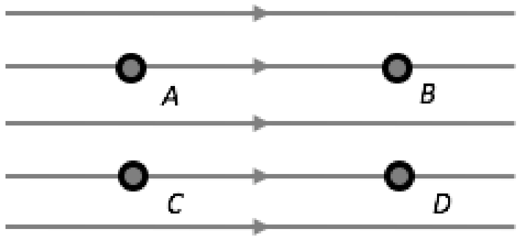
(iii) When the *1 kg* mass is moved released from *a* to fall to *c*, how does the kinetic energy at *c* compared to it is potential energy at *a*.

(iv) When the mass is moved from *d* to *b*, is the displacement *in the direction of*, *against the direction of* or *perpendicular* to the gravitational force?

(v) Is the work done by the gravitational field, in moving the mass from *d* to *b*, *positive*, *negative* or *zero*. Explain.

(vi) Is the work done by the gravitational field, in moving the mass from *a* to *b*, *positive*, *negative* or *zero*. Explain.

I. Work done by a field on a charge.



A positive charge (+*Q*) is placed in a uniform electric field, and is moved in the following paths.

(i) In which paths is the net work done on the charge, by the field, positive, negative or zero. Explain you reasoning.

A to *B*

C to *B*:

D to *C*:

A to *B* to *D* to *C*:

A to *C*:

A to *B* to *C* to *D:*

(ii) How does the <u>*net work done on the charge, by the field*</u>, compare when it moves from *A* to *B* as when it moves from *C* to *D*? Explain.

(Iii) How does the <u>*net work done on the charge, by the field*</u>, compare when it moves from *A* to *D* as when it moves from *A to C to D*? Explain.

Two people are considering what occurs when the charge in moved through the two paths outlined. Their understanding is shown below.

Person 1: *When we move the charge from A to D directly, there is less work done than moving it from A to C to D as we add up the work done moving from A to C directly to the work moving from C to D directly.*

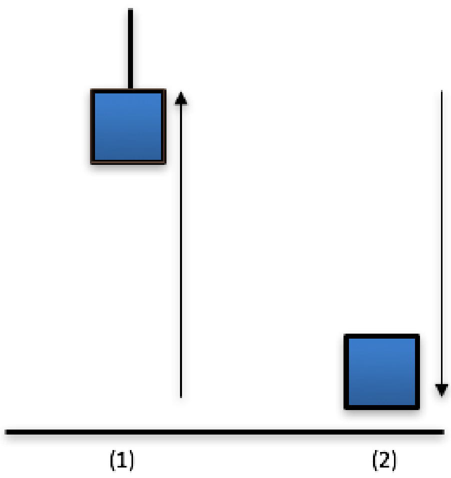
Person 2: *When the charge if brought from A to C and C to D, the displacement has a vertical component which gives zero work. This makes the work done independent of the path taken.*

(vi) Which person do you agree with?

What error in understanding did the other person make?

II. Work done on mass in gravitational field.

A 1 kg box is lifted a distance of 3 m into the air, as seen in (1). The box is then released so it falls to the ground (2). (W = Fs)



(i) Calculate the work done, by gravity, when the 1 kg box falls to to the ground.

(ii) Calculate the work done, by gravity, if the box had a mass of (a) 2 *kg* and (b) 3 *kg*.

(iii) Find the ratio (fraction) of *work done* per *mass of the box* for your answers in (i) and (ii), and write them in their simplest form.

(iv) What do you notice about all of the ratios?

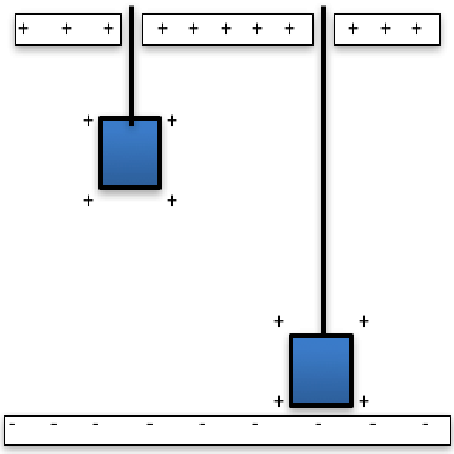
(v) How does the potential energy of the box in (1) compare to the kinetic energy of the ball just before it hits the ground in (2).

(vi) Express your answer from (v) mathematically.

*****Checkpoint****

III. Work done in a charge moving between plates.

A *+1 C* box is lifted a distance of *3 m* towards a positive plate (3) and then released towards a negative plate (4). The electric field between the two plates is uniform and has a magnitude of 2 $N\,C^{-1}$. ($W = Fs$, $F = qE$)



(i) Calculate the (a) force and then (b) the work done, by the electric field, when the *+1 C* box is brought to the negative plate.

(ii) Calculate the work done, by gravity, if the box had a charge of (a) +2 *C* and (b) +3 *C*.

(iii) Find the ratio (fraction) of work done per mass for your answers in (i) and (ii).

(iv) What do you notice about all of the ratios?

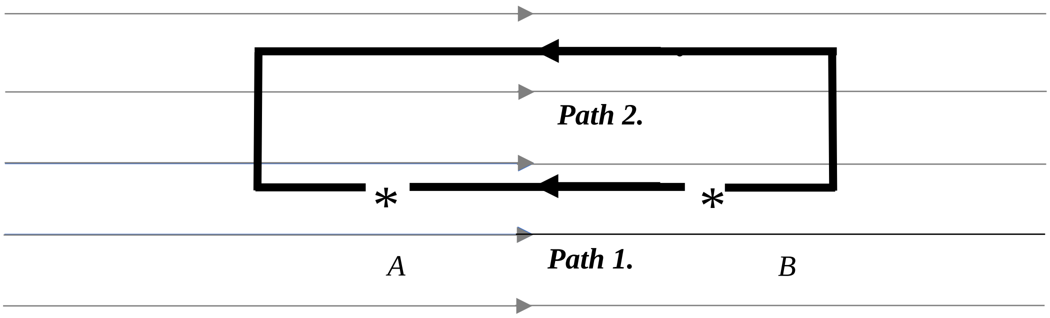
(v) How would you define this ratio, in your own words?

*****Checkpoint*****

In electricity, there is a special name given to the ratio you just calculated. This is called **potential difference (V),** and is defined as *the work done in moving a unit of charge (+1 C) between two points in an electric field or electric circuit.*

$$V = \frac{W}{q}$$

A uniform electric field is shown below. Two points, A and B, are highlighted. There are two paths between the points, also shown on the diagram.



(i) Explain how you can tell it is a uniform electric field.

(ii) It takes *6 J* of energy to move a -1 C charge from *B* to *A*, along *Path 1*. What is the potential difference between *A* and *B*?

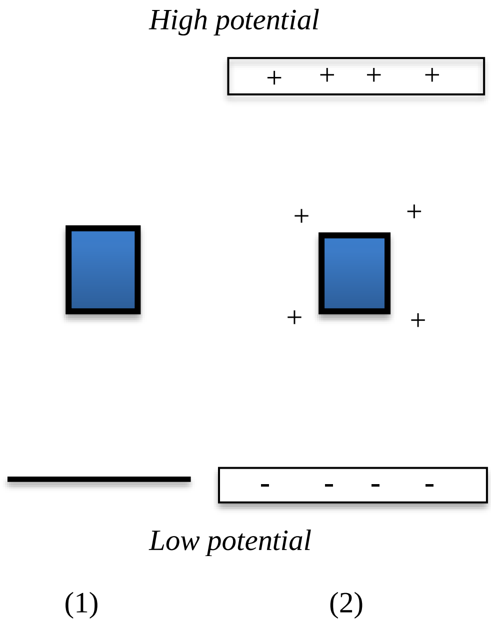
(iii) What is the work done in moving the *-1 C* charge from *B* to *A*, along *Path 2*? Explain your answer.

(iv) How much energy would it take to move a *-4 C* charge?

(v) If I use 36 J moving a charge from *A* to *B*, what is the magnitude of the charge moved?

(vi) If a charged particle of magnitude -*3 C* has a mass of *0.5 g*, determine the magnitude of the velocity it would have at *A* when it travels along *Path 1* from *B to A*. ($W = {}^{1}/_{2}\, mv^2, V = \frac{W}{Q}$)

I. Positive and negative charges in a potential difference.



A box held is held a height above the ground, as seen in (1).

(i) As it is height above ground increases, what change is seen in it is gravitational potential energy?

(ii) When it is released, will the box move up, fall down or remain at it is height?

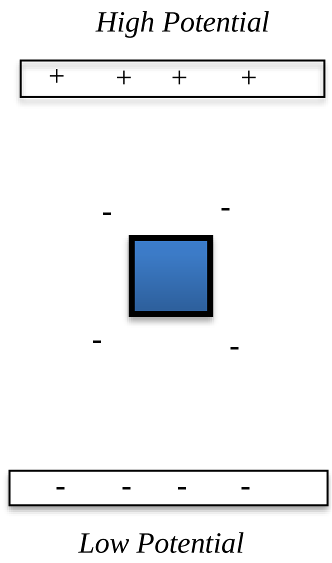
(iiI) From your answer in (i) and (ii), does the box move from (a) low to high potential (b) high to low potential or (c) is not affected. Explain.

A positively charged box is held between a postively charged plate and a negatively charged plate. We associate a high potential with positively charged plate and a low potential with the negatively charge plate.

(iv) When the box is released, will it move towards the positively charged plate, or the negatively charged plate. Explain your answer, ***referring to either the forces involved or the electric field between the plates.***

(v) In terms of potential, is the positively charged box moving from an area of high to low potential or low to high potential? Explain.

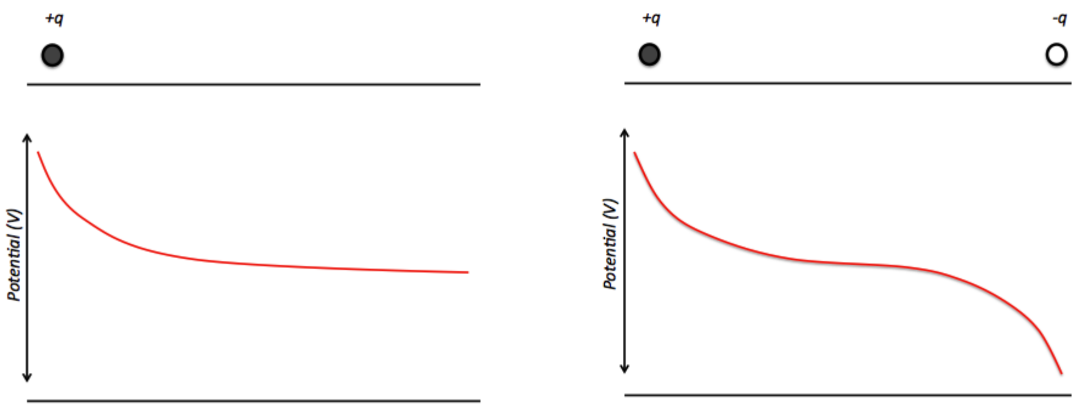
(vi) If the box was replaced with a negatively charged box, which way would it move?



Explanation (refer to potential):

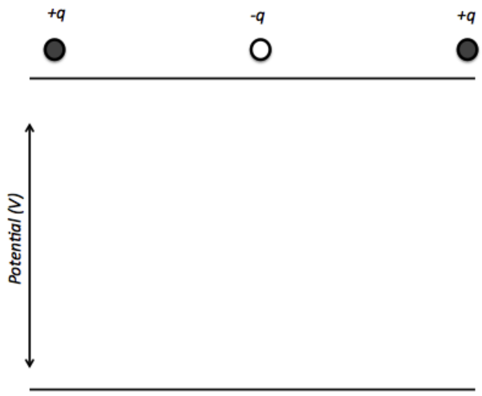
II. Graphing potential.

We have seen that regions of positive charge are considered to have high potential and areas of negative charge are areas of low potential. Consider the following graphs of potential for the setups shown, as you moves from left to right in each.

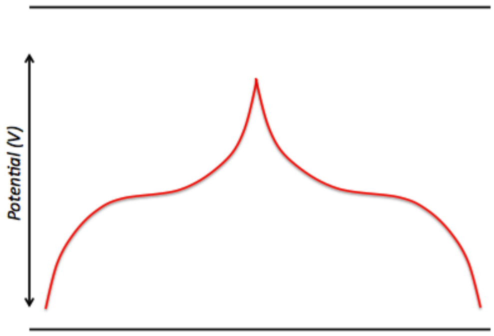


(a) *(b)*

(i) Explain the shape of both graphs as you move from left to right.



(c)



(d)

(ii) Draw on the graph how the potential varies from going from left to right for the setup shown in (c). Explain why you drew it as you did.

(iii) Draw the setup that produces the graph for potential as seen in (d). Explain why you drew the setup as you did.

III. Understanding a battery.

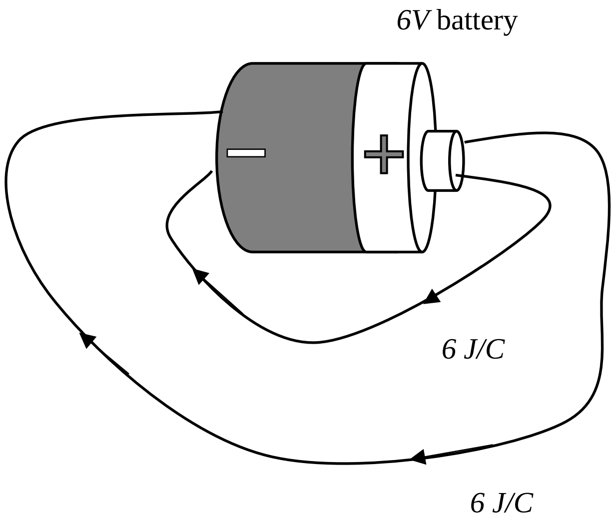
In a battery, there is a positive and a negative terminal. The potential difference that exists between these is usually printed on the battery (typically *1.5 V*, *6 V*, *9 V* and *12 V*). Use your understanding of some or all of the following:

- work,
- potential,
- electric fields,
- electric field lines,
- vectors,
- the behaviour of negative charges in electric fields,

- the behaviour of negative charges between a potential difference

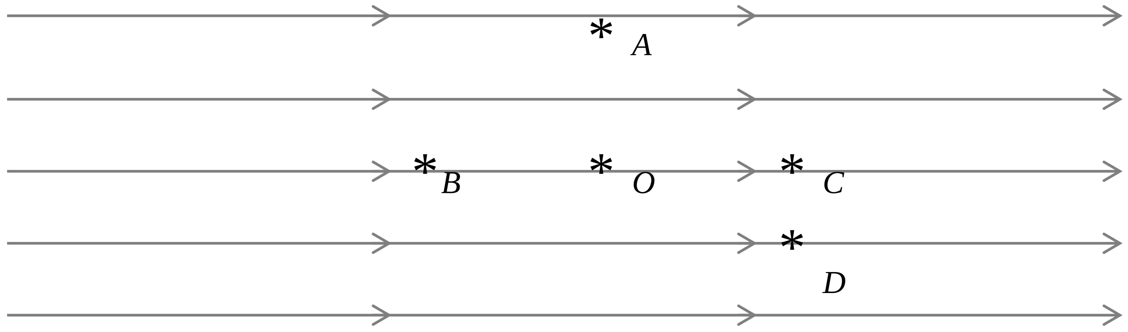
to explain, In the space below, explain why:

3. Current (which is moving negative charge) flows from the negative to the positive terminal.
4. The *work done per charge* in moving current from one terminal to the other is constant, regardless of the length / layout of the wire.



Relevant formulae: $E = \frac{F}{Q}$ $W = F.s$ $V = \frac{W}{Q}$

1. A ***negative charge*** is placed in an electric field as shown, at position *O*. It can be moved to any of the positions marked on the field as shown.



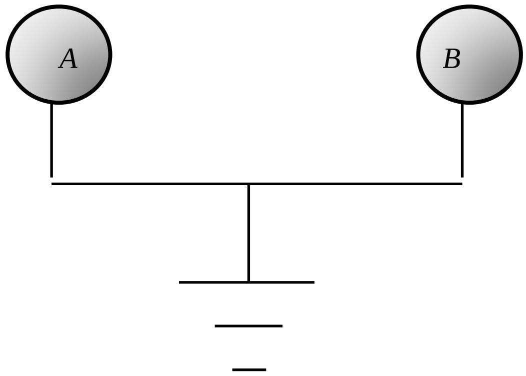
(i) Draw in the direction of the force acting on the negative charge, when placed at *O*.

(ii) State whether the work done in each case is *positive, negative or zero*, when ***the negative charge*** is moved from *O* to *A*, from *O* to *B*, from *O* to *C* and from *O* to *D*. Explain.

(iii) How does the work done, by the field, in moving the charge from "*O* to *C*" compare moving from "*O* to *D*" and "*O* to *C* to *D*?" Explain.

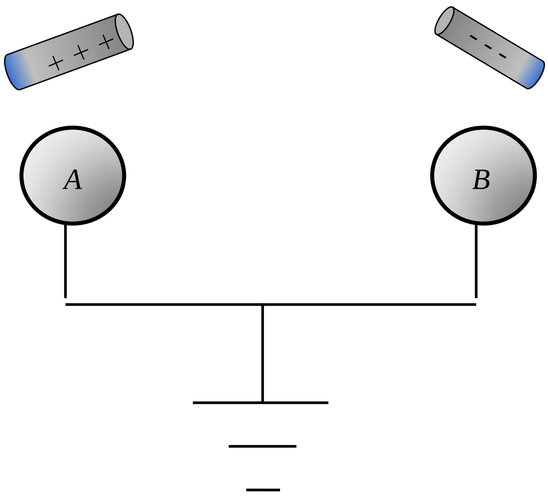
(iv) A student says that the potential difference between the B and O, is the same as the potential difference between O and C. Do you agree with this student? Explain why you think they are correct / incorrect.

2. Two conducting balls are connected to the ground as shown in the diagram.



(i) How does the potential at *A* and *B* compare to each-other and the ground. Explain.

A positively charged rod is held about *A* and a negatively charged rod is held above *B*.



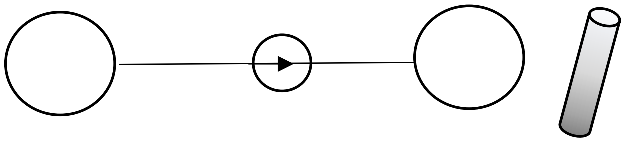
(ii) What effect does this have on the potential of *A*, initially.

(iii) After some time passes, what type of charge will build up on *A*. Explain, **referencing the potential** on *A* you gave in (ii).

(v) What effect does this have on the potential of *B*, initially.

(vi) After some time passes, what type of charge builds up on *B*. Explain, **referencing the potential** on *B* you gave in (iv).

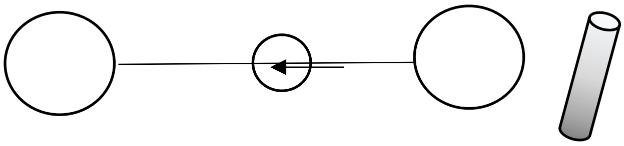
3. Two metal domes are connect with a conducting wire. A device with shows the direction of the movement of charge is placed on this wire.



When a charged rod is placed beside the rightmost dome, the negative charge begins to move, in the initial moments, in the direction as shown on the device.

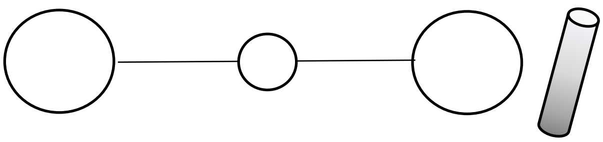
(i) Using this information, compare the potential on the left dome to the potential on the right dome. Explain your comparison.

The rod is taken away replaced with a rod that causes the negative charge to move, in the initial moments, in the direction as shown on the device.



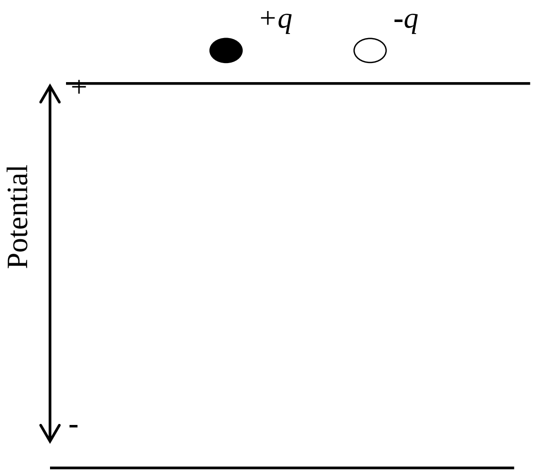
(ii) Using this information, compare the potential on the left dome to the potential on the right dome. Explain your comparison.

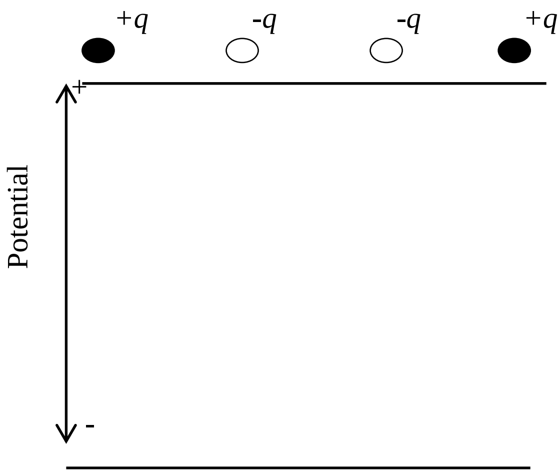
After a long period of time has past, the device registers that there is no electric charges moving, as shown in the diagram.



(iii) What does this tell you about the potential of both domes? Explain.

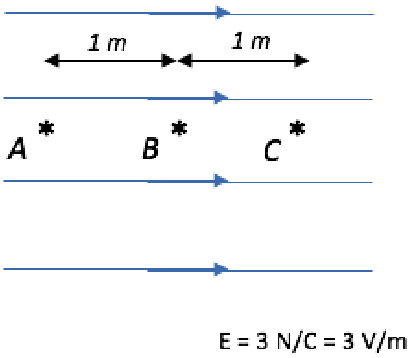
4. Draw on the graph how the potential varies from going from left to right for the setups shown. Explain why you drew it as you did.

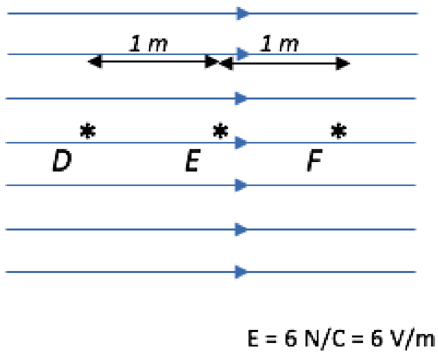




Explanation:

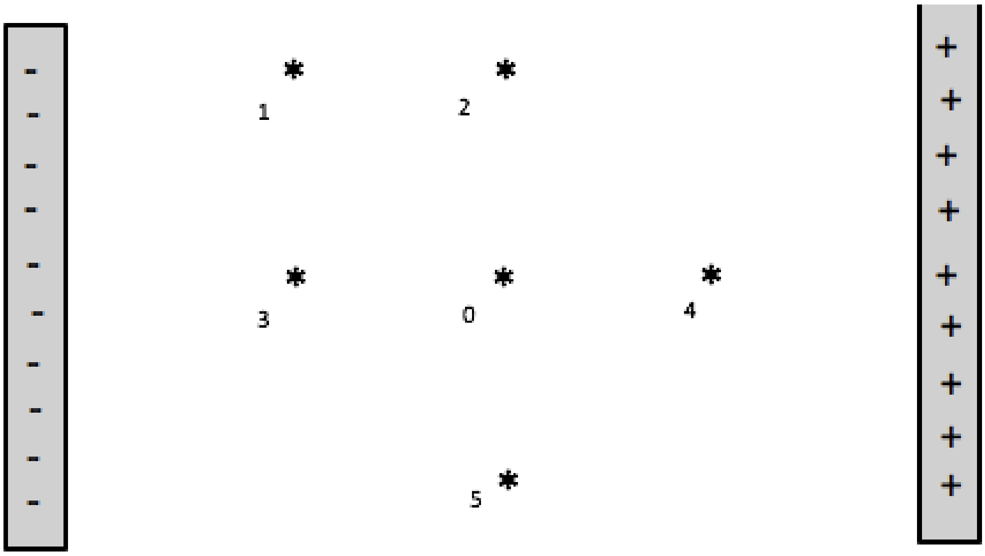
5. Two electric fields are shown in the diagram below. Both fields uniform. The field strength and distance between the points is shown on the diagram.





(i) Rank the Potential difference, from highest to lowest, between AB, AC, DE, and DF. Justify your answer using whatever reasoning you think is necessary (there are multiple ways to justify your answers correctly – you need only use one. If you choose to try use calculations, use a test charge of magnitude +1 C)

6. A positively and negative charged plate at held across from each-other. A uniform electric field exists between these plates. A number of positions are shown between them.



(i) If a positive charge is laid down at position 0, rank the magnitude, from lowest to highest, as it is from along the following paths:

[01], [02], [03], [04] and [05].

[01] means the particle starts at 0 and moves directly to 1. You may draw on the diagram in any manner you see fit.

Ranking:

Justification:

(ii) Rank the potential difference, from lowest to highest, between the points [01], [02], [03], [04] and [05], where [01] means the potential different between the point 0 and the point 1.

Ranking:

Justification:

Appendix F

Legend for codes used in line plots to display extent of conceptual change.

Code.	Learning outcome	Figure.
V 1	Ranking vectors based on magnitude.	4.47
V 2	Use of vector constructions.	4.47
V 3	Consideration of vector components in vector addition	4.47
In 1	Graphically representing inverse square relationship.	4.47
In 2	Reasoning for change in area using scale model.	4.47
In 3	Use of inverse square relationship mathematically.	4.47
Fl 1	Using field line density to determine relative field strength.	4.47
Fl 2	Drawing vectors are points based on field line diagram.	4.47
Fl 3	Reasonable sketches of trajectories of bodies under influence of a field.	4.47
V 4	Variation of electric field strength represented with vectors	5.43
V 5	Use of superposition principal using vectors and electric fields.	5.43
In 4	Use of inverse square relationship mathematically in electric fields.	5.43
Fl 4	Direction of force on negative charge in electric field.	5.43
Fl 5	Reasonable sketches of trajectories of charged bodies under influence of an electric field.	5.43
Fl 6	Using field line density to determine relative field strength.	5.43
W 1	Identification of positive, negative and zero work.	6.25
W 2	Application of work and displacement to electric fields.	6.25
PD 1	Association of relative high and low potential to areas surrounding positive and negative charges.	6.25
PD 2	The movement of charge under the influence of a potential difference.	6.25