EASING THE TRANSITION TO THIRD-LEVEL FOR FIRST-YEAR MATHEMATICIANS

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Abstract

Despite students' prior high achievements in mathematics, it was observed over a period of years among a first-year cohort of almost 80 students studying Pure and Applied Mathematics/Theoretical Physics in Trinity College Dublin that a large number struggled significantly with the transition to third-level. A number of factors were identified as being potentially responsible for this, including larger class sizes, a more abstract form of mathematics, and general difficulties settling into a class in which a noticeable proportion of the students did not possess good social skills.

To avoid students dropping out or failing first year, it was decided to introduce a carefully constructed problem-solving module into the students' timetable. This module, while addressing challenging areas from the general first-year programme, was delivered to students in small groups, once a week, with the focus on pair/group work. Attendance was compulsory, and all marks for the module were awarded based on attendance and participation, with no final examination. The idea behind this was to ensure that students maintained contact with their programme at least once a week, and that they were encouraged to participate in each class, despite their initial misgivings. The pace was considerably slower than in lectures, and students were encouraged to discuss their approaches to various problems with others in their group. Pairs of students working together was mutually beneficial, as weaker students often found it easier to learn from a peer; while stronger students developed techniques to better articulate their problem-solving approach. Students were deliberately paired differently every week, with the dual purpose of allowing them to experience working with students of various abilities, and also to encourage greater mixing among the class.

This module assisted students in developing general problem-solving skills, as methods of approaching previously unseen problems were examined on a weekly basis. In addition, it forced them to collaborate with classmates on problems and agree on a single solution; an approach with which some of the brightest students struggled noticeably. The ability to explain complex mathematics in simpler terms will be a vital one for many of these students once they become part of the workforce.

At the end of each term, students were given an anonymous questionnaire to complete, to ascertain the benefit of the module. The feedback obtained in this way, over a series of three years, was extremely favourable, with the vast majority of students acknowledging that, although they found it difficult initially, the pair-work proved to be invaluable to their learning and fostered academic co-operation among their classmates.

Keywords
Mathematics, transition to third-level, group-work, problem-solving skills.

1. INTRODUCTION

The transition from second-level to third-level is a challenging one for many new undergraduate students, and the reasons for this are complex and manifold. In this paper, we focus on the area of third-level mathematics. Much of the research undertaken in the field of undergraduate mathematics education deals, quite naturally, with the large proportion of students studying service mathematics for degree programmes such as engineering, science or business, rather than those focussing on mathematics as a subject in its own right. Here, we consider the latter group and the particular challenges faced by them while making the transition to third-level, based on observations made over a number of years in the School of Maths in Trinity College Dublin. We give details of a problem-solving module introduced to counteract some of these difficulties, and conclude with a qualitative analysis of the effectiveness of this module.

1.1 Description of student cohort

This study took place from October 2002 – January 2005. The students involved spanned three different programmes – a four-year honours degree in Pure and Applied Mathematics; a four-year honours degree in Theoretical Physics; and a four-year Two-Subject Moderatorship honours degree, consisting of Mathematics and one other subject of the student's choice. The total number of students involved per year ranged from 68 to 85, depending on the year. The minimum mathematics requirement for any student entering one of these programmes is a grade B (higher than 70%) in Higher Level Leaving Certificate Mathematics (the examination taken at the end of second-level education in Ireland, typically at 17-18 years of age). Mathematics is compulsory in the Leaving Certificate, but there are three different levels – Higher, Ordinary and Foundation. In 2006, for example, 18.32% of Leaving Certificate students completed the Higher Level Mathematics paper, with 48.8% of these achieving a grade B or higher [1].

1.2 Possible problem areas in first year

Despite the students' prior high achievements in mathematics, it became clear that a large portion of the cohort experienced significant difficulties adjusting to third-level. In conjunction with issues common to many new undergraduates, a number of additional factors were identified as being potentially responsible for this, such as larger class sizes, a more abstract form of mathematics and general difficulties settling into a class in which a noticeable proportion of the students did not possess good social skills.

Large class sizes in third-level settings cause difficulties in numerous programmes, but in a practical subject such as mathematics, they can seriously hinder learning. At a recent undergraduate mathematics education conference attended by the author [2], the audience was asked by one of the speakers to note the one change that would most assist them in doing a better job; almost all in attendance chose smaller class sizes! It is vital for mathematics students to have regular practice in constructing proofs, problem-solving and computation. However, in large group settings, it is far more challenging to effectively implement an interactive teaching style, particularly one in which all students are engaged. As observed by Marsh et al: *"Although large classes are cost effective for the institution, there is widespread concern about the quality of instruction"* [3]. In addition, many students are too intimidated to ask questions when they do not understand the material, and the enforced anonymity associated with being in a larger group is difficult for many first year students to come to terms with.

Unlike many other third-level cohorts considered when discussing maths education, these students have all been highly successful in mathematics to date, and chose to spend their four years in university studying the subject in detail. This strongly suggests that, prior to third-level, they felt engaged and stimulated by mathematics. Anecdotally, by the end of a single term studying third-level mathematics, many of these students began to show signs of "maths anxiety", a feeling hitherto unknown to the majority of them. This observation is echoed in the findings of Melissa Rodd, who states that: *"Students mostly attribute their original choice of mathematics as a specialist subject to enjoyment. Enjoyment is highly correlated with skill. When these students become unable to understand the mathematics presented, frustration, fear or bitterness often arise"* [4]. Indeed, research has shown that students suffering from such feelings of maths anxiety *"demonstrated smaller working memory spans"* [5], making it more difficult for them to learn. When students encounter these feelings in relation to mathematics for the first time at third-level, they are often unable to cope, and may end up dropping out of the programme completely.

Finally, the importance of developing a solid friendship group in university cannot be underestimated; and within the confines of a mathematics department, social skills are not always to the forefront! Simply getting to know their classmates can be difficult initially for many students in the observed class groups, and yet this can prove vital to their sense of "belonging" in the programme. As stated by Quinn et all, *"developing new friendships facilitates integration and peer support can be a key factor in a student's decision to continue studying or withdraw"* [6]. Most students entering these three programmes do not know any others studying the same topic, meaning that, until they begin to make friends within their programme, they lack this vital safety net.

2. PROBLEM-SOLVING MODULE

To offset these difficulties, it was decided to introduce a carefully constructed problem-solving module for all first-year students in the above-mentioned three programmes. This module had several aims, namely

1. to develop students' confidence in their abilities as mathematicians when faced with previously unseen problems;
2. to encourage greater academic co-operation between students;
3. to maintain students' interest in mathematics;
4. to keep students engaged with the overall mathematics programme;
5. to improve students' overall university experience.

It was important that the module be designed with these specific aims in mind, while also addressing aspects of the first-year programme likely to cause difficulties for the students.

2.1 Design of problem-solving module

The problem-solving module was delivered to students in small groups once a week, for an hour, so as not to overload their timetable. The smaller group sizes allowed the lecturer to get to know all the students' names within the first week or two, ensuring that students would feel less anonymous. Attendance was compulsory every week, as all marks for the module were awarded based on attendance and participation, with no additional assignments or final examination. The idea behind this was to ensure that students maintained contact with their mathematics programme at least once a week and also that there was an incentive for them to participate in the class from the very beginning of term. Therefore, if a student missed a class without a prior excuse, they would receive an email from the lecturer that same day, enquiring if they were well, and suggesting alternative classes they could attend that week if so.

Different challenging areas from the first year mathematics programme were addressed each week, after they had been covered in the students' other lectures. All problem-solving was done in pairs, meaning that students had to interact with at least one member of their class during that hour. The benefits of having students work in pairs in a supervised environment are numerous: weaker students often find it easier and less intimidating to learn from a peer, while stronger students develop a better ability to explain more complex maths in simpler forms – a skill that is very important in the workplace. Because a lecturer is also present while the pair works together, if either or both parties need help, it is readily available. Working in pairs also allows students to develop better inter-personal skills and to get to know their classmates, particularly in the early months of their programme.

3. IMPLEMENTATION OF MODULE

At the start of the academic year, the students were divided into groups of ten-fourteen, which comprised their problem-solving group for the duration of the module. Rather than assemble the groups based on any kind of diagnostic exercise or Leaving Cert grade, students were assigned to a group simply based on alphabetical order, with students from the same programme being kept in the same group. The reason for this was to provide a more realistic mix of students in each group and maximise the learning experience for all students.

The layout of each class was kept consistent, to allow students to become comfortable with this method of learning. At the beginning of the class, students were presented with an "information sheet" and an exercise sheet. The information sheet contained any definitions, theorems, simple explanations and tips they might need to address the exercises for the topic in question. This was to provide students who had not understood the material the first time around with a fresh look at the topic, while ensuring that students who may have missed the original lecture in which the material was covered could not use this as an excuse to opt out of the class. The first ten-fifteen minutes were devoted to an explanation of the topic in question, during which students were encouraged to ask questions about any aspect that was unclear to them.

At this stage, students were split into pairs. To encourage greater socialising within the class, students were usually separated from those whom they had naturally sat beside, meaning that they were obligated to talk to a new member of their class. Students were deliberately paired differently each week, with the dual purpose of allowing them to experience working with students of various abilities and encouraging greater mixing within the class. The pairs had to work their way through the exercise sheet, discussing each problem and agreeing on a common solution.

At the end of the class, a “solution sheet” was provided for all students, with full worked solutions to all the exercises, to enable better revision close to exam time. In the week following the class, these three sheets were made available on an internal webpage, so that students could print another copy if necessary.

The module assisted students in developing general problem-solving skills, as methods of approaching previously unseen problems were examined on a weekly basis in relation to some topic from their first-year maths programme.

3.1 Topics addressed

The problem-solving module ran for sixteen weeks, through the first and second term of first-year. Table 1 below shows the sixteen topics covered with students in the academic year 2003-2004 (slight variations were in place from year to year, to ensure that lecturers had covered material with students in class prior to its introduction to the problem-solving module).

Week	Topic	Week	Topic
1	Mathematical Statements	9	Equivalence Relations and Cosets
2	Symbols and Abbreviations	10	Inverting Square Matrices
3	Operations and Groups	11	Isomorphism and Sylow’s Theorem
4	Limits and Convergence	12	Differentiation and Taylor Series
5	Mappings and Permutations	13	Binary, Octal, Hex
6	Vectors	14	Rings and Fields
7	Subgroups and Conjugation	15	Eigenvalues and Eigenvectors
8	Problem-Solving for Fun	16	Problem-Solving for Fun

Table 1. Topics covered with students during the sixteen-week problem-solving module.

As can be seen in the table, almost every second week a topic from the Abstract Algebra module was chosen, as students have particular difficulty with this module, and it was beneficial for them to devote additional time to problem-solving in this area. The other topics were chosen from their modules in Analysis, Maths Methods and Computational Methods.

3.2 Issues surrounding pair-work

Each year that this module was implemented, the same issues arose in relation to pair-work. If two shy students were paired together, they often found it difficult to work together initially, as neither would take the lead. Indeed, such students frequently had to be reminded to introduce themselves to each other, even after five-ten minutes “working together”. However, given that the pair-work took place in supervised conditions, this problem was usually easily overcome.

Stronger students often found the pair-work particularly challenging, especially when paired with a student who was considerably weaker on a topic than them. These students were not used to having to justify or explain their thinking, or even examining the steps that led them to a certain conclusion. Occasionally, students had a tendency to react in an almost hostile manner to the partner who had asked for an explanation. Again, given that a lecturer was present, it was possible to recast the situation for them, and ask them to imagine that their partner was actually their manager, who had just asked them to explain their reasoning in regard to their latest work assignment. Most students recognised that they had a lot to gain from such an experience, as it forced them to develop greater clarity and accuracy in their explanations, and also helped them to think of different ways of describing their approach.

Finally, there were occasional serious personality clashes in certain pairings, resulting in an unsatisfactory experience for both members. Given that the pairs were changed every week, such unprofitable pairings were not repeated, insofar as was possible!

4. RESULTS OF STUDENT SURVEY

Because the aims of this module were not simply to boost students’ final grades, but rather to improve their overall university experience, through increased confidence and enjoyment of mathematics, we do not feel it is appropriate to consider students’ end-of-year results, and choose to focus instead on their own views of the success of the module. Because attendance was compulsory, the average numbers attending each week were very high, with an overall average rate of almost 96% attendance over the three years. As a result, we were ideally placed to obtain wide-ranging feedback on the module, as almost all students were in regular attendance. Therefore, at the end of each term, students were given a short, anonymous questionnaire to complete, in an attempt to ascertain the benefit of the problem-solving module from their point-of-view. A total of five different terms were assessed in this way, from October 2002 to January 2005, and the results of the 321 responses obtained have been amalgamated to give an overview of the students' opinions on various aspects. Some of the questions focussed on the presentation of the module or the explanation of the material and so are not included in this paper.

The first question students were asked related to how helpful they found the module to be. As can be seen from Figure 1, 95% of students found the module to be either "very helpful" or "helpful".

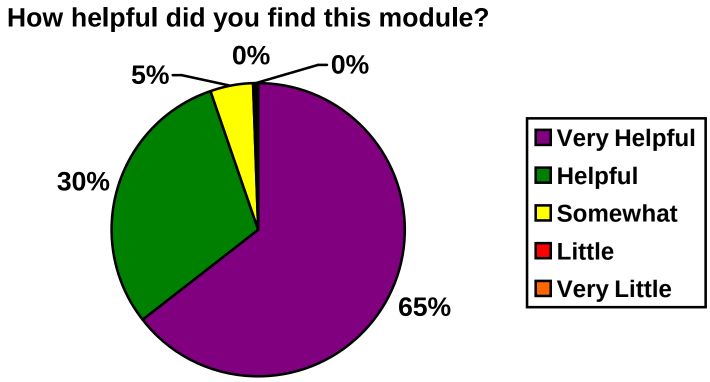

Figure 1. Amalgamated results of 321 anonymous student questionnaires done between December 2002 and January 2005, to assess students' opinions of the problem-solving module. This pie-chart details students' responses to the question "How helpful did you find this module?"

Students commented:

- "A great help in the transition from school to college"
- "Much easier to learn and ask questions in a smaller group"
- "Gave me confidence to do maths which I find very difficult"
- "Helped me to get used to approaching maths problems and ideas"
- "Problem-solving itself was fun"
- "Stuff I assumed difficult actually became quite simple"
- "It helped me to get to know people that I may not have spoken to otherwise",

indicating that many of the students in this group benefited, both academically and socially, from the inclusion of this module in their programme.

The next question asked students to consider how useful the problem-solving module had been in assisting them with their other modules. The results are shown in Figure 2, and again, it can be seen that students rated the module very highly, with 93% deeming it to be "very useful" or "useful".

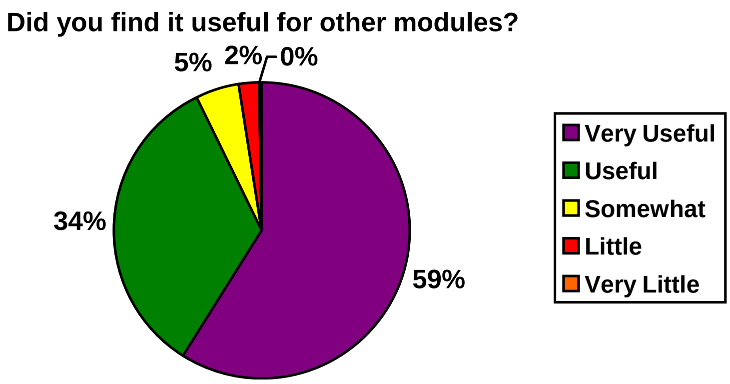

Figure 2. Amalgamated results of 321 anonymous student questionnaires done between December 2002 and January 2005, to assess students' opinions of the problem-solving module. This pie-chart details students' responses to the question "Did you find it useful for other modules?"

Comments included:

- "I understand my other modules more because of this module"
- "Simple numerical examples helped with difficult definitions"
- "Taught us how to approach questions"
- "Small class size helped"
- "Explains little things that lecturers take for granted and gives us practical tips"
- "Helps us deal with proofs and problems and reminds us to be more disciplined on the simple things"
- "Really helped in first few weeks of lectures by introducing notation well"
- "Solving problems rather than just being given the theory makes learning much easier",

showing that students appeared to be able to relate the material covered in the problem-solving module to the rest of their mathematics programmes, as was our aim.

As mentioned in the previous section, the most challenging part of the module for many students was the obligation to work in pairs on problems, and therefore, it was of particular interest to see the students' responses to the question: "Do you think it is good to work in pairs on questions and why?" As shown in Figure 3, 95% of respondents thought it was always or sometimes good to work in pairs, despite the initial reluctance of many.

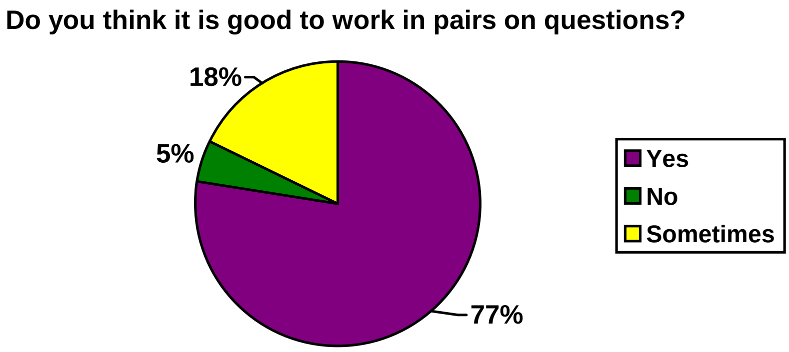

Figure 3. Amalgamated results of 321 anonymous student questionnaires done between December 2002 and January 2005, to assess students' opinions of the problem-solving module. This pie-chart details students' responses to the question "Do you think it is good to work in pairs on questions?"

Their comments on this question are even more telling; some of the most common sentiments are reproduced here:

- "Different people have different strengths and weaknesses and we can help each other"
- "Good for getting to know people; works as ice-breaker to develop good working relationship with new classmates."
- "Ensures you know what you are doing, especially if you have to explain your reasoning to a partner"
- "Encourages an active, vibrant, memorable class and allows social interaction"
- "Forces you to work at listening to what the other person is saying"
- "If you don't understand, you don't have to ask in front of the class"
- "Discussing problems can make them easier to understand"
- "There isn't as much pressure on you individually"
- "You don't feel so stupid if someone else doesn't understand something as well"
- "Develops a sense of class bonding, as many people are somewhat shy"
- "You see how someone else approaches a problem"
- "You can't get anywhere without working with other people, so shy, retiring, reclusive mathematicians should have to work with others in college"

It is also instructive to look at some of the comments of the 5% of respondents who did not agree with the idea of working in groups:

- "I didn't like being paired off as I always ended up with someone I didn't like"
- "We won't be working in pairs in our exams"
- "I like to try and figure things out for myself"
- "They only slow me down"
- "I find that I think better by myself"

It is clear that these students found the experience of working in pairs to be largely frustrating and unhelpful; although some such students recognised that, although they were not particularly partial to this method of working, it was an important skill to learn:

- "I don't always enjoy it because I prefer privacy when working on something. Unfortunately the modern workplace involves pair and team work, so in the long run, it is a good idea."

Finally, students were asked for any suggestions for improving the classes: by far the most common suggestion was to have more classes like this one. Other students suggested more frequent or longer classes, and several also asked for more focus on exam questions, reflecting the understandable concern of undergraduates about passing their examinations.

5. CONCLUSION

The problem-solving module was introduced in order to ease the transition to third-level mathematics for a group of approximately 80 first-year students of Pure and Applied Mathematics or Theoretical Physics in Trinity College Dublin. The module was implemented in small groups, with students working in pairs to solve problems on a variety of topics relevant to their first-year programme. The aims of the module were to increase students' confidence in their abilities to tackle previously unseen problems; encourage greater academic co-operation; improve their overall experience within the mathematics programme; keep them engaged and maintain their interest in mathematics. Based on the results of the anonymous questionnaires completed at the end of each term, in which 95% of students responded very positively to the problem-solving module, it would appear that the students considered this module to be of significant help to them in this process.

* Work undertaken while the author was in the School of Maths, Trinity College Dublin, Ireland.

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