Implementing an Inquiry Based Approach in First Year Undergraduate Physics Laboratories with Emphasis on Improving Graphing Literacy

Author
Thomas Wemyss B.Sc. (Hons)

Thesis submitted to Dublin City University for the degree of

Doctor of Philosophy

School of Physical Sciences

Centre for the Advancement of Science Teaching and Learning

Dublin City University

December 2009

Research Supervisor
Dr. Paul van Kampen

Declaration

I hereby certify that this material, which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work, that I have exercised reasonable care to ensure that the work is original, and does not to the best of my knowledge breach any law of copyright, and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

Signed: ________________________

(Candidate) ID No.: 51423711

Date: ________________________

Abstract

We have developed the first year undergraduate physics labs to incorporate both open and guided inquiry in a flexible approach. By doing this we have shown that there have been benefits in both the affective and the cognitive domain for our students.

One of the main aims for the labs was to develop necessary transferrable scientific skills. Developing graphing skills was an important part of the work that we did in the labs. Through our work we have developed assessments to test students' general graphing literacy and developed curriculum to tackle their difficulties. The difficulties addressed are with both qualitative and quantitative graphs.

Acknowledgements

I would first like to thank my supervisor Paul van Kampen. Paul has given me a massive amount of support, input and encouragement throughout the four years, without which my project would not have been possible. I would also like to thank my parents, Larry and Josephine Wemyss, for always being there when I needed them over the last four years and before.

My time in the School of Physical Sciences in DCU has been an overwhelmingly positive experience, and this is because of help and support from all the staff and technicians. These include Des Lavelle, Pat Wogan, Mike Aughey, Lisa Peyton and Alan Hughes, all of whom went beyond the call of duty and were endlessly generous with their time and suggestions. I would also like to thank the former and current heads of the School, John Costello and Greg Hughes, both of whom were very helpful and accommodating.

My Viva and transfer were personally two important events over the last few years. Odilla Finlayson, Enda McGlynn, Paul Swift and Ton Ellermeijer were enormously helpful giving their time and expertise to add great value to my thesis and project. Also thanks to J.J. Brennan, Jodie Dunne and David Smith along with all my friends and colleagues, whose help and support has to me and this project been very important.

Table of Contents

Table of Contents

List of Figures

List of Tables

Chapter 1: Integrating inquiry into first year undergraduate labs: Background

1.1: Introduction

In this chapter I present the background and research behind the development of the first undergraduate physics labs. Over the course of 10 lab experiments, we help students develop a "toolbox" of skills, which they use in developing investigative skills such as the ability to phrase and test scientific hypotheses. This approach is in keeping with building on students' abilities and experiences rather than hoping that students "adapt" to labs that are beyond their capability. It allows us to reduce the guidance and increase students' autonomy. Thus, their acquisition of inquiry skills is the result of the set of labs in its entirety, and not of any single lab.

At the same time, we use inquiry as a method of teaching to improve conceptual understanding. In later chapters I present examples of curriculum, with pre- and post-test data showing the efficacy of the labs in this area.

There was an overwhelming consensus from the staff and tutors in the department at arranged meetings that the existing labs were unpopular with students and staff alike, and that little meaningful learning took place in them. It was clear as a tutor in the lab, that students were not enjoying themselves (see Figure 2.6 for some relevant data). We set about researching and developing a set of 10 guided inquiry experiments to replace the first year undergraduate physics lab experiments. Participants in these labs take courses in the faculty of science, but were not taking physics as their primary degree. In this chapter I describe how we developed a set of labs that started prescriptively but transferred procedural autonomy to students as they gained necessary experience in the labs. This allowed us to develop labs that both developed necessary scientific skills and improve understanding of difficult topics.

The existing labs were very prescriptive, with emphasis on verifying formulas, and operating complicated experimental setups in cookbook fashion. Informal feedback suggested that experiments in the previous labs like the Hooke's law (1.3.2) experiment were unpopular with both staff and students. Too much of the tutors' time was spent not on helping students understand concepts or the approach of the experiment, but on helping with basic scientific and manipulative skills and tasks, like labelling and drawing graphs or making measurements.

In order to develop a relevant set of labs, our approach was focused around the following set of goals.

1. The labs should be an enjoyable and positive experience for students;
2. Students should develop general scientific skills such as hypothesis testing, control of variables, graphing and graph interpretation, tabulation, drawing conclusions and extracting mathematical relationships from observed data;
3. Students should clarify conceptual difficulties based on their observations in the laboratory;
4. Students should be able to carry out quasi-independent investigations.

Before implementing the labs we investigated what experience the students had with science labs in school. Based on their experiences and our goals we then considered what features of the existing traditional labs we needed to change.

1.2: Students' background

1.2.1 Students' experience of physics in formal education.

A survey among nearly 400 students, spread over three years, revealed that approximately 25% of students taking part in the lab had studied physics at a Leaving Certificate level (an upper secondary level two-year course, typically taken at age 16-18). The vast majority, 70%, had not taken physics at Leaving Certificate level, and had only experienced physics as a part of a general Science course at Junior Certificate level (a lower level three-year course, typically taken at age 12-15). All students attending the labs have taken at least one science subject for their leaving certificate in order to qualify to take a course in the faculty of science.

1.2.2 Students' prior experience of experiments - an Irish context.

In this section, I will describe the Junior Certificate Science course as it was taught up to a few years ago, i.e., to the cohort of students taking our labs. Theory would have been presented first, and experiments were described afterwards mainly to illustrate or verify the claims made. In practice, in many classrooms students would not carry out any experimentation themselves apart from about 30 so-called mandatory experiments, ten of which were in the area of physics. These mandatory experiments are written in cookbook style, step by step, in the textbook.

The teacher is clearly not precluded from taking alternative approaches to using experiments in the classroom [1,2]. However, the student interviews described in Section 1.1.3 suggest that, from a student perspective, the role of the experiments is to help them remember the theory.

Assessment of the experiments also seems to suggest that the view of these experiments tends to be narrow and content-led. For instance, in a Junior Certificate Science paper in 2008, students were given the setup of a mandatory experiment (shown in Figure 1.1) that they carried out in the labs. It involved measuring the volume of irregular shaped objects and then finding the densities of the objects [3].

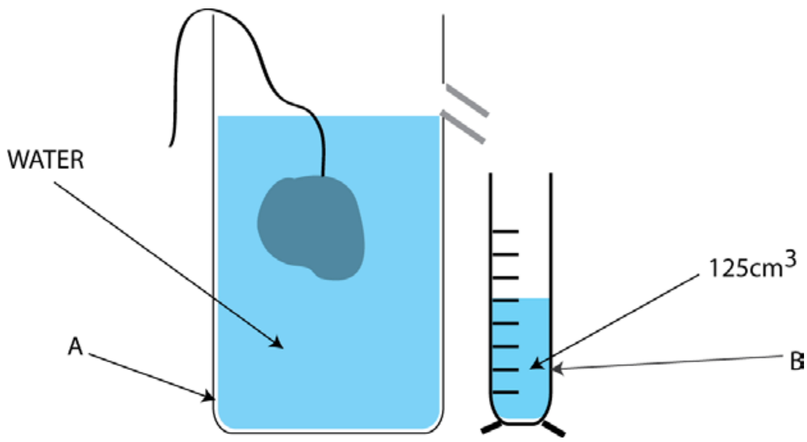


Figure 1.1: Examination question on a Junior Certificate experiment, 2008.

The first question was to name the items labelled A and B. Once the student had labelled them as being an overflow container and a graduated cylinder they are then given the question “The potato had mass 175 g and volume 125 cm^3. Calculate the density of the potato. Give the units of density with your answer.” The last question that the students are given is “Why did the potato sink in the water?”

The curriculum describes the experiment as “measure mass and volume of fixed quantities of a variety of solids and liquids and hence determine their densities” and “investigate flotation for a variety of solids and liquids in water and other liquids, and relate the results of this investigation to their densities”. While the curriculum leaves some room for inquiry-designed classes by a teacher, the assessment of the experiment only requires the student to recall outcomes of an experiment that could have been done in cookbook fashion, pieces of equipment used or to use a formula featured in an experiment.

1.2.3 Students' experiences of experiments in science

To gain insight into students' experiences in science labs we invited students to participate in interviews before the labs started. Over the course of two years, we interviewed just over 20 students. Below we present some of the pertinent responses from the interviews along with the questions asked.

I: What do you think the role is of experiments is in [school] science?

S1: They kind of give you more grounding to what you're studying. Because you're putting it into practice.

S3: In biology when we were doing the heart, when we were doing the experiment on the heart, you get to know the heart better, because you're physically seeing, while out of a book you might not see as much. When you're in the middle of doing an experiment, you are going to remember it. Out of a book you might not remember it, but when you're physically doing it, you might remember it more.

S4: I think if you do the experiments you can remember. I think if you do the experiments you can get a result. And the result is a little definition or *something, you could actually remember them more. You actually would remember them more if you did them.*

S6: For me it made learning the experiments easier.

S7: It increased the knowledge on the topics you were learning at the time. That's basically it.

When asked to identify what they felt the role of labs in the classroom was, nearly all students focused on using the labs to remember or learn information from the topic. Students 3 and 4 specifically used the word remember, while Student 6 said that it made it easier to learn the experiments. Student 4 specifically highlights that the results of the experiment are easier to remember if you have come to it in an experiment. These responses would also suggest that the experiment was something that they needed to know or learn off. The response given by Student 7 was the only response that suggests a wider role for experiments, like helping to understand the ideas behind the experiment or learning to think and approach problems like a scientist.

In a number of follow-up questions we probed some of the lab practices in schools, and students' attitudes to these practices.

I: What did you find positive and negative about [the labs]?

S1: All our equipment was dirty and all that. I didn't like that everything was broken. Nothing went right, maybe it was just me but nothing went right with all our experiments. They always failed miserably. That kind of put me off. I was just like ahhh.

I: Did you think it was important to get the right result?

S1: It's not really important.

I: Were you prepared before an experiment about what you were going to be doing and what you were going to find?

S1: [The teacher] would show you beforehand. She would stand up at the top and say you put this in here and it would work fine for her. And then everyone would go "Grand, we know how to this". Then things would break and then I don't know would blow up.

I: When you were doing school science, did the experiments always work?

S3: No, they barely ever… Usually sometimes only the teacher did the experiment while everyone was watching. So she would say it was at 3 cm, but it was at 5 and she would say, "oh, here you go, it's at 3." A lot of the time it didn't work but when it did work it was great

I: How did you feel when it didn't work? Did it bother you or not bother you?

S3: No not really. Like sometimes when you see an experiment in a book, and you go: "oh, this is going to be cool" and then you go and do it and it's nothing like what it's supposed to be. I wouldn't have been too bothered but I would have been like "ah sure, why didn't that work out."

I: Would it annoy you if you didn't get the right result for an experiment?

S6: It would. It would annoy me. It would kind of stop me because I did something… I wouldn't be able to think about the whole thing properly because I know something I did was wrong.

There are some mixed responses to whether the success of an experiment is dependent on a successful outcome. Student 3 describes a teacher's attempts to keep that side of science locked away by manipulating results. This student also compares the experiment in the book and the experiment in practice that did not work out. It seems that no matter how much the curriculum, book or teacher tries to plan and keep the experiment as simple as possible, experiments will never be as straight forward as planned. Keeping the experiment as a linear exercise seems to mean that for a lot of students the success of the experiment is focused on a collection of actions, all of which must work and give a pre-determined result.

A couple of students we interviewed had studied physics at Leaving Certificate level. We asked these students about their experience with a mandatory experiment they had completed, to measure the acceleration due to gravity (*g*). The question we asked aimed to see if these students had realistic hopes about the outcome of the experiment.

I: Can you remember the experiment where you had to find the acceleration due to gravity?

S6: Yeah, the one with the magnet and the ball falls through a trap door.

I: […] What would be considered a wrong result for that experiment? Would it have to be far out, or anything kind of close?

S6: I would have said, if you get 9.5 and 10, if you get anywhere between there, then I would say that would be accurate enough. The closer to 9.8 you are the better. So if it was off by .2 then it might not be. Well that's in my head, anyway.

I: So if you got a result of 9 m/s^2, you would not be happy with that.

S6: I suppose it is close enough. If I got 9 I would go back and see if I did something wrong.

I: What results would you have got and been happy with in terms of the experiment?

S7: Less than 9.8 or just a bit over 9.8, so 10 not acceptable and 9.7 just a little bit off.

When asked about acceptable accuracies in the experiment, Students 6 and 7 had unrealistic expectations of the results that they would find. They did not seem to consider experimental errors but seemed to pick a number they thought reasonable. Student 6 said that, if he got 9 m/s^2, he would go back and see if he had done something wrong. As shown in previous answers, the "success" of an experiment is strongly correlated with how close the value obtained is to a predetermined outcome.

Overall, the students had a narrow conception of the purpose of the experiment. Many students said that the purpose of the lab was to serve as an aid in being able to remember the content of the general curriculum. No students in any of the interviews identified the labs as a place where they learnt new science or a place where they developed scientific skills.

The interviews also support the view that, in the students' eyes, the quality of their experiment is strongly correlated with the closeness of their result to the pre-determined value. They have become accustomed to experiments "not working" and teachers showing the experiment and in some cases the teaching demonstrating expected outcomes beforehand.

1.3 Conventional Laboratories

1.3.1 Introduction

The laboratory is one of the most identifiable features of science teaching: *"Laboratory work is almost ubiquitously seen as being of great importance to science education, by some as almost the defining characteristic of this component of the school curriculum"* [4]. The uses of labs in science education are broad ranging from being motivational to challenging students' misconceptions in a practical setting [5]. Our interviews show that our students' own beliefs and ideas of the role of labs were narrower.

In practice, labs often emphasise systematic teaching of the subject while the students' own reasoning is bypassed [6]. The systematic cookbook approach often means that participants in the experiment are disconnected with the process they are carrying out [7]. This type of labs is narrowly focused on content, and their efficacy at developing conceptual understanding in students has been shown to be limited [8,9]. Though there have been many movements away from these types of lab, they still remain commonplace at both second and third level [10].

<u>1.3.2 A conventional approach to teaching Hooke's Law</u>

To illustrate the approach taken in the existing set of physics labs, we describe the experiment on Hooke's Law. The conventional cookbook approach to laboratory instruction quickly becomes apparent (see Appendix 1). At the start of each lab, each student was given a page of text to introduce them to the concept being taught, and "the point" of the experiment, as shown in Figure 1.2. They were then given a table to fill out, and were asked to tabulate the measurements for a rubber cord.

Each step required to find each figure in the table was given to the students. These steps included:

- How to make the measurements;
- How many readings to take;
- What graph to draw;
- What are the units of the slope measured from a graph?

Having carried out these steps, students are told to find the ratio g/k.

Analysis

When a mass M is suspended from a spring, the spring extends by an amount Δl so that the downward force (Mg) is balanced by the upward restoring force Fs of the spring (see Figure 1). The mass is at rest in its equilibrium position.

i.e. $Fs = -Mg$

Because extension is linearly proportional to load (as you discovered in Expt. 1 -- Hooke's Law) it follows that the restoring force Fs must be linearly proportional to extension. Thus,

$$Fs = -k\Delta l$$

$$\therefore \quad Mg = k\Delta l$$

equilibrium.

i.e. $\Delta l = (g/k)M$

k is a constant -- termed the <u>spring constant</u>

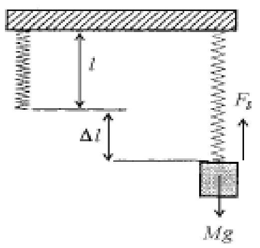


Figure 1. Spring at

Therefore the slope of the graph Δl vs M (as plotted in Experiment 1) enables the ratio g/k to be determined.

Figure 1.2: Background given to students in a Hooke's Law experiment.

In other experiments, students were given a list of formulae, and told which ones to use in different questions. In nearly all cases a graph is drawn and the slope of the line is used to obtain a physical quantity like the expansion coefficient of a bar, the voltage in the mains, the viscosity of water, etc. By following a set pattern of instructions, both in setting up the experiment and manipulating the results, more focus is on following each step correctly rather than understanding clearly the purpose of each step.

In addition to this standard cookbook approach, comprehension questions were asked after the introduction and before the experiment started as a way of getting students to engage with the theory of the experiment or the ideas that underpinned each experiment. For example, in the Hooke's Law experiment the students were asked to list three examples from everyday life where materials that exhibit elasticity are used, and to identify how making a change in a number of properties (length, diameter, number of turns, density of the wire, force applied) would affect the stiffness of the spring. This type of question however did not fundamentally change the structure or feel of the existing labs.

1.4 Defining inquiry in teaching

1.4.1 Inquiry and curriculum

Inquiry has been an important theme with new approaches taken by curriculum designers in many countries. Even though the benefits of the traditional labs are limited, the difficulty and cost of implementing inquiry-based labs often means that many aspects of a traditional approach are commonplace in the science education labs [8].

One complaint from some researchers is that the word "inquiry" is often ill-defined or too broad [11]. Inquiry in two different classrooms can appear to be very different because of the range of interpretations and approaches to inquiry. One of the broader interpretations is given by [7]:

> *"It also refers to more authentic ways in which learners can investigate the natural world, propose ideas and explain and justify assertions based upon evidence and in the process, sense the spirit of science."*

Two broad purposes of inquiry teaching are identified by Lunetta [12]:

- Inquiry science teaching where students learn how knowledge is developed;
- Science through inquiry where students gain conceptual understanding, with inquiry being used as a method of instruction.

1.4.2 Inquiry as a method

A strong example where inquiry is a predominantly, though not exclusively, a means of instruction is the *Physics by Inquiry* curriculum developed for teacher education by the Physics Education Group at the University of Washington [13-15]. This guided inquiry curriculum strongly emphasises developing a level of deep conceptual understanding. The focus is on inductive reasoning based on the students' own observations. The labs are heavily structured, with quasi-Socratic questioning preceding and following prescribed experiments. Large conceptual gains have been proven to be made in practically all areas of secondary school physics.

1.4.3 Inquiry as a goal of the lab

Inquiry can also be defined as a goal of instruction. Open inquiry can be identified as an approach that has as one of its overriding purposes that students develop inquiry skills and abilities. In open inquiry labs, developing conceptual understanding is often secondary. For example, Nott and Wellington state that they

> *"have little confidence that open investigations in school science lessons generate or would be allowed to generate much worthwhile new conceptual or procedural knowledge."* [16]

However, they identify as a potential benefit of open inquiry that it reveals

> *"the messy side of real science which suddenly appears clean and tidy when it becomes accepted and is packaged away in the black box."* [16]

An example of open type inquiry is given by Tuan *et al* [17]. In this case the teacher gives the student necessary laboratory skills before the lab for the students to explore. Then the teacher provides a problem for students in groups, and they must decide as a group how to solve it. After a class discussion, students carry out the experiment. Afterwards, the teachers discuss problems and the class' solutions. Students' reports are marked on what they are doing and why they did it.

This form of open inquiry showed some improvements in the motivation of students of different learning backgrounds. This type of open inquiry was made possible by carefully planned exercises and discussions before and after the lesson as well as careful assessment.

1.4.4 Autonomy in labs

Autonomy refers to the amount of freedom that is given to students in different aspects of the labs. Cognitive autonomy is common to both the open inquiry lab and guided inquiry labs, albeit in different ways. Both allow students to lead discussion of their views, though in the open inquiry environment students do not have the same degree of scaffolding from tutors that would be present in the guided inquiry labs. The difference between both inquiry labs lies in the amount of procedural autonomy students have. In open inquiry, students are allowed to design their own experiments, whereas in guided inquiry, the experimental procedures are prescribed. [18]

Traditional labs which feature cookbook lists for students to follow have little to no procedural or cognitive autonomy. Students who experience high procedural autonomy often have to contend with the messy side of science.

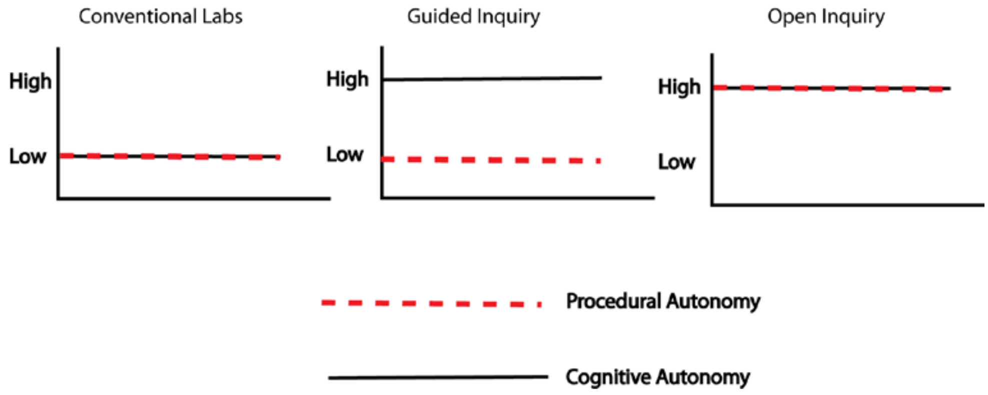


Figure 1.3: Cognitive and procedural autonomy in different types of labs.

1.5 Experience mismatch

The profile that we got of our students from surveys and interviews as described in Section 1.1 would suggest that the existing labs were set at a level that was beyond students' experience. For example, a standard set-up such as Searle's bar apparatus, described in many physics textbooks, in which the thermal conductivity of a metal bar is determined by heating one end while keeping the other in an ice bath, can be a very nice extension of an existing body of knowledge of secondary school physics, but is unlikely to be understood at all by a student with, at best, a very basic physics background carrying it out in the second week of their physics module. Instead, to improve the learning experience for most students, in our approach the starting level of the labs would be more or less that of the starting level of the upper secondary physics course.

The experiments in second level science are usually well defined and set out, with many of the procedural steps laid out in the books. The outcomes of the experiments in the curriculum are also well defined. From the interviews we also got a sense that experiments that students had completed at second level had been focused on content outcomes. Students' experimental skills and ability to plan and execute an experiment appear to be basic. In Chapter 3 I will show evidence that students often have a very basic ability for drawing and interpreting graphs, a skill assumed present in the existing labs.

The evidence points to both content of the existing labs and the skills required of the students being mismatched to their previous classroom experience in physics and science. This is backed up by results we present in chapter 4, in which students are asked to represent simple events with qualitative graphs. Overall we found that in many areas students had significant difficulties completing this task. We would have expected these questions to be manageable in this lab. To illustrate the importance of identifying the mismatch, in the previous set of labs students were asked to use graphs to complete theoretically more sophisticated tasks like error analysis, or using the slope of the line to find the unknown value in an equation, It is therefore not surprising that tutors often found that their time was split between helping students complete very basic tasks and skills like drawing graphs, drawing an appropriate trend-line, guiding students to use equipment or doing it for them, and helping students with more complicated skills like error analysis of a trend-line or defining a physical quantity from using a graph. The expectations of students starting were in many cases beyond the students' abilities. As a result, the labs were often a struggle for the students.

1.6 Our approach for moving towards inquiry as an outcome

The existing labs placed little emphasis on students planning their own experiments, or developing skills that are necessary to carry out quasi-independent investigations. Even guided inquiry labs are typically not designed to do this. Etkina et al used physics labs for some similar goals with the design of the investigative science learning environment (ISLE). Central to her curriculum was that students "design their own experiments to investigate new phenomena, test hypotheses, and solve realistic problems". [19]

We strongly believe that the skills required to carry out open or semi-open investigations are invaluable for any science student to acquire. In an attempt to achieve this, we decided we would start with quite tightly guided inquiry based labs, with little procedural autonomy (but as much as possible without the students having to rely on tutor support). As their skills base grows, students are given less and less guidance until they are able to carry out simple investigations with equipment they are familiar with. Thus, in this “reduced scaffolding” approach, we accept and acknowledge the need for stage management of practical work as suggested by Nott and Wellington in the initial phase, but gradually reduce the prescriptive elements and increase student autonomy. In this way the labs are more matched with students’ experiences, letting us move towards students successfully completing quasi independent investigations while also allowing them to develop an understanding of necessary physics concepts.

References

1. National Council for Curriculum and Assessment Junior Certificate (2007), *Guidelines for Teachers.* Government Publications.

2. National Council for Curriculum and Assessment Junior Certificate (2008), *Junior Certificate Revised Science Curriculum.* Government Publications

3. State Examinations Commission (2009) *Junior Certificate examinations 2009 higher level paper.* Government Publications

4. Hart C., Mulhall P., Berry A., Loughran J., Gunstone R. (2000), *What is the purpose of this experiment? Or can students learn something from doing experiments?* Journal of Research in Science Teaching (37) 7, 655-675

5. Euler M. (2003), *The role of experiments in the teaching and learning of physics*. Proceedings of the international school of physics: Research on Physics Education 175-221

6. McDermott L.C. (1991), *Millikan Lecture 1990: What we teach and what is learned -closing the gap*. American Journal of Physics (59) 4, 301-315

7. Hofstein A. and Lunetta V. N. (2003), *The laboratory in science education: foundations for the twenty-first century*. (88) 1, 28-54

8. McDermott L.C. and Shaffer P. (1992), *Research as a guide for curriculum development: an example from introductory electricity. Part1: investigating of student understanding*. American Journal of Physics (60) 11, 994-1002

9. McDermott L.C. (1984) *Research on conceptual understanding in mechanics.* Physics Today (37) 7, 24-32

10. Buckmaster H. A. (1998) Evaluating the effectiveness of an elementary undergraduate laboratory. American Journal of Physics, (54) 8, 702-704

11. Abd-El-Khalick F., BouJaoude S., Duschl R., Lederman N. G., Mamlok-Naaman R., Hofstein A., Niaz M., Treagust D., H. Tan (2004), *Inquiry in Science Education: International Perspective, Culture and Comparative Studies* .Science Education (88) 3, 397-419

12. Lunetta V. N., Hofstein A., Clough M. P. (2007). *Learning and teaching in school science laboratory: An analysis of research, theory, and practice*. Handbook of research on Science Education. Lawrence Erlbaum Associates, Inc.

13. McDermott L.C. and the Physics Education Group at the University of Washington (1996), *Physics by Inquiry Volume 1*. Wiley, New York.

14. McDermott L.C. and the Physics Education Group at the University of Washington (1996), *Physics by Inquiry Volume 2*. Wiley, New York.

15. McDermott L.C., Shaffer P., (2001) *Tutorials in Introductory Physics*. Prentice Hall.

16. Nott M. and Wellington J. (1996), *When the black box springs open: practical work in schools and the nature of science*. International Journal of Science Education. (18) 7, 807-818

17. Tuan H. L., Chin C.-C., Tsai C.-C. and Cheng S.-F. (2005), *Investigating the effectiveness of inquiry instruction on the motivation of different learning styles students.* International Journal of Science and Mathematics Education (3) 4, 541-566

18. Stefanou C, Perenevich K., DiCintio M. and Turner J. (1994), *Supporting autonomy in the classroom. Ways teachers encourage student decision making and making ownership*. The Education Psychologist, (39) 2, 97-110

19. Etkina E., Van Heuvelen A., White-Brahmia S., Brooks T. D., Gentile G., Murthy S., Rosengrant D., Warren A. (2006), *Scientific abilities and their assessment.* Physical Review Specal Topics-Physics Education Research , (2), 1-15

Chapter 2: Integrating inquiry into first year undergraduate labs: Implementation

Chapter 1 set out the background behind why we revised the lab, and the research that has led to their development. In this chapter we set how we organised the labs and give an overview of we approached their practical implementation. The labs were designed to start prescriptively and then became less prescriptive as students developed more necessary skills. Towards the end of the set of labs students were able to carry out their investigations when given some background information.

2.1 Overview of aims

2.1.1 Enjoyment in the labs

One of our main aims was that the labs should be enjoyable and worthwhile for our students. This should result in students being more motivated, and willing to engage with their tasks in a productive and meaningful way. This is further underlined as for many of these students; these labs will be part of their last formal educational experience of physics.

The existing set of labs was not a positive experience for students or staff. We discussed in Chapter 1 how they were not set at an appropriate level for the students participating in the lab. Another feature that added to their unpopularity was the type of tasks that we asked the students to complete.

For example, in a fairly standard introductory lab, where students determine the period of a pendulum by repeatedly measuring the time it takes to complete say 20 swings for different pendulum lengths, they end up counting many hundreds of pendulum swings in the space of a three hours lab. Also, students do not have any connection with the outcome of the experiment, as they do not make hypotheses or explore the setup before starting the experiment.

2.1.2 Developing useful skills.

The labs have been designed in such a way that over their course students develop investigative skills to a level that allows them complete simple quasi-independent investigations. These skills include manipulative, representational and reasoning/investigative skills. These investigations are not full open investigations as some context to what they are to investigate is provided to them.

Manipulative skills speak for themselves – students need to be able to use the equipment independently. We include graph construction, tabulation, scientific notation and unit conversion, and report writing among the representational skills. Interpretative skills consist of interpretation of graphs, control of variables, qualitative hypothesis testing (i.e., the ability to phrase a hypothesis and to design and carry out an experiment to support or falsify it), drawing inferences from data, establishing physical and mathematical relationships from experimental data, and metacognitive skills. These skills were identified as a result of the areas in the previous labs in which students needed the most help, and as skills that we felt were vital scientific skills for any scientist requiring to carry out an investigation.

While formal error analysis is an important part of scientific investigation, due to time constraints, we found it difficult to tackle both conceptual development and formal error analysis in one and the same lab. However these students are non-physics majors, so error analysis should be covered in other elements of their degree program. Informal error analysis, like estimating whether a deviation from a hypothesis is large enough that it cannot reasonably be ascribed to uncertainty in measurement, does form part of the labs.

2.1.3 Conceptual Development.

In the existing labs conceptual development usually was limited to two or three higher order comprehension questions that either preceded the experiments or came after the experiment had come to its conclusion.

We try to incorporate this type of question at all stages of the experiment. This allows us to help students develop conceptual models using inductive reasoning, and to make sure the students understand how the tasks they are carrying out are linked with these models. For example, in the uniform motion lab students are asked to take readings for a ball rolling along a track. They are then asked to interpret those readings and determine if the motion of the ball is constant or not. When they find it is not, they are asked to manipulate the setup so that they obtain constant motion with the ball. Students then write their own hypothesis for a different setup and investigate it using their own model.

In the previous labs there was little questioning or student involvement outside the list of procedures and the tables students were asked to fill in. We hope that by taking this approach to experiments that students will be encouraged to think independently outside the bounds of the followed procedures.

The conceptual development was verified using the standard pre-test/post-test method.

2.2 Practical Implementation

2.2.1 Equipment

The equipment in the labs was kept as simple and as hands-on as possible, avoiding the use of black boxes. By avoiding black boxes students are given some opportunity to explore the principles behind even such a simple device as a commercial spring balance (also called newtonmeter) before they go and use it. This also helped us implement non-rotational labs, with a new forty sets of each experiment, which in turn allows us to let all students do the same experiment at the same time, so that we can build up their skills base gradually. It would be hard to implement such a set-up with forty expensive pieces equipment.

2.2.2 Pre-labs

One concern that academic staff and tutors had before revising the labs was the amount of difficulties that students were having with simple routine tasks. Often the purpose of the experiment was lost as students struggled to tackle these tasks. To combat this problem, we adopted the pre-lab approach taken by Johnstone *et al* [1], with minor modifications. Johnstone *et al* found that the introduction of pre-labs had made the laboratory experience a better one for the students. Students felt like they understood the purpose of the experiment and had a better footing on the tasks they were to complete before going into the lab.

Therefore, before entering our labs students took not only an online pre-test and answered a four-question survey about the experience of the previous lab; they also read a pre-lab and did a short pre-lab assignment. The pre-lab lays out what students should expect when coming into the labs, following which they may be asked to use web based resources or easily accessible textbooks to carry out a simple assignment. In some cases students receive a set of instructions or help them in carrying out the pre-lab. These assignments are not conceptual in nature and focus on skills that students need to know before entering the labs. The skills covered in the pre-labs are generally are incorporated in the following lab. An example of a pre-lab that our students take is shown in Figure 2.1.

1. What should I expect to see in this experiment?

You will see how a spring stretches when different objects are attached to it.

2. What will I be doing?

To examine the way springs stretch you will hang masses from two different springs and observe the changes in each spring. You will use this information to construct a spring balance which will allow you to determine the mass of different objects.

3. What equipment will I be using?

An equipment list plus a photograph of the equipment is given: a retort stand, two springs, a ruler, a mass hanger, a slotted mass set, and a digital mass balance.

4. What should I know before I begin?

- The experiment will require you to take measurements with a ruler and to enter these measurements in a table.
- You will be asked to plot the data gathered on a graph. The graphs are provided with the axes drawn; they will just require the addition of data points.
- In addition to plotting the data points on a graph, you will be required to add a best fit line to the data. (See information below on Best Fit Lines)
- What is the meaning of ‘slope of a line’ for a line plotted on a graph?
- What is the meaning of ‘horizontal and vertical intersect’ of a line plotted on a graph?

5. The pre-lab assignment:

In your own words, give definitions of slope, intercept, and best fit line. You may of course use your textbooks or the Web, but do phrase the definitions yourself.

Figure 2.1: A pre-lab given to students before the making a spring balance experiment.

2.3 Assessment of labs

2.3.1 Learning environment/ Enjoyment

Since the early 1980s, the cognitive domain in general and conceptual change in particular has gained prominence in science education research, at the expense of research into the affective domain [2,3]. For example, in a similar context to ours, Simonson and Maushak [3] found that

"It is obvious that attitude study is not an area of interest or importance in mainstream instructional technology research. Of the hundreds of studies published in the literature of educational communications since [1979] less than 5% examined attitude variables as a major area of interest."

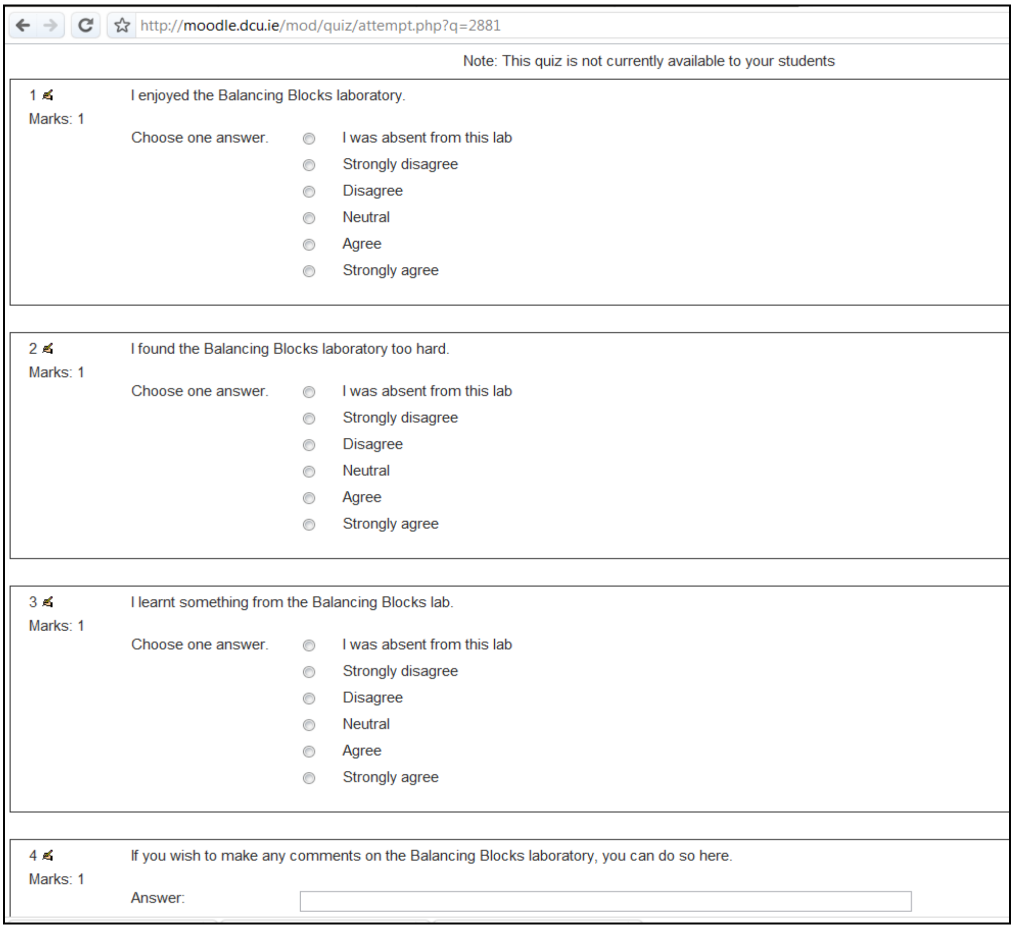
In the area of science education, research into attitudes and motivation has seen peaks and troughs in terms of interest over the past three decades. Interest has often been motivated by trying to address or understand the poor uptake of science at second level, and understanding the implications of new scientific approaches [3].

There are many valuable instruments to survey both the students' attitudes and motivation and what is happening in the classroom. Most contain many items that are Likert scale type answers. These include the What is Happening in the Classroom (WIHIC) survey [4] and the Colorado Learning Attitudes about Science Survey (CLASS) [5]. However, we found that the size and detail of the surveys was too great to allow research-based development of individual labs on a weekly basis. We decided that feedback from students was important and set ourselves the following goals for that feedback:

- The feedback should be a non intrusive method of giving students the opportunity to rate their experience on a week to week basis,
- The feedback should allow us to compare each lab with similar questions,
- Students should have the opportunity to provide open and qualitative feedback.
- At the end of the labs we would be able to compare their overall experiences.

To achieve our aims we used a web freeware application called Moodle. Using Moodle, students logged on each week and completed a survey. This allowed us to collect quantitative data for each lab on students' attitudes towards the lab. Typically, they were asked to rate on a five-point Likert scale (ranging from strongly agree to strongly disagree) whether they enjoyed the lab, had found it too hard, and felt they had learnt something from it. Also provided was an opportunity at the end of the survey for students to provide their own comments on the previous lab. An example of one of the surveys is shown in Figure 2.2. By gaining feedback on each lab, we were able to carry out a research based development of individual labs, especially where the affective domain is concerned, and not just the set of labs.

For the iterative design process we found that this stream of information was invaluable and helped us immensely in changing and tweaking the labs so that where needed students were properly supported. Especially in the first implementations we found a very strong correlation between enjoyment and being able to grasp the meaning of a question at once – a correlation that seemed stronger than how hard students perceived a question to be.



http://moodle.dcu.ie/mod/quiz/attempt.php?q=2881

Note: This quiz is not currently available to your students

1
Marks: 1
I enjoyed the Balancing Blocks laboratory.
Choose one answer.
I was absent from this lab
Strongly disagree
Disagree
Neutral
Agree
Strongly agree

2
Marks: 1
I found the Balancing Blocks laboratory too hard.
Choose one answer.
I was absent from this lab
Strongly disagree
Disagree
Neutral
Agree
Strongly agree

3
Marks: 1
I learnt something from the Balancing Blocks lab.
Choose one answer.
I was absent from this lab
Strongly disagree
Disagree
Neutral
Agree
Strongly agree

4
Marks: 1
If you wish to make any comments on the Balancing Blocks laboratory, you can do so here.
Answer:

Figure 2.2: The survey students use to give online feedback on the previous lab.

2.3.2 Assessment of student learning.

Conceptual development is assessed using the pre-test/post-test method. The pre-test is given on-line prior to the labs. The post-test usually takes the form of exams, though occasionally they are asked as part of the pre-test of the subsequent lab. Two exams are given: a mid-term exam, which examines conceptual development through open unseen theory questions, and the end-of-term exam, which includes the examination of the investigative and experimental skills.

The overall grade consists of three parts. Students get 5% for completing every pre-test, regardless of whether their answers are correct, incorrect or incomplete. The graded worksheets are worth 45%, and the mid-term and end-of-term exams are worth 50%. The initial weights were 5%-30%-65%, in line with our philosophy that the lab should be a learning environment and not an examination environment. However students themselves requested that more weight was given to the weekly worksheets and we believe a good balance has been found.

2.4 Implementing guided labs with a practical approach to autonomy

2.4.1 Experiment 1: Prescriptive guided inquiry.

In the first lab, students start by a free exploration of how the length of a spring changes when different masses are hung on it, and they have to write a few lines on what they did. After that initial episode, however, the guidance is quite tight. The students carry out the same experiment on two different springs. First they attach the spring to a retort stand, and measure the length of unloaded spring. Then they attach a mass hanger of unknown mass and attach up to six slotted 20 g discs. In doing so, they tabulate their data, graph the extension of the spring as a function of the total mass of discs, and they draw a best-fit line through the data points.

This activity forms the basis for three additional activities. The two graphs are used to help the students to think about the slopes and intercepts in a qualitative manner. Moreover the students are led to think about interpolation by adding unknown objects to the hanger (after removal of all slotted discs), finding the extension and hence read off the mass of the object. In the following weeks more freedom is given to students on collecting data and drawing their own graphs.

2.4.2 Experiment 4: More autonomy for students.

In the pre-lab to Experiments 2 and 3 students are introduced to hypothesis testing and controlling variables. By the time they get to Experiment 4, the students' procedural autonomy has been increased compared to Experiment 1 (described in Section 2.4.1).

At the start of the experiment, students are again given free rein to explore the equipment and to "play and observe". They are then given a brief description of the task: to investigate the effect of mass on the period of a pendulum. There is space in this investigation for some procedural autonomy, and we give it to students.

Table 1.1: Fixed quantities for the quantitative pendulum experiment of Section 2.

Quantity & symbol	Value
Number of swings for your experiment (between 1 and 15), N	
Fixed starting angle for your experiment, α	
Fixed length of the string, l_{string}	

Comment on your decisions.

Describe in some detail the control of variables in this experiment.

Figure 2.3: Part of the pendulum lab in which students have some hand in planning parts of their own experiment.

As shown in Figure 2.3, students are allowed to choose their own fixed starting angle, the length of string, the number of swings of the pendulum, and are required also to give a rationale for their choices. They also need to phrase in their own words how variables are controlled in the experiment. At this stage, it is little more than recapitulating their choices; in subsequent experiments, they will not be guided to the same extent.

2.4.3 Final investigations: Implementing new skills in the labs.

By Experiment 9, the students have acquired sufficient skills to successfully tackle a laboratory like the one shown in Figures 2.4 and 2.5 below. The investigation focuses on the max height a ping pong ball reaches after successive bounces. Students are given two guidelines for two investigations to carry out, and then they are asked to pick their own investigation. In their own investigation, students write their own hypothesis and decide what investigation they want to carry out. A list of possible investigations is provided, because we found that students otherwise get stuck. However, students are not restricted to this list, and a few choose their own investigation.

Guidelines are given to help student show what is expected of students, as shown in Figure 2.4.

Investigation 1:

Check that you have at your disposal: a ping pong ball, a meter stick, a retort stand with clamp, and a stopwatch.

I. General comments

This is the first investigation that you will carry out in the physics labs. You will first set up investigations into two prescribed aspects of the motion of a bouncing ball. In the last section, you will investigate any aspect you choose.

The structure of each investigation is a lot like what you have done in the labs so far. For each of the investigations you will carry out, think about the following issues:

- What do I need to measure to verify my hypothesis?
- How can I achieve control of variables?
- How many different measurements will I make?
- How many repeat measurements will I make?
- How can I make my experiment as accurate as possible?
- Is the accuracy of my experiment sufficient to falsify or confirm my hypothesis?

NOTES ON HOW TO WRITE YOUR REPORTS ARE GIVEN IN THE APPENDIX

Figure 2.4: The guidelines for a quasi-independent investigation.

To make the investigation as real as possible, the hypotheses chosen were the most popular answers on the pre-test to this lab. In the first version of this investigation, the students were not given the hints shown in Figure 2.5. As it turned out, from classroom observations it was clear that many students were not ready to carry out this investigation by themselves. With these additional clues, most students were able to carry out a satisfactory investigation with appropriate reporting.

Section 1: Multiple bounces

In this section you will investigate how the maximum height reached by the ball changes after successive bounces on the floor or table.

You are to investigate two hypotheses:

Hypothesis 1: “The maximum height reached by the ball decreases by the same amount after every bounce.”

Hypothesis 2: “The ratio of maximum heights reached by the ball on successive bounces is constant.”

A few hints to help you with the investigation:

1. Ensure that the height from which you drop the ball and the maximum height it reaches after the first bounce are approximately constant
2. Make measurements for a reasonable number of bounces
3. Repeat the experiment a reasonable number of times, and average your results
4. State whether you have falsified or confirmed the hypothesis
5. Plot the maximum height against the number of bounced (i.e. plot the maximum height on the vertical axis and the number of bounces on the horizontal axis). You may treat the release height as the height after zero bounces. Explain what sort of line you drew.

Figure 2.5: The first investigation students are asked to carry out.

2.5 Students’ experiences of the labs

2.5.1 Students’ experience in the laboratory space

After each lab students were asked if they enjoyed the lab, if the lab was too difficult, and if they felt they had learnt something. Our target, to get the undesired answers to below 20% and the desired answers to 50% or over, has now been met for each of the experiments. Some relevant statistics have been collected in Figure 2.6.

In the end-of-semester survey, we give a more substantive questionnaire. Some interesting trends have emerged. In the first year, one of the investigations was not as clear as desired, and more than 50% of students stated they preferred structured labs over the investigations. Having replaced the unsuccessful investigation, numbers are now consistently undecided: 40% prefer one over the other, while 20% are neutral.

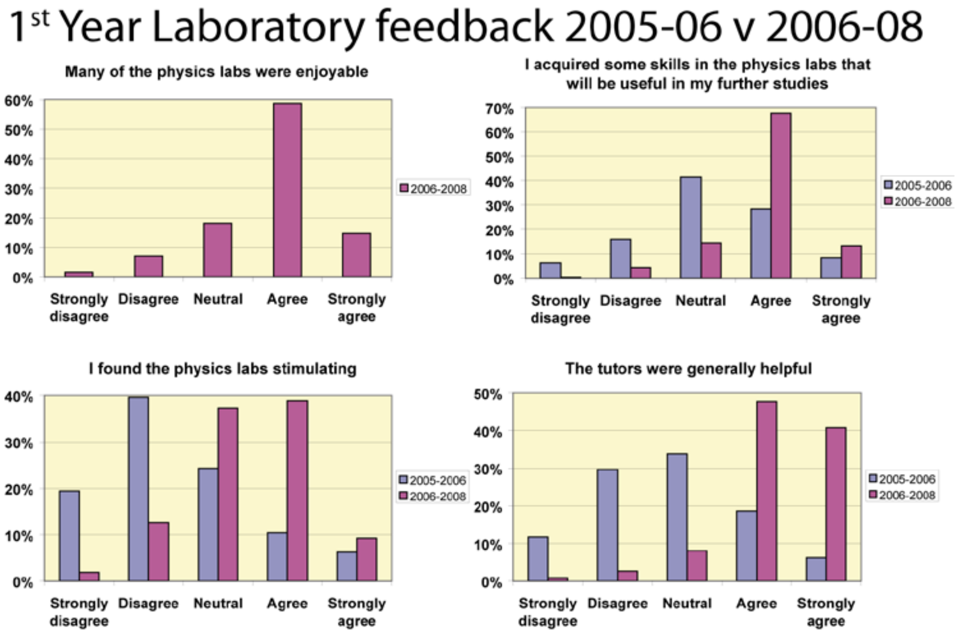


Figure 2.6: Sample survey responses on the new set of physics labs described in this paper ('2006-08', *N*~400) matched to closest corresponding question on 2005-2006 survey (*N*=150). The first question was asked only on the 2006-2007, 2007-2008 and 2008-2009 surveys.

It is clear that these labs have been transformed. Interestingly, it appears that not only do the students find the labs more stimulating and useful; the tutors are judged to be much more helpful than they were in the old labs – even though in many instances, these tutors were the same people.

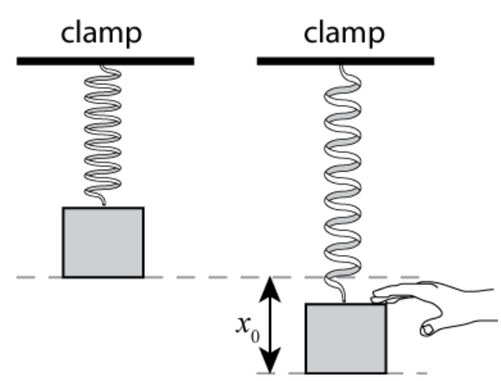
*We have found that the pre-test results year on year are consistently within 5% of each other, and while also acknowledging some variations between groups of students we felt confident in rounding figures to the nearest 5%, and in doing so not present false accuracy. We also round off our data to the nearest 5% in later chapters.

2.6 Investigative skills

Students were to carry out a graded investigation at the end-of-term exam (week 11). The exam question is given below.

1. **You have approximately ONE HOUR to complete this experimental question.**

You have at your disposal a retort stand, a clamp, a metre stick, a slotted mass set, and a stopwatch. The mass of the mass hanger is 50 g; the masses of the different slotted disks are 5 g, 10 g, and 20 g.



If an object hanging from a spring is displaced from its equilibrium position by some distance x_0 and then released, it will oscillate about the equilibrium position. The object will move back and forth between positions of minimum and maximum height in a regular, periodic motion. The period of this motion is defined as the time it takes to complete one cycle, i.e. for the object to go from minimum height to maximum height back to minimum height

You are to investigate your own scientific hypothesis on how the period of the motion of an object attached to a spring depends on the mass of that object. ***Please note the following:***

- *The total mass you hang on the spring should be not be less than 70 g, and not more than 150 g.*
- *You may use the stopwatch only as a clock, measuring time in seconds.* Alternatively, you may use your own watch as a clock, again measuring time in seconds.

Carry out your investigation, and write a brief report on it in the space below and the next page. Your report should have a similar format to that you used in Experiments 9 and 10. The introduction however may be brief, merely stating the hypothesis you are testing. Graph paper is provided in your exam booklet, just before the theory questions.

Figure 2.7: An overview of an exam question in which students had to complete an investigation.

We note here that the mass range was restricted because otherwise the likelihood of the mass hanger and discs coming off or banging on the table was too great. The hint about using only the stopwatch essentially as a clock accurate to seconds was put in there predominantly to avoid continuous beeping in the lab. We were initially concerned that this would push students towards making measurements on more than one swing. This does not appear to have been a problem in practice.

2.6.1 Ability to phrase a scientific hypothesis (N=197):

Some 80% of students were able to give a clear, testable hypothesis, such as "the period increases when the mass increases". Another 15% gave a more vague yet still testable hypothesis (e.g., "the period depends on the mass"), while some 10% gave unclear answers (such as "we will investigate the effect of the mass on the period") or gave no hypothesis at all. We deemed this outcome satisfactory.

2.6.2 Understanding the experiment (N=197):

Despite defining the period of the oscillatory motion in the text, merely 75% understood the experiment. Some 10% thought that the period is time it takes for object to come to rest; another 10% thought that the period is the difference in minimum and maximum height. A further 5% of students gave unclear or ambiguous answers. In the remainder of this section, we will only consider the 147 students who clearly understood the experiment. This ensures that we investigate a more homogeneous group, while still retaining a high enough number of students to extract meaningful data from.

2.6.3 Control of variables (N=147 out of 197):

Results here were quite encouraging. Half the students gave written evidence that they kept x_0 constant. As we counted explicit statements only, it is likely that the number of students who did this was actually higher. Just under 5% kept the initial height above table constant –a variable a professional physicist is unlikely to keep constant, but showing evidence of a desire to control variables nevertheless

More than 90% of students kept the number of oscillations constant, but this high percentage includes those who measured the time to complete just one bounce. It is impossible to judge whether these students would have thought of keeping the number of bounces constant. Interestingly, some 5% of students kept the total time constant and varied the number of oscillations in that period – an entirely valid procedure.

2.6.4 Number of oscillations, masses, and repeat measurements (N=147 out of 197):

All students who kept the time interval constant obtained a reasonable number of oscillations. However, of the other students, only 15% chose 10 or more oscillations, another 15% chose between 2 and 9 oscillations, and 65% of the students chose to measure the time it takes to complete just one oscillation. Of the latter group, many complained that the instructions made it impossible to make clear measurements; some ignored the instructions, and used silent stopwatches on their own clocks to measure the period to hundredths of a second. It is interesting to note that students could see what others were doing in the lab, and still two-thirds of the students did not think to measure the time needed to complete many oscillations, despite having done something similar in a pendulum lab.

The situation is not much better when it comes to the number of different masses the students chose. As the period versus mass graph is curved, one would hope that the students would make more measurements than required for a straight line graph. Given the mass range set in the question, it is perhaps not surprising that 65% of students chose five different masses (corresponding to 1, 2, 3, 4 or 5 slotted discs of

20 g mass). Only 20% used the 10 g or 5 g slotted discs as well, and made 6 or more measurements. A surprisingly high fraction of almost 20% made measurements for 4 or fewer different masses. A more encouraging result came from analysing what percentage of students made repeat measurements, with 55% of students doing so.

2.6.5 Tabulation and graphing (N=147 out of 197):

Over 90% of students drew up acceptable tables, though only 40% gave both table headings and caption. The results for the graphing skills were positive. While only 55% of students drew a big enough graph (defined as using more than one quarter of the area of an A4 sheet), 90% drew scaled and properly labelled axes and gave units on both axes. Some 85% drew a good best fit line; as most students had never seen the formula linking the period of an oscillating spring with the mass, it is entirely acceptable for them to draw a straight line (that does not go through the origin) through five data points. As we are only assessing graphing skills here, we do not consider the fact that students should ideally have taken more data points.

2.7 Findings

Overall we found that our flexible approach to guided inquiry labs, by allowing students more control with time on how they setup their experiments, had some benefits with the development of important scientific skills for students. Also by setting the labs an appropriate level and coupled with our approach, the labs have been transformed from being a chore for both students to tutors, to being a positive and valuable experience for students.

References:

1. Johnstone A. H., Watt A. and Zaman T.U. (1998), *The students' attitude and cognition change to a physics laboratory,* Physics Education. (33) 22–29

2. Hofstein A. and Lunetta V. N. (2003), *The laboratory in science education: foundations for the twenty-first century*. Science Education. (88) 1, 28-54.

3. Simonson M. and Maushak N. (2001), *Instructional technology and attitude change.* In D. Jonassen (Ed.), *Handbook of research for educational communications and technology* (Lawrence Erlbaum Associates,) 984–1016

4. Doorman J.P. (2003), *Cross-National Validation of the What is Happening in this Class? (WIHIC) Questionnaire using Confirmatory Factor Analysis,* Learning Environments Research (6) 3, 231-245

5. Adams W.K., Perkins K.K, Podolefsky N.S., Dubson M., Finkelstein N.D. and Wieman C.E., (2005), *New instrument for measuring student beliefs about physics and learning physics: The Colorado Learning Attitudes about Science Survey*, Physical Review Special Topics. (2) 1, 1-14,

Chapter 3: Overview of Graphing in Physics Labs

3.1 Overview

One notable difficulty that was identified before introducing the new labs was that of students' abilities to functionally use graphs. This inability to use graphs was one significant block to students independently carrying out their own experiments. There appeared to be difficulties connecting the graph and the experiment through proper interaction or patterns of reasoning. A difficulty that became clear during the first year of the newly introduced labs was the inability of many students to choose a suitable trend line to represent their data. One particular difficulty we identified was a tendency to draw a straight line through the origin regardless of the data students had collected.

Assessments were designed to test students' abilities to draw simple trend lines to represent observed events. The assessments were given as part of pre- and post-tests in the physics labs. Significant difficulties and patterns were identified in students' pre-tests and post-tests. These are set out in Chapter 4. Chapter 5 highlights the approaches that were made in the labs to help develop students' understanding and abilities in the labs. In Chapter 7 the efficacy of the lab is described by comparing the pre- and post-test results.

3.2 Motivation

We put strong emphasis on developing graphing abilities and skills in the physics labs. Graphing transcends subject boundaries and is essential for a practicing scientist, whether working in the field of chemistry, biology or physics. As stated in Chapter 1, one of our main goals was to allow students to independently carry out their own investigations. To achieve this, students must acquire functional abilities with graphs. These abilities can be classed as both analytical and communicative.

This was summarized well by Jackson *et al* [1]:

> *"Graphs are one of the most important tools in the practice of modern science, as a means of both exploration and communication. Graphing has been recognized as an important process skill in science education"*.

Graphs can also be used to solve problems, analyze multiple events and pictorially show intervals or co-variation. Interpolation and extrapolation are made easier with graphs, and it is possible to get a global view of trends from a series of plotted data points. All of this is more difficult to do with tabulated data. The use of graphs in textbooks is widespread at all levels of science instruction, and students are exposed to graphs from a very early stage of formal science education. They are extensively used by physics educators [2], almost as a second language. As a result mastery of these different graphing abilities is an important aspect of scientific literacy. PISA [3] contends that the comparison with language is appropriate,

> *"as language implies that students must learn the design features involved with mathematical discourse"*

and they are able to

"solve non-routine problems in a variety of situations defined in terms of social functions".

An example where graphing can be used as a tool of instruction is the development of the concept of instantaneous motion [4]. In this approach, students had previously come to associate uniform motion with linear (distance-time) graphs, and non-uniform motion with curved graphs. By zooming in on a curved line, considering increasingly smaller pieces as shown in Figure 3.1, students observe that the smaller pieces strongly resemble a straight line. This helps them associate a very small interval with constant motion. With more work students come to an understanding of instantaneous motion as the limit of constant motion over an infinitesimally small time interval.

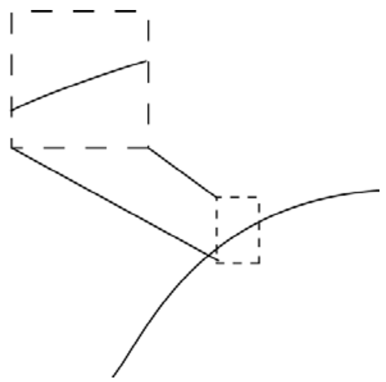


Figure 3.1: Breaking down a curved graph into smaller pieces to make a link with linear graphs.

The graphing ability of students at third level often falls short of expectations. The number and variety of difficulties with graphing for students in science have been well documented. Beichner [5] and McDermott *et al* [6] have found that students entering college have a deficiency in their graphing capabilities and do not necessarily pick up graphing skills spontaneously. Beichner [5] found that students in college and at high school level showed no statistical difference in their abilities to use or interpret kinematics graphs. McDermott *et al* [6] found that students at third level often have difficulties in choosing which trend line to draw after plotting points on a graph. McDermott *et al* [5], Clement [7], and others have found that many students read the graph as if it was a picture, or pick the height of the graph when required to pick the slope of the graph.

3.2.1 The importance of context

One of the important factors that affect how students approach and interpret a graph is the context in which the graph is set. Åberg-Bengtsson and Ottosson [8] designed a large graphing survey and found that there is a strong content-related factor in the success rate of interpreting the graph. They conclude:

> *"However it may, for example, not be assumed that having learned the handling a particular type of graph in math class provides sufficient background knowledge for an intended reading of the same type of graph in a new context (e.g. a particular situation in science class)"*

Roth [2] points out that in order to answer a question, students need to both have an understanding of the context and a general ability to interpret graphs. Figure 3.2 illustrates this with an example from our own student cohort. Three different groups were to figure out the rate of change for each of the three graphs.

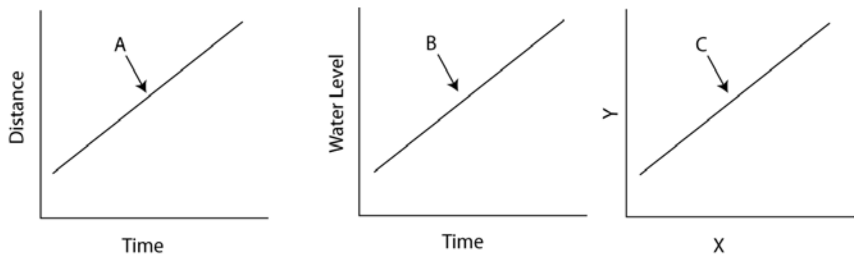


Figure 3.2: Similar graphs in different contexts. a) distance-time graph; b) water level-time graph; c) context-free graph.

The first is a distance-time graph, the second is a water-level versus time graph and the third is a purely numerical graph. When asked to find the speed of the object at point A, how fast the water level is changing at point B, or the slope at point C, many students incorrectly use a single point to answer each question. It is quite common for students to use the incorrect formulae speed = distance / time, or slope $=y/x$, but students use the formula rate of change=water level/time much less frequently[1]. Moreover, we find that after instruction students are much more likely to abandon slope$=y/x$ and use slope $\Delta y/\Delta x$ than to abandon speed=distance/time and use (average) speed = change in distance/ change in time.

As this example shows, testing general graphing skills is difficult because of the influence of context on how students answer questions. Different instruments that have tried to test general graphing abilities have taken similar approaches. They ask graphing questions in various contexts from everyday life. An example of this approach is Test of Graphing in Science (TOGS) [9]. This test was aimed at middle school students and used contexts such as the amount of gasoline used for a car journey and the amount of heartbeats experienced by a jogger. A similar approach is taken by Åberg-Bengtsson and Ottosson [8] who developed a 21 item test with questions about e.g. the differences in temperature and amounts of rainfall. They concluded that

> *"The importance of interest and familiarity with the content domain of the graph interpreted cannot be overemphasized"*.

[1] For qualitative data backing up this statement, see Chapter 7.

As a third example, the PISA test also uses everyday context related situations as

> "*they demand the ability to apply those skills in a less structured context, where the directions are not so clear, and where the students must make decisions about what knowledge may be relevant, and how it might be usefully applied*".[3]

3.2.2 The format of the questions

Three question formats are often used: multiple choice, interviews and open questions. Each format has its own advantages and disadvantages. Interviews afford the ability to follow up on a question and allow the interviewer to be more exploratory with student responses. A strong disadvantage with interviews is that it becomes too time consuming to produce quantitative reproducible data. However, as a prelude to designing open or multiple choice questions, interviews can be very useful.

Open questions afford the researcher less flexibility, but well constructed questions can probe students' understanding effectively. The format allows for activities such as graph construction. One downside is that analysis can be very cumbersome, especially when many different answers are possible.

The multiple choice format allows for very quick analysis. However, probing reasoning can be problematic. By necessity each answer must contain some implicit or explicit reasoning, which the students may not have arrived at spontaneously. For example, Berg and Smith find that students are more susceptible to the misconception of treating the graph as a picture in multiple choice tests [10]. They suggest that students often answer multiple choice questions with low-level cognitive engagement, whereas in open questions they often make sketches and have a chance to take other approaches in analyzing and answering the question.

3.3 Graphing difficulties

3.3.1 Difficulties in constructing a trend line

Line graphs are the most common type of graph in physics as they show co-variation most readily. In this thesis, I discuss only graphs with two linear axes. When constructing such line graphs, most students can draw axes and mark in the values from a table of figures. However, many students have difficulties with constructing a trend line, which demonstrates a lack of understanding of the role of different components of a graph.

For example, McDermott *et al* [6] found that many third level students construct graphs without adding a trend line, or else join each dot with a straight line as shown in Figure 3.3. Leinhardt *et al* [11] suggest that students might do this as a result of early childhood practices of joining dots.

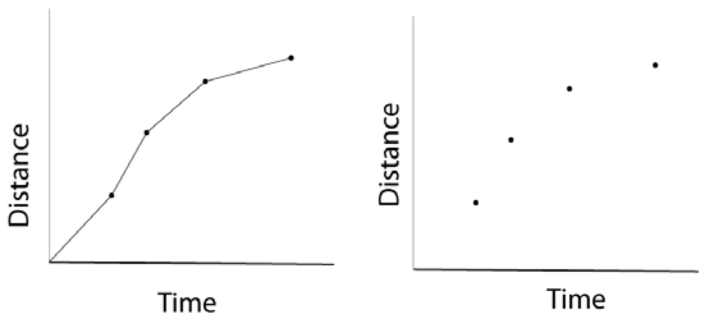


Figure 3.3: Typical incorrect or incomplete graphs: a) connect-the-dots graph; b) no trend line on the graph.

However, depending on context, other graph construction problems can emerge. Mevarech & Kramarsky [12] did a study with eighth grade students and found that some *"students conserve the form of an increasing function under all conditions"*. For example, some students represented a decreasing function with an increasing line. In order to do this, the students re-organized the x-axis so that the numbers on it decreased in value. Other difficulties occur when students represented an entire graph with one point, or a series of points each on a separate graph. Others drew linear graphs under all circumstances by making the scale on the graph non-linear. *"Students' tendency to conserve the linear function may be explained by negative transfer: being exposed to positively sloped lines led some students to generalise that knowledge to all situations even to those where it is not accurate, as one student explained: I constructed the graphs this way because that's what popped in my mind"*

Mevarech & Kramarsky [12] suggest that alternative conceptions of graphs are *"rooted in the quality of instruction that may have led students to confuse, for example, process and product, or to apply strategies that are accurate to one situation but not in another."* They go on to suggest that *"an overemphasis on one kind of function (e.g., an increasing function), or one kind of graph (e.g., histograms) may lead students to conceive all graphs as having that form."* Overall they found that *"the transition from verbal description to graphic representation is associated with various kinds of alternative conceptions that were robust in resistance to traditional instruction about graphing"*.

Goldberg and Anderson [13] found that students often had problems with negatively sloped graphs in kinematics, and suggest that students should be able to handle graphs with negative slopes because they have covered graphs with positive slopes.

We have found that third level students have similar problems. In an experiment where students investigate the focal length of a lens, they are required to draw a graph of 1/*u* versus 1/*v*, where *u* is the distance from the lens and *v* is the distance from the image to the lens. The correct graph is a negatively sloped linear graph. However some 45% of 115 students label the axes as shown in Figure 3.4. In addition, many appear to have manipulated their experimental data to allow their graph to become positively sloped; this is also true for many students who used linear axes[2].

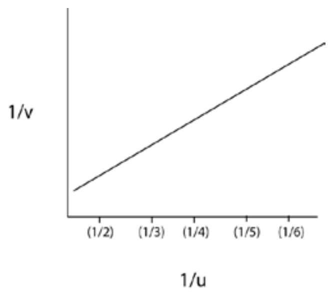


Figure 3.4: Graph used to find the focal length of a lens

3.3.2 Difficulties with interpreting a graph

The first step in interpreting graphical data is reading the values from the graph. As we will show in Chapter 4, even at third level not all students are able to do this. Aside from this problem, many students struggle with interpreting a graph. An important reason for this is that in school [6] students are *"customarily taught and expected to practice the low level mechanics of plotting a particular kind of graph when given a table of information"*. Teachers often do not *"focus on such issues as interpreting graphs, selecting information to be included in a graph, judging which*

[2] We have no direct proof for data manipulation, but it is almost impossible to get positively sloped data with raw data.

kind of graph is most suitable for a given purpose or purposefully modifying existing graphs."

We focus on two important broad classes of interpretation errors: the graph as picture error and the slope for height error. The graph as picture error has been reported in numerous papers [5,7,10,14]. Clement suggests that students are mixing up a "*figurative correspondence between shape of the graph and some visual characteristics of the problem scene."*[7] He identified two types of graph as picture errors: global and local correspondence errors.

The adjective "global" is used when the entire shape of the line is considered, while the "local" correspondence error refers to a point, for instance when two lines are intersecting. Examples of a global correspondence error are given in Figure 3.5a and b. In a multiple choice question, some students choose Figure 3.5a when asked to pick a graph that represents a person first walking away from, and then walking towards a wall [14].

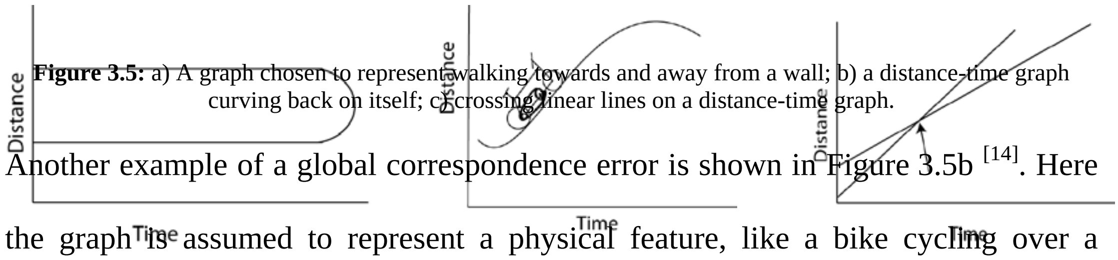


Figure 3.5: a) A graph chosen to represent walking towards and away from a wall; b) a distance-time graph curving back on itself; c) crossing linear lines on a distance-time graph.

Another example of a global correspondence error is shown in Figure 3.5b [14]. Here the graph is assumed to represent a physical feature, like a bike cycling over a terrain. Some students mistake the shape of the trend line for a representation of the terrain that the bike is cycling over, despite the labelled axes.

Figure 3.5c shows an example of a local correspondence error. In this diagram a student picks the point of intersection as being the point where two objects are travelling at an identical speed. Clement [7] states that *“in both cases, however, the figurative matching process producing the error contrasts with the more complex process of metaphorical and functional symbolization*”.

Another error commonly reported is the so-called slope-height error. The slope-height error is commonly made when a student is required to figure out the greatest slope on a graph. For example, on a distance-time graph the students may be required to find the instant at which the object had the greatest speed, which corresponds to the greatest slope. Often students choose the point of greatest height. In Figure 3.6 below students who make slope height error choose A as the point of greatest speed and B as the point of lowest speed.

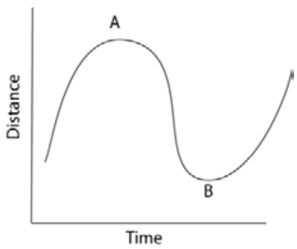


Figure 3.6: Curved distance-time graph. When a slope-height error is made, students think the object had the highest speed at point A.

Clement [7] proposes that the reason for the error is the misplaced link between a successfully isolated variable and an incorrect feature on the graph.

3.4 Graphing in the science lab

Wavering [15] found that there was a strong correlation between the development of students' reasoning process and their ability to construct line graphs. Research on graphing in the science lab can be broken into the efficacy of high-tech lessons (see e.g. [16,17]) and low-tech lessons.

In the past 3 decades there has been huge activity with micro-computers and data loggers. Their convenience and ability to allow the students the opportunity to see the production of a graph close to or during real time production makes the graph more "concrete". Brasell [18] finds that an important feature of the calculators is the immediacy of the action and the appearance of the graph on the screen. Students are immediately able to connect the important features of the graph to the motion. She found, that with a 10 to 20 second delay between the graph appearing on the screen and the action from which the graph resulted, there was a significant drop in achievement in pre- and post-test questions.

Hofstein and Lunetta [19] point out the possible benefits of computers and graphing technology to inquiry materials design.

> *"By using associated software they can examine graphs of relationships generated in real time as the investigation progresses, and examine the same data in spreadsheets and in other visual representations. When inquiry empowering technologies are properly used by teachers and students to gather and analyze data, students have more time to observe, to reflect, and to construct conceptual knowledge that underlies the laboratory experiences."*

However Wavering [15] argued against the use of graphing calculators as students may short circuit the logical development and understanding of graphing. He believes that students should develop graphing skills from a young age using their own data from their own experiments. McDermott *et al* [6] contend that students need to experience graphs in different contexts like temperature with depth of the ocean etc. She found students gain from carrying out *"the same reasoning and procedures in different texts"*.

Mokros and Tinker [14] found that only a few self-drawn graphs are needed to overcome large misconceptions, though their results are disputed by Berg and Smith [10]. Mokros and Tinker used the MBL calculators for a number of labs and, like McDermott *et al* [6], across a number of different contexts (including motion based labs) with different sensors. They found that students improved on their ability to answer multiple choice questions on kinematics.

Zollman and Fuller [20] describe the use of curriculum that allows the students to predict simple motions and represent them in different formats. The curriculum used was based around prediction and observation, with students gradually developing a link between the graph and the observed event. It is clear that the number of predictions and observations can be increased greatly by using graphing technology. *We decided not develop high tech labs, for a number of reasons: the higher cost, and our philosophy that labs should not have black boxes and that graphing should be integrated into all labs and not be the focus of just one or two.*

It is clear that the of difficulties students have with graphing are diverse, with a main difficulty being students' inability to move away from the patterns they learn with a curriculum that uses a mechanical approach to graphing in the classroom. In chapter 4, we set out how we approached testing students' abilities to represent events using graphs. In chapter 5, how we approached developing a general graphing literacy based on our curriculum is set out.

References

1. Jackson D. F., Edwards B. J. and Berger C. F. (1993), *Teaching the design and interpretation of graphs through Computer-Aided Graphical Data Analysis.* Journal of Research in Science Teaching (30) 5, 483-501

2. Roth W. M. (1997), *Cascades of Inscriptions and Re-Presentation of Nature: How Numbers, Tables, Graphs, and Money Come to Re-Present a Rolling Ball.* International Journal of Science Education (19) 9, 1075-1091

3. PISA (2003), *The PISA 2003 Assessment Framework: Mathematics, Reading, Science and Problem Solving Knowledge and Skills,* OECD Publishing.

4. Rosenquist M. L. and McDermott L. (1987), *A conceptual approach to teaching kinematics.* American Journal of Physics (55) 5, 407-415.

5. Beichner R. J. (1994), *Testing Student Interpretation of Kinematics Graphs,* American Journal of Physics (62) 8, 750-762

6. McDermott L. C., Rosenquist M. L. and van Zee E. H. (1986), *Student difficulties in connecting graphs and physics: Examples from kinematics*. American Journal of Physics. (55) 6, 503-513

7. Clement J. (1989), *The Concept of Variation and Misconceptions in Cartesian Graphing.* Focus on Learning Problems in Mathematics, (11) 1-2, 77-87

8. Åberg-Bengtsson L. and Ottosson T. (2004), *What Lies behind Graphicacy? Relating Students' Results on a Test of Graphically Represented Quantitative Information to Formal Academic Achievement.* Journal of Research in Science Teaching (43) 1, 42-63

9. McKenzie D. L. and Padilla M. J. (1986), *The Construction and Validation of the Test of Graphing in Science,* Journal of Research in Science Teaching (23) 7, 571-579

10. Berg C. A. and Smith P., *(1994) Assessing Students' Abilities to Construct the Interpret Line Graphs: Disparities between Multiple-Choice and Free-Response Instruments.* Science Education (78) 6, 527-554

11. Leinhardt G., Zaslavsky O. and Stein M. K. (1990), *Functions, Graphs and Graphing: Tasks, Learning, and Teaching.* Review of Educational Research (60) 1, 1-64

12. Mevarech Z. R. and Kramarsky B. (1997), *From Verbal Descriptions to Graphic Representations: Stability and Change in Students' Alternative Conceptions*. Educational Studies in Mathematics (32), 229-263

13. Goldberg F.M. and Anderson J. H. (1989), *Student Difficulties with Graphical Representations of Negative Values of Velocity*. The Physics Teacher (27) 4, 254-260

14. Mokros J. R. and Tinker R. F. (1987), *The Impact of Microcomputer-Based Labs on Children's Ability To Interpret Graphs.* Journal of Research in Science Teaching. (24) 4, 369-383

15. Wavering M. J. (1983), *Logical Reasoning Necessary To Make Line Graphs*. Journal of Research in Science Teaching. (26) 5, 373-379.

16. Nemirovsky R., Tierney C. and Wright T. (1998), *Body Motion and Graphing.* Cognition and Instruction. (16) 2, 119-172

17. Thornton R. K. (1987), *Tools for scientific thinking-microcomputer-based laboratories for physics teaching.* Physics Education (22), 231-238

18. Brasell H. (1987), *The effect of real-time laboratory graphing on learning graphic representations of distance and velocity*, Journal of Research in Science Teaching (24) 4, 385-395

19. Hofstein A. and Lunetta V. N. (2003), *The Laboratory in Science Education: Foundations for the Twenty-First Century,* Science Education, (88) 1, 28-54

20. Zollman D. A. and Fuller R. G. (1994), *Teaching and Learning Physics with Interactive Video.* Physics Today. April.

Chapter 4: Assessing graphing literacy in the labs.

4.1 Introduction

To investigate students' qualitative graphing abilities in an unfamiliar but instantly understandable context, we asked students to graph how the level of water inside a number of different beakers would vary with time when water was added at a constant rate.

The context is familiar to students from everyday life but unlikely to have been used in a science class, and it is unlikely that formula-based reasoning or rote learning would influence their answers, as it may do when answering kinematics questions. The questions are analogous to qualitative kinematics questions, in that students need to use reasoning that is based on intervals, i.e. continuous changes with respect to time. These questions have three aims:

- To help us understand students' difficulties and their approaches when drawing qualitative trend lines.
- To help us intelligently incorporate graphing as a theme throughout the labs and specifically in the uniform motion and non-uniform motion labs to help us to teach interval reasoning and kinematics (see Chapters 5 & 6).

4.2 Chronology

In the first set of pretest/post-test questions, students were required to draw water level versus time graphs for the five equally tall beakers shown in Figure 4.1. Beaker A is empty. Beaker B has identical dimensions but is half filled with water. Beaker C is an identical beaker with a cone. Beaker D is wider than the other three beakers. Beaker E has the same width as Beaker D at the base but then narrows to the width of beaker A at the top.

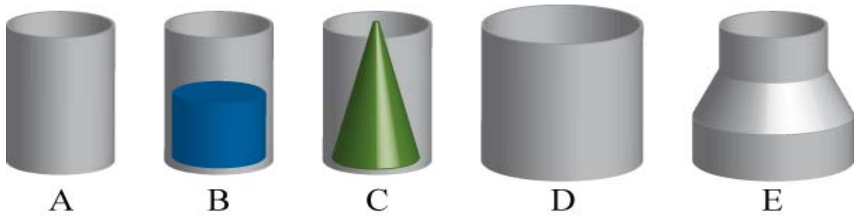


Figure 4.1: Five beakers filled at constant rate. Students are to draw water level versus time graphs for (i) A and B, (ii) A, D and E, (iii) A and C. All three qualitative trend lines are drawn on blank water level versus time graphs.

Water is poured into each beaker at the same constant rate, starting at the same time. Students are asked to draw trend lines for two or three beakers on the diagram shown in Figure 4.2.

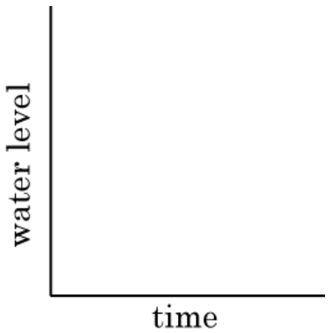


Figure 4.2: Blank graph for use with the questions of Figure 4.1.

Comparing beakers A and B requires the same interval reasoning as for a kinematics problem where two objects move at the same constant speed but start at different points. The kinematics counterpart of comparing beakers A and C would require students to draw a graph where one object moves with constant speed, while another object accelerates from rest until it reaches the same speed as the first object.

Analysis of the students' answers made us realize that some of the questions highlighted difficulties but were unsuitable for pinpointing specific difficulties in students reasoning or approach. We have since developed similar but different questions that have allowed us to identify and differentiate between difficulties students have with drawing qualitative graphs. In Section 4.3, we explore difficulties when the water level rises at a constant rate; in Section 4.4 we discuss those where the water level rises at changing rates.

4.3. Student difficulties representing a constant rise in water level

4.3.1. Overview

Almost all questions contain an empty water beaker being filled at a constant flow rate. Students draw the water level versus time graph for this beaker mainly for reference purposes. Some 400 answers reveal that 85% of students correctly represent this process by a straight line through the origin. It seems likely that many more would do so if the question were asked in isolation: as shown in the remainder of this section, under some circumstances students abandon the notion of a straight line graph to represent other characteristics.

4.3.2. Two beakers, same flow rate, same width, different height

The first pre-test question we discuss features two beakers, A and B, with different height but identical widths, as shown in Figure 4.3. Students sketch their trend lines on a graph with labelled axes as shown in Figure 4.2.

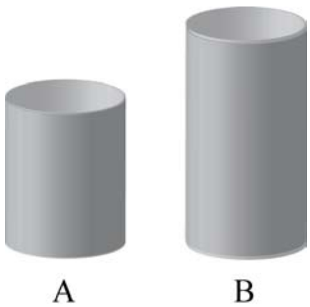


Figure 4.3: An experiment students needed to represent graphically: two empty beakers of the same width but different heights, filled at the same flow rate.

Since we are interested in finding out whether students can represent the rate at which the water level changes by correct relative slopes, we consider an answer correct if both graphs are straight lines through the origin with the same slope, and if line B is longer than line A. Ideally, students would show that the water level remains constant once the beaker is full by means of a sharp bend in the graph as shown in Figure 4.4a.

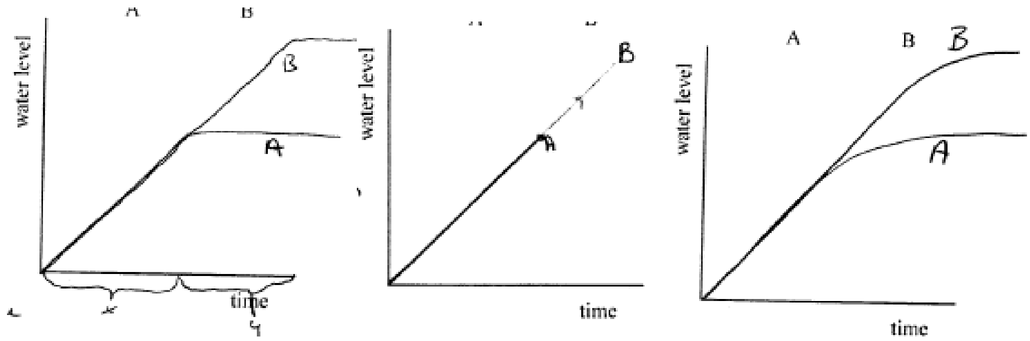


Figure 4.4: Correct and nearly correct graphs for the experiment of Figure 3.3. a): 25% of the students drew a horizontal line representing a full beaker; b) 20% did not draw lines levelling off; c) 20% drew smoothly levelling curve to represent a full beaker.

This total of 65% consists of students who drew correct graphs with correct reasoning. Another 10% drew a correct graph but did not provide reasoning with their answer. Examples of what we considered a correctly drawn answer are shown in Figure 4.4. Our preferred answer, given by 25% of the students, is given in Figure 4.4a. They drew a sharp bend to show the trend lines levelling off. Twenty percent had the graphs not levelling off. The 20% of students who drew a graph like Figure 4.4c appear to represent the water “levelling off” in a fashion that corresponds to the more colloquial meaning of a gradual levelling off. The answers are summarized in Table 4.1.

Table 4.1: Most common answers to the question for two beakers, one taller than the other, both same width and water is poured in at the same constant rate, as given in Figure 4.3.

Answer	*Percentage (N=116)*
Correct with correct and complete reasoning	**55%**
Correct with incorrect or no explanation	10%
A steeper than B	10%
B steeper than A	15%
Coincident curved lines	5%
Same lines for A and B	<5%
Other	<5%
No answer	<5%

Among the incorrect answers, 5% drew lines A and B with identical slopes, but with the lines curved either up or down. Some 10% of students drew two straight line graphs where A was steeper than B, as in Figure 4.5. These students seem to focus on the time it takes to fill the beakers. Typical explanations are: *"A will fill quicker because it has a smaller volume than B"*, and *"It would take less time to fill beaker A as it is smaller"*. Note the ambiguity in the first quote: "quicker" in everyday parlance may mean "in less time" or "at a greater rate". This ambiguity is persistent in many of the questions that feature linear graphs and non-linear graphs.

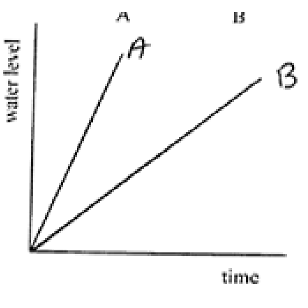


Figure 4.5: Common incorrect answers to the question of Figure 4.3.

Another 10% of students drew two straight lines with B steeper than A, as shown in Figure 4.6. Typical explanations are: *"Beaker B is larger than beaker A therefore it has a greater water level than beaker A", "B takes more time to fill than A and also has a higher water level"* and *"the water level will remain almost the same until A is full and B continues to change and increase the water amount".* All of these students recognized that the line should finish at a greater water level, but seemed to neglect time or rate. Eight out of these 14 students terminated line A at the same time or at a later time than line B. Reasoning seems to be led by the time it takes for the beakers to fill, with little focus on how each beaker fills.

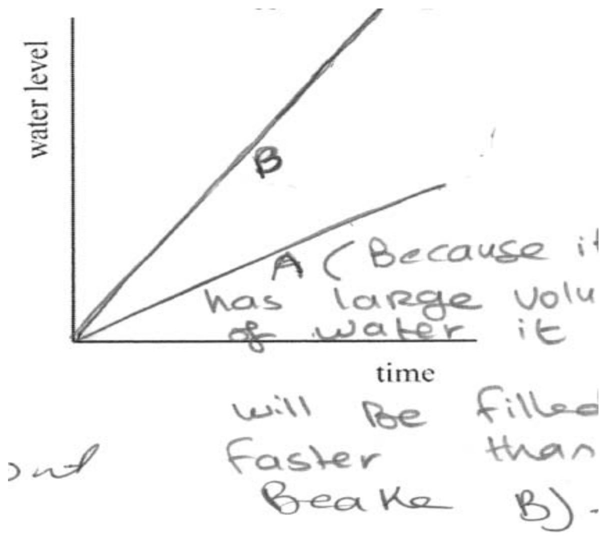


Figure 4.6: Common incorrect answer to the question in which water fills two beakers with identical widths and at an identical rate, but beaker B is taller than beaker A. Line B is drawn with a greater slope in this case.

We tentatively conclude from this analysis that at least one-third of the students do not think of the slope as representing a rate, when answering this question. There also seems to be a difficulty when more than one feature of the line needs to be taken into consideration, e.g., rate of change and finishing level. To probe deeper, we asked a number of questions where the water level changes at different rates, described in sections 4.3.34.3.5.

4.3.3. Two beakers, same flow rate, different width, same height

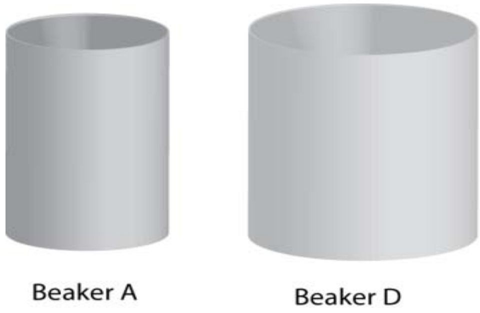


Figure 4.7: Beakers A and D. D is wider than A, but has the same height.

The next pre-test question asks students to draw the water level for beakers A, D and E of Figure 4.1 on the same graph. In this section, only answers for uniformly changing water level intervals are discussed, so we consider only the answers for beakers A and D here (shown again in Figure 4.7 above). Students' answers for beaker E will be discussed in Section 4.4.2. As shown in Table 4.2, 60% out of 194 students answered the question correctly, with D having a smaller slope than line A. Again, we only looked at rates – not all of these finished correctly at the same level.

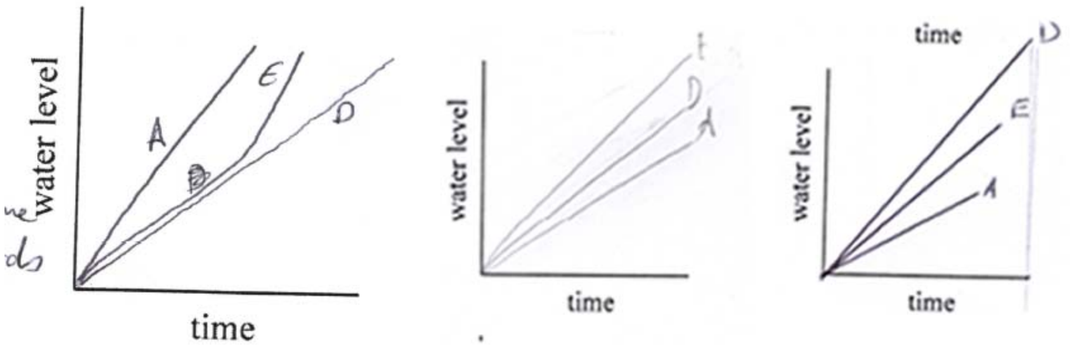


Figure 4.8: Common student answers comparing water levels in beakers A, D and E. a) A correctly drawn graph; b) A, D and E are all straight line graphs finishing at a near identical time; c) D finishes with a greater time and slope as demonstrated with the vertical line sketched from the end of D to the time axis.

The most common incorrect answer (given by 15% of students) had line D with a greater slope than A, as in Figures 4.8b and c. As with the question of Section 4.3.2, the emphasis in these students' answers was on the time it takes for D to fill. The student that drew the graph in Figure 4.8b has the lines ending at about the same time, despite writing: *"D - needs more water and more time to fill"*. The graph of Figure 4.8c appears to use both slope and abscissa to represent the time taken to fill the beakers: *"D will take the longest time to fill because it is the biggest container; […] A is the smallest out of the three so it takes the least time to fill"*. The line sketched from the end of line D to the time axis shows that the student is indicating that line D takes the greatest time to fill. However, a similar horizontal line is not drawn, which suggests that the final water levels are overlooked.

Thus, we again find that students often do not focus on more than one feature of the graph when drawing a trend line. In this case the finishing time took precedence over accurately representing the slope of the line and the finishing level. A complete overview of the frequencies of different answers is given in Table 4.2.

Table 4.2: Most common answers comparing beakers A and D of Figure 3.1.

Answer	*Percentage (N=194)*
Correct with correct and complete reasoning	**55%**
Correct no explanation	5%
D steeper than A	15%
A steeper but lines not straight	10%
Identical slopes for D and A	<2%
Other	<5%
No answer	<5%

For some students (5%) who focus on the time it takes for D to fill, the length of the line they draw is a significant feature of the graph. Figure 4.9a and b show graphs where lines A, D and E coincide, apart from their different lengths. Despite inventing a numerical scale on the vertical axis, the final water level does not appear to be the feature that was in focus in Figure 3.9a. The explanation is nearly identical to that of the student who drew Figure 4.8b: *"Beaker A would fill up the quickest, because it's thinner than D and E. Beaker D will fill up at the same rate as E initially, but then slows down due to E being thinner at the top"*. The explanation given with Figure 3.9b: *"A will take the least time to fill, D will take the most time and E will be in between"*.

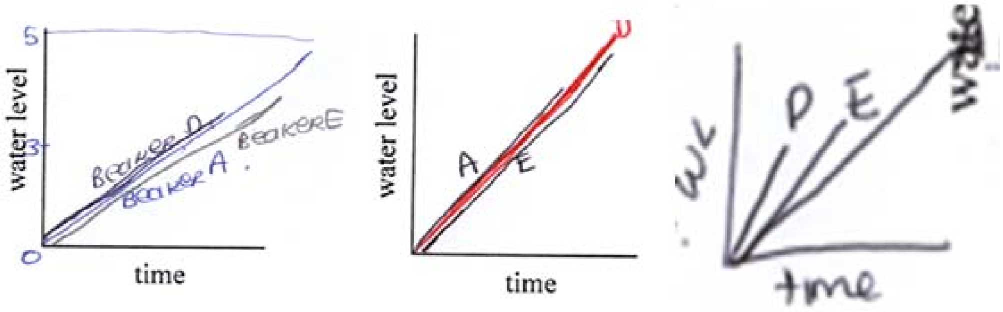


Figure 4.9: a) A, D and E finish at a near identical time; b) D finishes with a greater time and slope as demonstrated with the vertical line sketched from the end of D to the abscissa. c) In the written explanation, this student puts a time limit on the experiment by saying that when A is finished filling D or E stop filling. A is allowed to fill to the greatest level while E and D do not as they are bigger than A.

Some students drew line D shorter than A and E because they put a time limit on the experiment, as if all beakers ceased to be filled when A is full. For example, in the answer given with Figure 4.9c: *"Because there is an increase in water in all 3 but A will be full at the end. However the other two won't as there greater in size than A, D and E."*

The findings for this question corroborate what we found in Section 4.3.2. Moreover, it now appears that for some students, the length of the graph can be an important feature.

4.3.4. Two beakers: same flow rate, different width, different height

The question featuring beakers B and C as shown in Figure 4.10 was specifically designed to test if students focus on the time taken for the beaker to fill, or on the rate of change of water level. Beaker C is wider than beaker B, but fills before beaker B. Note that we do not tell students explicitly that the volume of beaker C is less than the volume of beaker B. Thus, students focusing on the time taken to fill would be expected to draw C steeper than B. The most common answers are given in Table 4.3.

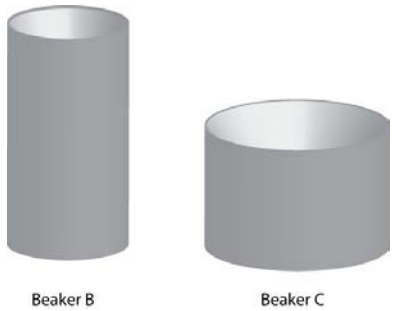


Figure 4.10: Pre-test question designed to test if students use the slope or the total filling time to represent how the beakers fill. Water is poured into each beaker at the same constant rate.

Table 4.3: Most common answers to the question for two beakers, different heights, and the shorter beaker wider. Water is poured into both beakers at the same constant rate. The shorter, wider beaker fills first.

Answer	*Percentage (N=102)*
Correct graph drawn	**30 %**
C steeper than B	25%
Both with the same slope	15%
Either/both curved	15%
No answer	15%

A correct answer for this question was judged to be two straight lines, with beaker C having a smaller slope than beaker B and finishing in a shorter time, as shown in Figure 4.11. As Table 4.3 shows, 30% of 102 students correctly drew straight lines for B and C, with B having a greater slope than C, with a correct explanation. Another 5% drew the same graph with no explanation. Some 15% drew B with a greater slope, but had one or both of the lines curved.

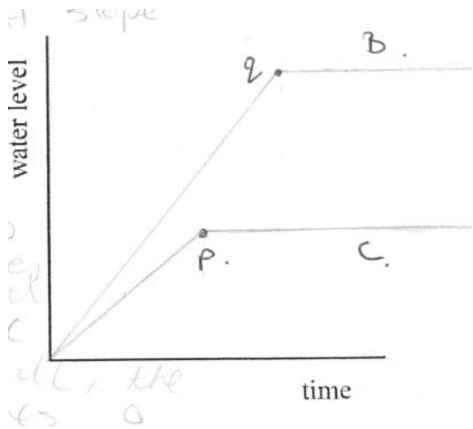


Figure 4.11: Correctly drawn graph for two beakers, different heights, and the shorter beaker wider. Water is poured into both beakers at the same constant rate. The shorter wider beaker fills first.

We found that 25% of students drew the line for beaker C with a greater slope than the line for beaker B. In these answers the focus is on the time it takes for C to fill. For example, the explanation given with Figure 4.12a: *"As C fills up before beaker B does but beaker C is wider than beaker [B]"*. In some cases there seems to be an ambiguity between the total time taken for the beakers to fill and how quickly the beakers fill. The explanation for Figure 4.12b explicitly states that the slope represents the filling time: *"The water level in C raises quicker than the water in B, therefore it takes less time to fill so the curve is much steeper than that of B"*. Likewise, explaining the graph of Figure 4.12c: *"As C fills up quicker the curve will be more vertical than B which is slower in time"*.

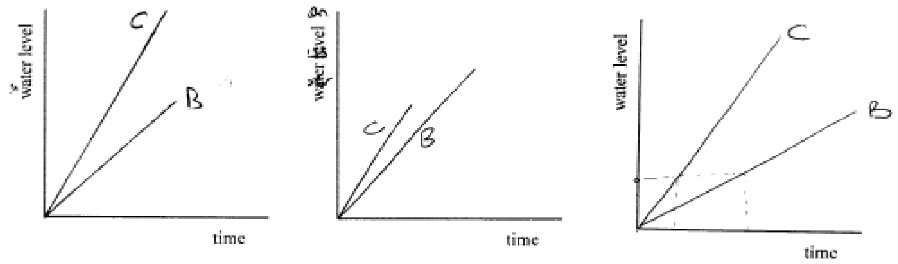


Figure 4.12: Graphs where C is steeper than B. a) Line C has a greater slope, a greater finishing level and a near identical finishing time, b) line C has a smaller finishing time but a finishing level, c) Line C has a greater finishing level and finishes before line B.

Some 15% of answers drew B and C with the same slope. All but one of these students drew C both at a lower finishing level and finishing before B. The common theme for students who drew lines with the same slope was not to focus on time or the rate of change of water level, but rather to focus on the rate at which the water enters each beaker. The explanation that accompanies the graphs of Figure 4.13 features both lines rising, but once C is full B continues to rise: *"Because beaker C isn't as tall as beaker B, the water level won't be marked as high. Beaker C fills up more quickly than beaker B and because they are at the same constant rate both beakers B and C share the line but C is cut off earlier than B."*

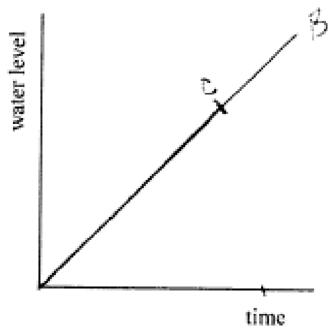


Figure 4.13: Incorrect answer to the question of Figure 3.10: same steepness, different length.

The question proved more difficult than any other question featuring uniformly changing water levels. The beakers have different end levels, different finishing times and different rates at which the water rises. Hence an unusually high fraction, 15%, didn't attempt the question and only 30% got the question correct. This is lower than other graphs in which uniformly changing intervals are present.

A picture is emerging that, the more difficult a setting, the more students tend to focus on one feature of the graph. In some cases the finishing point or finishing level seems to be more important than the slope and can determine the type of line drawn, while in other cases the slope determines the shape of the line.

4.3.5. Two beakers: different flow rate, same width, different height

Another pre-test question uses the same two beakers as in Section 4.3.3, with different heights but the same widths, but in this case the beakers fill at the same time. Students must infer that the filling rates are therefore different. The criterion for a correct answer is that two straight lines are drawn, with line A having a smaller slope than B, and A and B finishing at the same time but at a higher level for B. Figure 4.14 shows the two beakers.

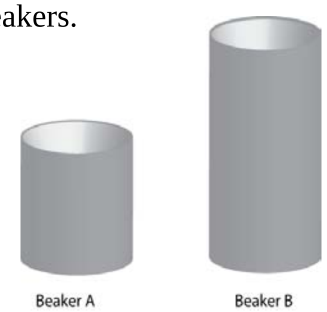


Figure 4.14: Beakers A and B. Beaker B is taller than beaker A and both containers are the same width. Water is poured into each beaker such that they fill in the same amount of time.

As can be seen in Table 4.4, 50% of students correctly answer the question, with correct reasoning. A further 15% of students drew line A with a smaller slope but drew one or two curved lines to represent the identical filling times.

Table 4.4: Most common answers to the question of Figure 4.14.

Answer	*Percentage (N=116)*
Correct with correct and complete reasoning	**40%**
Correct with incorrect or no explanation	10%
A steeper than B	5%
Lines finish at the same point	10%
Same Lines for A and B	5%
Other including curved lined answers	20%
No answer	5%

Some 10% of all students finished both lines at the same point. Some examples are shown in Figure 4.15. Even though it seems that the student who drew the graph of Figure 4.15a must have focused on the end point, the reasoning does not highlight this: "*A seems to be getting to the top quicker because it is a smaller beaker. B takes longer to get to the top it is taller.*" Nevertheless, we think the point represents the event that both beakers are full. In the similar graph of figure 4.15b, another student has marked "full" on the water-level axis.

In Figure 4.15b, the trend lines for beakers A and B are curved in opposite directions. While it is difficult to interpret this, one possible explanation may be that they both are curving so that they can intersect at a point. The point on the graph for this student may be a more important detail than the slopes. The lines drawn appear to function merely as a path to the point rather than being anything significant in them.

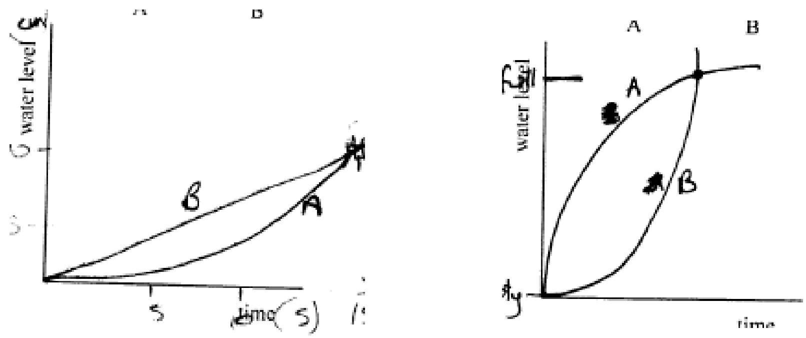


Figure 4.15: a) B is represented by a straight line, A is represented by a line curving upwards to meet B at the same finishing point; b) two curved lines, with line A curving downwards and line B curving upwards. Both lines intersect at a point that is marked as full on the *y*-axis.

The explanation for the graph shown in Figure 4.15a supports this interpretation. This student has turned the question into something semi-quantitative (note that no numbers were given in the question): *"A+B start off empty therefore zero (0,0) and are full at the same time Pt(15,10). B measures fuller each time interval on it is narrower. When A is measured at same time and intervals the water level is less than B. Since each beaker has same volume they are full at same time and correspond to same end pt on graph e.g. (15,10)*". It is clear that (15,10) represents the event that both beakers are full and the students does not consider the water level.

4.3.6 Two beakers: same flow rate, same width, same height, different starting levels

In all preceding questions the beakers initially contained no water. In the pre-test question of Figure 4.16, beaker B is initially half full. Some 55% of 103 students drew a straight line for A through the origin and a parallel straight line for B that starts at the water level axis. Almost all of these gave explanations that tended to focus on the time it would take to fill both beakers.

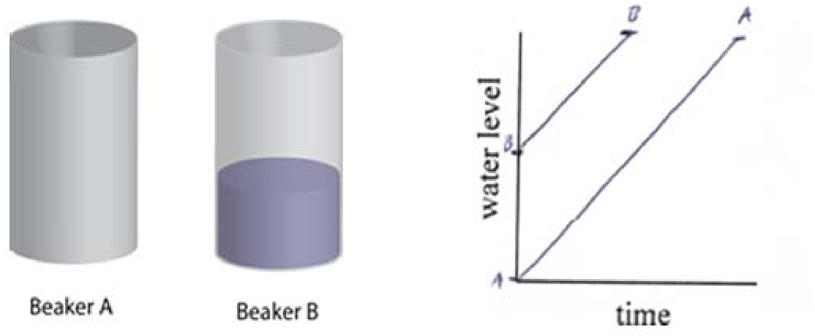


Figure 4.16: a) An empty beaker A, and a half full beaker C; b) Correct answer showing correct finishing and starting levels, and both lines are parallel.

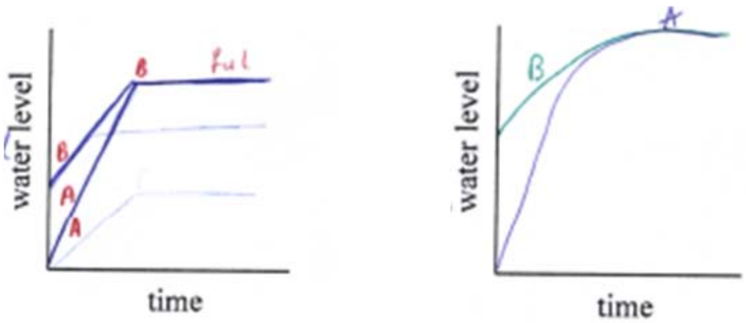


Figure 4.17: Common incorrect answers to the question of Figure 4.16a indicating a tendency towards a point on the graph. a) B represented by a straight line

We identified a tendency to treat the end point of graph A as representing an event where both beakers are full. Some 10% of students correctly started A at the origin and B half-way up the water level axis, but drew A with a greater slope than B. Half of these students drew the line B such that it finished at the same point as line A. The explanation that accompanies Figure 4.17a: *"A fills quickly and the slope of the line is steep, B is already half full and levels off the same time as A"*. Figure 4.17b comes with an explanation where the focus is not on the time but on the level that both finish: *"Beaker A was empty at the beginning so the water level starts at 0 and rises gradually until it is full. Beaker B was half full so the water level starts at the half* *mark and rises until it is full"*. As in Section 4.3.2, the filling of the beaker appears to be represented by a smooth curve rather than an abrupt change in slope.

No fewer than 25% of the 194 students who answered this question started line B at the origin. Some 10% drew a line B with a greater slope than B; 5% drew a trend line overlapping line A; nearly 5% drew line B less steep than line A. The remaining 5% who drew lines starting at the origin, drew lines that were not straight.

As before, students who drew B steeper than A tend to confuse a shorter time to fill up with a greater rate. The explanation accompanying Figure 4.18a was *"B is already filled with water so the water-level will rise at the same rate as A but B is filled already so the level will be quicker to reach the top in beaker B"*. The student correctly identifies that the beaker B will be filled quicker than A and also reiterates relevant information given in the question that both beakers fill at the same rate. However, the graph appears to go through the origin by default: there is no mention of the starting level of the water in beaker B.

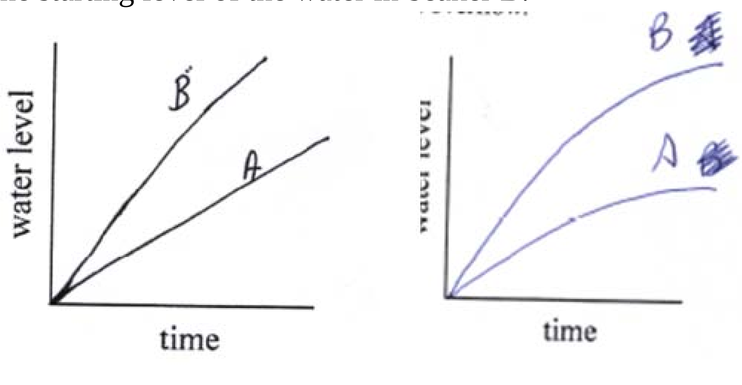


Figure 4.18: Trend line B drawn with a greater slope than A to show that the water level fills at with a quicker time in beaker B.

The explanation given for Figure 4.18b is that *"B is already half full, so it only takes another half load of water to fill to the beaker. In the amount of time it takes B to become full, A is only half full"*. As is often the case, the written explanation is correct, if incomplete. It is not possible to ascertain whether the curved graph represents levelling off when the beakers are full. If this is the case, and the student assumed that when B is full, water is stopped being poured into A, then the student gets the end-level right, and correctly represents both lines as finishing at the same time. Even in this generous interpretation, the focus is again on the overall time and the finishing levels, not on the rate.

Table 4.5: Most common answers comparing beakers A and B of Figure 4.16a.

Answer	*Percentage (N=194)*
Correct with correct and complete reasoning	**55%**
Correct with incorrect or no explanation	<5%
B starting at the origin but parallel	5%
B starting at water level, but greater slope	10%
B starting at the origin but different shaped line	5%
Total B starting at the origin	*25%*
No answer	

4.3.7 Summary of findings for linear water level graphs

A variety of water level questions have revealed a number of student difficulties. For most questions, about half the students give answers that are consistent with treating the slope of a line as representing the rate at which the water level changes, and hence seeing the filling of the beakers as a dynamic process. However, depending on context a number of common errors are revealed. Despite typically giving a correct written description of how the water level changes with time, when drawing the graphs many students appear to focus on one or two static features. The most prominent of these is the time taken to fill the beaker, but starting and end points also play a role. Moreover, it seems that some students use the length of the graph to indicate the duration of the process. We also identified a tendency to rely on familiar forms of a graph even when their use is inappropriate, such as using a straight line through the origin for a graph with a non-zero intercept.

4.4. Student difficulties with changing rate of change of water level

Many of the trends we identified for questions where the water level changed at a constant rate were present, and even amplified, when we investigated questions where the rate of changed of water level varied.

4.4.1. Cone in beaker

The first question designed to test students' understanding of changing rates asked students to compare the water levels in beakers A and C of Figure 4.1. The beakers are shown again in Figure 4.19a and b below. As in all of our questions, water is poured in at a constant rate; here, beaker A is empty but beaker C contains a cone. A correct answer would consist of a graph with a straight line through the origin for beaker A, and a line for C that initially rises more steeply but curves downwards (i.e., gets less steep) until it is parallel with line A at the same end level.

Even when accepting graphs where the slope of C gets smaller but not exactly parallel at the end, and accepting end levels are not too different from each other, merely 20% of students drew a graph that we considered correct, with correct reasoning. A further 5% drew the correct graph but failed to give a correct explanation. The explanation given for the essentially correct graph of Figure 4.19b states: *"A increases as normal, C initially fills faster as there is less area at the bottom of the beaker where the cone is wider as the cone gets thinner near the top it takes longer for the beaker to fill beaker there is a greater area to fill."* The use of the word “area” strongly suggests that this student is thinking of small cross sectional areas that are being filled sequentially, as would be required to obtain a correct well-reasoned answer.

Figure 4.19: a) Empty beaker A and C with a cone inside are being filled at the same constant flow rate; b) a typical correct graph for water entering the containers.

Table 4.6: Most common answers comparing beakers A and C of Figure 4.19a.

Answer	*Percentage (N=194)*
Correct with correct and complete reasoning	**20%**
Correct with no explanation	5%
Line with an increasing slope	5%
Straight line graph through the origin steeper than A	45%
No answer	<5%
Other	20%

Straight line graph.

The trend identified in some of the linear graphs of Section 4.3 to represent a process that takes less time by a steeper straight line through the origin now becomes prominent. The most popular answer consisted of students drawing a straight line for graph C with steeper slope than A, as shown in Figure 4.20. As Table 4.6 shows, 45% of students gave this answer. The explanation given with Figure 4.20a: *“A is* *empty thus it fills quickly hence its steep gradient, Beaker C has a cone in it thus it fills quicker as it has less space to fill and beaker C fills up first due to its lesser capacity of water"*. The student who drew Figure 4.20b stated: *"C fills up quicker than A as the cone in C takes up some of the volume in the beaker, meaning the water fills up beaker C quicker"*. Dynamic entities such as interval changes or the changing speed of the water level in the beaker do not feature in the students answer; only static entities such as the time taken, average speed, and total volume are considered. This answer echoes the incorrect reasoning identified in Section 4.3.6 for the graph of Figure 4.18a.

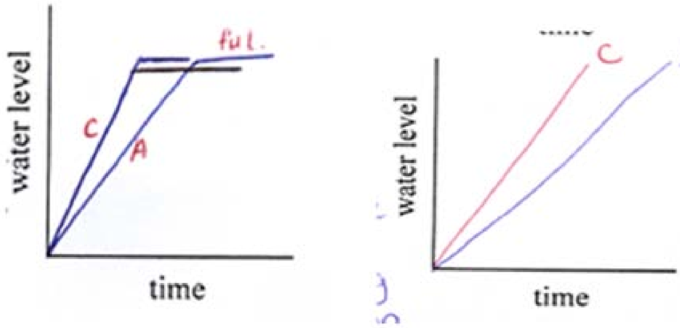


Figure 4.20: Examples of the most common answer to the question of Figure 3.19: two straight lines, with C steeper than A.

Length of a line

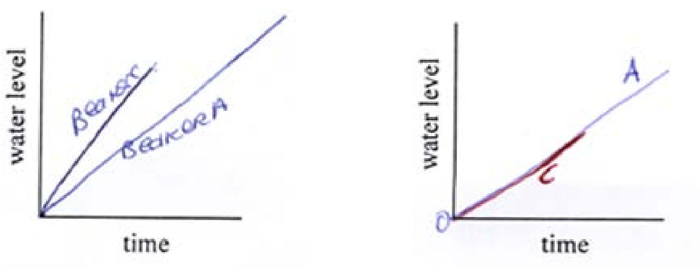


Figure 4.21: Incorrect answers to the question of featuring figure 3.19a that focus on the length of the line. a) Straight line graph through the origin, the length of line C is far shorter than line A; b) Line C is shorter than A and has the same slope.

Some 10% of students drew a much shorter straight line through the origin for C. This can often be ascribed to the tendency identified in Section 4.3 to let the length of a line describe the duration of the process. Thus, the graph of Figure 4.21a appears to reflect an attempt to represent the shorter duration of a process taking place at changing rate by a shorter straight line through the origin. The explanation given with Figure 4.21a: *"Because C fills up quicker than A, due to the cone, the volume of this beaker is less."* The amount of water entering the beaker may play a part in the length of the line that the student drew, as the lower water level may be a way of representing a smaller volume entering the beaker.

The explanation given with Figure 4.21b was: *"A will take more time to fill than beaker C."* In this case, the length of the line is the only difference between lines A and C, and it seems clear that for this student the length of the line represents the time it takes for the beakers to fill.

The most striking finding from this question is the number of students who draw a straight line graph through the origin, along with the low percentage of those who got a correct answer. Rather than reasoning on how the beaker is filling, and representing this with an appropriate trend line, the answers and the reasoning focus around an average speed or a total filling time for each of the beakers

4.4.2. Changing shape of vessel

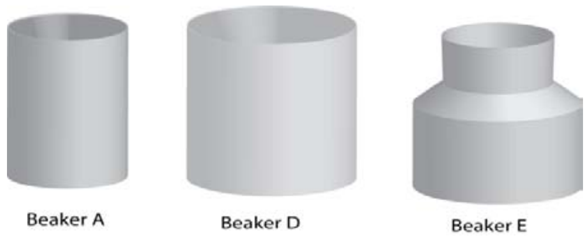


Figure 4.22: Three beakers A, D and E filled at the same constant flow rate. E has the same width as D at the base, and narrows to the same width as A at the top.

The next question under discussion asked students to draw water level versus time graphs for the set-up in Figure 4.22. The results for beakers A and D have already been discussed in Section 4.3.3. Students are informed that beaker E has the same width at the base as beaker D, and then narrows into the same width as beaker A. trend lines for the water level in all three beakers are drawn on the same blank graph.

Some 10% of students represented E as consisting of two parts: a straight line coincident with D, then curving upwards with a steeper slope. Another 30% drew a straight line traced over D, but drew the second part of the line straight, with steeper slope. These answers showed some focus on intervals when drawing the graph.

Table 4.7: Most common answers for line E of Figure 4.22.

Answer	*Percentage (N=194)*
2 parts, 1 straight, 1 curved upwards	10%
2 straight parts	30%
Straight line through origin, steeper than A	25%
Straight line through origin, parallel or less slope than A	15%
Curve line upwards	5%
Other/no answer	5%

Some 25% of students drew a straight line for E with a slope in between that of A and D, as shown in Figure 4.23. Similar reasoning is given to this as with the last question which focused on a changing interval.

The focus again appears to be almost exclusively on time. The student who drew Figure 4.23a gave the explanation: *"A is the smallest volume-fills first. E is the 2nd smallest and D takes the longest to fill as it has a larger volume."* Some students, like the one who drew the graph of Figure 4.23b, consider the static feature of the end level as well.

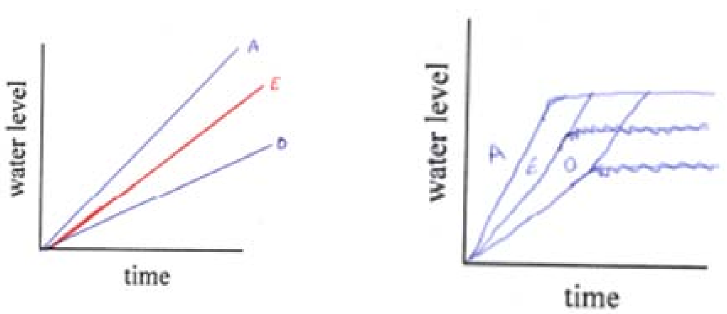


Figure 4.23: Straight line graph through the origin to represent beaker E. a) Ending at different levels; b) Correctly ending at the same level.

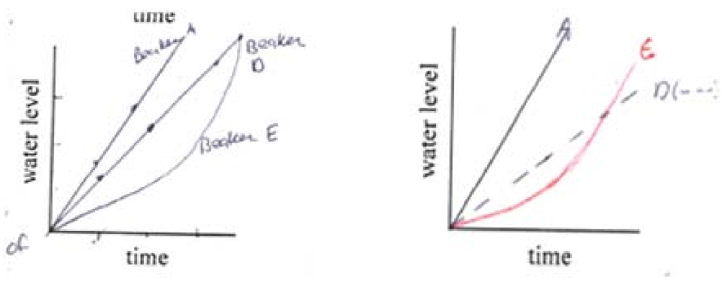


Figure 4.24: a) E curved upwards and finished at the same point as D; b) line curved up but finishes between the lines for beakers A and D

Some 5% of students draw a single slope curved up as shown in Figure 4.24. The student who drew Figure 4.24a gave the explanation: *"A would rise fastest hence the steeper slope. D would be steeper as its wider than A. E would be curved given that it tapers inwards thus being filled faster the further it rises"*. This student does not focus on the identical widths of beaker D and the base of beaker E, or the identical widths of beaker A and the top of beaker E. Rather, there is a focus on the how the trend of the graph changes globally.

Figure 4.24b shows a similar line to Figure 4.24a, with the explanation: *"Beaker A will fill up first as it is the smallest of all the beakers followed by beaker D. Beaker E will follow a curve as it with differs than that of both A and E at different points"*. Even though the graphs of A and E are matched well, this student appears unable to match up graph D correctly with E. In the explanation, again a global trend is identified and represented, but the student lacks the correct level of reasoning to break down how the water level changes at different times when the beaker is filling.

4.4.3. Stepped cylinder

The next container requires an understanding of abruptly changing intervals to fully and correctly answer the question. As shown in Figure 4.25a, one of the beakers now contains a stepped cylinder, rather than a smooth cone. It was given as a post-test question after students completed the labs. An important point to note is that this question was given as a post-test, and students had not drawn lines that had stepped changes in slopes as part of their instruction.

An almost correct answer is shown in Figure 4.25b: it has three straight lines with progressively decreasing slope pieced together. Ideally both lines would finish at the same water level, however in this example they do not. Overall, 15% give the correct answer with a slope that is stepped downward.

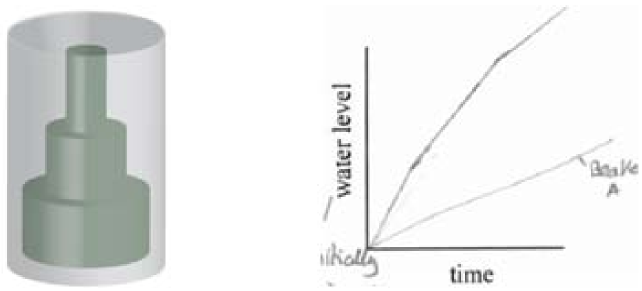


Figure 4.25: a) A beaker with a stepped cylinder in it. b) The correct graph drawn, with incorrect finishing heights.

The correct explanation for the shape of line C given with this graph was: *"Beaker A yet again will be filling with a constant rate, i.e. the water level will rise at a constant (constant slope/linear graph) whereas beaker C will initially increase at a constant (more quicker than A) then the slope will become less when the first step is reached, yet water level will yet again increase with a constant slope before the next step is reached and water level will continue to increase but slower again. It will still move at a constant during this step. At no time will beaker A have a greater slope than beaker C. i.e. beaker C will always be increasing faster"*.

Interestingly, another 30% of students drew a smooth curve that bends downwards as shown in Figure 4.26. The shape of the graph and the answers accompanying the graphs would suggest that the students understand the general trend of the intervals, but are unable to analytically identify changes in the rate or isolate patterns of change for the intervals. Instead, it seems that a curved graph which looks like a graph they have drawn for period versus length in a pendulum experiment, and for the distance-time graph of a decreasing ball, has become the default graph for a process in which the rate of change is variable.

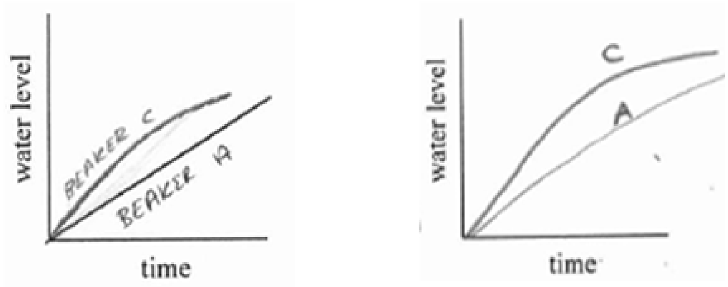


Figure 4.26: a) A curved line representing C, with both lines finishing correctly at the same height; b) A curved line for C finishing at a greater height than A.

The student who drew Figure 4.26a gives the explanation: *"A will fill at the same rate as before but this time C will fill only slightly quicker. The graph of C gets to the different levels of the stepped cylinder gradually takes longer to fill"*. This student understands that the rate is decreasing. The answer also makes clear why this is the case and directly links to the shape of the cylinder. A similar explanation is given for Figure 4.26b: *"A will fill at a slower rate than beaker C because it's completely empty. C will fill quickly but will begin to slow down because the cylinder is stepped"*.

Another 5% of students represented the changing water level in beaker C with a single curved line sloping the upwards, as shown in Figure 4.27. In the explanation for the graph of Figure 4.27a, there is mention of rate of change: *"Again C will be steeper than graph A as beaker A is empty and beaker C contains a stepping cylinder which will cause the beaker to fill at a quicker rate"*. Likewise, Figure 4.27b is explained by: *"Beaker C fills up a lot quicker than beaker A. It takes less time for the water level to rise as shown in diagrams. However beaker C does not fill up as* *quickly as beaker B"*. Neither answer is analytical, in terms of describing how the rate changes abruptly change. The former alludes to rate but does not elaborate, and the latter only mentions the overall time.

It seems from the answers given that these students had no general strategy for interpreting how the rate of changed in the water level beaker, or how to represent these rates on a graph.

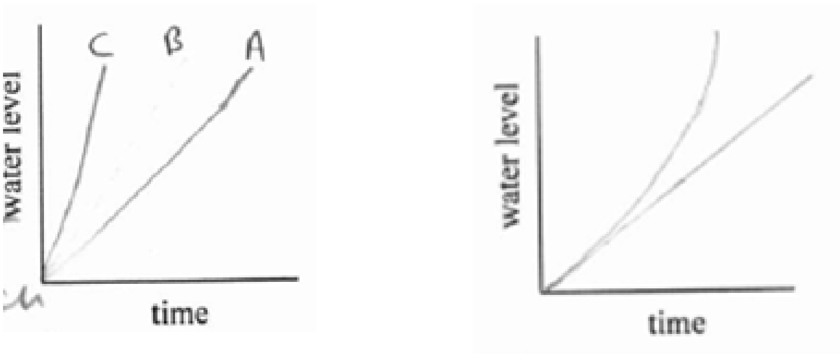


Figure 4.27: Examples of single line curving up.

Figure 4.28: a) a straight line graph with the slope of C greater than that for an empty beaker; b) a straight line graph with the slope of C smaller than that of an empty beaker.

As with the pre-test question that described how the water level changed when a cone was present in the beaker discussed in Section 4.4.1, a large number of students drew a straight line graph through the origin, as shown in Figure 4.28. In this case, 30% did so. The explanation given with Figure 4.28a: *"Similar to beaker B, beaker*

C will fill up faster than A due to the space that up by cylinders present in the beaker. The only difference is that the stepped cylinder in beaker C takes up less space than the one in beaker B or C would fill up faster than A but slower than B. We can show this on graph by drawing the line B or C at a more steep angle to A but less steep than B." The beaker B the student refers to in his answer contains a regular cylinder, which means that the water level changes uniformly for the entire time that the beaker is filling. However, the student treats the stepped cylinder in beaker C as equivalent to the regular cylinder in beaker B. In Figure 4.28b, there is a focus on the total time it takes to fill the beaker: *"As before it will take more time to fill beaker A than beaker C"*. What counts is the total volume to be filled or the time it takes to fill that volume; the distribution of this volume is inconsequential for the students.

Table 4.7: Most common answers comparing beakers A and C of Figure 4.25.

Answer	*Percentage (N=116)*
Correct with correct and complete reasoning	**15%**
Line with an increasing slope	5%
Straight line graph through the origin	30%
Curve line downwards	30%
Other/no answer	20%

4.5 Findings

4.5.1 Approach to changing intervals

We have found that the students' approaches to intervals can fit into three distinct categories.

The first category is students who focus on either the total time taken for a beaker to fill. Typically students who answer a question in this way draw straight line graphs through the origin for beakers in which the water level does not change uniformly. Answers usually centre on the time taken to fill the beaker. A startling example is the 45% who draw a straight line graph through the origin when a cone is present inside a beaker and filled at a constant flow rate

The second category is students who consider intervals, but do not or cannot break down how the water level changes at different times over a span of time. Rather, these students recognize a global pattern to how the water level changes. An example of this are students who draw a curved line to represent he changing water level in beaker E, in Section 4.4.2, or draw a curved line sloping upwards, for a beaker that abruptly changes from being wide at the base to being narrow at the top and for a beaker with a stepped cylinder as shown in Section 4.4.4.

The third category is students that are able to successfully and analytically break down abrupt changes in interval and successfully represent them on a graph. However, in the case of a stepped cylinder in the beaker, as little as 15% of students could successfully complete this task after the labs.

4.5.2 Focus on features on a graph

Many students however do not appear to focus on intervals. We find that about 10% of students focus on a single feature of the graph such as the length of the line, or the finishing point of the line. For instance when both beakers fill at the same time, but are different heights, students often draw one or two curved lines, so that both lines intersect at a point, that represents the time it takes for both these lines to fill. The length of the line seems to be linked with time or the amount of water being poured into the beaker also.

4.5.3 Tendency towards preconception or a prototype

No fewer than 25% of students started the trend line at the origin when required to start the trend line on the water level axis to show that water was already contained in the container. This strong tendency for students drawing graphs through the origin is in line with our experience of students' graphing experience in school labs, and our pre-test results show that a considerable number of students draw trend line graphs inappropriately through the origin prior to taking the labs.

As much as 45% of students drew a straight line graph through the origin when required to draw a curved graph to show a changing rate of change. Many of these students maintained that the slope of this straight line represented the time it took the beaker to fill, *i.e.* the greater slope, the less time it takes to fill. One possible explanation for this is the tendency to draw lines inappropriately. This may be traced to the narrow range of graphs that students experience at many levels of science education. In both the Leaving Certificate and Junior Certificate few graphs were not a straight line graph through the origin.

4.5.4 A link between time and slope

As we have discussed before, the term speed is often used as shorthand for how much time it takes for something to get done. We find in nearly all of the questions featuring linear graphs that students have a strong tendency to focus on the time it takes for the beakers to fill as opposed the rate of change the beakers fill at.

The time it takes to fill the beaker has also been nearly uniformly used as justification for drawing straight line graphs that represent changing intervals. The slope of the lines linked to the overall time it takes for the beakers to fill. Both examples from 4.4.1 with the cone in the beaker and 4.4.2 with a differently shaped beaker show that students often consider only the time and not the speed at which the process takes place.

4.5.5 Inability to represent multiple features of a line

In many of the questions there were a number of important features to be represented by a single trend line (i.e. finishing time, finishing level, rate of change). However, many students were not able to represent more than one feature with the trend line. The post-test results presented in Chapter 7 provide additional support for this statement, and show how the problem persists after instruction.

Chapter 5: Approach to developing graphing literacy

5.1 Overview

Based on the difficulties we identified in Chapters 3 and 4, we developed laboratory exercises designed to help students more effectively interpret and work with graphs. In this chapter we present the approach that we take through a series of four guided inquiry experiments. Pre and post-test assessment of their efficacy is presented in Chapter 7.

The development of graphing skills is centred on four experiments: making a spring balance, the pendulum, uniform motion and non uniform motion. In this chapter we focus only on aspects of these experiments that are designed to help students develop their graph construction and graph interpretation abilities. The complete experimental worksheets as run in 2009-2010 are included in Appendix A.

5.2. Overview of experiments and approach to developing graphing skills

The general approach that we took to the curriculum was to develop the amount of autonomy that students had as they built on their experience in the lab. This was similarly the case with the graphs that they draw in the labs. The first lab is very controlled in that students are given detailed instructions on what measurement to make and how to make them. As the labs progress, students are given more freedom with the trend lines they draw, scaling axes, deciding how many readings to plot and the size of the graph.

5.2.1 Making a spring balance

Graphing issues addressed:

- interpretation of slope and intercept of linear graphs
- straight lines not through origin
- the role of time in drawing a graph

Verification of Hooke's law has always featured in our physics labs. In our approach, students discover Hooke's Law in the first lab (see Appendix A). From a graphing point of view, students were given tables to fill out and given detailed instructions of what measurements to make, along with guidance on how to make them. The graphs that they draw have familiar straight lines.

In this lab we focus on how the slope of the line relates to the stretchiness of the spring, that the slope and the height of a line are independent, that the slope of an extension versus mass graph is independent of the rate at which mass is added to the spring, and what the horizontal and vertical intercepts of these graphs signify.

5.2.2 Uniform motion

Graphing issues addressed:

- straight lines not through origin
- interval reasoning
- re-creating motion from a graph

The second lab, in which students discover the law of the lever, does not feature any graphs. In the third lab[1] however, students obtain and record uniform motion. One of the most important exercises in terms of the development of graphing skills has students draw two graphs for the same motion. One of the graphs is a straight line that passes through the origin, as the students record the time taken for the ball to

[1] In the first three years of implementation, the pendulum lab was given before the uniform motion lab. We changed the sequencing so as to tackle the development of linear graphing skills before moving on to non-linear graphs.

travel from the bottom of the ramp to each equidistant cube on a track that they have angled so as to compensate for friction. The first of these distances is marked as x_1 in Figure 5.1. The second graph is drawn for the same times recorded, but the distances are now measured from each cube to the back of the ramp (d_1 in Figure 5.1).

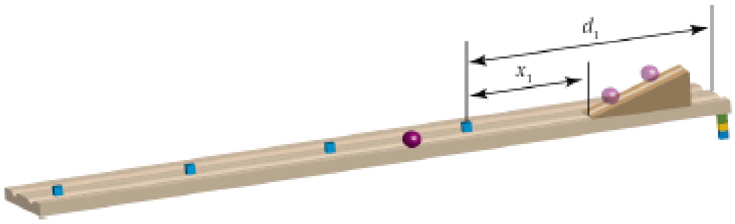


Figure 5.1: Setup for uniform motion lab. On a track there is a ramp to help the balls start moving. Cubes are used both as markers on the track, and also to angle the track to attain uniform motion.

As a final exercise, we require the students to recreate the motion of two balls rolling simultaneously along two parallel tracks.

5.2.3 Pendulum lab

Graphing issues addressed:
▪ curved graphs
▪ interval reasoning
▪ re-creating motion from a graph

In the fourth experiment, students phrase and check their own hypotheses on how the period of a pendulum varies with mass and length. In this lab students are required to draw their first graph that is not a straight line when they represent how the period of the pendulum changes with length.

As in the uniform motion lab, students are given a graph and asked to recreate a pendulum-like motion of the bob on the string, thus reversing the process of graphing and experiment.

5.2.4 Non uniform motion

Graphing issues addressed:

- curved graphs
- interval reasoning
- creating a graph from hypothetical motion

The last lab in which graphing features significantly deals with non-uniform motion. Students get a ball to slow down as it rolls up a track, and draw an appropriate graph. Links again are made between the shape of the graph and the speed of the ball based on interval reasoning. Students need to piece together a distance-time graph for a hypothetical track.

5.3 Developing skills to interpret slope

As shown in Chapter 4, many students find it difficult to relate slope to a rate or co-variation. This section focuses on how we tried to develop students' reasoning skills when it comes to slope.

5.3.1 Linking slope and shape of a graph with a process

In the existing labs, graphs focused on the mechanics of plotting data points and a best fit line, and finding a numerical value for the slope. Little attention was paid to how the shape of a graph is related to the events they represented. As evidenced in Chapter 3, students have not yet developed an ability to link the shape of a line with the events despite their previous science experience in schools, and this linkage is something students need to learn. In our approach students were asked to make explicit links between the shape of the graphs and the observations in the experiments that they were carrying out.

In the first lab, two extension versus mass graphs are drawn for masses attached to two different springs. Slotted discs are attached to a mass hanger. Students are told to plot the extension of the spring, measured from the length when neither mass hanger nor discs were attached, as a function of the mass of the discs added. While this is somewhat contrived, it allows us to get students to think about slopes and intercepts. (It is interesting to note that not one out of the ~1000 students who have taken the labs have asked us why we follow this procedure. We think this is because they are used to doing cookbook experiments without thinking about the procedures followed.)

One spring is notably stretchier than the other. Using two different springs allows us to ask students to identify which of the lines represents the stretchier spring and why. This link is asked through a series of two questions: *"Examine the graph you have drawn in Section 4 and describe in your own words the 'steepness' of the slopes for Spring 1 and Spring 2"*, as shown in Figure 5.2.

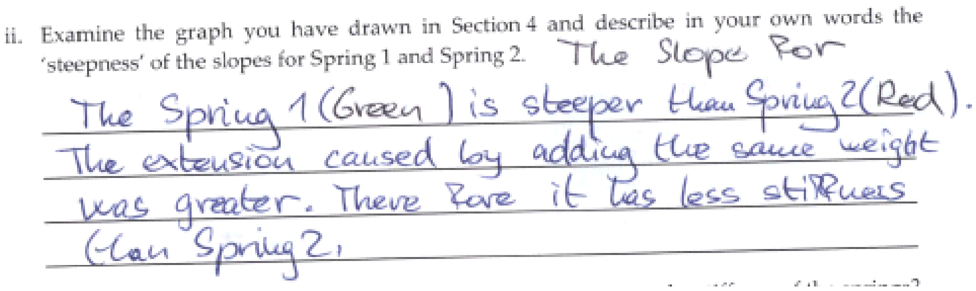

ii. Examine the graph you have drawn in Section 4 and describe in your own words the 'steepness' of the slopes for Spring 1 and Spring 2. The Slope For The Spring 1 (Green) is steeper than Spring 2 (Red). The extension caused by adding the same weight was greater. There Fore it has less stiffness than Spring 2.

Figure 5.2: A student's answer for comparing the steepness of the slopes of the lines for spring 1 and 2.

We also ask: *"How can you use the slopes of the two best fit lines to compare the stiffness of the spring?"* Questions such as these are rarely encountered in Hooke's Law experiments.

More substantial effort is needed on the part student for an exercise in the uniform motion lab. We ask our students to re-create the motion of two balls on a track simultaneously from the graph of Figure 5.3. Here students have to consider the slope and the intercepts (or starting points) for both lines on the graph.

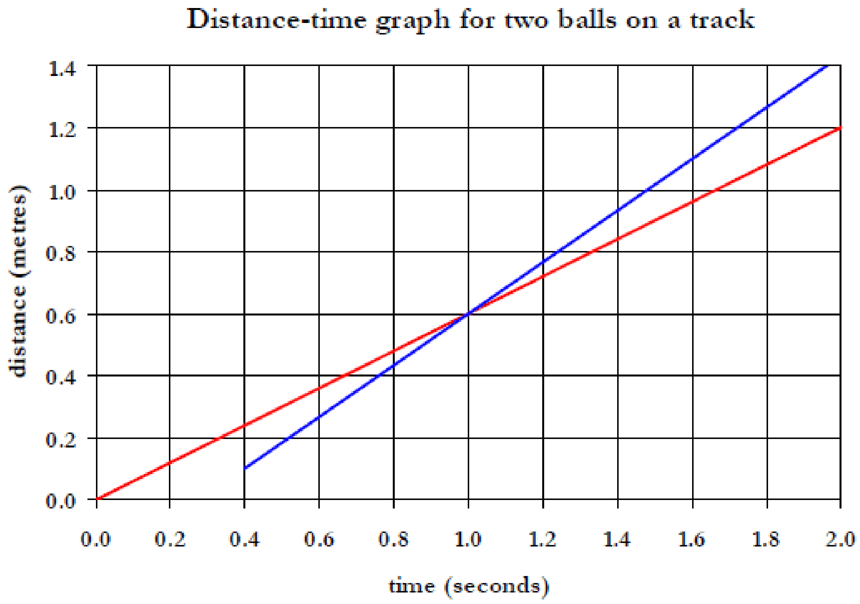


Figure 5.3:A graph showing two balls, one starting later, travelling faster, and overtaking the other.

A similar approach is taken in the pendulum lab. We ask students to re-create the motion of the pendulum bob, which is similar to but clearly different from the motion students have qualitatively investigated before for a pendulum bob released from a small angle. Aided by questions about the motion of the pendulum during intervals A to E. Figure 5.4a features the change of angle both positive and negative as the pendulum is physically moved left and right.

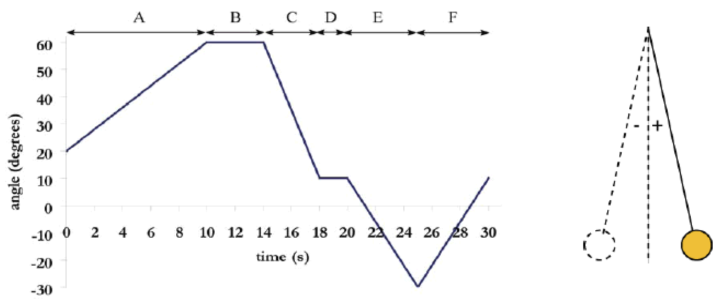


Figure 5.4:a) An angle versus time graph; b) the bob to be moved left and right.

Having completed this exercise, we then require students to consider which graph (A, B or C) in Figure 5.5 appropriately represents the motion of the pendulum as it freely moves side to side.

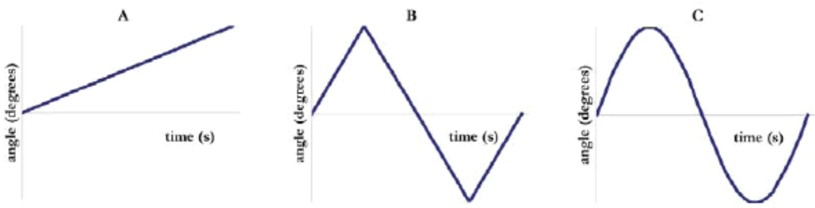


Figure 5.5: Three possible graphs A-C that represent the motion of a pendulum swinging freely.

Finally, in the non uniform lab, we ask the students to pick out the appropriate graph for each of the five segments as the ball rolls along the track as shown in Figure 5.6. The task requires the students to break down the motion of the ball into five distinct segments and draw to pick an appropriate graph for them. Then the student represents the total motion of the graph on a blank graph. We found it necessary to put in the five different segments to select from. In previous versions of the lab, which only had students drawing the motion of the ball on the blank graph, students found this exercise so difficult that little or no learning took place.

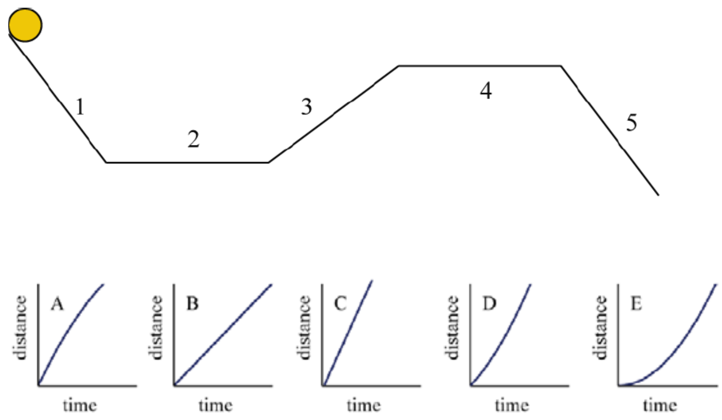


Figure 5.6: Top: a hypothetical ball and track broken into five segments; bottom: the five graphs that students are asked to match to each segment.

5.3.2 Relating rate of change to the shape of a line using interval reasoning

Early versions of the labs showed that students find it difficult to choose an appropriately shaped trend line. In one early version of the pendulum lab, students were given the series of graphs shown in Figure 5.7. Students were asked: *"Which line best represents how the period changes as the length is changed? Explain how your data […] helped you determine your answer"*. This question was unexpectedly troublesome for a lot of our students, with a lot of tutor support needed for many students to successfully attempt to answer it.

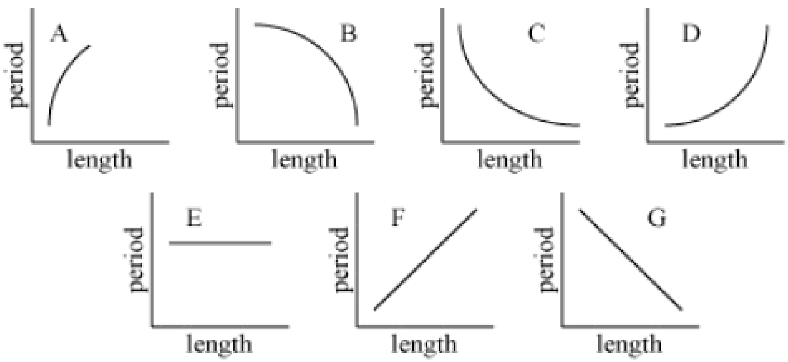


Figure 5.7: Seven possible lines to show how the period of the pendulum changes with length.

The students’ difficulties centred around the inability to, or lack of experience with, considering the line as a series of interval changes. Details of these difficulties are given in Chapter 4. The problems encountered frustrated students and tutors alike.

We set about adding questions which centred on intervals to offer support to the students when choosing of trend line. In the revised version of the pendulum lab, students first manipulate their data to complete the table of Figure 5.8a. Consideration is then given to four intervals in each of the four different graphs A-D shown in Figure 5.8b.

l_1 (cm) to l_2 (cm)	Δl (cm)	ΔT (___)
15 to 25	10	
25 to 35		

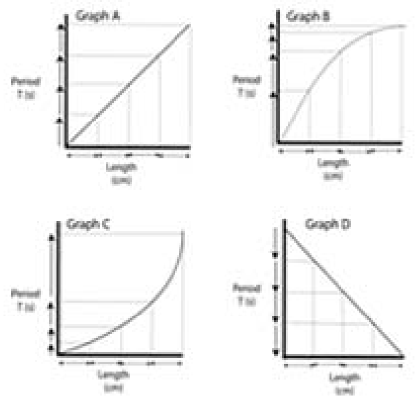


Figure 5.8 a) Table showing how the change in length effects a change in period, b) Four possible graphs that represent the data in the table.

Each line is broken into equal length intervals indicated with arrows on the horizontal axis. The corresponding change in period is shown on the vertical axis. In all cases students are to consider how the “length of the period arrows” change in direction and size with equal changes in the length intervals. Comparisons can then be made with the changes in the period for the equal changes of length in the table. The modifications to the lab have decreased the amount of guidance needed from tutors. More importantly, they have allowed many students to figure out the answer for themselves based on interval reasoning.

A similar approach was taken in the uniform motion experiment. To put these findings on firmer ground, students are shown the similar distance-time graphs of

Figure 5.9a. Changes in distance are shown for equal time intervals. Students are asked initially which of these graphs represent uniform motion. Then students have to compare the motion represented by each of the lines. This approach confronted students with how the straight line graphs in all four cases represented uniform motion in different ways. The exercise is designed to illustrate how different graphs with straight lines all represent uniform motion and interval reasoning is used to come to this conclusion. (In Chapter 6 we describe how questions based on the same setup and graphs are used to help us compare methods of quantifying speed.)

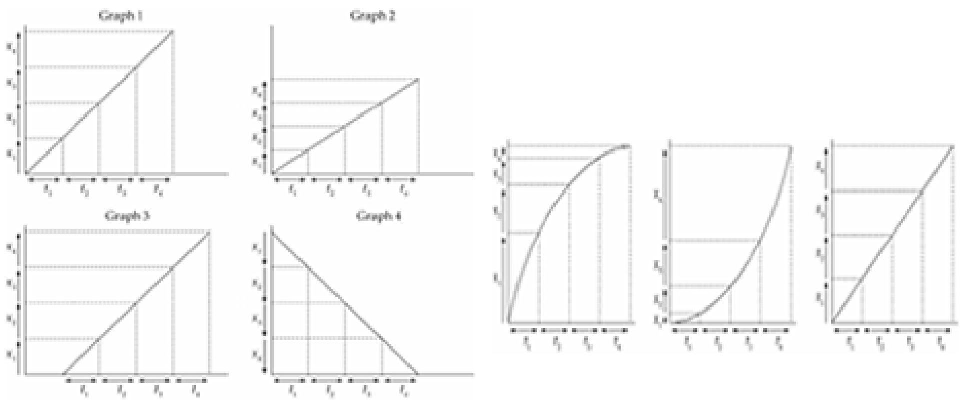


Figure 5.9:a)Four different trend lines that all represent uniform motion; b) three differently shaped trend lines that could represent the motion of a ball slowing down.

<u>*5.3.3 Tackling time dependencies of slope. Slope and height of a graph.*</u>

In Chapter 4 we found that there is a strong association with slope and time, rather than slope and rate. Also, we found that the height of the line or length of the line seemed to be commonly mixed with the slope.

To help develop an understanding of what the slope of a straight line represents, it was crucial that we confronted students with their previous misconceptions. In the course of the pendulum lab, some exercises were set for the students to help them distinguish between the slope and other features of the line.

Figure 5.10 is shown to students in the spring balance lab. They are asked to compare the slopes and heights of the two hills. Hill A has the smallest height but the same slope as B; B has the same height as C a smaller slope. Students don't find this question hard at all, but it appears to be a useful step as it helps to prepare them for what is to come.

Two questions are asked shown also shown along with a student's answer in Figure 5.10: *"Rank the three hills from steepest to gentlest slope"* and *"In your own words, explain the difference between the height and the slope of a hill"*. Each question requires students to consider the difference between the height of the hills and the slopes of the hills.

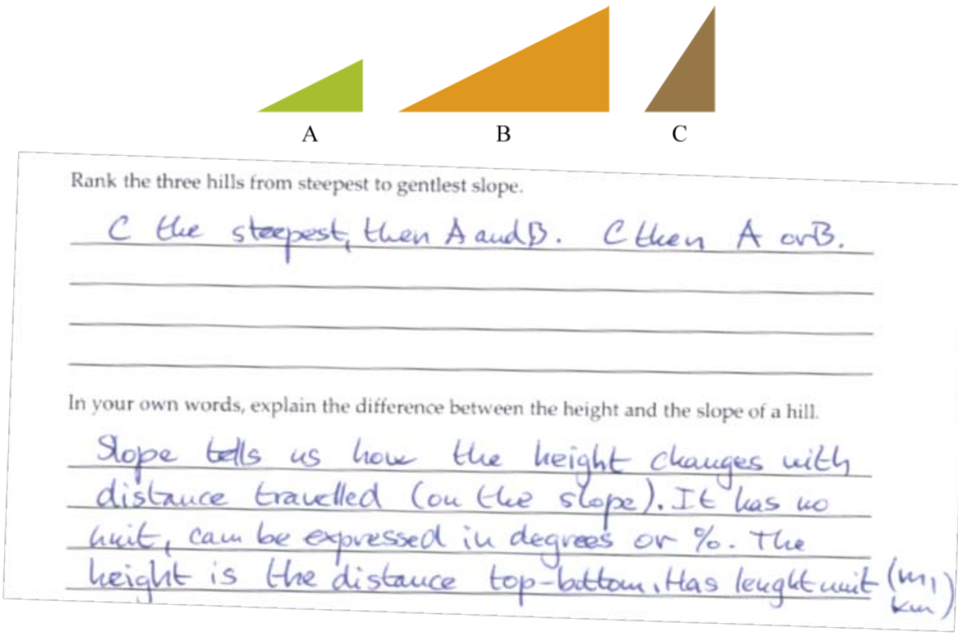


Figure 5.10: Three slopes A, B and C. The slopes of A and B are identical, and the height of B and C are identical. Two related questions are shown under the three hills.

In the same spring balance lab, another question is set after the student's graph for extension versus mass has been drawn. The exercise is a thought experiment and often needs some tutor attention and guidance. In the question, the supposition is put forward that, rather than adding 20 g slotted masses to the mass hanger, sand is added slowly (as shown in Figure 5.11). We then ask students to sketch on their extension versus mass graph (which has data points only on it at this stage, not yet a trend line) and ask how the extension changes as sand is added.

In a pretest, only 55% of students (*N*~500) pick the correct trend line; 25% pick a line with shallower slope, and 15% pick a steeper line. This supports the findings of Chapter 3, which shows that many students associate the slope with the time taken by an event rather than the rate of change of one quantity with respect to another.

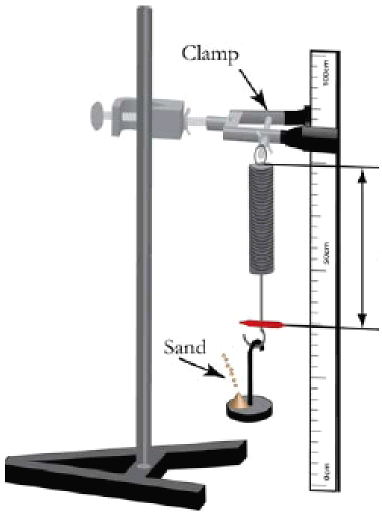


Figure 5.11: Sand is added slowly to a mass hanger instead of 20gram mass disks.

The reasoning that the tutors use to help with this question often involves asking:*"What would the extension be if a total of 20 grams of sand were added?"* or *"What if you started adding discs of only 5 g or 10 g?"* The reasoning can be then taken to sand on a smaller scale, to bridge the gap from discrete data to continuous data. The student then is confronted with the inconsistency that was present in their reasoning, and their misconception at least in this instance is satisfactorily resolved.

5.4 Intercept and the curriculum

Chapter 4 showed that many students did not understand how to link the intercept of a line and the event that was being graphed. Often students tended towards drawing the line through the origin. In our first lab, a strong emphasis is put on interpreting how the intercept of the graph relates to the physical event. Students also use the intercept to figure out the mass of the mass hanger.

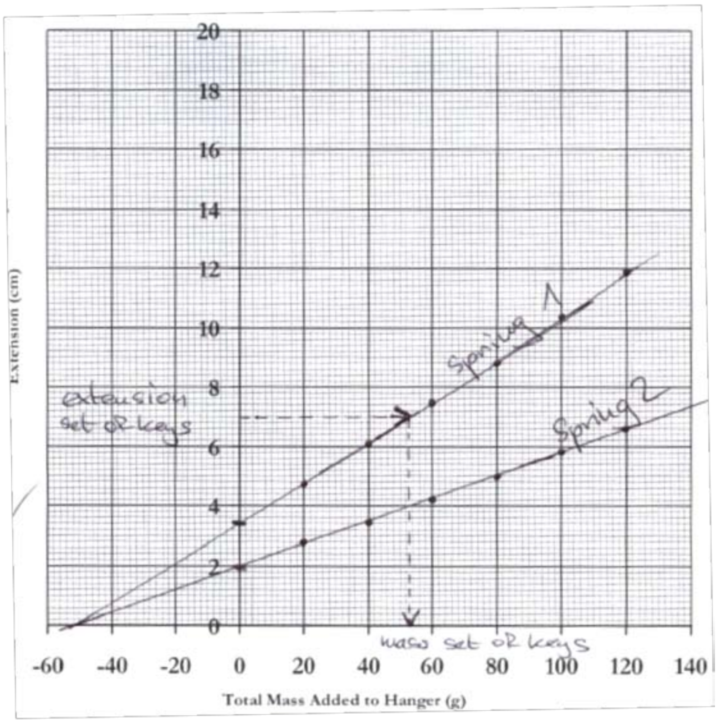


Figure 5.12: A scan of a student's answer showing the graph representing the extension of two springs in the spring balance lab.

In the spring balance lab, we design the experiment such that the extension versus mass graphs for both springs does not pass through the origin. This is done by making the first reading the extension due to the mass hanger and the 20 g disk. The students only graph mark the mass of the 20 g disks on the horizontal axis initially resulting in the graph of Figure 5.12.

The first link between the vertical intercept and the extension due to the mass hanger is made when students compare the value for the vertical intercept with the measured extension of the spring when no 20 g discs were added. This gives students some guidance before being asked explicitly to compare the vertical intercept and its relevance to the experiment.

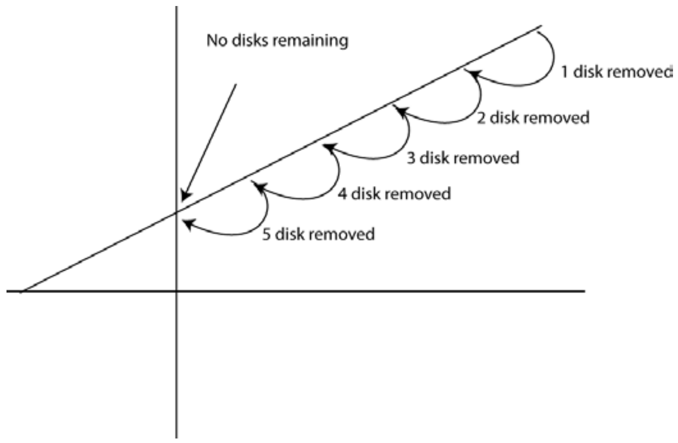


Figure 5.13: A graph showing some of the reasoning used with students to show the significance of the y intercept to the experiment. The arrows show the mass disks as they are removed from the graph.

Then students are asked what happens if the mass hanger is removed from the spring balance. By following the pattern shown in Figure 5.13, they observe that they must go "back" another 50 g on the horizontal axis for there to be no extension to the spring. This helps students make the step that the horizontal intercept at minus 50 g represents the plus 50 g mass of the mass hanger. We have found that the approaches we take to developing graphing have been useful in engaging students with the labs, by addressing known difficulties addressed in chapters 3 and 4. The effectiveness of these approaches is presented in chapter 7.

Chapter 6: Intervals and rate of change

6.1 Overview of the labs

We have found through our own pre-tests that students cannot (or do not) apply interval reasoning to solve questions in a number of basic kinematics questions. While there are no mandatory kinematics experiments at Junior Certificate level, distance, time, average speed and average acceleration are addressed.

An example of the level of kinematics that students experience at second level is the examination question featuring a graph of a stone that is falling to the ground over five seconds (Figure 6.1). The graph plots the velocity of the stone and goes through the origin.

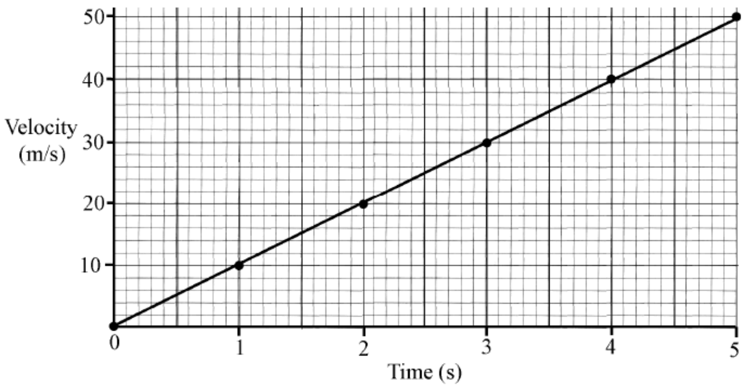


Figure 6.1 A graph representing a stone falling to Earth
(Junior Certificate examination problem).

In their examination, students are asked:

1. Define velocity.
2. Use data from the graph to estimate the acceleration of the stone as it fell. Give the units of acceleration with your answer.
3. Name the force that caused the stone to fall.
4. The stone had a mass of 2 kg. What was the weight of the stone on earth? Give the unit.

It is clear that the sophistication of the ideas and concepts behind the motion are not examined by this question. Instead, reliance on rote learning and algorithmic problem solving would suffice to obtain a good score on this question. Moreover, we note that all kinematics graphs in textbooks and examinations at Junior Certificate level go through the origin.

Although almost all of our students have been exposed to Junior Certificate physics at least, we find that as little as 20% of students coming into the labs can calculate the speed of an object from a simple straight line distance-time graph which does not go through the origin. Alarmingly, the numbers are independent of whether the students have taken Leaving Certificate physics or not.

Beichner [1] and Woolnough [2] found similar results for students at college level with students applying incorrect or incomplete formulae to questions which could be solved easily without one.

6.2 Testing students' ability to use intervals

Some of the questions that we use to pre-test and post-test students feature graphing questions like simple straight line graphs of distance versus time. In order to investigate if students' reasoning is transferable to other situations, we also pre-test and post-test with an equivalent rate of change question in a context that is not familiar to students, and in a context-free setting. These questions feature a graph in which the water level in a swimming pool is changing at a constant rate, and a numerical graph of dimensionless *y* versus *x* values.

To broaden our understanding of interval reasoning, we also ask a question that requires students to compare the relative motion of two objects in motion. This kind of question adds a number of complications to the analysis. Additional considerations include the relative positions of the cars, e.g. one car is passing another, or the two cars are side by side. The question is represented using a strobe diagram showing two cars at positions at four different intervals (Figure 6.2).

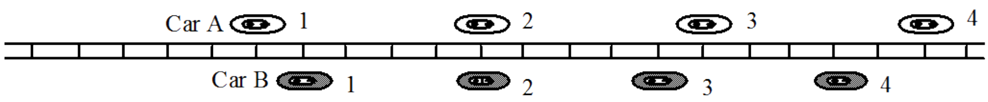


Figure 6.2: Car A and Car B are both traveling with constant speed down a track. The position of each car is shown at four different instances.[3]

All questions are designed so that there is a different level of familiarity with the context of each question. However, universal to all of our questions is that interval reasoning can be used as complete reasoning to answer all questions successfully, either quantitatively or qualitatively, if students choose to answer the question this way.

6.3 Difficulties with interval reasoning in different contexts

6.3.1 Overview

Depending on the context of question, students have different success rates with our pre and post test questions. For instance in a question that requires the student to obtain the quantitative speed from a distance-time graph, there is a tendency to apply a familiar formula like $s=d/t$. Unfortunately, throughout their secondary school science this equation has been translated into words as *"speed equals distance over time"* rather than *"average speed is the change in distance over the change in time"*.

If water level is graphed against time, rather than distance, intervals are more likely to be used in the rationale for answering the question because most students have not seen this problem before. That said many, but by no means all students spontaneously associate the rate at which the water level changes with a "speed". Some even explicitly use the same formula. Finally, even in the context-free setting, only 45% of students calculate the slope as $\Delta y/\Delta x$.

6.3.2 Difficulty with formulae

Misuse of a formula is the most common difficulty for students who try to find the speed at a point from a distance-time graph. In one question, we ask students to determine the speed of a ball at t = 6 s from the graph of Figure 6.3. Complete and correct reasoning involves the recognition that the speed of the motion is constant, and that therefore the speed at any instant is the same as the average speed.

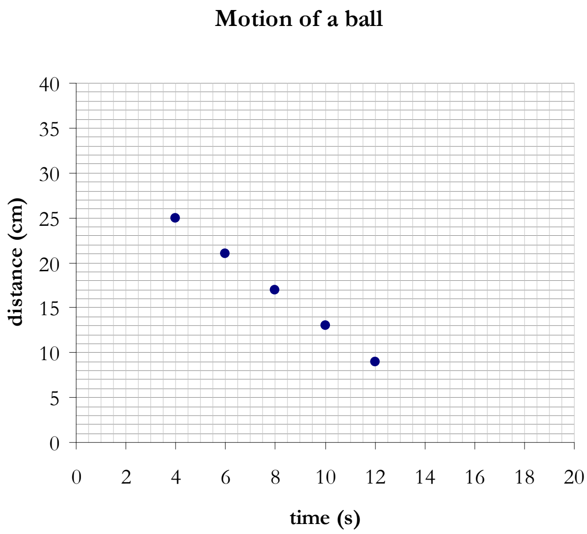


Figure 6.3: The motion of a ball moving with constant speed towards a point *P*, recorded in a distance-time graph.

A typical example of a student incorrectly applying a formula is shown in Figure 6.4. In this case the student incorrectly calculates the speed of an object from a straight distance-time graph as the object does not start at zero. The triangle shown in Figure 6.4 is often taught as an aid to help students memorise the formula.

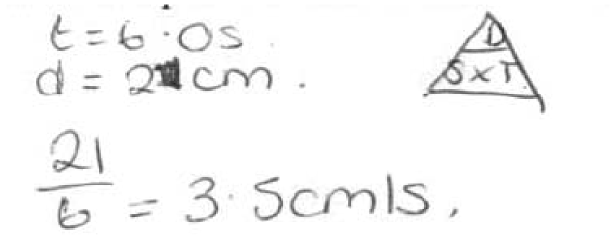


Figure 6.4: A student answer in which a formula is used incorrectly. The formula only features a ratio of the position and the time, not the change in distance divided by the time taken.

When asked to find how quickly the water level changes at a particular time from a straight water level versus time graph that does not pass through the origin, a different student applies a similar equation (see Figure 6.5). While the context was unfamiliar to the student, a formula was used which consisted of a simple ratio nearly identical to the formula that the student used in Figure 6.4.

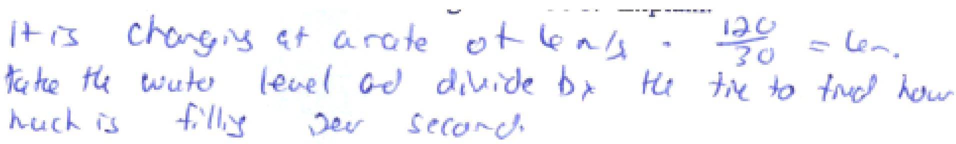


Figure 6.5: Using an incorrect formula to determine the rate of change of water level in a swimming pool.

In Figures 6.4 and 6.5 both students used formulae that used the position of the water or the object and a single time, and not the interval changes they were undergoing. A correct interpretation of the formula would have yielded the correct result.

Another example in which the formula not only gives a quantitative figure for the motion of the car, but also determines the qualitative description of the motion, is shown in Figure 6.6 below. The student uses the formula incorrectly to calculate the speed of the ball at three different times coming to the conclusion that the ball is moving with a decreasing speed.

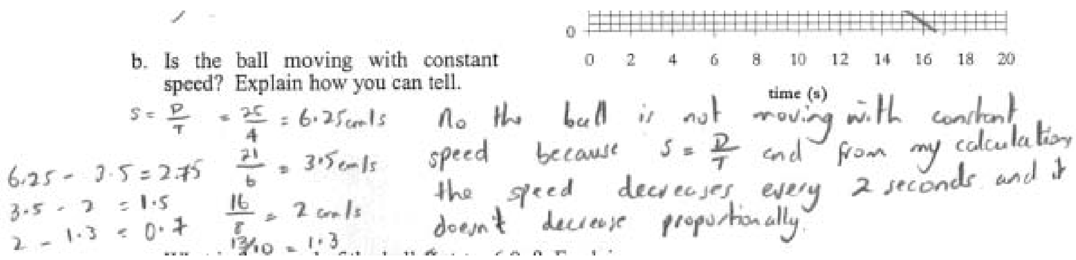


Figure 6.6:A formula is used to incorrectly find different speeds at different points on a straight trend-line on a distance-time graph. The student comes to the conclusion that the motion of the object is non-uniform.

6.3.3 Other distractions from interval reasoning

Another difficulty that students often have is applying incorrect reasoning to determine the relative motion of two objects. An inability to apply (or lack of experience with) interval reasoning results in the student using the relative distance between two cars as the relative speed. For one car passing out another a student gives the answer shown in Figure 6.7.

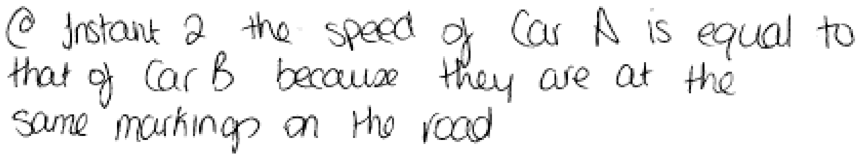

@ Instant 2 the speed of Car A is equal to that of Car B because they are at the same markings on the road

Figure 6.7: Neither using a formula or interval reasoning, this student uses the relative positions of the cars to come to the conclusion that they are traveling with equal speed.

In order to correctly answer the question, the student must recognise that both cars are travelling with constant speed, and that car A is moving more spaces in each time interval than car B. (Alternatively, they could see that the separation of cars changes by a constant amount over equal time intervals.) Thus, at all instances, including instance 2, car A is travelling with a greater speed than car B. In total 50% of students identify the cars as travelling with equal speeds when side by side in pre-tests.

6.4 The labs

<u>6.4.1 Investigating motion using interval reasoning</u>

The previous sections have shown that students often use formulae, be it correctly or incorrectly, to solving kinematics problems. Use of formulae often bypasses a more fundamental understanding of motion.

In both the uniform and non-uniform motion labs, in order to strengthen students' model and their ability to apply it, we have adopted an approach that focuses on interval reasoning. We use three simple investigations to allow students to become familiar with and understand a simple approach of breaking down the motion of objects into relevant intervals. Because graphing was integrated into the labs, there is some duplication of exercises discussed in Chapter 5, but this time shown in a more general context.

6.4.2 The uniform motion lab: setting up uniform motion

Interval issues addressed:
▪ Using interval reasoning to classify the type of motion of an object ▪ Distinguishing between the type of motion and the magnitude of the speed ▪ Use interval reasoning to investigate uniform motion of an object in two dimensions ▪ Investigating the use of an appropriate formula to describe uniform motion

At the start of the lab, students use a simple model for testing whether an object is travelling with uniform motion. They are asked to predict if a ball rolling down a flat track would roll with constant speed. Students verify their prediction quantitatively by measuring the time it takes to traverse the first and second halves of the track, as shown in Figure 6.8.

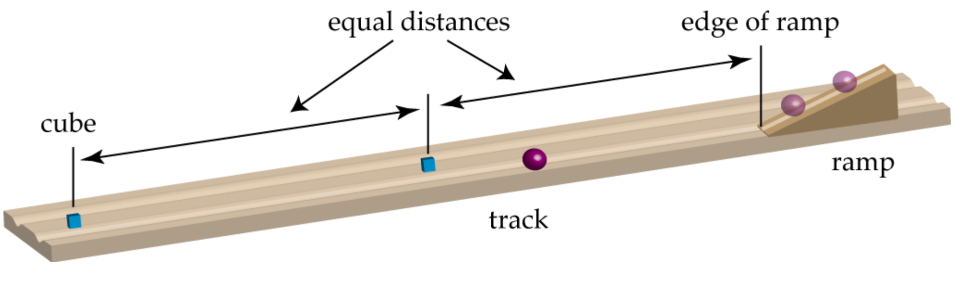


Figure 6.8: Setup for uniform motion lab, with a track, ramp and a ball.

A large number of students neglect to consider that the friction of the track is going to slow down the ball when making predictions such as "*We predict that the speed will remain constant once it reaches the track*". As a result, many students prove their hypothesis incorrect. However, even after they have obtained motion that is clearly non-uniform, students often ask "if it is ok to ignore friction". This is disconcerting at first, but understandable in the light of their experience in school discussed in Chapter 1.

The students then adjust the slope of the track until the friction is compensated for and uniform motion is indeed obtained. They are then asked to test whether the position that the ball is released on the ramp will affect whether the motion of the ball is uniform or not, as shown in Figure 6.9. (The difference in rolling friction is so small that the motion remains uniform.) To answer this question successfully, the student must differentiate between the magnitude of the motion and the type of motion (i.e. uniform or non-uniform)

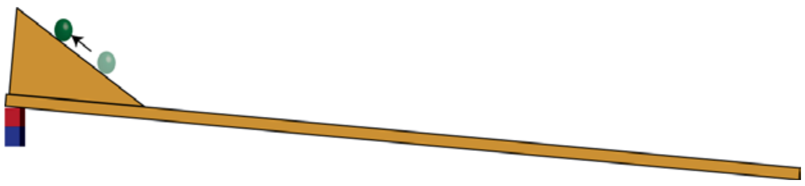


Figure 6.9: The side view of a ball on a ramp. The ramp sits on the track. The ball has been moved further up the ramp. The student investigates whether the motion of the ball on the track now changes from being uniform.

6.4.3 The uniform motion lab: obtaining values for speed

Four measurements are made for position as the balls rolls down a track with uniform motion. As shown in Figure 6.10, two separate variables are used to record the position of the ball: x_i is the position of cube i measured from the bottom of the ramp, and d_i is measured from the end of the track.

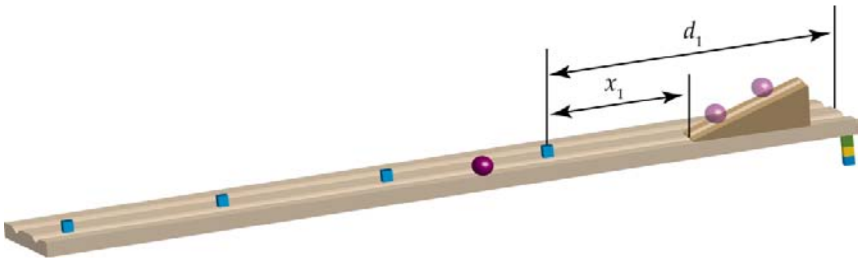


Figure 6.10: The track is divided into four equal lengths using cubes. The motion of the ball is considered from the instant the ball reaches the bottom of the ramp. Two graphs are drawn: one with x versus time and the other with d versus time for the same motion.

The measurements are recorded in a table that is provided to the students (Figure 6.11). Thus, two sets of data are obtained for the same uniform motion of the ball. The students are then asked to construct two different graphs as shown in Figure 6.12.

Table 2.1: ____

cube	x(cm)	d (cm)	t_1 (s)	t_2 (s)		t_{av} (s)
1						
2						
3						
4						

Figure 6.11: The table provided to students to fill in. t_1 stands for the first measurement of time.

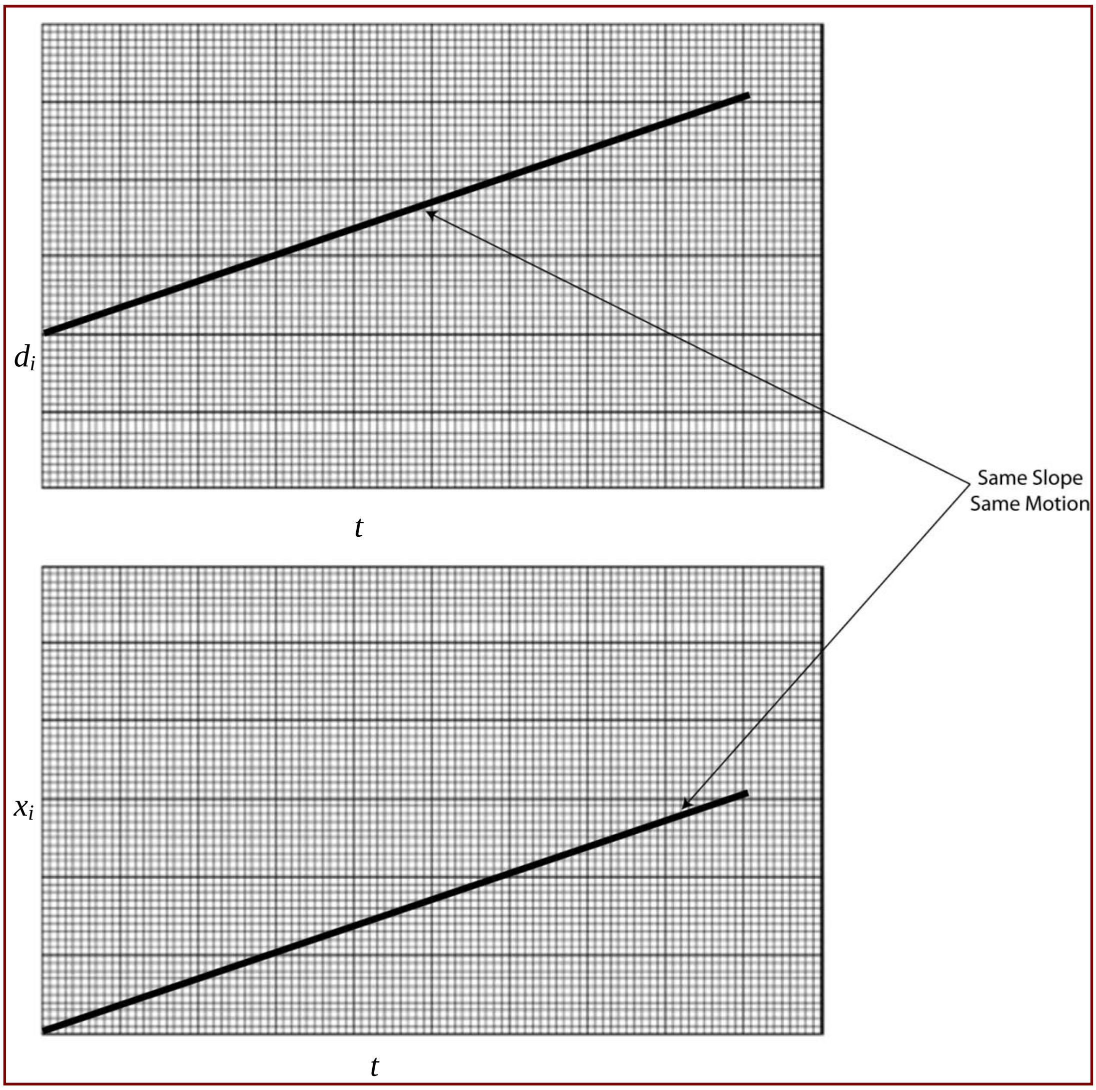


Figure 6.12: The motion of Figure 6.11 represented by distance-time graphs.

Written descriptions of approaches to measure speed are given by hypothetical Students 1 and 2 in Figure 6.13. Students are explicitly asked to write out the two calculations proposed by Student 1 using some or all of the variables x_A, x_B, t_A, and t_B, and then using some or all of the variables d_A, d_B, t_A, and t_B. They then carry out the calculations proposed by Student 2.

Table 2.2: ____

Quantity	*value*
x_A	
x_B	
t_A	
t_B	
d_A	
d_B	

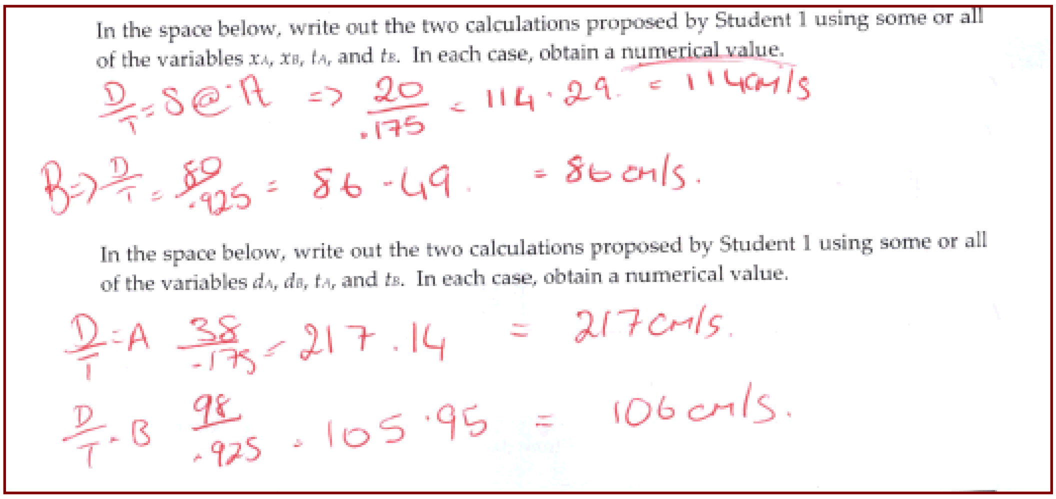
Student 1: *"The speed of the ball is constant. I can calculate this speed either by dividing distance over time at point A, or at point B – it does not matter."*

Student 2: *"The speed between A and B is given by the distance travelled between points A and B, divided by the time taken to travel from A to B."*

Student 3: *"I think you're both right – you're saying the same thing in a different way."*

Figure 6.13: Hypothetical student discussion of how to obtain speed from the distance-time graphs of Figure 6.12.

In doing so, they are confronted with the fact that Student 1's formula gives different values for the speed at different times (Figure 6.14), whereas Student 2's formula does give a constant value (Figure 6.15). In this way we hope the students discover the inadequacy of the formula that many of them use to quantify the speed of an object from a graph.


In the space below, write out the two calculations proposed by Student 1 using some or all of the variables x_A, x_B, t_A, and t_B. In each case, obtain a numerical value.

$\frac{D}{T}$ = S@ A => $\frac{20}{.175}$ = 114.29 = 114cm/s

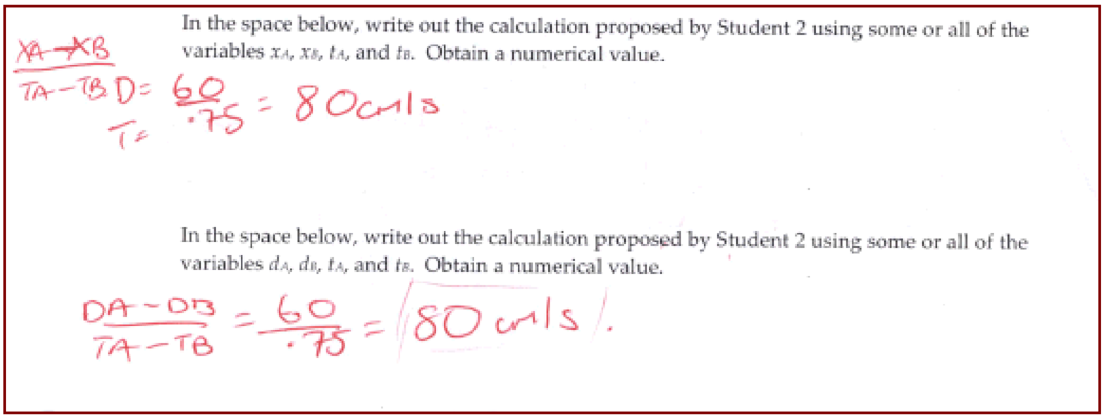
B=> $\frac{D}{T}$ = $\frac{80}{.925}$ = 86.49 = 86 cm/s.

In the space below, write out the two calculations proposed by Student 1 using some or all of the variables d_A, d_B, t_A, and t_B. In each case, obtain a numerical value.

$\frac{D}{T}$: A $\frac{38}{.175}$ = 217.14 = 217cm/s.

$\frac{D}{T}$: B $\frac{98}{.925}$ = 105.95 = 106 cm/s.

Figure 6.14: A scanned student's answer in which they use the approach of Student 1 (see Figure 6.13.


In the space below, write out the calculation proposed by Student 2 using some or all of the variables x_A, x_B, t_A, and t_B. Obtain a numerical value.

$\frac{\cancel{XA - XB}}{TA - TB}$ D = $\frac{60}{.75}$ = 80cm/s

T =

In the space below, write out the calculation proposed by Student 2 using some or all of the variables d_A, d_B, t_A, and t_B. Obtain a numerical value.

$\frac{DA - DB}{TA - TB}$ = $\frac{60}{.75}$ = 80 cm/s.

Figure 6.15: A scanned student's answer in which they use the approach of Student 2 (see Figure 6.13.

Students are asked to describe explicitly why the values for the speed should be the same, and are then asked to choose which of the students appears to have used a correct method. Some students still hold on to the belief that *"speed is distance over time"* ,because *"that is the correct formula"*. However, most now articulate that Student 2 uses a correct method. It is common in the lab that students' are surprised or disconcerted when they find that student's 1 method of using the simple ratio does not give the same results for the two different graphs.

We have found it useful to insert two tutorial-type exercises at this point. First, students have to discuss which of the graphs of Figure 6.16 represent uniform motion. Then they are to determine the speed of the ball undergoing the motion represented by the graph of Figure 6.17. In this question, they are again aided by a hypothetical student conversation.

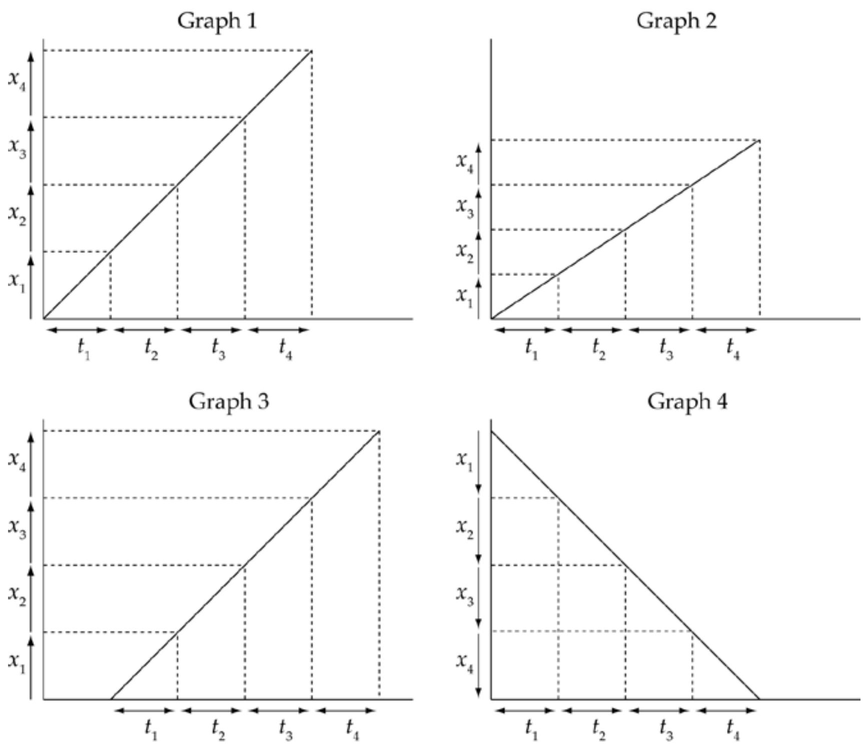
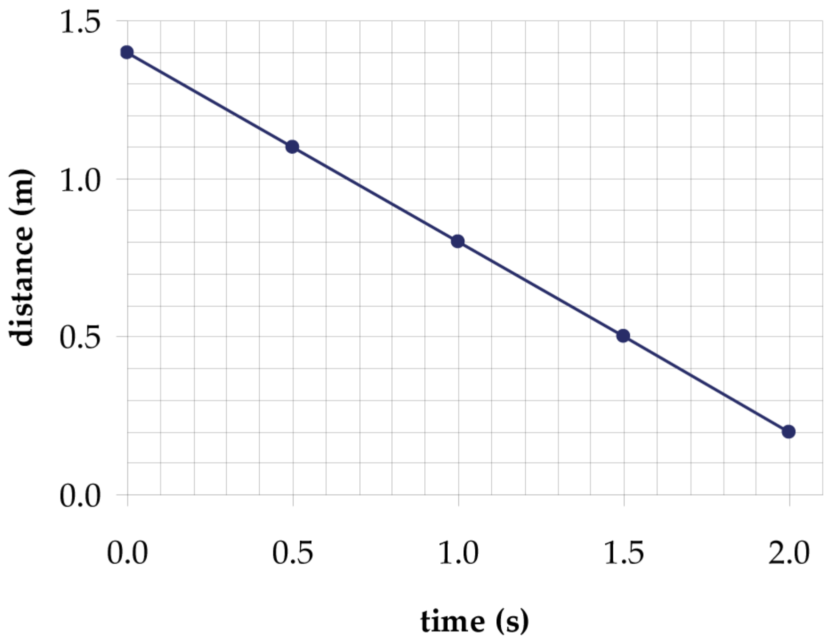


Figure 6.16: Four graphs representing uniform motion.



Student 1: *"I know that the speed of the ball was constant, because we got a straight line graph."*

Student 2: *"I agree. You can see that the ball gets closer to the edge by 30 cm every 0.5 seconds, so the speed of the ball is 0.6 m/s."*

Student 3: *"I think the ball is slowing down. Speed is distance over time. After half a second, the ball was 1.1 m from the edge, so the speed was 2.2 m/s. Then after one second, the ball was 80 cm from the edge, so the speed was 0.8 m/s."*

Figure 6.17: Hypothetical student discussion of how to obtain speed from the distance-time graph shown. Students are to discuss each statement.

6.4.4 The uniform motion lab: two-dimensional motion

In Section 3 of the uniform motion lab, students are given a setup on which a ball rolls down track that has been laid diagonally across a piece of paper (Figure 6.18). Students are asked to consider the following hypothesis: "While the ball is rolling with constant speed along the track, it will take equal amounts of time to traverse each segment on the paper".

They are given a sheet of marked paper and told to set up their experiment. While the experiment is very close to the experiment they performed earlier, this is the first instance in which students are given the freedom to design their experiment.

Drawing a table and comparing the time it takes for the ball to travel each segment allows for the hypothesis to be tested. Again an understanding of intervals and how the times will compare for each traversed segment are used to strengthen this approach to analysing simple motion.

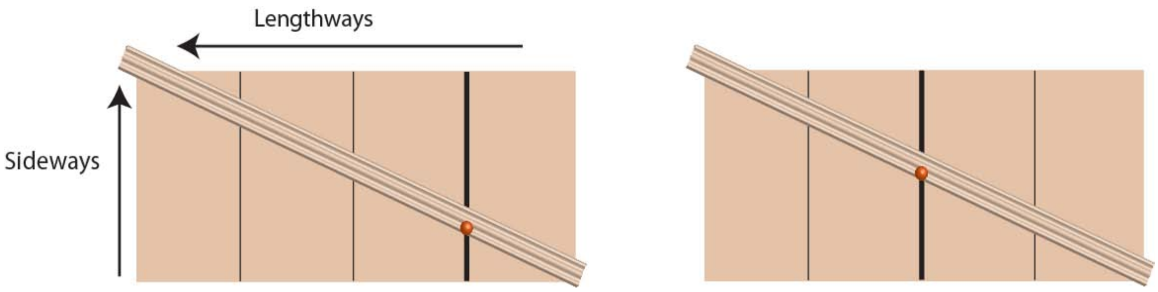


Figure 6.18: A track is laid out across a piece of paper. The paper is marked so that the lengthways direction is divided into four equal lengths.

<u>6.4.5 The uniform motion lab: from graph to lab</u>

In the final section of the lab, students further strengthen their graph reading skills by recreating the motion of two balls given in the graph of Figure 6.19. They are guided to finding the right set up by being asked whether the balls travel at constant speed, at the same speeds, start at the same time; which one finishes first, and whether the balls overtake each other.

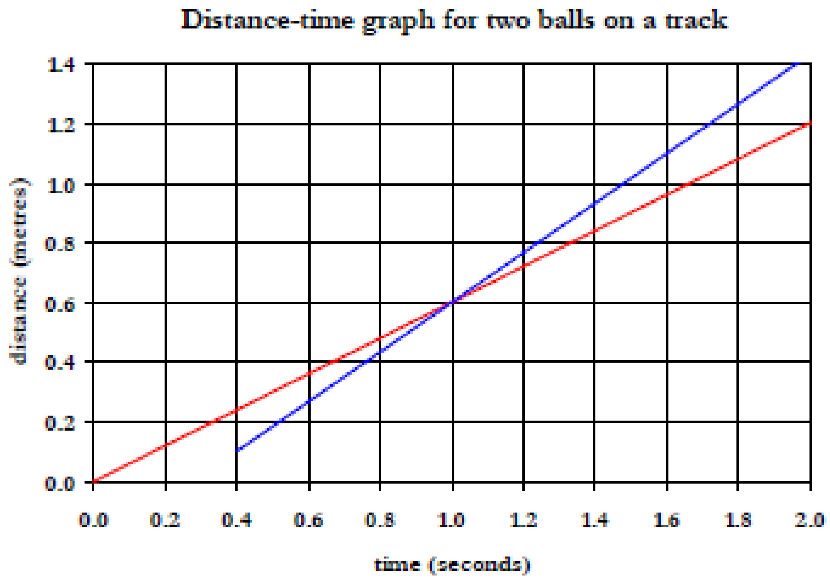


Figure 6.19: Distance-time graph for two balls rolling along two tracks.

6.4.6 Non-uniform motion labs: one- and two-dimensional motion

Interval issues addressed:
▪ Using interval reasoning, quantitatively verify non-uniform motion ▪ Applying interval reasoning to non-uniform motion in two directions

In the non-uniform motion lab, the equipment is manipulated by the students so that the ball slows down rolling up a ramp in a one-dimensional motion. The motion takes place over about two seconds, which is enough to show quantitatively that the ball slows down without a need to use formal error analysis. They apply interval reasoning to select the right graph to represent the motion, as discussed in Chapter 5.

In the second part of the lab, students first set up a board so that the motion along the long side of the board is uniform by putting *N* 1 cm tall cubes under two corners of the board, as shown in Figure 6.20a. They then set up the board so that the motion along the short side is accelerated, by putting 3 cubes under two of the corners, as shown in Figure 6.20b. Finally, they arrange the board so that there are *N*, *N*+3, three, and zero cubes under each of the four corners in such a way that the two previous setups are superposed, as shown in Figure 6.20c.

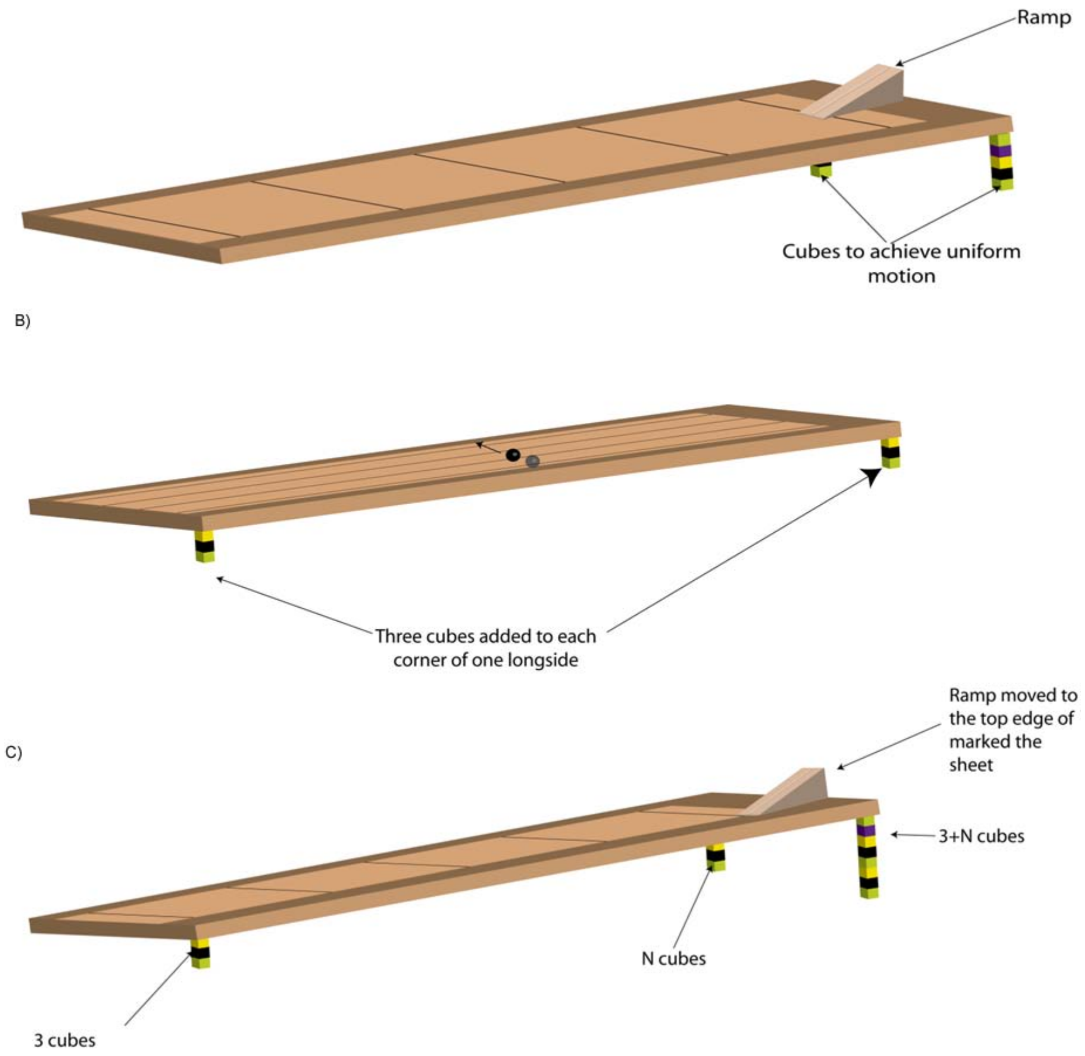


Figure 6.20: a)Cubes added to achieve uniform motion in lengthways direction. b) Cubes added to achieve uniform motion in lengthways direction. c) Superposition of the setups of Figures a and b.

In the "combined" set-up of Figure 6.20c, students observe that the motion lengthways is still uniform, and the motion sideways is still accelerated. As a result, the ball takes a curved path. This experiment aims to get students to see for themselves that the motions in two dimensions are independent of each other.

6.4.7 Non-uniform motion labs: from track to graph

The last section of the non-uniform motion lab again gives students the opportunity to relate laboratory set-up to graph. Students are told that a ball is released from rest on the hypothetical track of Figure 6.21. They are asked to consider the following:

- On what, if any, of the segments does the ball travel with constant speed? If there is more than one such segment, how do the speeds on these segments compare?
- On what, if any, of the segments does the ball travel with increasing speed? If there is more than one such segment, how do the initial speeds and the accelerations on these segments compare?
- On what, if any, of the segments does the ball travel with decreasing speed? If there is more than one such segment, how do the initial speeds and the accelerations on these segments compare?

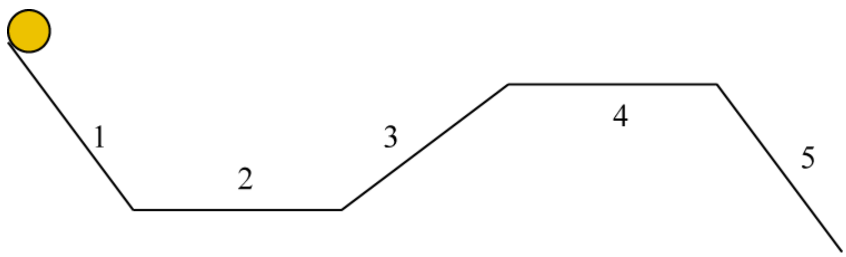


Figure 6.21: Ball on hypothetical track.

Based on their answers, students are then to identify which of the five distance-time graphs of Figure 6.22 correspond to the motion of the five parts of the track.

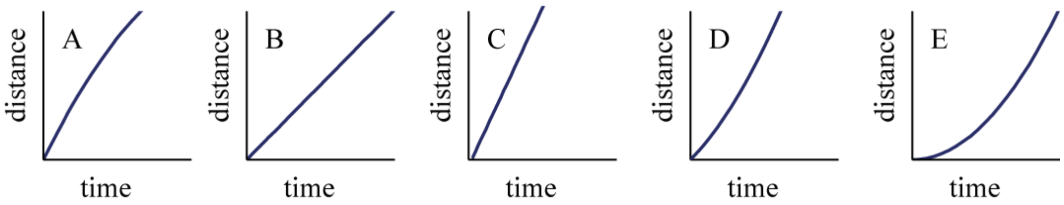


Figure 6.22: Five distance-time graphs to be matched up with the motion of Figure 6.21.

Finally, the students patch the five segments together in one distance-time graph.

Throughout the labs we have tried to engage students by asking questions that force them to use interval reasoning and thereby tying the shape of the graph to the experiment. Also we use questions that are specifically aimed to address difficulties that we found in chapter 4 and were already known from chapter 3.

References

1. Woolnough J.,(2000), *How students Learn to Apply Their Mathematical Knowledge to Interpret Graphs in Physics?* Research in Science Education, (30) 3, 259-267

2. Beichner R. J. (1994), *Testing Student Interpretation of Kinematics Graphs*, American Journal of Physics (62) 8, 750-762

3. Rosenquist M. L. and McDermott L. (1987), *A conceptual approach to teaching kinematics.* American Journal of Physics (55) 5, 407-415.

Chapter 7: Results

7.1 Linear qualitative water level graphs

7.1.1 Overview

As discussed in Chapter 4, a set of qualitative assessments was designed and delivered to students in which they had to draw appropriate trend lines to represent water level changing against time as a beaker is filled. The trend lines are drawn on blank water level versus time graphs (provided in the question) with labeled axes that have no numbers. All questions were set around water being poured in at a constant rate into two beakers. The beakers either differed in shape or height or the objects that they contained. The difficulties that we have found most students commonly have are laid out in Chapter 4. The approaches that are taken in the labs to specifically improve students' general ability in this area are set out in Chapter 5.

In this section the pre-test questions set before students enter the labs and post-test results for linear graph questions asked after completing the labs are compared. The pre- and post-test questions are similar or even identical, but the same question is never asked pre and post of the same group of students.

We should point out from the start that no instruction dealt with qualitative water level questions at all. Hence any improvement is purely due to transfer from the taught materials. Hence, while improved performance can be taken as a strong indicator of successful teaching and learning, failure to achieve this is not necessarily an indication that the teaching sequence did not work.

All questions were given in the academic years 2008/2009 or 2009/2010. The labs are broken into four groups, A, B, C and D. Each group are made up of students taking different courses in the Faculty of Science and Health, and are all taking identical courses as part of their first year in college. With all students answering the same pre-test question in section 7.1.2, the post-test question was designed to be a variation of the pre-test question and of equivalent difficulty. For all other water level questions, the pre-test question is given to two groups and the same question is given as a post-test to the other two groups. This allows us to verify that the pre and post-test questions are indeed equivalent.

7.1.2 Two beakers: same height, same rate, different width.

In the 2008-2009 academic years we set a pre-test question in which water is being poured into beakers A and beakers D (Figure 7.1). The pre-test was given to all students. Water is poured into each beaker at the same constant rate. The two beakers have the same height, so the two lines should end at the same level on the water level axis. As beaker D is wider than A, D fills up more slowly than A, so a correct answer will show D with a less steep slope than A. It also fills up after A, so it should finish further to the right on the time axis.

We also set a post-test question which consisted of a beaker B with the same dimensions as beaker A, but this beaker has a cylinder contained in it. Water again is poured into this beaker at the same constant rate as A. The question requires similar reasoning to the reasoning behind beaker D filling, but this time the water in B rises at a faster constant rate than in A. The line for B should show that both beakers fill to the same level, but B fills up before A.

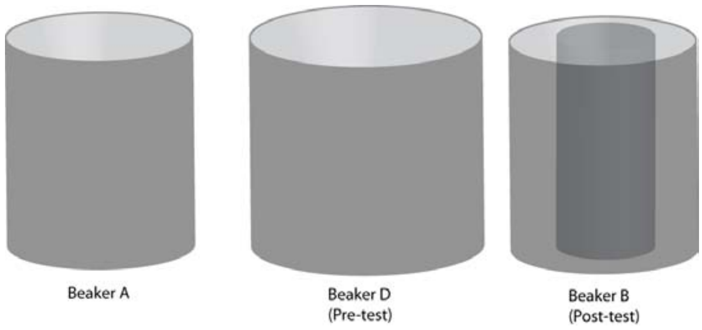


Figure 7.1: Beaker A is empty. Beaker D has the same height but a different width. Beaker B is the same height as beaker A and contains a regular cylinder.

In Table 7.1 the pre and post-test results for the slope of the line, the relative end time and the relative end level of each trend line are given for both the pre and post test questions.

While there was some improvement in the percentage of students that drew the slope correctly, it was notable that the focus in both the pre and the post test was entirely on the slope of the line and the time it took to fill, and not on the finishing level, with far fewer students representing these quantities accurately. The emphasis in answering the question appears to be on representing the differences between how the two beakers fill, while neglecting to represent the similarities (i.e. both finish at the same level).

In the pretest, 60% of students correctly constructed graphs with the slope of A steeper than D. In the post-test, 75% correctly constructed graphs where the slope of B was greater than that of A. Given the similarity in required reasoning, we are taking this to be a small but real improvement. In Table 7.1, we have expressed this improvement as a so-called Hake gain *h*, defined by

$$h = \frac{\text{correct \% post-test} \; - \; \text{correct \% pretest}}{100\% \; - \; \text{correct \% pretest}}$$

In Chapter 4, we saw that some 10% of students drew line D with identical slope to A but longer, to represent the different times taken to fill beaker D. The corresponding answer for the post-test would see line B shorter than, but as steep as, A. This problem seems to have been addressed, as the percentage of students that did this dropped from 10% in the pretest to <2% in the post-test.

The number of students that drew both levels D and A finishing at an appropriate level was low in both the pre and the post with 20% in the pretest and 25% in the post-test. We do not think that all other students necessarily got it wrong; we suspect that many just didn't think of drawing the graphs to the correct end level. This is something that will be investigated in further studies.

Some 75% in the pre-test, and 80% in the post-test, gave correct finishing times. It is unclear whether students deliberately focused on the finishing time. We also must consider that a greater slope would likely result in an earlier finishing time, and a less steep slope would likely result in a later finishing time. This means we cannot definitively say that the higher number of students representing the correct finishing time deliberately focused on doing so.

7.1.3 Two beakers: same rate, same width, different heights.

Figure 7.2 below shows the same setting as the pretest question of Figure 4.3. Beakers A and B are both the same width but beaker B is taller than beaker A. Beaker A fills up at the same rate as beaker B so both lines have the same slope. As beaker A is smaller, it fills up before beaker B and to a lower level than Beaker B. The results for the finishing time, level and position of beaker B are shown in Table 7.2. The question was given as both a pre and a post-test in the 2009-2010 academic years. The pretest was given to groups A and C, while the post-test was given to groups B and D. Next year, the pre and post test questions will be reversed.

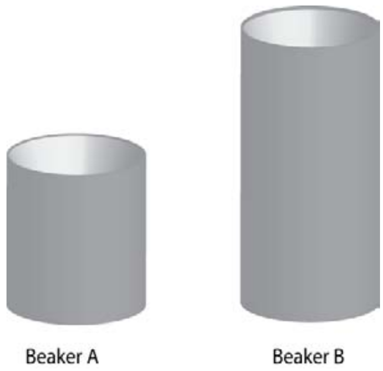


Figure 7.2: Beaker A is the same width as B, but less tall. Both beakers are filled at the same constant flow rate.

It is interesting to note that this question was answered much more successfully than the previous question, both as a pre- and as a post-test. In fact, this question was answered correctly by a higher percentage of students than any other question. It is probably significant that in this question the rates at which the water level changed were equal. This presumably allowed students a greater opportunity to consider other aspects of the graph.

An increase from 70% to 80% was seen from pre to post-test for correctly constructing a straight line graph with identical slopes for A and B. There were similar increases from 90% pre to 95% post for students that represented correct relative end levels, and 85% pre to 95% for students that represented correct relative finishing times for beakers A and B.

7.1.4: Different height, same width, fills at the same time.

We gave a variation on the setup discussed in Section 7.1.3. The beakers are filled at constant but different rates, in such a way that A and B are full at the same time. Students are to infer that to make this possible, the water level in beaker B must rise at a greater rate than beaker A. So line A should a have a smaller slope than B, and finish at the same time. Beaker A should also finish at a lower level than Beaker B.

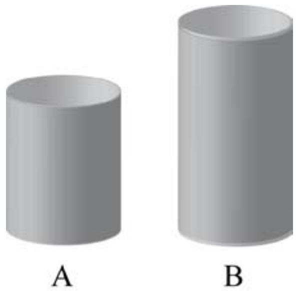


Figure 7.3: Beakers A and B are different heights but the same width. Both fill such that they are full at the same time.

A comparatively low number of students drew a correct slope for this question in the pre and the post-test. With more aspects to consider, students tended to focus on one aspect of the line (finishing time and level mostly in this case). Finding the balance between the correct finishing levels, time and slope requires some coordination and good reasoning. Pre and post-test results are shown in Table 7.3.

No notable increase was found in the percentage of students that correctly draw slope A with a less steep slope than B after instruction. One area of improvement was the drop in the number of students that drew curved lines for A or B – down from 30% to 15%. In the pre-test the high number of students that drew curved lines instead of straight lines seemed to result from trying to make both lines finish at the same point.

In the post-test there was a noticeable increase in the number of students that drew lines for A and B with equal slopes (up from 5% up to 20%). This increase of students drawing equal slopes seems to be as a result of an increased focus on the finishing level and finishing times. Correct finishing levels improved from 70% up to 85%, and finishing times improved up from 70% up to 80%.

7.1.5 Two beakers: different height, different width, same rate, C fills before B.

In a pretest detailed in Chapter 4, a large number of students used the time it took a beaker to fill to justify their answers. In all previous questions, the shorter time also meant a greater rate of change. It was impossible in these cases to conclusively prove a significant number of students were distinguishing between the time it took the beaker to fill and the rate of change in the beaker as it was filling.

Students were asked to draw a qualitative graph to show the water level changing in beakers B and C shown in Figure 7.4. Each beaker has water poured into it at the same constant rate, and beaker C fills before beaker B. Students were therefore required to distinguish between the time it takes for the water to fill up (time C > time B) and the rate the water level changes in beaker (rate C< rate B). A complete piece of reasoning to answer the question is shown in Figure 7.5. A full table of results is given in Table 7.4.

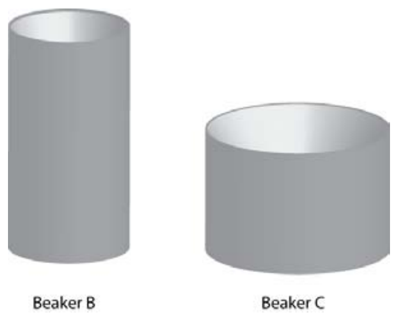


Figure 7.4: Beaker C is wider with less height than beaker B.

As Table 7.4 shows, 30% of students in the pre-test and 45% of students after instruction constructed a trend line with an appropriate slope for B and C. There was also variation between the different groups with post-test group A in particular drawing both the lines with identical slopes. Both pre- and post-test scores were lower than any other graph on which the correct answer consisted of two straight lines.

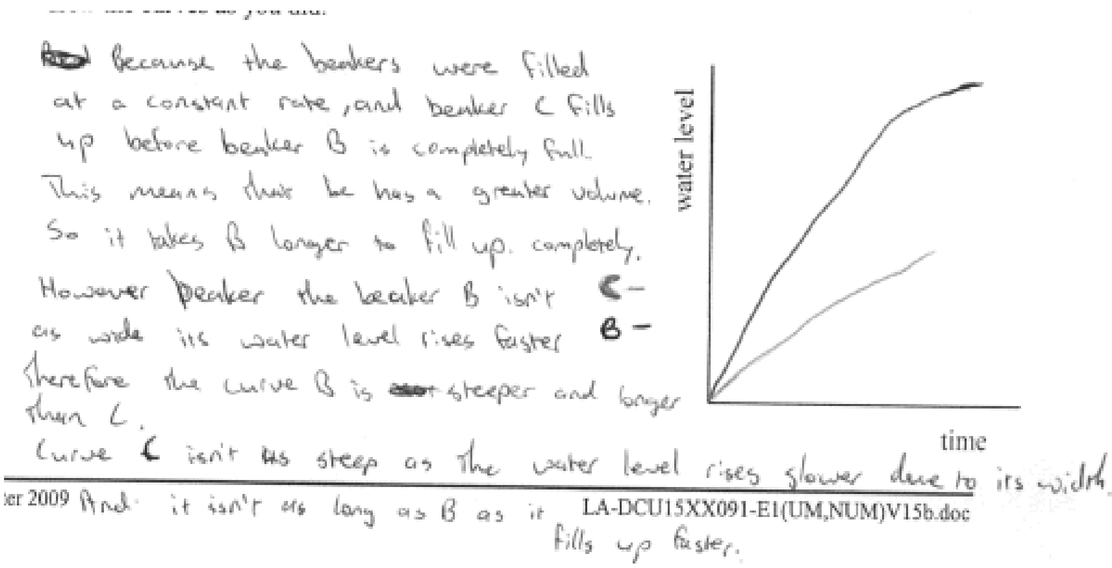


Figure 7.5: Correct reasoning though the answer is not completely correct.

There were no gains for pre- and post-test finishing levels, while there were small gains for the relative finishing levels and times. Students had significant difficulties in finding the balance between the features of the event. The theme of one feature on the graph improving after instruction and others not improving seems to be emerging from this and previous questions – see Sections 7.1.2 and 7.1.4.

The main aim of the question was to evaluate how many students choose to draw the slope based on the time it takes a beaker to fill rather than the rate at which it fills.

We found that 20% draw trend line C with a greater slope focusing on the time it took for beaker C to fill rather than the rate C filled when deciding to draw the slope this way. We cannot conclusively prove though that 80% of students connect the trend line and the slope of the line as many students emphasized finishing level and time over the rate of change in their answer. For instance, 25% of students in the post-test and 15% pre-test drew both lines with the same slope. These students tended to focus on drawing a correct finishing time and level rather than constructing an appropriate slope.

Table 7.1: Pre and post test results for water being poured into beakers A and D and A and B respectively (Figure 7.1). D is wider than A but has the same height, and B is identical to A but has a cylinder inside. Water is poured into each beaker at the same constant rate. The correct interpretations are shown shaded in on the table.

Pretest	**Group A**	**67**	**Group B**	**68**	**Total**	**135**	**Corresponding post-test**	**Group A**	**61**	**Group B**	**62**	**Total**	**123**	***h***
slope D>slope A	15%	11	10%	6	15%	17	slope B<slope A	15%	9	10%	5	10%	14	
slope D=slope A	20%	12	5%	2	10%	14	slope B=slope A	0%	1	5%	2	0%	3	.33
slope D<slope A	65%	42	60%	42	60%	84	slope B>slope A	75%	46	75%	46	75%	92	
no answer/other	5%	2	25%	18	25%	31	no answer/other	10%	5	15%	9	10%	14	
A straight, D curved	5%	5	5%	5	5%	10	A straight, B curved	0%	0	0%	0	0%	0	
level D>level A	15%	10	20%	15	20%	25	level B>level A	50%	31	45%	27	45%	58	
level D=level A	25%	16	20%	14	20%	30	level B=level A	20%	13	30%	19	25%	32	0.05
level D<level A	50%	32	55%	37	50%	69	level B<level A	20%	12	25%	16	25%	28	
no answer/other	15%	9	5%	2	10%	11	no answer/other	10%	5	0%	0	5%	5	
time D>time A	80%	55	70%	48	75%	103	time B<time A	75%	47	85%	52	80%	99	
time D=time A	5%	2	10%	7	5%	9	time B=time A	0%	1	5%	4	5%	5	0.18
time D<time A	0%	1	15%	11	10%	12	time B>time A	15%	8	10%	6	10%	14	
no answer/other	15%	9	5%	2	10%	11	no answer/other	10%	5	0%	0	5%	5	

Table 7.2: Pre and post test results for water being poured into beakers A and B (Figure 7.2). B is taller than A, but both have the same width. Water is poured into both at the same constant rate. The correct interpretations are shown shaded in on the table.

Pretest	**Group A**	**65**	**Group C**	**39**	**Total**	**104**	**Corresponding post-test**	**Group B**	**53**	**Group D**	**67**	**Total**	**122**	***h***
slope A >slope B	10%	6	0%	0	5%	6	slope A >slope B	10%	5	5%	3	10%	8	
slope A=slope B	60%	40	80%	32	70%	72	slope A=slope B	75%	39	80%	55	80%	94	.33
slope A <slope B	20%	11	0%	0	10%	11	slope A <slope B	5%	3	10%	7	10%	10	
no answer	0%	0	5%	2	<5%	2	no answer	0%	0	0%	0	0%	0	
either/both curved	10%	8	15%	5	15%	13	either/both curved	10%	6	5%	4	10%	10	
level A>level B	10%	6	<5%	1	5%	7	level A>level B	0%	0	5%	4	5%	4	
level A=level B	2%	1	5%	2	<5%	3	level A=level B	5%	2	0%	0	<5%	2	.50
level A<level B	90%	58	90%	34	90%	92	level A<level B	95%	50	95%	65	95%	115	
no answer/other	0%	0	5%	2	2%	2	no answer/other	<5%	1	0%	0	1%	1	
time A > time B	85%	55	80%	32	85%	87	time A > time B	95%	50	95%	64	95%	114	
time A =time B	10%	7	15%	5	10%	12	time A =time B	5%	2	1%	1	<5%	3	.66
time A<time B	<5%	2	0%	0	<5%	2	time A<time B	<5%	1	5%	4	5%	5	
no answer/other	0%	0	5%	2	<5%	2	no answer/other	0%	0	0%	0	0%	0	

Table 7.3: Beaker A and B are the same width, and different heights. Water is poured into each beaker at a different rate so that both beakers fill at the same time.

Pretest	Group C	53	Group D	63	Total	116	Corresponding post-test	Group B		Group A		Total	109	*h*
slope A >slope B	10%	4	2%	1	5%	5	slope A >slope B	0	0%	6	10%	6	5%	
slope A=slope B	5%	3	10%	5	5%	8	slope A=slope B	10	25%	14	20%	24	20%	
slope A <slope B	50%	26	55%	35	55%	61	slope A <slope B	27	60%	34	50%	61	55%	0
no answer	0%	0	15%	8	10%	8	no answer	0	0%	0	0%	0	0%	
either/both curved	40%	20	20%	14	30%	34	either/both curved	7	15%	11	20%	18	20%	
level A>level B	5%	2	2%	1	<5%	3	level A>level B	0	0%	2	5%	2	<5%	
level A=level B	20%	9	15%	10	15%	19	level A=level B	7	15%	8	10%	15	15%	
level A<level B	75%	39	70%	42	70%	81	level A<level B	37	85%	55	85%	92	85%	.50
no answer/other	5%	3	15%	10	10%	13	no answer/other	0	0%	0	0%	0	0%	
time A > time B	15%	7	5%	3	10%	10	time A > time B	1	2%	19	30%	19	20%	
time A =time B	65%	35	75%	48	70%	83	time A =time B	43	100%	45	70%	88	80%	
time A<time B	15%	8	<5%	2	10%	10	time A<time B	0	0%	2	3%	2	<5%	.33
no answer/other	5%	3	15%	10	10%	13	no answer/other	0	0%	0	0%	0	0%	

Table 7.4: Beakers C is wider and shorter than beaker A. Water is poured into both at the same constant rate. Beaker C fills before beaker A.

Pretest	**Group C**		**Group D**		**Total**	**102**	**Corresponding post-test**	**Group A**	**61**	**Group B**	**62**	**Total**	**123**	***h***
slope B>slope C	40%	15	30%	18	30%	33	slope B>slope C	35%	22	60%	33	45%	55	
slope B=slope C	15%	5	20%	12	15%	17	slope B=slope C	40%	25	15%	7	25%	32	0.25
slope B<slope C	25%	10	20%	12	20%	22	slope B<slope C	30%	11	20%	12	20%	23	
no answer	10%	3	20%	14	15%	17	no answer	0%	0	<2%	1	1%	1	
either/both curved	15%	6	10%	7	15%	13	either/both curved	10%	7	5%	3	10%	10	
level B>level C	70%	27	50%	32	60%	59	level B>level C	30%	18	75%	41	50%	59	
level B=level C	10%	3	15%	10	15%	13	level B=level C	5%	4	10%	5	10%	9	0.00
level B<level C	15%	6	10%	6	10%	12	level B<level C	30%	18	15%	7	20%	25	
no answer/other	10%	3	25%	15	20%	18	no answer/other	2%	1	5%	3	5%	4	
time B>time C	70%	27	60%	39	65%	66	time B>time C	65%	42	70%	39	65%	81	
time B=time C	15%	5	5%	4	10%	9	time B=time C	5%	2	15%	9	10%	11	0.00
time B<time C	10%	4	10%	5	10%	9	time B<time C	30%	20	10%	5	20%	25	
no answer/other	10%	3	25%	15	15%	15	no answer/other	2%	1	5%	3	5%	4	

7.2 Water level, non-uniform intervals

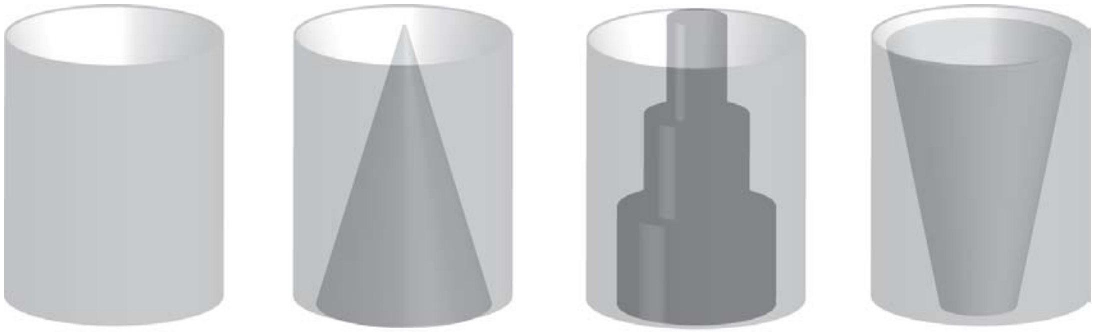


Figure 7.6: An empty beaker A, pre-test beaker C that contains a cone, post test beaker C contains a stepped cone and post test beaker D contains an inverted cone.

In Section 7.1 all interval changes in water level in the beakers were equal resulting in trend lines that were straight. To test our students' abilities to represent varying rates of change we pre-tested students using beaker C of Figure 7.6. Water is poured into each beaker at the same constant rate. For the post-test question students were shown post-test a different beaker, also labeled C in the post-test (see Figure 7.6) in which there is a stepped cone, and a beaker D with an inverted cone.

In the pre-test 45% of students drew a straight line to represent pre-test beaker C. 25% of students correctly drew a line that slopes downwards, with 5% incorrectly sloping the beaker in the other direction. These results also showed that there were significant difficulties for our students to answer this question, with 25% answering the question with no answer or an answer that defies categorisation.

Post-test beaker C had 15% of students answering the question correctly by drawing a line that with three line segments getting consecutively less steep. A further 30% of students drew a curve sloping downward to represent the water level filling post-test beaker C. The number of students drawing a straight line through the origin also dropped by 15% to 30% of students. The number of students that are focusing on representing how the water level is changing and not just on the finishing time has increased.

The stepped beaker in post-test beaker C did show that, while more students were able to represent the general trend of a changing rate, there seemed to be difficulty representing stepped changes in the rate. We note that, while students did encounter curved graphs in their instructions, stepped graphs were something they had not seen.

For post-test beaker D, 45% correctly drew the line curving upwards, and 15% incorrectly drew the line curving downwards. The number of students that drew a straight line to represent post-test beakers C and D encouragingly dropped from 45% pre-test to 15% post-test. In both post-test questions there was a noticeable number of students that focused on changes in water level and attempted to represent these changes using the shape of the line, compared to the large numbers of students in the pre-tests that focused in on the time taken for the beaker to fill.

Table 7.5: Pre and post test results for non uniform changes in water level in beakers

Pretest	Group A	67	Group B	68	Total	135	Corresponding post-test A	Group A	61	combined	61	Corresponding post-test B	Group B	62	Total	123	h
straight line	45%	29	45%	32	45%	61	straight line	30%	18	30%	5	straight line	20%	12	15%	17	
curved up	5%	4	5%	5	5%	9	curved up	5%	4	5%	6	curved down	25%	14	15%	20	
							stepped up	5%	2								
curved down	30%	20	25%	17	25%	37	curved down	30%	19	50%	29	curved up	40%	26	45%	55	0.24
							stepped down	15%	10								
no answer/other	20%	14	25%	17	25%	31	no answer/other	15%	8	15%	10	no answer/other	15%	10	15%	20	

7.3 Interval reasoning and uniform motion of two objects

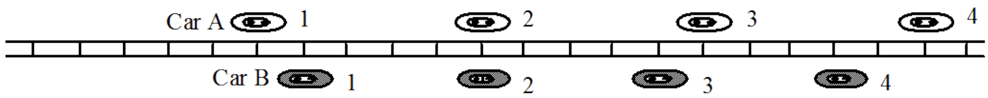


Figure 7.7: The position of cars A and B is shown at four times marked one to four different times.

Parts of the uniform motion lab and the non-uniform motion lab focused on helping students to develop an understanding of interval reasoning to describe motion. A question based on Figure 7.7 was given to pre- and post-test students over the past two years. The same question was given to different groups. Students were asked to describe if car A was moving at constant speed, and to compare the speeds of both cars A and B at instants 2 and 3.

There is significant progress to report on all questions. The number of students that were able to identify that car A was travelling with a constant speed improved from 70% in the pre-test to 90% in the post-test. Incorrect answers tended to be because students used the relative positions of the cars as an indication of the type of motion, i.e. car A is passing out car B so it must be speeding up.

There was also improvement in the number of students who identified that car A was travelling with a greater speed at instant 2: 35% in the pre-test, 55% in the post-test. Again the focus was on interval reasoning rather than on the relative positions of the cars. There was also an improvement for the number of students that identified that car A was travelling with a greater speed at instance 3.

Table 7.6: Answers to the questions whether car A was travelling at a constant speed, and how its speed compared to that of car B at instants 2 and 3.

	Pretest Speed	PS 153 2008 (132)		Post-test Speed (Corresponding Post-test)	2009 (N=67)		2007 (N=67)		*h*
Speed of A	Car A is speeding up	20%	30	Car A is speeding up	<10%	7	10%	7	.66
	Car A is slowing down	0%	0	Car A is slowing down	0	0%	0%	0	
	Car A is moving at constant speed	70%	97	Car A is moving at constant speed	90%	59	90%	60	
	It is impossible to tell from the information	10%	10	It is impossible to tell from the information	< 2%	1	0%	0	
Interval 2	Car A Speed < Car B Speed	5%	8	Car A Speed < Car B Speed	<5%	2	5%	1	.30
	Car A Speed > Car B Speed	35%	47	Car A Speed > Car B Speed	55%	36	55%	36	
	Car A Speed = Car B Speed	50%	69	Car A Speed = Car B Speed	40%	28	40%	29	
	No Answer/Other	10%	11	No Answer/Other	5%	1	<2%	1	
Interval 3	Car A Speed < Car B Speed	5%	9	Car A Speed < Car B Speed	5%	4	<5%	2	.66
	Car A Speed > Car B Speed	85%	112	Car A Speed > Car B Speed	90%	60	95%	62	
	Car A Speed = Car B Speed	5%	6	Car A Speed = Car B Speed	0%	0	<5%	2	
	No Answer/Other	5%	9	No Answer/Other	< 5%	3	<2%	1	

7.4 Rates of change and speed

7.4.1 Overview

Parts of the uniform motion and non-uniform motion labs required students to categorise different types of motion as uniform and non uniform, and gave them methods of quantifying motion. We set equivalent pre and post-tests that required students to analyse a straight line graphs with scaled axes. To test whether students had developed a general understanding of the concept of rate of change two types of questions were asked.

- Unfamiliar context: The first graph is one that is unlikely to have been encountered by students before. On this graph, water level changes against time to represent a pool filling with water.
- Familiar context: The second, which is likely to have been encountered by students, has distance graphed against time to represent the distance of a ball to a fixed point. These ideas and concepts have been covered as part of the Junior Certificate cycle.

The term speed is not used for any question that features the graph in an unfamiliar context. This means that the student is not led to use learnt-off formulas for speed where he or she does not make the connection spontaneously. For each context, students are asked if the rate of change is constant or not, and to calculate the rate of change at one instant.

Both questions can be answered using different methods of reasoning. Some of these are listed below with examples:

1. An example of interval reasoning is shown in Figure 7.8. Changes in the *x*-axis are correlated with constant changes in the *y*-axis. The student can then deduce by comparing these changes that the motion is constant while also quantifying the motion itself.

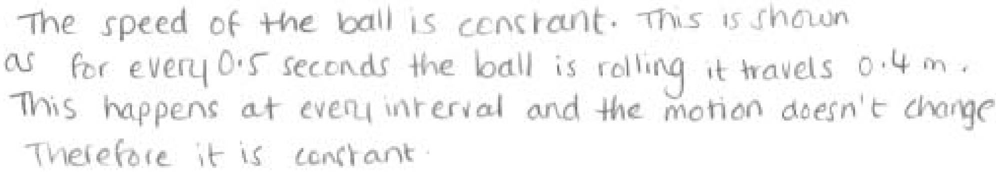

The speed of the ball is constant. This is shown
as for every 0.5 seconds the ball is rolling it travels 0.4 m.
This happens at every interval and the motion doesn't change
Therefore it is constant.

Figure 7.8: Interval reasoning to identify that motion of a ball is constant.

2. Proportionality: similar to interval reasoning, quantitative steps are circumvented as the student understands the significance of the shape of the line to the interval changes.

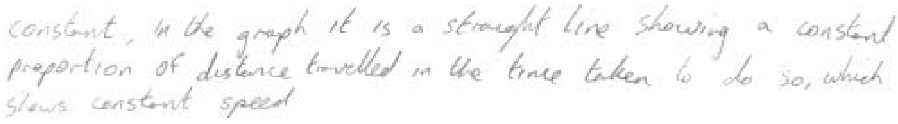

constant, in the graph it is a straight line showing a constant
proportion of distance travelled in the time taken to do so, which
shows constant speed

Figure 7.9: Proportionality of the changes in distance against time used to reason that the ball is travelling with a constant speed.

3. Shape of the line: The graph is a straight line, so the process occurs at a constant speed. This is usually learnt off or familiar as the student recognises that the rate of change is equal.

The straight line graph which goes through the origin shows that the speed of the ball is constant. The slope doesn't change along the graph and the slope = $\frac{\text{distance}}{\text{time}}$ = Speed

Figure 7.10: The student identifies that the slope is equivalent to the speed of the ball and the slope doesn't change, then making the connection that that the ball is moving constant speed.

4. Students often apply formulas to calculate the speed or rate of change. The two most common are *y*/*x* and Δ*y*/Δ*x*, both shown in Figures 7.11a and 7.11b.

$$\frac{1.2}{1} = 1.2\ \text{m/s}$$

At t = 1s on x-axis I draw a line vertically upwards until it hits the graph. At this point I draw a line horizontal to the left until it hits the y-axis. This shows distance at this point = 1·2 m Speed = $\frac{\text{Distance}}{\text{Time}}$ = Slope of graph = $\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

$(x_1, y_1)\ (x_2, y_2) = (2{\cdot}0, 2{\cdot}0)\ (1{\cdot}0, 1{\cdot}2)$

Winter 2009 LA-DCU15XX091-E1(UM)V16ab.doc

$$\frac{2{\cdot}0 - 1{\cdot}2}{2{\cdot}0 - 1{\cdot}0} = \frac{{\cdot}8}{1{\cdot}0} = {\cdot}8\ \text{m/s}$$

Figure 7.11: a) incorrect and b) correct formula is applied to find the speed for the same question.

The difficulties that students have, and our strategy, are detailed in Chapters 5 and 6.

7.4.2 Quantitative motion: Unfamiliar Context

Four groups took part in the pre and post test each doing the same course and at an equivalent level. Group 1 and 2 took the pre-test while groups 3 and 4 took the post-test. For both the pre and the post test one group took the negative slope question while the other took the positive sloped question.

Table 7.7 shows the number (and type) of correct answers for the question “how quickly does the water level change at t=30 seconds”, Table 7.8 shows the number and type of answers for “Is the water level changing at a constant rate? How can you tell?”

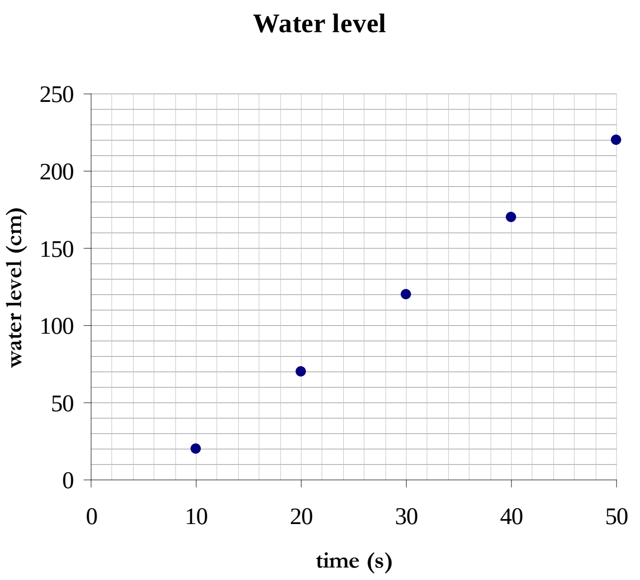


Figure 7.12: The water level of water being poured into a swimming pool.

After instruction a negligible gain was found between the pre and post-test, as 45% could correctly calculate the rate of change at 30 seconds in the pretest and 50% in the post-test questions. Students tended to change patterns with a reduction in the number of students having no answer or “other” answers and an increase in the number of students that incorrectly use the ratio y/x.

Table 7.8 shows a breakdown of all the answers and approaches taken by students who were asked if the pool was filled at a constant rate. There is some improvement from nearly 90% pre to 100% post for students that say that the pool is being filled at a constant rate. The number of students that use interval reasoning increases from 40% pre to 75% post showing that there is a greater increase in the quality of reasoning than the pre and post test correct answers.

For all the correct answers in Table 7.8, a correlation between the reasoning used to come to those correct answers and the reasoning that they used in order to find how quickly the pool fills at t = 30 s is given in table 7.9. Of the 60 students that used interval reasoning to identify the pool as filling with a constant rate, only 50% correctly identified the rate of change at t = 30 s while 15% of these students divided both the coordinates. There seems little change in this context from pre to post in how students approached finding the rate of change at t = 30 s after using interval reasoning to examine if the pool was filling at a constant rate. In Section 7.4.3 we see that in a more familiar context, this approach does seem to change.

Table 7.7: Pre-test and Post-test results for students when asked to find “How quickly does the water level change” at a particular time.

	Pretest	**Group 1**		**Group 2**		**Total**	**Groups 1 and 2 (116)**		**Groups 3 and 4 (109)**	***h***
Correct solutions	Correct	30%	16	30%	16	30%	32	35%	39	
	Correct apart from a sign error	2%	1	0%	0	1%	1	0%	0	
	$\Delta x/\Delta y$	0%	0	5%	1	1%	1	0%	0	0.17
	Correct but not 5 cm/s (e.g. 50 cm/10 s)	20%	9	10%	6	15%	15	15%	17	
	total correct	**50%**	**26**	**45%**	**23**	**45%**	**49**	**50%**	**56**	
Incorrect solutions	*y*/*x* or similar	20%	11	20%	11	20%	22	35%	37	
	unclear	25%	12	15%	7	15%	19	10%	13	
	other	10%	5	5%	2	5%	7	5%	3	
	no answer	20%	9	20%	10	15%	19	0%	0	

Table 7.8 : Pre-test and post-test results for students when asked if the pool was filling at a constant speed. There is an increase from 90% correct pre to nearly 100% post. In the pre 40% of students used interval reasoning but in the post 75% of students use interval reasoning.

		Pretest 2009 Groups 1 and 2			**Post test 2009 Groups 2 and 4**		***h***
Correct or almost correct	intervals	27	40%	intervals	79	75%	1
	slope, straight line	20	30%	slope, straight line	18	20%	
	proportional	2	5%	proportional	2	<2%	
	other constant	9	15%	other constant	4	5%	
	total correct	**58**	**90%**	**total correct**	**103**	**100%**	
Incorrect answers	decreasing	7	10%	decreasing	0	0%	
	increasing	0	0%	increasing	1	<1%	
	no answer/unclear	0	0%	no answer/unclear	0	0	

Table 7.9: Pre and post- test results linking what approach students who correctly stated that the water level is changing at a constant rate took to find the a value for the rate of change at t = 30 s. The second column shows that of the 60 students who used interval reasoning to determine that the water level changes at constant rate in the pretest, 50% correctly determined the rate of change using a correct method; 15% divided the two coordinates, 20% gave uncategorized answers, and 15% gave no answer.

Approaches to correctly identifying constant rate	**intervals**				**slope**				**slope + interval**				**other**				**total**			
Approaches to finding the rate at *t* = 30 s	**Pre (*N*=60)**		**Post (*N*=60)**		**Pre (*N*=35)**		**Post (*N*=18)**		**Pre (*N*=6)**		**Post (*N*=20)**		**Pre (*N*=12)**		**Post (*N*=6)**		**Pre (*N*=113)**		**Post (*N*=104)**	
$\Delta y/\Delta x$ or similar	50%	(30)	55%	(32)	35%	(12)	40%	(7)	50%	(3)	60%	(12)	35%	(4)	50%	(3)	45%	(49)	50%	(54)
y/x or similar	15%	(8)	30%	(18)	30%	(11)	50%	(9)	0%	(0)	30%	(6)	10%	(1)	35%	(2)	20%	(20)	35%	(35)
other	20%	(12)	15%	(8)	25%	(8)	5%	(1)	15%	(1)	10%	(2)	40%	(5)	15%	(1)	25%	(26)	10%	(12)
no answer	15%	(10)	5%	(2)	10%	(4)	5%	(1)	35%	(2)	0%	(0)	15%	(2)	0%	(0)	15%	(18)	5%	(3)

7.4.3 Quantitative motion: familiar context of speed

In pre- and post-test questions students were given the graph of Figure 7.13.

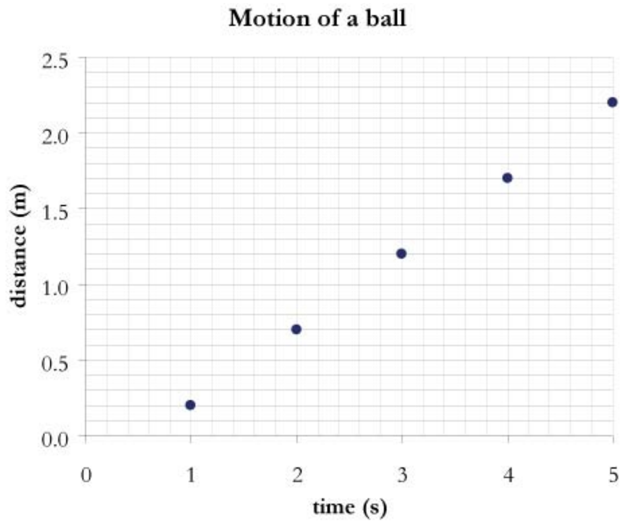


Figure 7.13: The distance of a ball from a fixed point *P*.

In the academic year 2009/2010, four groups took part in the pre- and post-test. Each group took take the same labs at different times. All students were participants in the revised first year physics labs. Group 1 and 2 took the pre-test while groups 3 and 4 took the post-test. For the pre-test groups 1 and 2 were given questions on Figure 7.13. They were post tested with a similar graph with negative slope. Pre-tests and post-tests were reversed for groups 3 and 4.

Table 7.10 shows the pre and post-test assessment results for students asked to find the speed of the ball at t = 1 s. The pre-test shows that there is an improvement in the pre-test data from a low of 20% to 40%. Both low pre and post test figures are as a result of 70% of students in the pre-test and 60% of students in the post test persisting to calculate speed by using the y/x or d/t formula. The results demonstrate the level of difficulty that the question had for the students in the labs.

However, Table 7.10 also shows a significant gain from 70% pre-test to 100% post-test saying that the graph represented a ball travelling with uniform motion. This gain was not entirely unexpected as a part of the labs focused entirely on straight line uniform motion graphs.

There was also a more encouraging shift in the reasoning that the student used to come to the conclusion that the ball was travelling with a constant motion. In the pre-test 20% of students used interval reasoning, but in the post-test 80% used interval reasoning as at least part of their answer. 40% of the pre-test answers consisted of an explanation that focused on the straight shape of the line, this drops to 20% post-test. Overall it was encouraging to note how more students after instruction used interval reasoning to not only analyse the type of motion but also to quantify the speed of motion.

Table 7.12 details how students that correctly analyse the motion of the ball as being constant and categorises their correct answers under the headings of intervals, slope, intervals & slope, other, total. For each heading, the approaches taken by these students to finding the speed of the ball at $t = 1$ s are analysed. For students who used interval reasoning to say that the ball was moving with constant motion, 20% went on to correctly calculate the speed of the ball at $t = 1$ s in the pre-test and 45% went on to successfully calculate the speed of the ball at $t = 1$ s in the post-test. Students were more likely to use interval reasoning in both questions after instruction.

Table 7.10: Pre and post test results with reasoning for finding the speed of the ball at t=1second

		Pretest 2009 (*N*=194)		**Post-test 2009 (*N*=123)**		***h***
Correct reasoning	Correct	3	<2%	42	35%	0.25
	Correct apart from a sign error	37	20%	0	0%	
	$\Delta x/\Delta y$	0	0%	0	0%	
	Correct but not 5 cm/s (e.g. 50 cm/10 s)	2	<1%	5	5%	
	total correct	**42**	20%	**47**	40%	
Incorrect approaches	y/x or similar	134	70%	76	60%	
	unclear	14	10%	0	0%	
	other	4	<2%	0	0%	
	no answer	**42**	20%	**47**	40%	

Table 7.11: Pre and post test results for the question " is the motion constant or not constant"

			Pretest 2009 (*N*=192)		**Post-test 2009 (*N*=104)**	**h**
Correct answers and reasoning	constant	10%	17	5%	4	.99
	straight	40%	75	20%	18	
	interval	20%	43	60%	60	
	proportional	<5%	4	<5%	2	
	sl + int	0%	0	20%	19	
	total correct	**70%**	**139**	**100%**	**103**	
Incorrect answers	increasing	<5%	3	<5%	1	
	decreasing	25%	49	0%	0	
	don't know	<5%	1	0%	0	

Table 7.12: Pre and post- test results linking what approach students who correctly stated that the speed of the ball is constant took to find the speed of the object at t = 1 s. The second column shows that of the 31 students who used interval reasoning to determine that the speed of the ball is constant in the pretest, 20% correctly determined the speed of the ball using a correct method; 70% divided the two coordinates, and a further 10% gave uncategorized answers.

Approaches to correctly identifying constant speed	**intervals**				**slope**				**slope + interval**				**other**				**total**			
Approaches to finding the speed at *t* = 1 s	**Pre (*N*=31)**		**Post (*N*=47)**		**Pre (*N*=58)**		**Post (*N*=34)**		**Pre (*N*=1)**		**Post (*N*=17)**		**Pre (*N*=15)**		**Post (*N*=2)**		**Pre (*N*=108)**		**Post (*N*=100)**	
$\Delta y/\Delta x$ or similar	20%	(6)	45%	(21)	35%	(20)	55%	(18)	0%	(0)	40%	(7)	15%	(2)	0%	(0)	25%	(28)	45%	(46)
y/x or similar	70%	(22)	55%	(26)	60%	(35)	45%	(16)	100%	(1)	60%	(10)	65%	(10)	100%	(2)	65%	(71)	55%	(54)
other	10%	(3)	0%	(0)	5%	(2)	0%	(0)	0%	(0)	0%	(0)	15%	(2)	0%	(0)	5%	(7)	0%	(0)
no answer	0%	(0)	0%	(0)	0%	(1)	0%	(0)	0%	(0)	0%	(0)	5%	(1)	0%	(0)	0%	(2)	0%	(0)

7.4.4 Context-free slope

We asked students to calculate the slope of the line in *y*,*x*-graphs like those of Figure 7.14. We find that in the pretest, merely 45% of 205 students correctly calculate the slope (excusing sign errors). A further 25% calculate *y*/*x*, 15% give other answers and 25% cannot answer the question at all. Many of the students who did not give an answer at all commented that it is not possible to calculate the slope at a point.

Post-test results for a similar question but with a positive slope show a very high gain, to over 85% correct answers by 191 students, a Hake gain of 0.8. Less than 10% still adhere to the procedure *y*/*x*. These results show that our teaching of interval reasoning has been very successful. However, there is a strong suggestion that incorrectly assimilated prior knowledge gets in the way of this reasoning when applying it to a setting involving speed, distance, and time, either implicitly or explicitly.

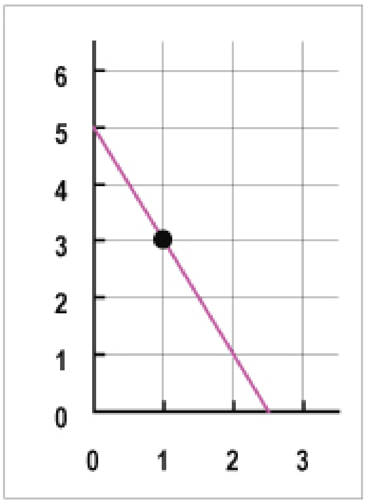


Figure 7.14: Context-free slope question. Students are asked to give the slope of the line drawn at the point indicated.

Chapter 8: Conclusion

Organising our labs so that they incorporate elements of open and guided inquiry in a flexible approach has broadened the aims of the lab to include both conceptual and skills development. It is possible that a guided inquiry course that targeted conceptual development only would yield better pre/post test data, and without the focus on conceptual development students could have arrived at open inquiry earlier and gained more experience with it. However, our approach has consistently and reproducibly achieved gains in both inquiry aspects and conceptual development, in an environment that students find enjoyable and stimulating.

Having assessment in the affective domain appears to have been mutually beneficial for both ourselves and the students. From our point of view, a set of labs was developed which would most likely have been further away from a correct level for these students without this type of assessment. Students in return had a more enjoyable and motivating experience in the labs and were able to fully benefit from the materials that we developed. It is hard to imagine that research-based development in which we only relied on assessment in the cognitive domain would have been as successful when developing this set of labs.

Understanding and developing graphing skills from different perspectives was an important part of the work that we carried out in the labs. One of the problems we identified and tried to remedy was students calculating speed or rate of change with the simple ratio of y/x or d/t. It was surprising to find that of the science students coming into the labs, only 20% of students could calculate the slope of a linear distance-time graph at a point, and only 45% in a context-free setting. Through focusing on interval reasoning, we managed to improve the post-test scores in the context-free setting to almost 90%, but much more modest gains were made in the questions pertaining to uniform motion and another numerical question on rate of change, that of a swimming pool filling up or being drained.

The context-free post-test results show that students had the ability to answer these questions completely and correctly, but a formula-led approach to solving these problems appears to have been a block. It is remarkable that nearly three quarters of students use interval reasoning to correctly identify a straight line graph as representing a process that takes place at a constant rate in post-tests, and yet seeing half of these students abandon their reasoning when asked to find a numerical value for the rate of change or speed at a point on the same graphs both in a familiar context and unfamiliar context. How students learnt kinematics from an introductory stage in school appears to be an important part of how they develop their reasoning approaches and strategies.

It would seem that strongly emphasising formulas in early introductory physics curriculum before students develop intuitive and robust reasoning models for kinematics is an area that needs to be carefully looked at. In our labs we have implemented an engaging inquiry lab experience that we feel could be built on and adapted to second level teaching.

Another aspect of graphing that we found important was that of testing and developing a general graphing literacy. We did this by getting students to represent simple situations using qualitative graphs as both pre and post-tests. We have found that students have significant difficulties with changing rates, a focus on time, a preconceived shape of a graph as being a straight line through the origin and often become unnecessarily distracted by other physical features of a graph. It is almost certain that a contributory factor to these difficulties is the limited use for and narrow range of graphs that students have encountered before they enter university. Most graphs at second level are straight line graphs through the origin, and interaction with graphs (as with general experimentation) is limited to cookbook instructions. These approaches appear to do little or nothing when it comes to helping students to interpret graphs, or use them beyond the lab.

Even if a more traditional approach to teaching is maintained, these issues could be addressed in part by students experiencing a wider range of graph types (different shaped trend lines throughout their instruction). More focused exercises that deal with students' misconceptions and ideas can be also incorporated in lab and general instructional design.

The instruction we have developed has shown some improvements with developing graphing literacy. It follows from our research that other areas could be focused on and enhanced, such as abrupt rates of change, finishing points of a line, and how multiple features of a line can be used to represent multiple features of an event. However we feel that our instructional approaches are a good example of how labs can be used to empower students with a better graphing literacy. Clearly the concept or idea of graphing literacy is an incredibly rich area for research, both in terms of how we evaluate the impact of our research, and how we develop it.

Future work will continue to reinforce pre and post-test data discussed in this thesis while making further improvements to the materials. It is hoped that other institutions will implement our labs to obtain to cement the validity of the research beyond the setting in which it was obtained. We feel there is wide scope for extending many aspects of this project, especially in the area of interval reasoning, to second level students.

EXPERIMENT P4

INVESTIGATION OF ELASTICITY

Objectives

1. To investigate the elasticity of various materials by examining to what extent they obey Hooke's Law.
2. To investigate periodic motion for a spring.

Background

Elasticity is a physical property of material objects that determines how easily an object may be deformed by stretching, bending, or compressing and still return to its original shape. It is said to be more elastic if it restores itself more precisely to its original configuration. A rubber band is easy to stretch, and snaps back to near its original length when released, but it is not as elastic as a piece of piano wire. The piano wire is harder to stretch, but would be said to be more elastic than the rubber band because of the precision of its return to its original length. A real piano string can be struck hundreds of time without stretching enough to go noticeably out of tune.

A spring is an example of an elastic object - when stretched, it exerts a restoring force which tends to bring it back to its original length. Automobile suspensions, playground toys and even retractable ball-point pens employ springs. Most springs have an easily predicted behaviour when a force is applied i.e. as the spring is extended or compressed. Hooke's Law, as commonly used, states that the force F a spring exerts on a body is directly proportional to the displacement Δl of the system (extension of the spring).

$$F = -k\Delta l$$

where k is the spring constant and the magnitude depends on the spring, being large for stiff springs and small for easily stretched springs. For wires or columns, the elasticity is generally described in terms of the amount of deformation (strain) resulting from a given stress (Young’s Modulus).

Hooke's Law applies as long as the material stress (applied force) does not pass a certain point known as its proportional limit. Beyond this point there is no longer a linear relationship between the applied force and the spring extension, but up a point called the *elastic limit* the spring will still return to its original length once the force is removed. However, if the spring is stretched <u>beyond</u> its elastic limit, it does not return to its original length upon removal of the applied force but remains permanently deformed (like bending a paper clip).
Application concept: Tendons in the human body connect muscles to your bones. These tendons can stretch and contract similar to an elastic band and are what causes our limbs to move.

Now answer questions A1 through A2 on the answer sheet

Experiment 1: Hooke's Law

Materials for which the deformation is proportional to the applied force are said to obey Hooke's Law. It is the objective of this present experiment to examine a range of materials generally classified as elastic and to investigate to what extent they obey Hooke's Law.

Apparatus

Rubber cord	Metre stick	mass hanger
Steel spring	Retort stand	10 g and 100 g masses

Procedure

1. Fix one end of the rubber cord to the retort stand and the other to a mass hanger.
2. Measure the relaxed length *l* of rubber cord with no mass attached.
3. Determine a suitable range of masses over which to measure the extension of the rubber cord.
4. Successively apply the masses (*M*) onto the rubber cord and record the length *l* and the extension (Δl) in each case to get at least 8-10 readings.
5. Take care not to overload the cord otherwise permanent deformation can result.
6. Plot a graph of extension (in mm) versus mass (in kg) for the rubber cord. Indicate on the graph the region (if any) where Hooke's Law is satisfied. For this region, measure the slope (S) in mm kg^{-1}.
7. Connect the steel spring to the retort stand and repeat steps 1-6 for the steel spring.

Note: Graphs should be plotted in accordance with the guidelines given in Appendix 1.

Now answer questions A3 through A9 on the answer sheet

Analysis

When a mass *M* is suspended from a spring, the spring extends by an amount Δl so that the downward force (Mg) is balanced by the upward restoring force F_s of the spring (see Figure 1). The mass is at rest in its equilibrium position.

i.e. $F_s = -Mg$

Because extension is linearly proportional to load (as you discovered in Expt. 1 -- Hooke's Law) it follows that the restoring force F_s must be linearly proportional to extension. Thus,

$$F_s = -k\Delta l$$

$$\therefore \quad Mg = k\Delta l$$

i.e. $\Delta l = (g/k)M$

k is a constant -- termed the spring constant

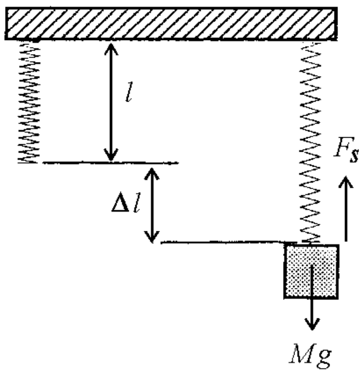


Figure 1. Spring at equilibrium.

Therefore the slope of the graph Δl vs *M* (as plotted in Experiment 1) enables the ratio *g/k* to be determined.

Experiment 2: Periodic Motion

If an elastic object is stressed and released it will oscillate periodically about its equilibrium or rest position. Examples of such objects are a musical string instrument, a saw blade clamped at one end or a mass attached to a spring.

It is the purpose of this experiment to investigate how the periodic time of an oscillating spring may be used to deduce a value for *g*, the acceleration due to gravity.

Apparatus

Steel spring	Stopclock	mass hanger
Metre stick	Retort stand	100 g and 10 g masses

Procedure

1. Set up the steel spring on the retort stand and attach a 100 g mass. Measure the time for 30 oscillations and take the average value to get the periodic time T. Include the mass of the hanger in your value of M.
2. Gradually increase the mass M, measuring T in each case. Record 8-10 readings taking care not to over-stretch the spring.
3. Plot a graph of T^2 (in s^2) vs M (kg) on graph paper.
4. From the slope of the T^2 vs M graph, determine *g* the acceleration due to gravity using Equation 4. (S is already known from Experiment 1).
5. From the intercept determine the effective mass m of the spring. Pay particular attention to units here.

Now answer questions A10 through A17 on the answer sheet

Analysis

The motion of a body that oscillates back and forth is defined as *Simple Harmonic Motion* if there exists a restoring force F that is opposite and directly proportional to the distance x that the body is displaced from its equilibrium position. If Hooke's Law holds for a spring, then the motion of masses vibrating up and down on the spring should be simple harmonic motion. If the mass, when hanging from the spring, is given a small additional displacement *x* from its equilibrium position and then released, the spring will exert a net force $F = -kx$ which tends to restore the mass to its equilibrium position. The constant of proportionality k is called the spring constant and can be found by subjecting the spring to an applied force and measuring the amount that the spring stretches.

It is clear that at all positions of the mass's motion, the net force on the mass *M* is directed towards the equilibrium position. As a result the mass *M* undergoes repetitive vertical oscillations about the equilibrium position. The periodic time for these vibrations may be determined in the following way.

$$F = M\, d^2x/dt^2 \qquad \text{(Newton's 2nd Law)} \quad \text{and} \quad F = -kx$$

$$\therefore \quad M\, d^2x/dt^2 = -kx$$

$$\text{or} \quad M\, d^2x/dt^2 + kx = 0 \qquad (1)$$

We can show, by direct substitution, that x = A sinωt satisfies Equation 1 if A (maximum Amplitude) and ω (angular frequency) are constants.

$$dx/dt = \omega A\cos \omega t$$

and

$$d^2 x/dt^2 = -\omega^2 A \sin \omega t$$

Therefore Equation 1 becomes

$-M\omega^2 A \sin \omega t + kA \sin \omega t = 0$

If this is to be true for all t we must have

$$\omega^2 = k/M \qquad \text{i.e. } \omega = \sqrt{\frac{k}{M}}$$

We can help to visualise the motion by plotting x against t as in figure 2.

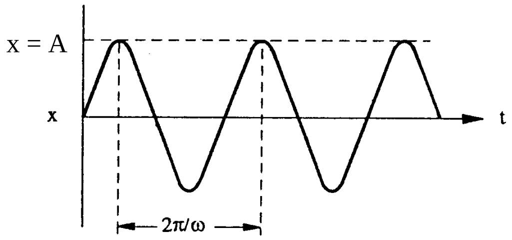


Figure 2. Simple Harmonic Motion

We see that the motion repeats itself with a periodic time, with the time for one oscillation called the period T, given by:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{k}} \qquad (2)$$

The frequency f of the oscillations is the number of oscillations per unit time and is the reciprocal of the period, f=1/T, and is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

We have shown already that the slope S of the Δl vs M graph is equal to g/k in Experiment 1.

Thus, $k = g/S$ and $T = 2\pi\sqrt{\frac{MS}{g}}$

We have ignored the mass of the spring itself in the above analysis. This may be taken into account by writing:

$$T = 2\pi\sqrt{\frac{(M+m)S}{g}} \qquad (3)$$

where m is the "effective mass" of the spring.

On squaring Equation we obtain

$$\boxed{T^2 = \left(\frac{4\pi^2 S}{g}\right)M + \frac{4\pi^2 Sm}{g}} \qquad (4)$$

This equation is in the form y = mx + c. Therefore, if the periodic time T is measured for various masses M, a graph of T^2 vs M should give a straight line

- the slope of which is $4\pi^2$ S/g (knowing S enables g to be determined)
- the intercept of which is $4\pi^2$mS/g (knowing S and g enables m to be determined.)

Reference

Giancoli, Physics, Fifth Edition, Chapter 9.

Young and Freedman, University Physics, Ed. 9, Chapters 6 and 13.

Name:	Class:	Group:	Date:

EXPERIMENT P4 – ANSWER SHEET

Elasticity of Materials: (1) Hooke’s Law (2) Periodic Motion

A1. List three (other) examples in everyday life where materials that exhibit elasticity are used.

____

____

____

A2. Keeping everything else the same, how do the following changes affect the ‘stiffness’ of a spring?

- Increasing the length of the spring? ( ) increases ( ) decreases ( ) no effect
- Increasing the diameter of the spring? ( ) increases ( ) decreases ( ) no effect
- Increasing the number of turns in the spring? ( ) increases ( ) decreases ( ) no effect
- Increasing the density of the wire in the spring? ( ) increases ( ) decreases ( ) no effect
- Increasing the force applied to the spring? ( ) increases ( ) decreases ( ) no effect

A3. Tabulate your measurements for the rubber cord.

Mass M (kg)	Length l (mm)	Extension Δl (mm)

A4. Plot a graph of Extension in mm versus Mass in kg for the rubber cord on graph paper.

A5. What is the slope of the graph drawn in part A4 above? ______________mm kg^{-1}?

How does this value relate to the "stiffness" of the rubber cord used?

____

What is the spring constant k of the rubber cord, including units?

____

A6. Tabulate your measurements for extension and for the steel spring.

Mass M (kg)	Length *l* (mm)	Extension Δl (mm)

A7. Plot a graph of Extension in mm versus Mass in kg for the steel spring on graph paper.

A8. What is the slope of the graph S drawn in part A7 above? _______________ mm kg^{-1}?

What is the spring constant k of the steel spring, including units?

____

A9. State what you deem to be the most important sources of error **other than human error** in order of importance. Include an estimate (in percents or absolute value) of its effect.

Source of error	**Estimate (% or SI units)**

A10. Make a sketch of an oscillating mass on a spring and indicate the following positions:
a. At which point(s) does the mass on a vibrating spring have the greatest acceleration?
b. At which point(s) does it have the least acceleration?
c. At which point(s) does the mass have the largest force exerted on it?
d. At which point(s) does the mass have the smallest force exerted on it?

A11. Keeping everything else the same, how do the following changes affect the period of oscillation?

- Increasing the length of the spring? ( ) increases ( ) decreases ( ) no effect
- Increasing the spring constant of the spring? ( ) increases ( ) decreases ( ) no effect
- Increasing the amplitude of the vibration? ( ) increases ( ) decreases ( ) no effect
- Increasing the force applied to the spring? ( ) increases ( ) decreases ( ) no effect
- Increasing the density of the wire in the spring? ( ) increases ( ) decreases ( ) no effect

A12. Tabulate your measurements for mass M and Period T for the steel spring.

Mass M (kg)	*Time for 30 oscillations* (s)	Period T (s)	Period Squared T^2 (s^2)

A13. Plot a graph of Period Squared T^2 in s^2 versus Mass in kg for the steel spring on graph paper.

A14. What is the slope of the graph drawn in A13 above, including units? ________________

Using the value of S from A8 above, determine the value of g, the acceleration due to gravity?

____

A15. What is the y-intercept of the graph drawn in part 11 above, including units? __________

Using the value of S from A8 and the value of g from A14 above, determine the value of effective mass m of the spring?

____

A16. Measured mass of spring in kg (using balance) ______________________________

Calculated effective mass of spring in kg (from A15)______________________________

Ratio of effective mass to actual mass ______________________________

Why do you think the effective mass of the spring varies from actual mass of the spring?

____

____

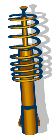
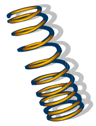
____

A17. State what you deem to be the most important sources of error **other than human error** in order of importance. Include an estimate (in percents or absolute value) of its effect.

Source of error	**Estimate (% or SI units)**

Name: ______________________________________ Class: ______

Experiment 1: Making a spring balance

There are many common examples of springs used in our everyday lives, such as the springs used in cars to ease the shock of the bumps on the road.

In the following experiment you will examine springs and use your observations in the construction of a spring balance.

Section 1: Experimental apparatus

Check that you have **two** different springs, a retort stand with clamp, a metre stick, a mass hanger and six 20 gram disks; if not, notify a tutor.

Before you begin the experiment, take some time to discover some properties of the springs. For example, you may want to try the following:

- Holding one end of the spring in your hand, hang some objects from the other end and observe what happens.
- Add more objects and again observe the results.
- Compare the two springs to each other.

i. Make notes on your investigations in the space below. Use the space at right below to draw an illustration.

____

____

____

____

____

____

____

____

____

____

____

____

Section 2: Preparing the experiment

i. Set up the equipment as shown in Figure 2.1 at right. Take one of the springs (which we will call Spring 1 from now on) and attach it to the retort stand by hooking the spring onto the clamp.

   While the spring is attached to the retort stand as shown in the diagram at right, measure the position of the top of the spring and the position of the pointer. From these data, calculate the initial length of the spring in centimetres. Enter the values in Table 2.1 below.

ii. Remove Spring 1. Replace it with Spring 2 and make the same measurements. Enter the values in Table 2.1 below.

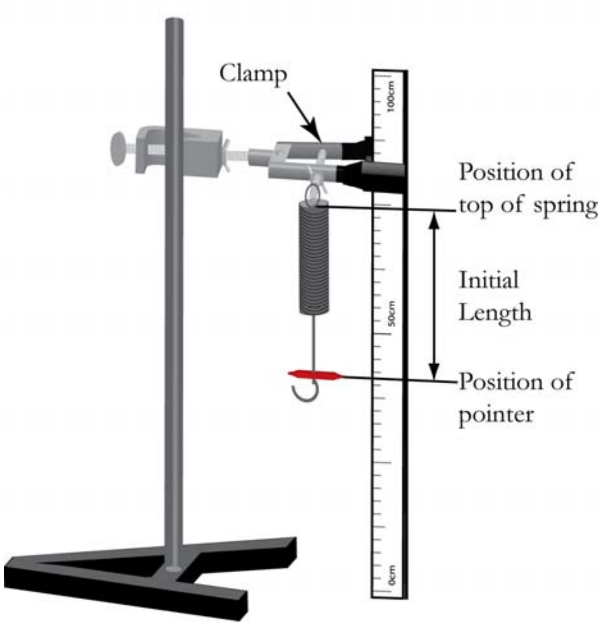


Figure 2.1: Retort stand with spring and ruler.

Table 2.1: Initial length of springs.

	Spring 1	**Spring 2**
Position of top of spring		
Position of pointer		
Initial length		

Section 3: Experimental procedure

To follow on with the construction of your spring balance it is necessary to investigate the stiffness of each spring. The following steps will allow you to achieve this and enable you to compare the springs to each other.

i. Attach Spring 1 to the retort stand as in Section 2, and hook the mass hanger onto the other end of the spring as shown in Figure 3.1 at right.

   Measure the new length of the spring as you did in Section 2, and calculate the extension (change in length) of the spring. Enter your measurements for Spring 1 in Table 3.1 below.

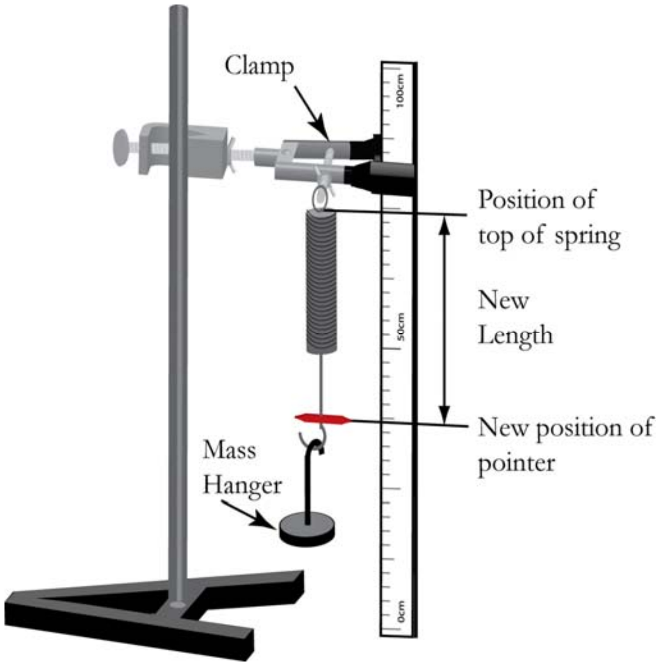


Figure 3.1: Retort stand with spring and mass hanger.

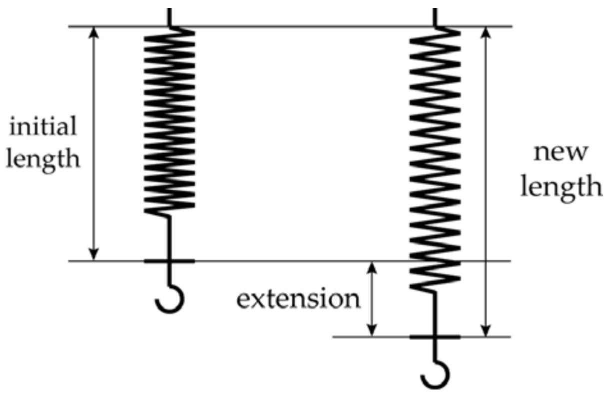


Figure 3.2: Illustration of the extension of a spring.

Table 3.1: Length of springs with mass hangers attached.

	Spring 1	**Spring 2**
Position of top of spring		
New position of pointer		
New length of spring		
Extension		

ii. Add one 20 gram disk to the mass hanger. Measure the new position of the pointer accurate to 1 mm, and calculate the new length of the spring and its extension (change in length). Enter your values in Table 3.2 below.

Note: The column 'extension' should contain the difference between the initial length of the spring without the mass hanger from Table 2.1, and the new length.

iii. Add five more 20 g disks, one-by-one, to the mass hanger and record the new lengths (accurate to 1 mm) for each disk in Table 3.2 below. In the column labelled 'total mass added to mass hanger', calculate the total mass added due to the 20 g disks.

Table 3.2: Measurements for Spring 1.

object added	total mass added to mass hanger (g)	new pointer position (cm)	new spring length (cm)	extension (cm)
disk 1	20			

iv. Starting from Section 2, repeat the same procedure with Spring 2. Record the data in Table 3.1 on the previous page and in Table 3.3 below.

Table 3.3: Measurements for Spring 2.

object added	total mass added to mass hanger (g)	new pointer position (cm)	new spring length (cm)	extension (cm)
disk 1	20			

Make sure you discuss your answers with a tutor before you proceed.

Section 4: Graphical analysis

This section deals with graphing the data you have gathered.

i. Add the data you have gathered for both springs to the graph below. You should plot the change in length of the spring in centimetres on the vertical axis and the total mass added to the hanger in grams on the horizontal axis. Draw the best fit line for the data plotted for each spring. Clearly label each line.

Experiment 1: Extension vs Total Mass Added to Mass Hanger

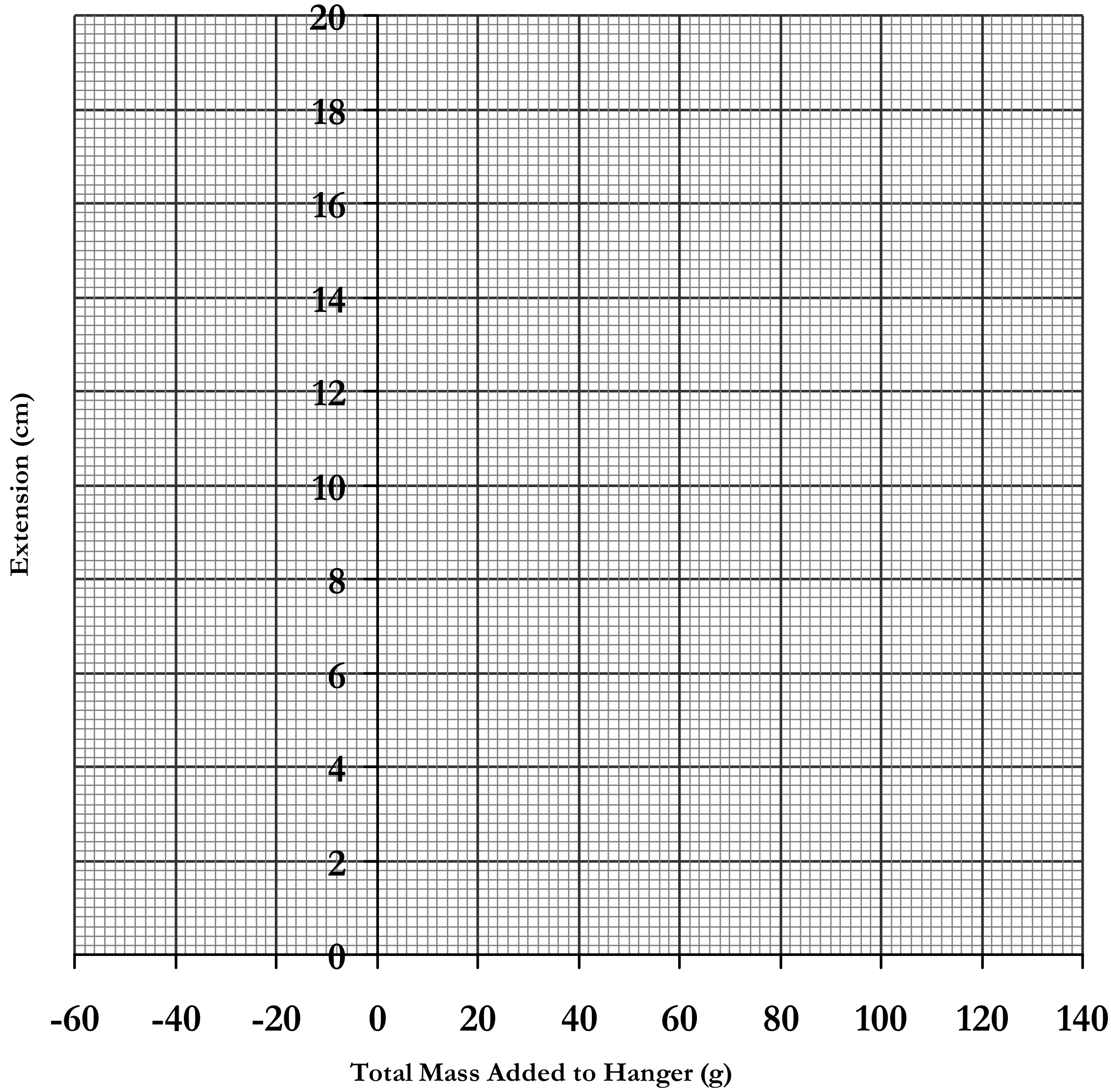


Figure 4.1: Graph representing the extension of the two springs for different masses added to the hanger.

ii. Suppose you were to carefully add a continuous stream of sand to the mass hanger when there are two disks attached to it. Use a pencil to indicate in your graph how the length of the spring changes. Explain.

____

____

____

____

____

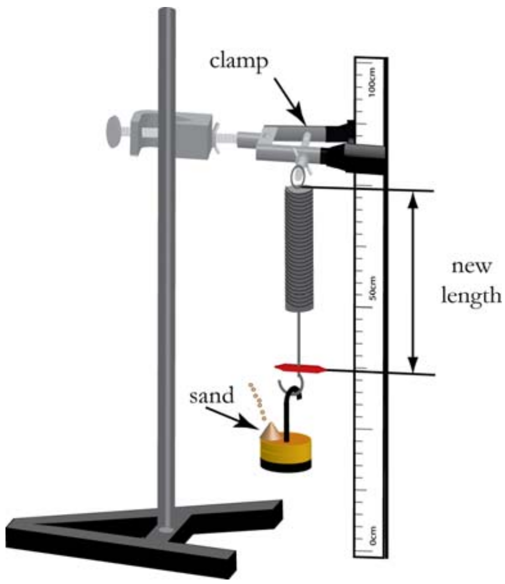


Figure 4.2: Adding sand to the mass hanger.

As you were drawing your graph, did you consider the following points?

- What quantities are plotted in the graph?
- What if you added a total of 20 g of sand?
- What if you added the sand more quickly?

Adjust your graph if necessary.

Section 5: Slopes

In everyday language we may use the word 'slope' to describe a property of a hill or a mountain. For example, we may say that a hill has a steep or a gentle slope.

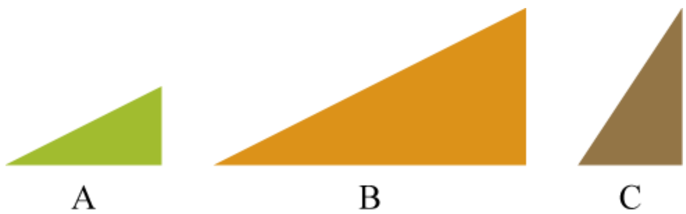


Figure 5.1: Three hills.

i. Rank the three hills of Figure 5.1 from greatest to smallest height.

____

____

____

____

Rank the three hills from steepest to gentlest slope.

In your own words, explain the difference between the height and the slope of a hill.

Generally, the **slope** tells you by how much the value on the vertical axis changes for a certain change of the value on the horizontal axis. The slope of a graph is used to highlight important properties of a specific experiment.

ii. Examine the graph you have drawn in Section 4 and describe in your own words the 'steepness' of the slopes for Spring 1 and Spring 2.

How can you use the slopes of the two best fit lines to compare the stiffness of the springs?

Section 6: Making a spring balance

The investigations you have carried out in the previous sections can now be used to construct your spring balance.

i. Set up the same experiment as in Section 2 with the mass hanger attached to the spring.

   Attach an object that is not heavier than the six slotted disks combined (e.g., a key, or a pen) to the mass hanger.

   Measure the extension of the spring and record your data. Think carefully about how you calculate the extension: what value should you use for the initial length?

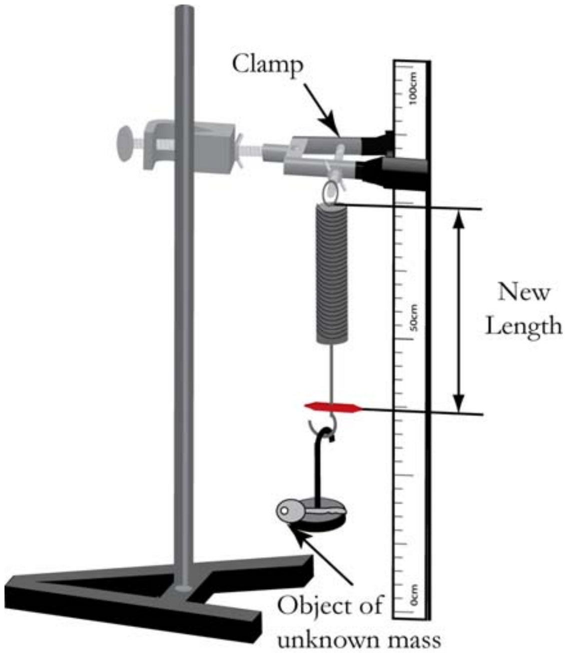


Figure 6.1: Measuring the mass of an unknown object.

____

____

ii. Use your graph of Figure 4.1 to determine the mass of the object you put on the mass hanger. Describe how you did this.

____

____

____

____

iii. To obtain the most accurate value for the mass of your object, which spring should you use? Explain. If necessary, use the other spring to do this.

____

____

____

____

____

iv. Measure the mass of your object on a digital spring balance in the laboratory. Enter the value in Table 6.1 below and compare this value to the mass you found using your balance.

Table 6.1: Mass of the object attached to the mass hanger.

Mass of object	
Spring Balance (g)	**Digital Spring balance (g)**

Do you think the spring balance you made is a good one?

____

____

Section 7: Further analysis

To analyse graphs further we can consider the following case:

> On a nice summer's day Dan decides to walk up the mountain nearby his house, which has a constant slope. Being an eager climber Dan decides to bring his altimeter so he can tell his height above sea level. However he doesn't make any readings until he has walked 100 m from his house and his altimeter tells him that he is 200 m above sea level. After 200 m he is 250 m above sea level, after 400 m he is 350 m above sea level and after 500 m he reaches a height of 400 m above sea level.

i. Complete the graph below. Add a best fit line which you feel represents the case above.

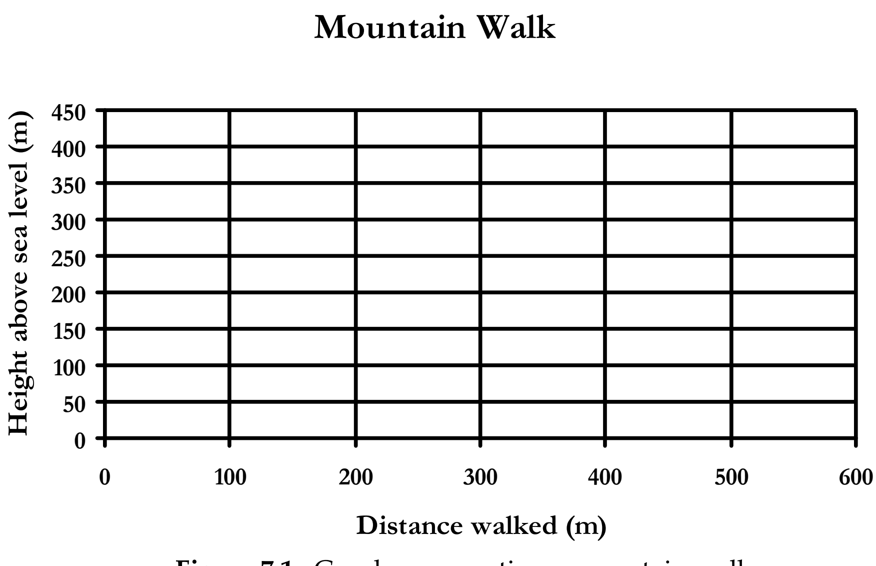


Figure 7.1: Graph representing a mountain walk.

ii. Dan walks back home. If the slope is steady, can you use the graph to find the height of Dan's house above sea level? If so, how?

____

____

____

The points where the best fit line meets the horizontal and vertical axes are known as the **intercepts** of the graph. Like the slope of the graph, intercepts often relate useful information about an experiment.

iii. Refer to the graph that both of you drew in Section 4 on page 5. If you have not done so already, extend the best fit lines for each spring until they intersect both axes.

iv. Suppose you took one disk off the mass hanger. How could you use the graph to find the new extension of the spring?

____

____

____

If all 20 g disks were removed, would the spring attain its initial length? Explain how you can tell from the graph. Is there still mass attached to the spring?

____

____

____

Make sure you discuss your answers with a tutor before you leave the lab.

v. Enter the values at which the best fit lines for Spring 1 and Spring 2 intersect the vertical axis in Table 7.1 below.

Table 7.1: Intercepts with vertical axis.

Spring	Vertical intercepts (cm)
1	
2	

Does the measurement for each spring correspond to any measurement that you have previously taken? If so, state the measurement. (**Hint:** Take a look at your measurements from Section 3, pages 3 & 4.) For this experiment, what does this value tell you about the experiment?

vi. Now look at where the best fit lines of Springs 1 and 2 intersect the horizontal axis. When analysing the graph you saw that the extension varies regularly as disks are removed from the hanger. Imagine removing the hanger. How would you trace the change in length of the spring on the graph?

Write down the values where the best fit lines for Spring 1 and 2 intersect the horizontal axis in Table 7.2 below. What units should you use?

Table 7.2: Intercepts with horizontal axis.

Spring	**Horizontal intercepts (__)**
1	
2	

vii. How much mass is attached to the spring at the horizontal intercept? Discuss in your own words what the horizontal intercept tells you about this experiment. Make sure to discuss the sign (positive or negative) of the intercept.

Make sure you discuss your answers with a tutor before you leave the lab.

Experiment 4: Making a grandfather clock

Check that you have at your disposal: a retort stand, a piece of string, four different metal cubes, a cork, a protractor, two spring balances, and a stopwatch.

Figure 1.1: Experimental set-up.

Section 1: Exploration

i. Set up the string, one of the metal cubes, the cork, the protractor and the retort stand as shown in Figure 1.1. Get the cube to swing over a range of angles and string lengths.

   You do not need to make accurate measurements here – just observe the motion in a qualitative way. Describe the motion below. Try different cubes.

   ____

   ____

   ____

   ____

   ____

   ____

   ____

ii. In the remainder of this experiment, you will investigate the time it takes for a block to swing a set number of times under different conditions. For example, you will investigate the effects of having cubes with different mass attached to the string. Based on your observations, decide on the values for the length of the string, number of swings and starting angle you will use in this experiment. (One swing counts as the bob swinging from the position on one side over to the other and back to its original position.)

Figure 1.2: Illustration of a swing.

Table 1.1: Fixed quantities for the quantitative pendulum experiment of Section 2.

Quantity & symbol	**Value**
Number of swings for your experiment (between 1 and 15), N	
Fixed starting angle for your experiment, α	
Fixed length of the string, l_{string}	

Comment on your decisions.

Describe in some detail the control of variables in this experiment.

iii. Many students in the class are likely to have chosen different values for the number of swings, N, the starting angle, α, and the length of the string. Suppose you had chosen the same starting angle and length of string as another student, but had picked a different number of swings.

Based on your explorations thus far, do you think you would have found the same or different values for the time t_N it takes to complete N swings? If the times are different, could you manipulate your data in a straightforward way that allows you to compare the experimental results?

Section 2: Graphs and motion

Figure 2.1 below charts the position of a ball on a taut string in an experimental set-up like that of Section 1. The angle between the string and the vertical is considered positive when the ball is to the right of the vertical and negative if it is to the left.

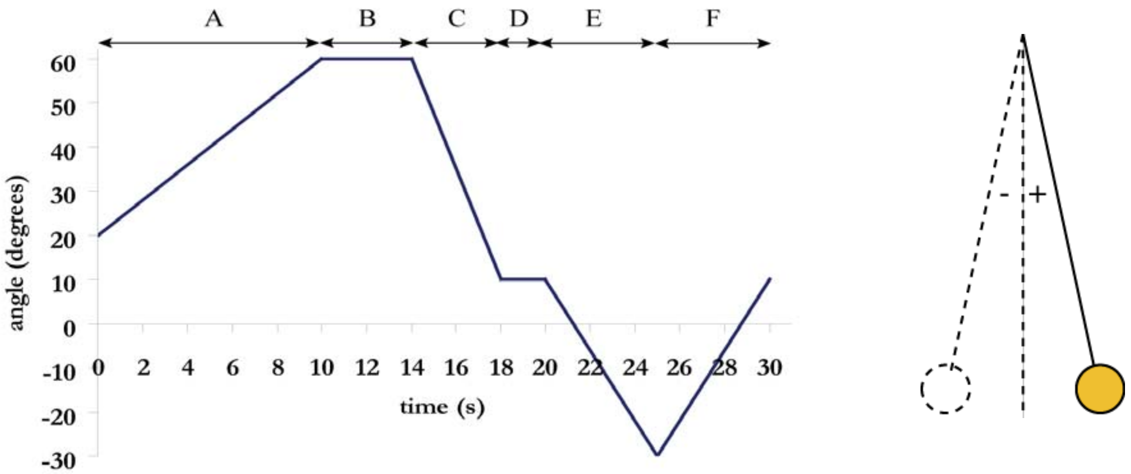


Figure 2.1: Graphical representation of the motion of an object on a taut string.

i. Could this graph represent the motion of the object of Section 1? Explain how you can tell.

____

____

____

ii. Using only the graph of Figure 2.1, compare the motion of the ball in terms of direction (clockwise or anticlockwise) and how quickly the object moves during the following pairs of intervals. (**Hint:** You may find it useful to attach one of the blocks to the string and execute the motion while answering the questions below).

a. Interval A and interval B

____

____

____

____

b. Interval A and interval C

c. Interval A and interval F

d. Interval C and interval E

iii. Now pull the object through a small angle while keeping the string taut. Release the object and let it swing freely. Which of the three graphs below best represents how the angle between the string and the vertical changes with time? Explain briefly.

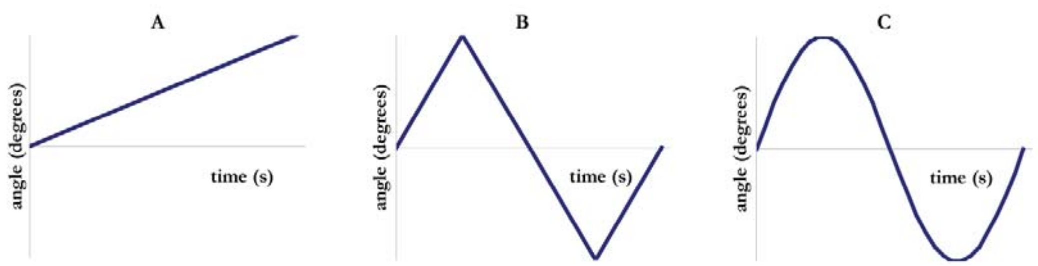


Figure 2.2: Three possible graphical representations of the motion of a freely swinging object on a string.

Section 3: The effect of mass on the swing of a pendulum

The **period** of a pendulum is defined as the time it takes to complete one swing. In this section you will investigate the effect of mass on the period.

i. Put forward a scientific hypothesis that predicts how mass affects the period of a swing. It is not important whether your hypothesis turns out to be correct; what matters is the process of checking your hypothesis. You should therefore not change your hypothesis after you carry out the experiment.

___

___

___

What aspects of your hypothesis make it scientific?

___

___

Plan an experiment to test your hypothesis.

___

___

___

___

___

___

___

Make sure you discuss your plans with a tutor before you continue.

ii. Set the pendulum swinging, and record the time t_N it takes to complete N swings. Repeat the experiment for the four cubes of different mass m. Use the spring balances to measure the mass of each cube as accurately as possible. Record your results in Table 3.1 below, then use your data to calculate the period T of the motion.

Table 3.1: Measurements taken at fixed length and starting angle.

m (g)	m (kg)	t_N (___)	T (___)

It is unlikely that you got the exact same period for each of the four blocks. Do you think that the differences are significant? Explain.

iii. Consider the following two hypotheses:

Hypothesis A: *"The greater the mass, the greater the period of the swing."*

Hypothesis B: *"Mass does not affect the period of the swing."*

Do the results of your experiments prove either hypothesis? Explain.

Do the results of your experiments support either hypothesis? Explain.

Do the results of your experiments falsify either hypothesis? Explain.

Section 4: The effect of length on the swing of a pendulum

In this section you will investigate how the length of the pendulum affects the time taken for a set number of swings.

i. Put forward a scientific hypothesis that predicts how length affects the period of a swing.

Plan an experiment to test your hypothesis.

Make sure you discuss your plans with a tutor before you continue.

ii. Adjust the experiment such that the pendulum length is 15 cm. All lengths should be measured from the centre of the cube to the pivot (i.e., the bottom of the cork). Set the pendulum swinging, and record the time t_N it takes to complete N swings in Table 4.1 below. Increase the length in steps of 10 cm and repeat the experiment for as many different lengths as you can.

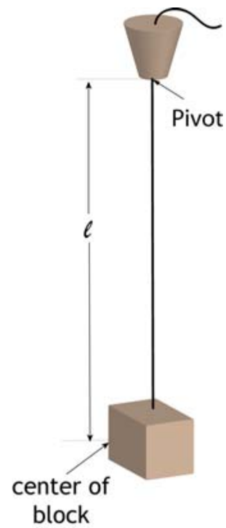


Figure 4.1: Measuring the length of a pendulum.

Table 4.1: Pendulum data taken at fixed mass and starting angle.

l (cm)	l (m)	t_N (___)	T (___)

Do your results support or falsify your hypothesis? Explain.

____

____

____

iii. Use your data from Table 4.1 to find the *change* in period ΔT for every change in pendulum length. For example, if you recorded a period of 0.78 s for a 15 cm pendulum length and a period of 0.98 s for a 25 cm pendulum length, you would enter '0.20' in the first row of the last column of Table 4.2.

Table 4.2: Changes in period with changing length.

l_1 (cm) to l_2 (cm)	Δl (cm)	ΔT (___)
15 to 25	10	
25 to 35		

Does the period change by the same amount for each 10 cm increase in the length of the pendulum? If not, do you think the differences are significant? Explain.

____

____

____

iv. In an experiment similar to yours, a student only took four data points. She changed the length of the pendulum by equal amounts each time.

Figure 4.2 at right shows four possible graphs for her data. Each graph shows how the period of the pendulum changes when the length is changed.

Assuming her data follow a similar pattern to yours, which graph best represents her data? Explain how the arrows help you obtain an answer.

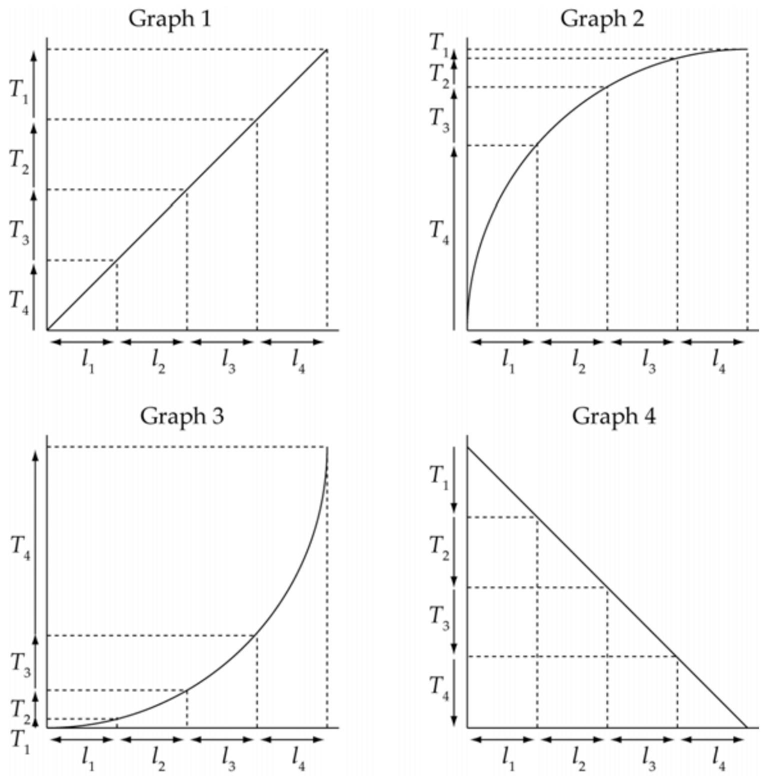


Figure 4.2: Possible best fit lines to show how the period of a pendulum varies with its length.

____

____

For the other three graphs, describe what data you would need to obtain them.

____

____

____

____

____

____

____

v. Plot your data in Figure 4.3 overleaf. Use your graph to estimate how long you would need to make the pendulum to get a period of 1.0 s, and verify your estimate experimentally. You have then put together some of the essentials of a grandfather clock.

Predicted length: __________
Experimental length: __________

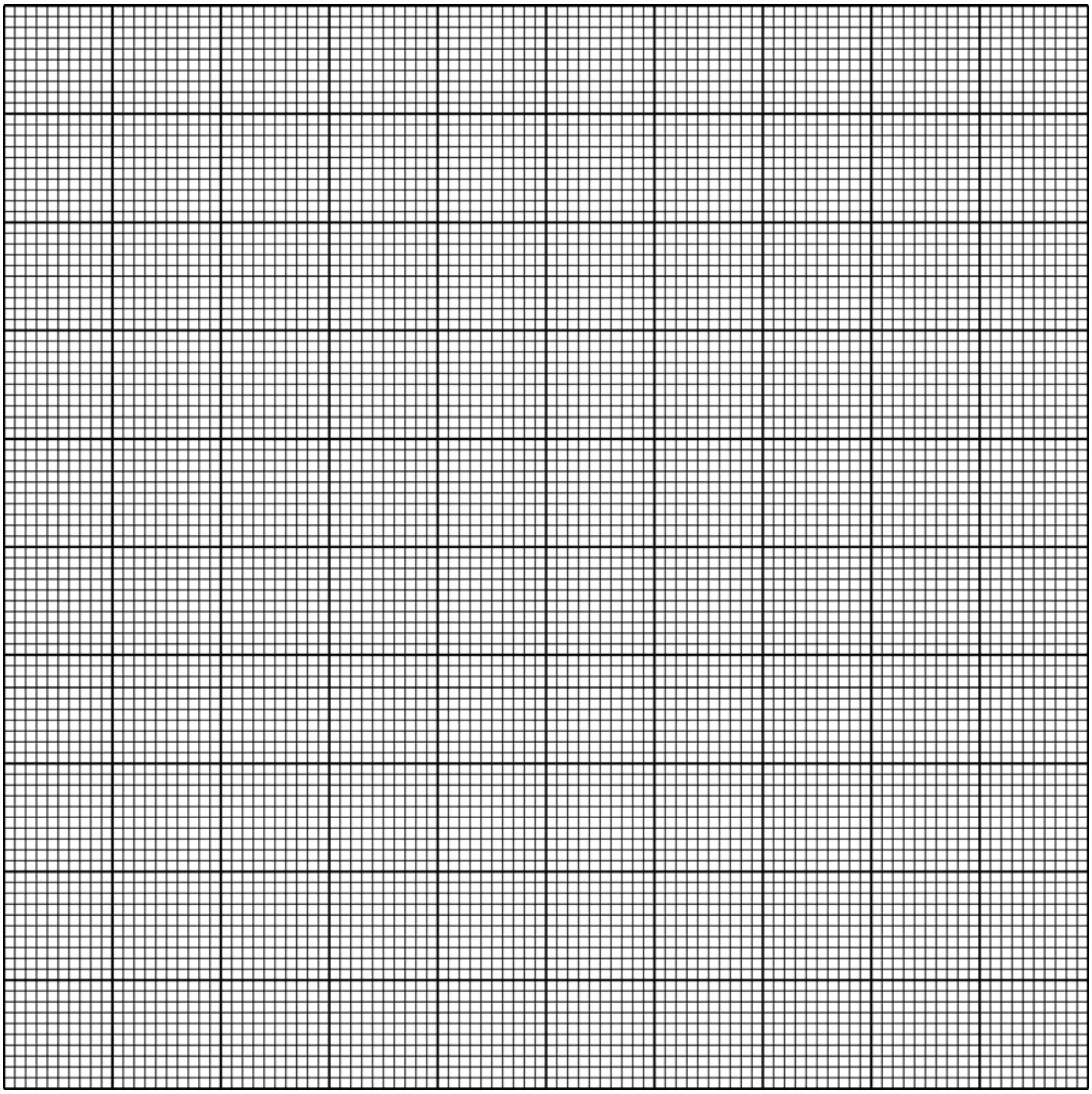


Figure 4.3: Variation of the period of your pendulum with length.

Experiment 3: Uniform motion

Make sure that you have at your disposal: a double track, a metre stick, two balls (marbles), two ramps, ten cubes, and a stopwatch.

Section 1: Exploration

Set up the track as in Figure 1.1 below. You will use only one ball and one of the ramps. Use the ramp to set the ball in motion and let it roll along the track. (**Hint:** It is useful to block the ball at the end of the track.)

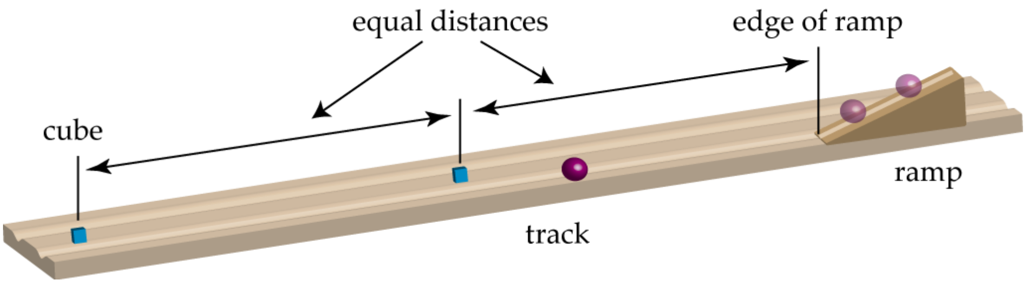


Figure 1.1: Flat track with ramp for uniform motion experiment.

i. Predict if the ball will speed up, slow down or travel at constant speed as it rolls along the track after is released from a point on the ramp. **You need only consider the time when the ball is on the track, not when it is on the ramp.**

____

____

____

____

____

Use the metre stick to divide the portion of the track the ball rolls on into two segments of equal length. Use two of the cubes as markers; **do not use pen or pencil to mark the track.** Allow the ball to run from a point at least halfway up the ramp.

Starting the stopwatch **when the ball gets onto the track,** record the time it takes for the ball to reach each of the cubes in Table 1.1 on the next page. Start the stopwatch when the ball gets onto the track. Then calculate the time it took the ball to get from the first to the second cube.

Note: <u>You can record both times with one roll of the ball if you follow this procedure:</u>

1. To start the stopwatch, press the right button. The timer will start.
2. When the ball passes the first cube, press the left button. The timer seems to stop but does not.
3. When the ball passes the second cube, press the right button.
4. To read the time when the ball passed the second cube, press the left button again.
5. To reset the stopwatch, press the left button once more.

Table 1.1: Time it takes the ball to traverse different segments of the track.

Time to reach first cube	
Time to reach second cube	
Time to get from first to second cube	

ii. Check if your prediction was correct. If not, give a likely reason why your prediction was incorrect.

iii. Plan an experiment to investigate the effect of the angle of the track on the motion of the ball on the track. As before, divide the track into two equal segments. Use some of the other cubes to change the angle of the track (see Figure 1.2).

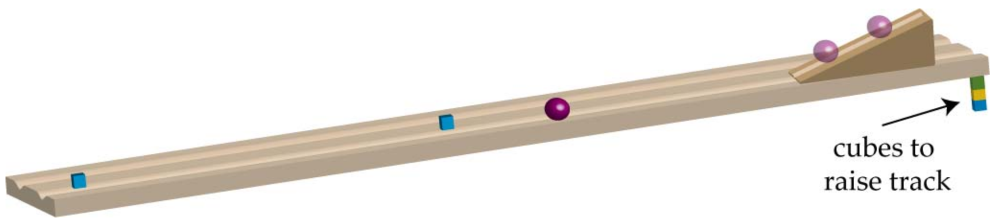


Figure 1.2: Angled track with ramp for uniform motion experiment.

Describe how you plan to control the variables as much as possible, and how you made your measurements as accurate as possible.

Adjust the angle of the track by changing the number of cubes under one end of the track (see Figure 1.2). With the track divided in two, make measurements to determine for which number of cubes you get closest to uniform motion.

iv. Release the ball from 1 cm below the original position on the ramp. Predict if the ball will travel in uniform motion, will accelerate, or will slow down along the track. Also predict whether it will travel faster, slower, or with the same speed.

Repeat the experiment using the same set-up, but release the ball from its new position on the ramp. Enter your results in Table 1.2 below.

Table 1.2: Time it takes the ball to traverse different segments of the track from a different position.

Time to reach first cube	
Time to reach second cube	
Time to get from first to second cube	

Does the position from which you release the ball affect whether it travels with constant speed along the track?

____

____

Section 2: Motion at constant speed

i. Set up the track such that the ball is in uniform motion (i.e., the ball rolls with constant speed).

Again using cubes, mark off four segments of equal length between the start and end points on the track as shown in Figure 2.1. If you are to release the ball from the same point on the ramp as before, is it possible to predict how much time it will take for the ball to reach each cube? If so, calculate the length of each time interval.

____

____

____

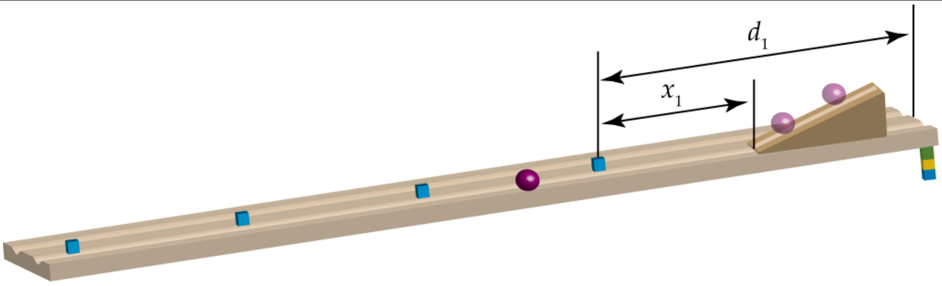


Figure 2.1: Angled track with five cubes. The distances x and d are shown for the first cube.

Measure the distance from each of the blocks to the ramp, x, and the distance to the edge of the track nearest the ramp, d. Carry out the experiment, and record the time it takes for the ball to reach each of the four cubes in Table 2.1, which you should give an appropriate title. Carry out your experiment at least twice to obtain an average for each of the times you measured.

Table 2.1: ________________________________

cube	x (cm)	d (cm)	t_1 (s)	t_2 (s)		t_{av} (s)
1						
2						
3						
4						

ii. Plot the motion recorded in Table 2.1 in two different ways in the distance-time graphs of Figure 2.2 below. In distance-time graphs, time is plotted on the horizontal axis while distance is plotted on the vertical axis. In the top graph, plot a distance-time graph for the distance from the ball to the ramp (i.e., plot x against t). In the bottom graph, plot the distance from the ball to the edge of the track nearest the ramp (d against t). **In both graphs, t=0 when the ball leaves the ramp.**

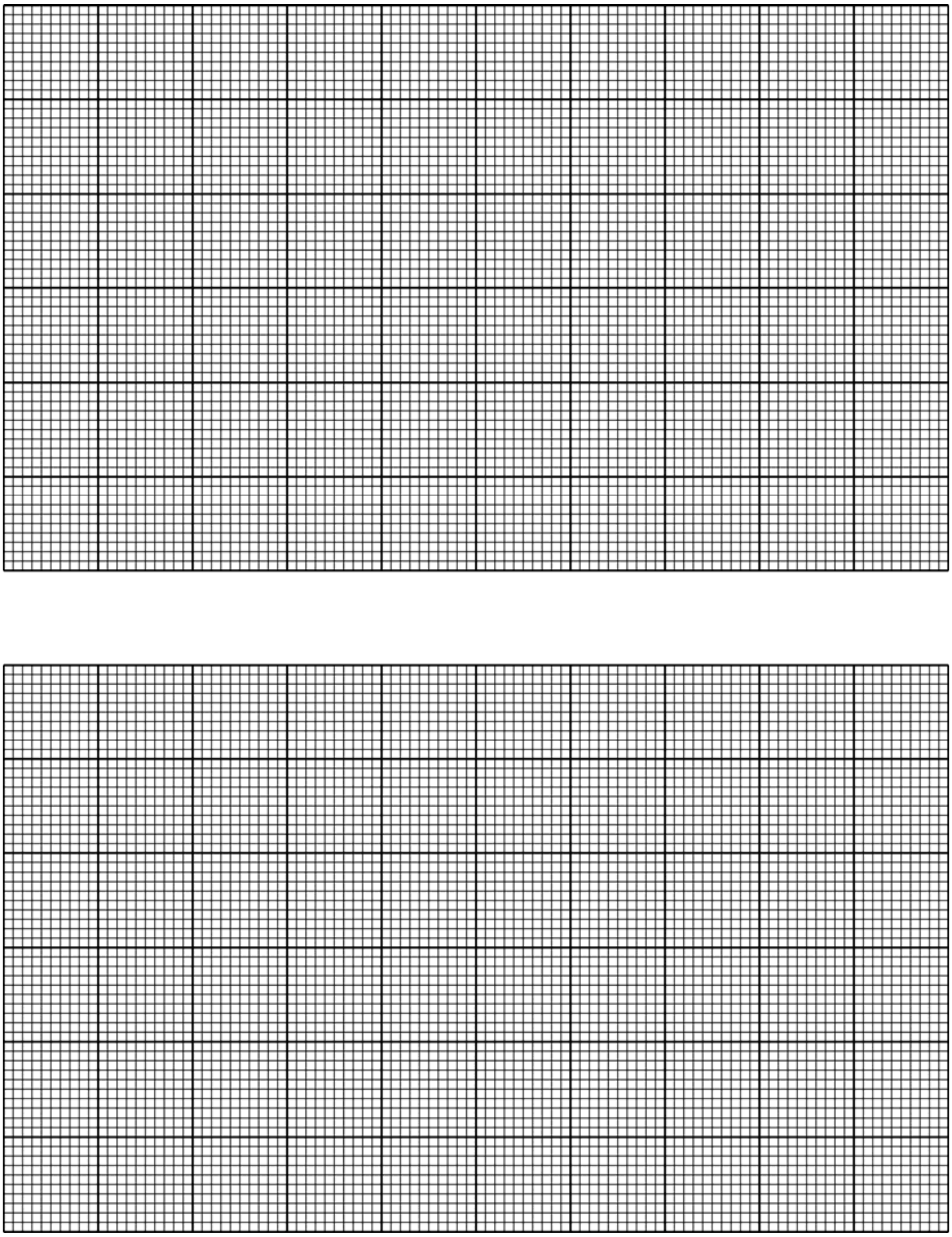


Figure 2.2: Distance-time graph for ball rolling along a track with constant speed. Top: measured from the ramp. Bottom: Measured from the edge of the track nearest the ramp.

iii. Take two points on the top graph of Figure 2.2 that are quite far apart. Label those points A and B, and enter the values for the distance, x_A and x_B, and time, t_A and t_B, in Table 2.2 below.

Read values for the distance to the edge of the track at times t_A and t_B, d_A and d_B, from the bottom graph, and enter your values in Table 2.2.

Table 2.2: ______________________________

Quantity	value
x_A	
x_B	
t_A	
t_B	
d_A	
d_B	

iv. Consider the following student conversation.

Student 1: *"The speed of the ball is constant. I can calculate this speed either by dividing distance over time at point A, or at point B – it does not matter."*

Student 2: *"The speed between A and B is given by the distance travelled between points A and B, divided by the time taken to travel from A to B."*

Student 3: *"I think you're both right – you're saying the same thing in a different way."*

In the space below, write out the two calculations proposed by Student 1 using some or all of the variables x_A, x_B, t_A, and t_B. In each case, obtain a numerical value.

In the space below, write out the two calculations proposed by Student 1 using some or all of the variables d_A, d_B, t_A, and t_B. In each case, obtain a numerical value.

In the space below, write out the calculation proposed by Student 2 using some or all of the variables x_A, x_B, t_A, and t_B. Obtain a numerical value.

In the space below, write out the calculation proposed by Student 2 using some or all of the variables d_A, d_B, t_A, and t_B. Obtain a numerical value.

Explain why the two graphs of Figure 2.2 must represent motion with the same speed.

Which of the calculations above give(s) you the numerical value of the speed of the ball? Explain briefly.

With which, if any, of the three students do you agree? Explain.

v. In Graphs 1-4 in Figure 2.3 below, t_1, t_2, t_3, and t_4 represent four equal time intervals, while x_1, x_2, x_3, and x_4 represent the corresponding change in distance during each time interval. Which of the graphs represent(s) uniform motion? Explain.

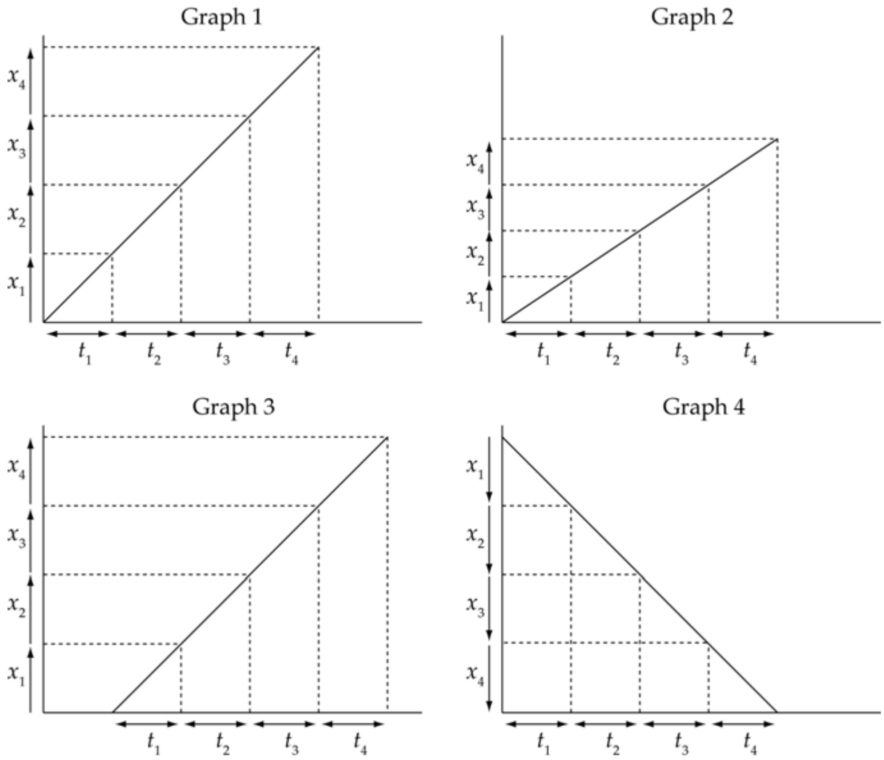


Figure 2.3: Four graphs representing motion of an object.

What is different about the motions each graph represents? Explain.

vi. Three students have carried out an experiment similar to yours. They have plotted their results in the graph shown in Figure 2.4 below.

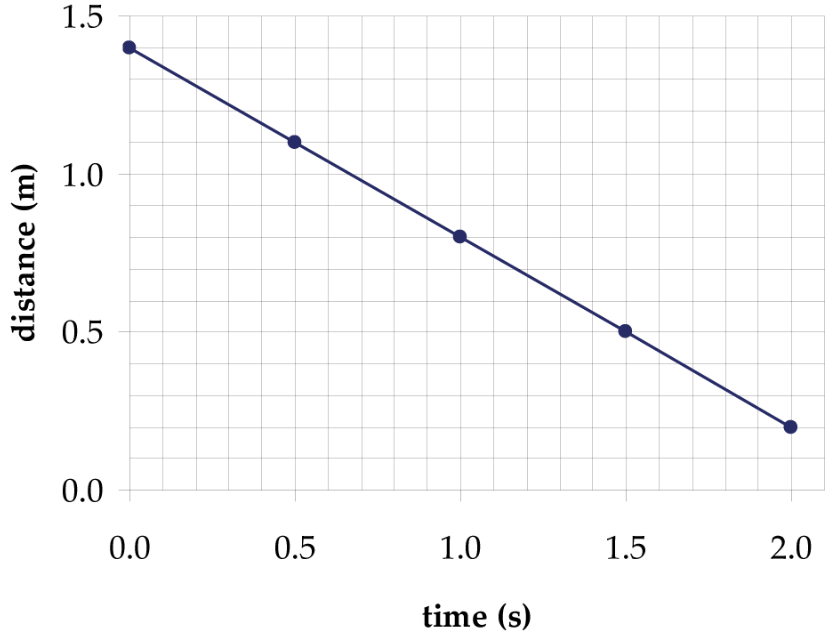


Figure 2.4: Distance-time graph for ball rolling along a track with constant speed obtained in an experiment similar to yours.

Consider the following three statements made by the students:

Student 1: *"I know that the speed of the ball was constant, because we got a straight line graph."*

Student 2: *"I agree. You can see that the ball gets closer to the edge by 30 cm every 0.5 seconds, so the speed of the ball is 0.6 m/s."*

Student 3: *"I think the ball is slowing down. Speed is distance over time. After half a second, the ball was 1.1 m from the edge, so the speed was 2.2 m/s. Then after one second, the ball was 80 cm from the edge, so the speed was 0.8 m/s."*

With which student(s) do you agree? Explain.

Section 3: Two-dimensional uniform motion

i. Obtain a sheet of paper divided lengthways into four segments of equal length. Place the long side of the sheet parallel to the long side of the table, and place the track on the sheet such that the location where ball gets onto the track lies directly above one corner of the sheet, and the end point of the track is above the diagonally opposite corner. **Make sure the ball travels in uniform motion along the track.**

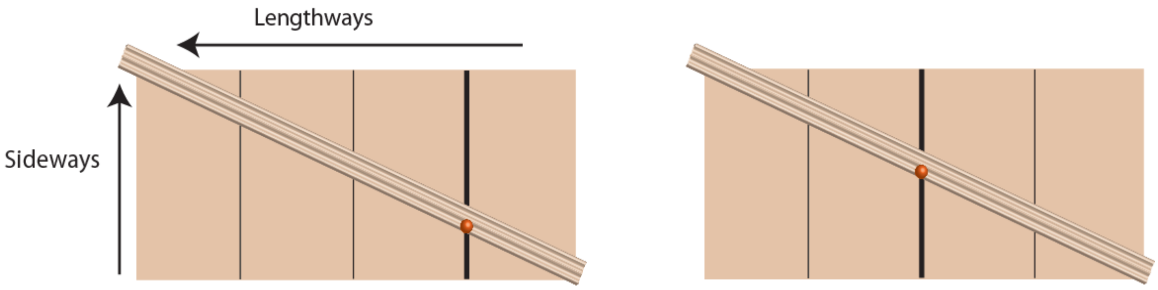


Figure 3.1: Grid for investigation of motion in two dimensions.

ii. Consider the following hypothesis:

> *"While the ball is rolling with constant speed along the track, it will take equal amounts of time to traverse each segment on the paper."*

Set up an experiment to test this hypothesis. Record your results in Table 3.1 below.

Table 3.1: ____

iii. Do the results of your experiment confirm of falsify the hypothesis?

____

____

____

When the track is placed diagonally across the sheet, you can think of the motion of the ball as consisting of two components: one component parallel to the long side of the sheet ("lengthways"), and one component parallel to the short side of the sheet.

iv. While the ball is rolling with constant speed along the track, can you consider it to be moving with constant speed in the lengthways direction also? Explain.

____

____

____

v. Turn the sheet over to the side that is divided sideways into three segments of equal length. Investigate whether the ball travelling at constant speed along the track is travelling at constant speed in the sideways direction. Enter your data in Table 3.2 below.

Table 3.2: ____

What conclusion do you draw from your investigation?

____

____

____

____

____

Section 4: Two balls

i. Imagine two tracks, A and B, are set up beside each other. Figure 4.1 shows distance-time graphs for two balls moving on the tracks **in a single experiment.**

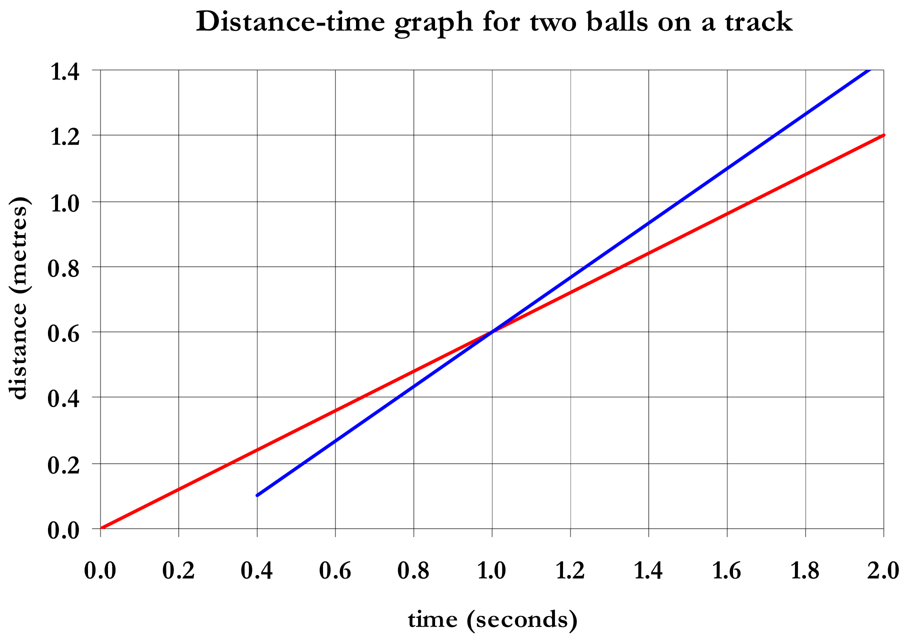


Figure 4.1: Distance-time graph for two balls rolling along two tracks.

ii. Using Figure 4.1 only, answer the questions below with a brief explanation.

a. Do both balls travel with constant speed?

____

____

b. Do both balls travel with the same speed?

____

____

c. Do both balls start at the same time?

____

____

d. Which ball passes the end point on the track first?

____

____

e. Does one of the balls overtake the other?

iii. Set up an experiment that allows you to reproduce the motion of the two balls displayed in Figure 4.1 as accurately as possible. Report on your work below.

Name: ____ Class: ______

Experiment 5: Non-uniform motion

In this experiment you will investigate non-uniform motion in one and two dimensions. Make sure that you have at your disposal: a 2 foot by 4 foot board, a ball (marble), a ramp, a sheet of paper, about twenty cubes, and a stopwatch.

WARNING!

You will be asked to support the board in various ways. The board is quite heavy – **make sure your fingers do not get caught under the board.** You may find it useful to put a small (dispensable) object under the board for safety.

Section 1: Getting started

i. Place the ramp and the sheet of paper (pre-marked with lines 25 cm apart in the lengthways direction) on the board as shown in Figure 1.1 below. Use the board, the cubes and the ramp to set up an experiment to investigate non-uniform motion.

When the ball gets onto the board, it should **clearly** slow down as it travels along the length of the board. **The ball should reach the 100 cm line and take between 1.5 and 2.0 seconds to do so.** Adjust the number of cubes under the board if necessary.

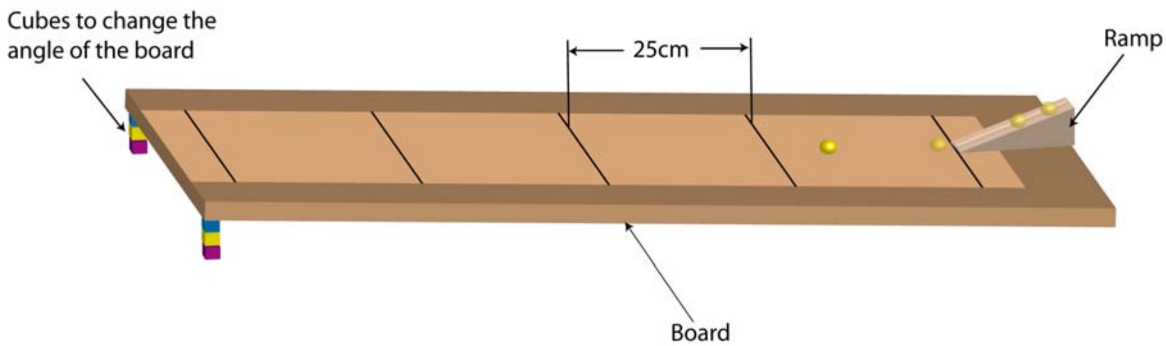


Figure 1.1: Arrangement of ramp, board and sheet of paper to investigate non-uniform motion.

Describe your set-up in some detail. What are the variables over which you have control that affect the motion of the ball along the track? How did you ensure your experiment can be carried out repeatedly and give reproducible results?

____

____

____

____

____

____

____

____

____

____

ii. Record the time it takes the ball to reach the lines at 50 cm and 100 cm from the ramp. Then calculate the time it took the ball to get from the 50 cm line to the 100 cm line. Repeat the experiment and calculate the average times in each case.

Table 1.1: Time it takes the ball to traverse different segments of the board.

	Attempt 1	**Attempt 2**	**Average**
Time to reach 50 cm			
Time to reach 100 cm			
Time to get from 50 cm to 100 cm			

Section 2: Measuring change in motion

i. If you are to release the ball from the same point on the ramp as before, is it possible to predict how much time it will take for the ball to reach each of the lines at 25 cm, 50 cm, 75 cm, and 100 cm from the ramp? If so, calculate each time. Explain.

____

____

____

Record the time it takes for the ball to travel 25 cm, 50 cm, 75 cm, and 100 cm in Table 2.1 below. You should give the table an appropriate title.

Table 2.1: ____

ii. Shown in Figure 2.1 below are three possible best fit lines for your data.

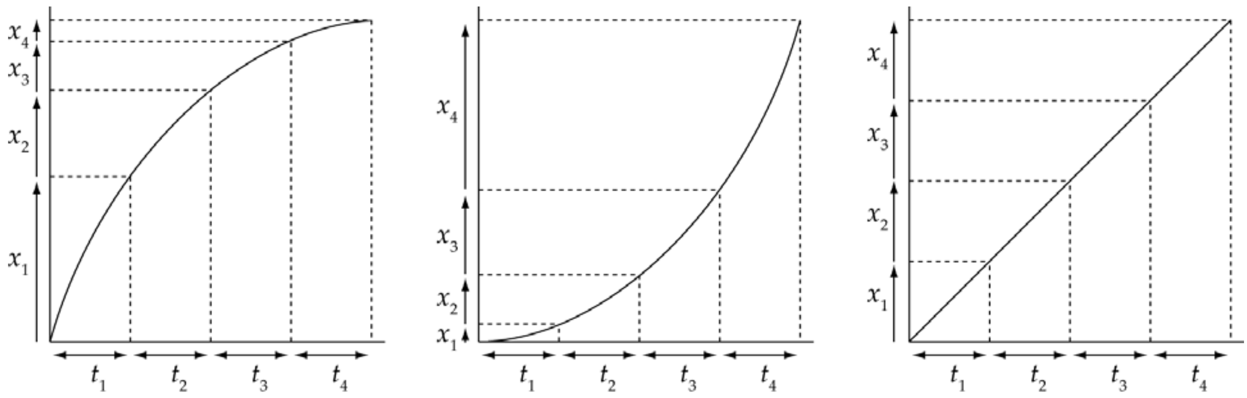


Figure 2.1: Possible best fit lines to the data in Table 2.1.

Which of the graphs above best represents the motion of the ball as it travels up the board? Explain.

____

____

____

iii. Plot the motion you just recorded in a distance-time graph in Figure 2.2 overleaf. Time should be plotted on the horizontal axis, distance on the vertical axis. Decide whether to draw a straight-line through the points or a smooth curve. Explain your choice.

____

____

____

____

____

____

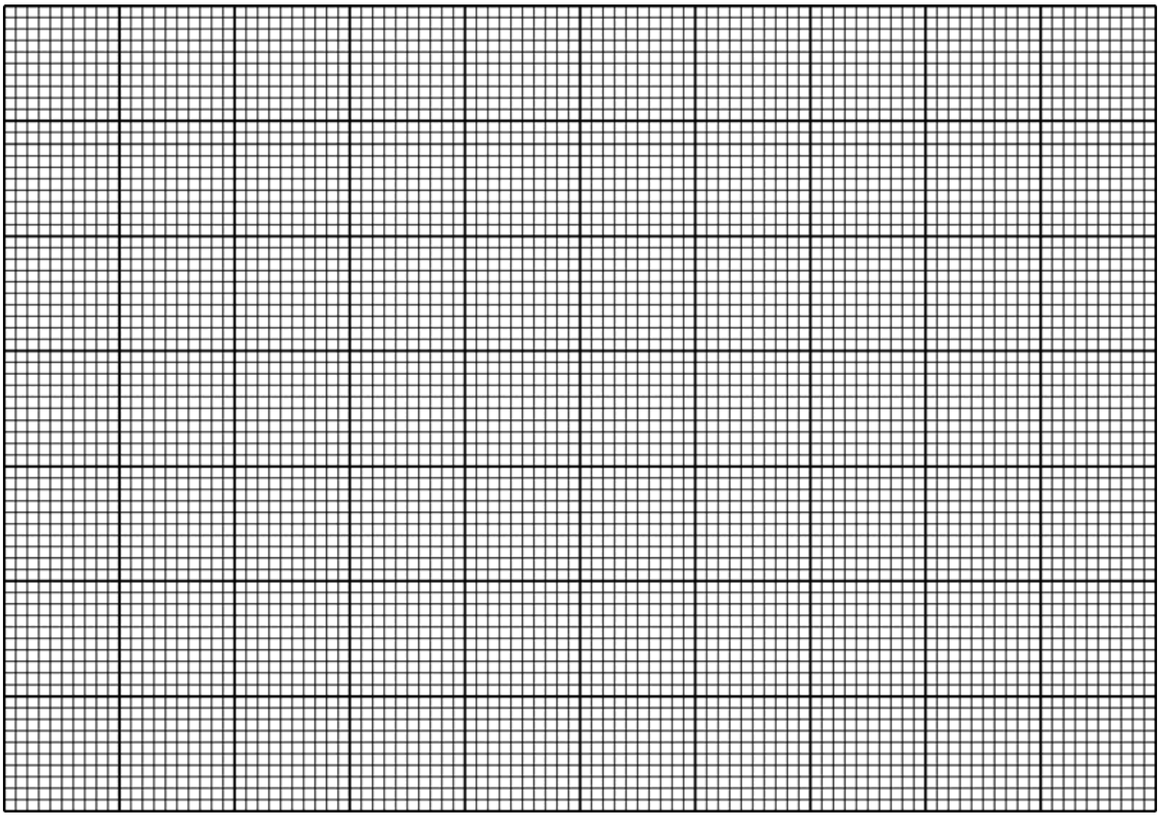


Figure 2.2: Motion of a ball on an incline.

iv. In your prelab you practiced the use of tangents to determine the slope of a curved line at a specific point. Use this technique to determine the speed of the ball at four different points along the track. (They do not have to be the points at which you measured the times).

You can determine two of the speeds while your partner determines the other two. Record the speeds you obtained along with their corresponding time in a suitably titled table below. Include your partner's results in your table.

Table 2.2: ____

Section 3: Two-dimensional motion

i. Set up the equipment as in Figure 3.1 below in such a way that uniform motion is attained when the ball is released from the ramp. (**Hint:** You will probably need to raise the short side of the board by three to six cubes to achieve uniform motion. You will need to verify how many cubes exactly you need.)

   Adjust the height from which you release the ball so that it takes about 1.6 seconds for the ball to travel between the 0 cm and 100 cm lines. Taking measurements at two points, e.g. the 50 cm and 100 cm lines, will suffice. Enter your data in Table 3.1 below.

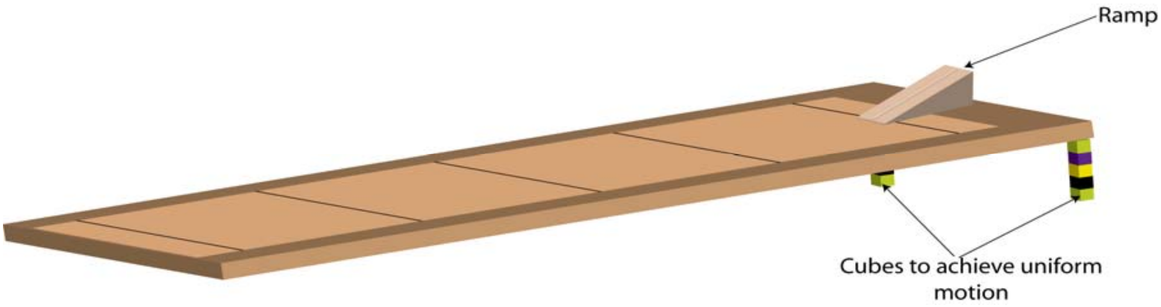


Figure 3.1: Cubes added to achieve uniform motion.

Table 3.1: Parameters when the ball is in uniform motion.

	Attempt 1	Attempt 2	Average
No. of cubes needed for uniform motion, *N*			
Time to reach 50 cm			
Time to reach 100 cm			
Time to get from 50 cm to 100 cm			

ii. Remove the *N* cubes. Now raise the long side of the board by placing two three-cube stacks under the board as shown in Figure 3.2.

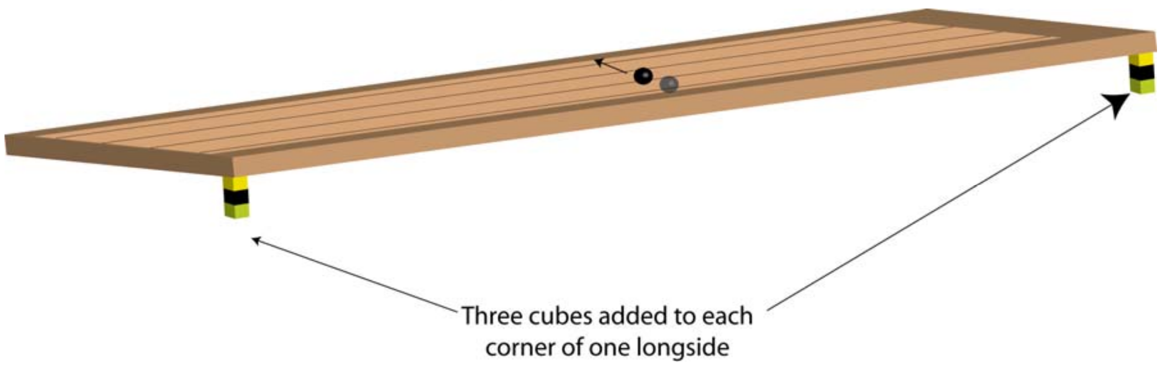


Figure 3.2: Cubes used to raise the board for accelerated motion.

Release a ball from rest near the high side of the board. **Do not use the ramp here.** Investigate qualitatively the motion of the ball on its release.

You have seen that by raising the short side by *N* cubes, uniform motion is attained in the lengthways direction. When you raised the long side by three cubes, the ball undergoes a different type of motion in the sideways direction. Now you will investigate what happens when we do both simultaneously.

iii. Combine the two effects achieved in Parts 1 and 2 of this section by adding the number of cubes (*N*) needed to achieve uniform to the same corners as in Figure 3.1. As a result, you should have one corner with no cubes under it, one corner with 3 cubes, one corner with *N* cubes, and one corner with *N*+3 cubes; see Figure 3.3 below.

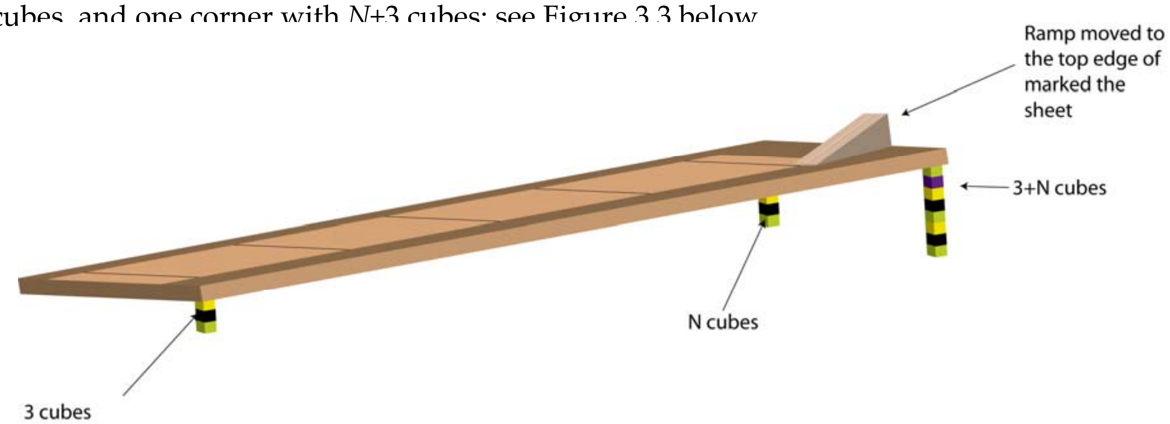


Figure 3.3: Adding *N* cubes to two corners the set-up of Figure 3.2.

Move the ramp to the high side of the paper as shown. Release the ball from the same point on the ramp. Sketch path the ball takes in Figure 3.4 below.

Figure 3.4: Top view of the observed path of the ball on the board.

Investigate quantitatively whether the ball speeds up, slows down, or moves with constant speed in the lengthways direction. Explain.

Investigate quantitatively whether the ball speeds up, slows down, or moves with constant speed in the sideways direction. (You need to flip over the sheet of paper.) Report on your investigation.

Does it seem to be possible to change the motion of the ball in the lengthways direction without changing its motion in the sideways direction? Explain.

iv. A student makes the following statement:

> *"I measured that it took the ball less time to traverse the second segment in the sideways direction than the first, but my measurements are not exact. It is impossible to tell if the motion of the ball in the sideways direction is uniform or not."*

Do you agree with this student? If not, explain how you would try to persuade the student.

Make sure you discuss your answers with a tutor before you leave the lab.

Section 4: Representing non-uniform motion in a graph

i. Figure 4.1 shows a ball on a **hypothetical** track consisting of five segments 1-5 of equal length. The ball is released with zero initial speed from the top of segment 1.

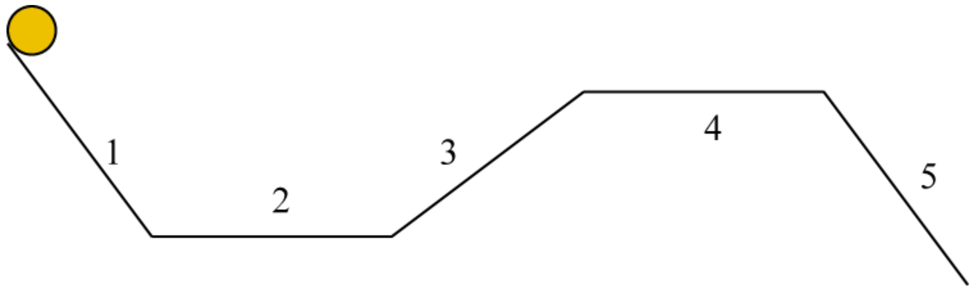


Figure 4.1: Side view of a hypothetical track and a ball.

a. On what, if any, of the segments does the ball travel with constant speed? If there is more than one such segment, how do the speeds on these segments compare? Explain.

b. On what, if any, of the segments does the ball travel with increasing speed? If there is more than one such segment, how do the initial speeds and the accelerations on these segments compare? Explain.

c. On what, if any, of the segments does the ball travel with decreasing speed? If there is more than one such segment, how do the initial speeds and the accelerations on these segments compare? Explain.

ii. Figure 4.2 below shows five different distance-time graphs A-E. All graphs have the same scale, and each represents the motion of the ball on one of the segments.

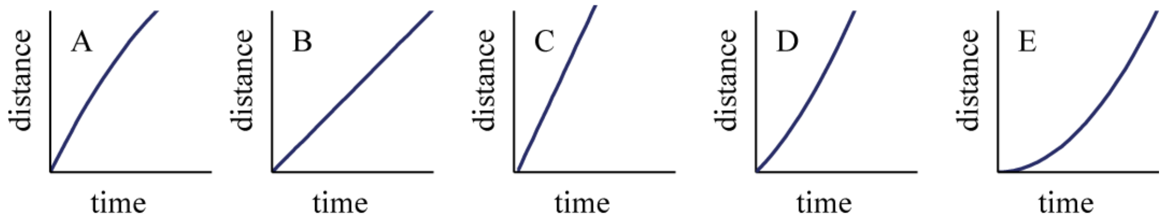


Figure 4.2: Five distance-time graphs. The scales on each graph are the same.

Use your answers to part i above to identify which graph represents the motion on what segment of the track. Explain briefly.

____

____

____

____

____

____

____

____

d. Sketch the shape of the distance-time graph for the entire motion along the track in Figure 4.3 below.

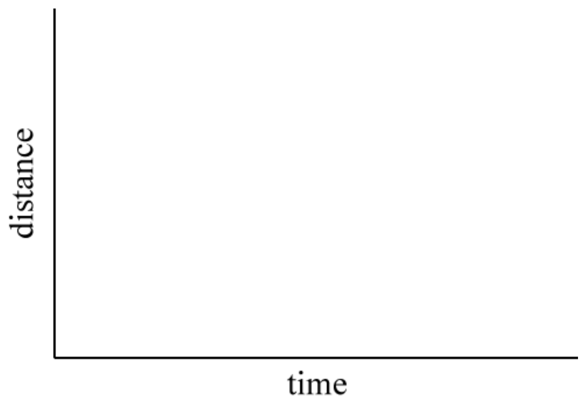


Figure 4.3: Distance-time graph for the ball on the track of Figure 4.1.

Make sure you discuss your answers with a tutor before you leave the lab.

Name: ______________________________________ Class: ______

Experiment 9: Bouncing balls

Check that you have at your disposal: a pingpong ball, a meter stick, a retort stand with clamp, and a stopwatch.

Section 1: General comments

This is the first investigation that you will carry out in the physics labs. You will first set up investigations into two prescribed aspects of the motion of a bouncing ball. In the last section, you will investigate any aspect you choose.

The structure of each investigation is a lot like what you have done in the labs so far. For each of the investigations you will carry out, think about the following issues:

- What do I need to measure to verify my hypothesis?
- How can I achieve control of variables?
- How many different measurements will I make?
- How many repeat measurements will I make?
- How can I make your experiment as accurate as possible?
- Is the accuracy of your experiment sufficient to falsify or confirm my hypothesis?

Notes on how to write your reports are given in the Appendix.

Section 2: Multiple bounces

In this section you will investigate how the maximum height reached by the ball changes after successive bounces on the floor or table.

You are to investigate two hypotheses:

Hypothesis 1: *"The maximum height reached by the ball decreases by the same amount after every bounce."*

Hypothesis 2: *"The ratio of maximum heights reached by the ball on successive bounces is constant."*

A few hints to help you with the investigation:

- Ensure that the height from which you drop the ball and the maximum height it reaches after the first bounce are approximately constant
- Make measurements for a reasonable number of bounces
- Repeat the experiment a reasonable number of times, and average your results
- State whether you have falsified or confirmed the hypothesis
- Plot the maximum height against the number of bounces (i.e., plot the maximum height on the vertical axis and the number of bounces on the horizontal axis). You may treat the release height as the height after zero bounces. Explain what sort of line you drew.

Section 3: Drop time and rise time

When you drop a ball from a height, it takes a certain time to reach the floor or table (which we will call the *drop time*), before it bounces back up and rises to a different height. We will call the time it takes the ball to get from the floor or table to its highest point the *rise time*.

You are to investigate the following hypothesis:

> *"The rise time is greater than the drop time."*

A few hints to help you with the investigation:

- Ensure that the height from which you drop the ball and the maximum height it reaches after the bounce are approximately constant
- Measure the drop and rise times in such a way that e.g. your reaction time when using the stopwatch does not impede interpretation of your results. For best results, place the metre stick in a retort clamp on the table and measure the drop time to the **floor**.
- Repeat the experiment a reasonable number of times, and average your results
- State whether you have falsified or confirmed the hypothesis

Section 4: Further investigations

In this section, you are free to carry out any investigation on bouncing balls that you like – provided it has something to do with physics, and it doesn't interfere with others – with the materials available in the lab. To give you some ideas, you could investigate the following questions:

- How does the drop time change when the ball is released from different heights?
- How much time elapses between successive bounces?
- Do balls bounce differently off different materials?
- Does the horizontal speed of a ball affect the drop time?
- Does a drop affect the horizontal speed of a ball?
- Is there a relationship between the incident and rebound angles?

There are many more experiments you can do, and many variations to the experiments suggested above. Remember that this is a two-and-a-half hour lab: you must finish your individual reports on your investigation(s) before leaving the lab.

You will be graded on the quality of your scientific hypothesis, your data, the way you represented your data, your conclusions, and the clarity of your report.

Appendix: Reports

Your report on your investigations should contain the following elements:

- A brief **introduction** which includes the hypothesis that you wish to put to the test
- A description of the **experiment.** To help you describe the experiment consider the following points:
  - Use of diagrams to show the set-up is recommended!
  - Include a discussion of the control of variables.
  - How many different measurements will you make?
  - How many repeat measurements will you make?
  - How did you make your experiment as accurate as possible?
- Your **results,** including tabulated data and graphs. Write a brief discussion considering the following points:
  - Is your best-fit line a smooth curve or a straight line? Why did you choose one over the other?
  - Can useful information (e.g., slope, intercept, etc.) to be gleaned from the graph?
- A **conclusion** – can you confirm or reject your hypothesis?
  - How does the shape of the graph or your data verify or falsify your hypothesis?
  - Is the accuracy of your experiment sufficient to decide either way?

Your **individual** report is due at the end of the lab session. Give **each** investigation a title and use headings for the different sections within each investigation (e.g., introduction, experimental set-up, results and discussion, conclusion).

Experiment 10: The Egg Bungee

Check that you have at your disposal: a spring, a slotted mass set, a metre stick with clamps and a retort stand.

Section 1: Introduction

In this experiment, the challenge is to get an egg that is attached to a spring (bungee) to fall as close to the table as possible **without breaking.** You will only get the egg after you have collected data that allows you to **predict** from what height you should release the egg. All eggs in the lab have a mass between 50 and 95 g. You will measure the mass and the dimensions of the egg after you have collected all other data.

Your investigation falls into three sections:

- Preliminary observations & questioning
- Collection of data that will allow you to predict how close the egg will get to the table without hitting it
- The egg bungee jump and evaluation

All questions should be answered on blank sheets of paper, each with your name and date on it.

Notes on how to write your reports are given in the Appendix.

Section 2: Preliminary observations

As always, you are encouraged to plan your experiment and explore your experimental set-up before making measurements. Make sure your investigations include the following points:

- Drop the mass hanger from an **unstretched** spring. Observe what happens when you change some of the variables. Discuss what variables you changed and what you observed.
- Discuss the challenges in accurately recording the minimum distance of the mass hanger to the table. What is the easiest way for you to record the distance?
- Draw free body diagrams to describe the forces on the mass hanger just before you let go of the mass hanger, just after you let go of it and it is in free fall, and when the spring is at its maximum extension.

Section 3: Collection of data

In this section, you should plan an experiment in which you collect data that allows you to predict how close the egg will get to the table without hitting it when released from n unstretched spring. (**Hint:** You may find it useful to represent the data in graphical form.) Your description that you hand up should include:

- Your experimental set-up, and how you used it to record your data
- Why the set of data that you recorded is suitable in allowing you to get the egg as close as possible to the table without breaking
- Why you need to know the mass and the dimensions of the egg (you will measure these in Section 4)
- All relevant data presented in suitable form (tables, graphs, etc.)

Section 4: Egg Bungee and Evaluation

For this part of the investigation you will be given your egg. Record any necessary measurements of the egg that you need. Vernier calipers and a mass balance are available.

Before you make the egg do a bungee jump, explain in detail the setup that you think will get the egg as close to the table as possible without breaking it.

Carry out your experiment. How far was the egg away from the table or did the egg hit the table? Evaluate your prediction and discuss any ways in which you could have improved on it in hindsight.

Appendix: Reports

Your report on your investigations should contain the following elements:

- A brief **introduction** which includes the hypothesis that you wish to put to the test
- A description of the **experiment.** Consider the following points:
  - Use of diagrams to show the set-up is recommended!
  - Include a discussion of the control of variables.
  - How many different measurements will you make?
  - How many repeat measurements will you make?
  - How did you make your experiment as accurate as possible?
- Your **results,** including tabulated data and graphs. Considering the following points:
  - Is your best-fit line a smooth curve or a straight line?
  - Can useful information (e.g., slope, intercept, etc.) be gleaned from the graph?
- A **conclusion** – can you confirm or reject your hypothesis?
  - How does the shape of the graph or your data verify or falsify your hypothesis?
  - Is the accuracy of your experiment sufficient to decide either way?

Your **individual** report is due at the end of the lab session. Give **each** investigation a title and use headings for the different sections within each investigation (e.g., introduction, experimental set-up, results and discussion, conclusion).